

## Indexing

## Cost Model for Data Access

- Data needs to be accessed fast
- How should we estimate the costs for accessing data?
  - ☆ number of I/Os: number of pages that need to be read from disk
- Why I/O and not other costs?
  - ☆ Real systems consider also CPU but I/O has definitely more weight
- Simplifications
  - ☆ only consider disk reads (ignore writes)
  - ☆ only consider number of I/Os and not the individual time for each read (ignores page pre-fetch)
  - ☆ Average-case analysis; based on several simplistic assumptions.

➡ Good enough to show the overall trends!

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## Typical Operations

- Scan over all records
  - ☆ `SELECT * FROM Skaters`
- Equality Search
  - ☆ `SELECT * FROM Skaters WHERE sid = 100`
- Range Search
  - ☆ `SELECT * FROM Skaters WHERE age > 5 and age <= 10`
- Insert
  - ☆ `INSERT INTO skaters VALUES (23, 'lilly', 10, 8)`
- Delete
  - ☆ `DELETE FROM Skaters WHERE sid = 100`
  - ☆ `DELETE FROM Skaters WHERE age > 30 AND age <= 50`
- Update
  - ☆ Delete+insert

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## File Organization

assume each relation is a file:

- Heap files:
  - ☆ Linked, unordered list of all pages of the file
  - ☆ Is it good for:
    - scan retrieving all records (`SELECT *`)
      - ▲ yes, you have to retrieve all pages anyways
    - equality search on **primary key**
      - ▲ not great: have to read on avg. half the pages to return one record
    - range search or equality search on non-primary key
      - ▲ not great: have to read all pages to return subset of records.
    - insert
      - ▲ yes: Cost for insert low (insert anywhere)
    - delete/update
      - ▲ same as for equality/range search -- depends on WHERE clause

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## File Organizations II

- Sorted Files:
  - ☆ Records are ordered according to one or more attributes of the relation
  - ☆ Is it good for:
    - scan retrieving all records (`SELECT *`)
      - ▲ yes, you have to retrieve all pages anyways
    - equality search on sort attribute
      - ▲ good: find first qualifying page with binary search in  $\log_2(\text{number-of-pages})$
    - range search on sort attribute
      - ▲ good: find first qualifying page with binary search in  $\log_2(\text{number-of-pages})$ ; adjacent pages might have additional matching records
    - insert
      - ▲ not good: have to find proper page; overflow possible
    - delete/update
      - ▲ finding tuple same as equality/range search depending on WHERE clause
      - ▲ update itself might lead to restructuring of pages
    - Sorted output: (`ORDER BY`)
      - ▲ good if on sorted attribute

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## Indexes

- Even a sorted file only support queries on sorted attributes.
- Solution: Build an index for any attribute (collection of attributes) that is frequently used in queries
  - ☆ Additional information that helps finding specific tuples faster
  - ☆ We call the collection of attributes over which the index is built the **search key attributes** for the index.
  - ☆ Any subset of the attributes of a relation can be the search key for an index on the relation.
  - ☆ Search key is not the same as primary key / key candidate

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## Creating an index in DB2

### Simple

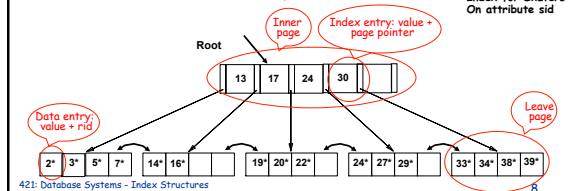
- ☆ `CREATE INDEX ind1 ON Skaters(sid);`
- ☆ `DROP INDEX ind1;`

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## B+ Tree: The Most Widely Used Index

- Each node/leaf represents one page
  - ☆ Since the page is the transfer unit to disk
- Leafs contain *data entries* (denoted as  $k^*$ )
  - ☆ For now, assume each data entry represents one tuple. The data entry consists of two parts
    - Value of the search key ( $k$ )
    - Record identifier ( $rid = (page-id, slot)$ )
  - ☆ That is: data entry is NOT a tuple but a pointer to a tuple
- Root and inner nodes have auxiliary *index entries*

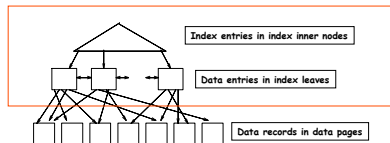


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## B+ Tree (contd.)

- *height-balanced.*
  - ☆ Each path from root to tree has the same height
- $F$  = fanout = number of children for each node (~ number of index entries stored in node)
- $N$  = # leaf pages
- Insert/delete at  $\log_F N$  cost;
- Minimum 50% occupancy (except for root).

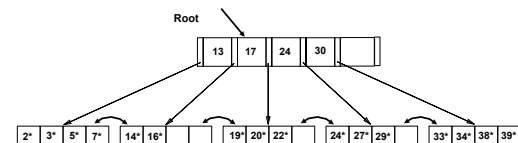


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## Example B+ Tree

- Example tree has height 2
- "Select \* from Skaters where sid = 5"
  - ☆ Search begins at root, and key comparisons direct it to a leaf
  - ☆ Number of pages accessed:
    - three: root, leaf, data page with the corresponding record
  - ☆ Number of I/O:
    - depends on how much of tree in main memory
    - rough assumption: root always in main memory; index leaves and data pages not in main memory upon first access

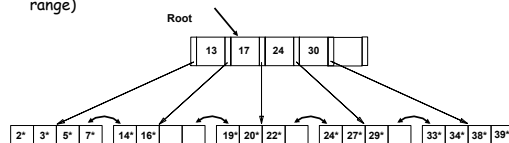


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## Example B+ Tree

- "Select \* from Skaters where sid = 5"
  - ☆ I/O costs:
    - one for leaf page with data entry, one for data page with data record
- "Select \* from Skaters where sid >= 33"
  - ☆ I/O costs:
    - one for leaf page
    - four for data pages with records
- Good for equality search AND range queries (depending on the range)



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## Inserting a Data Entry

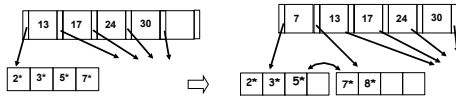
- Find correct leaf  $L$ .
- Put data entry onto  $L$ .
  - ☆ If  $L$  has enough space, *done!*
  - ☆ Else, must *split*  $L$  (into  $L$  and a new node  $L2$ )
    - Redistribute entries evenly, *copy up* middle key.
    - Insert index entry pointing to  $L2$  into parent of  $L$ .
- This can happen recursively
  - ☆ To split index node, redistribute entries evenly, but *push up* middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - ☆ Tree growth: gets *wider* or *one level taller at top*.

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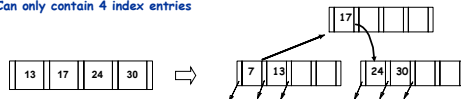
## Inserting 8\* into Example B+ Tree

### Insert into Leaf with leaf split



### Insert into internal node with node split

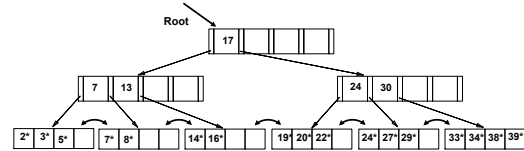
Assume that inner pages  
Can only contain 4 index entries



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## Example: After Inserting 8\*



❖ Notice that root was split, leading to increase in height.

❖ In this example, we can avoid split by redistributing entries; however, this is usually not done in practice.

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## Data Entry Alternatives: indirect Indexing

### Indirect Indexing I

- ☆ so far:  $\langle k, \text{rid of data record with search key value } k \rangle$  (indirect indexing)
- ☆ on non-primary key search key: (10, rid1), (10, rid2), (10, rid3), ...
  - several entries with the same search key side by side

### Indirect indexing II

- ☆  $\langle k, \text{list of rids of data records with search key } k \rangle$  (indirect indexing)
- ☆ on non-primary key search key: (10, rid1, rid2, rid3)...

### Comparison:

- ☆ first requires more space (search key repeated)
- ☆ second has variable length data entries
- ☆ second can have large data entries that span a page

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## Direct Indexing

- ❑ Instead of data-entries in index leaves containing rids, they could contain the entire tuple
  - ☆ data-entry = tuple
  - ☆ no extra data pages
- ❑ This is kind of a sorted file with an index on top

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## Index Classification

- ❑ Primary vs. secondary: If search key contains primary key, then called primary index.
  - ☆ Unique index: Search key contains a candidate key.
- ❑ Clustered vs. unclustered:
  - ☆ clustered:
    - Relation in file sorted by the search key attributes of the index
  - ☆ unclustered:
    - Relation in heap file or sorted by an attribute different to the search key attribute of the index.
  - ☆ A file can be clustered on at most one search key.
  - ☆ Cost of retrieving data records through index varies *greatly* based on whether index is clustered or not!

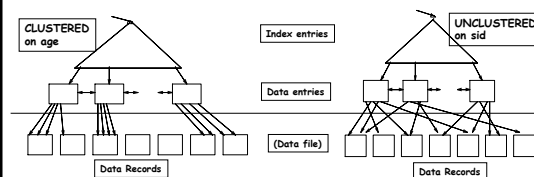
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## Clustered vs. Unclustered Index

### Example for Skaters:

- ☆ clustered on age
- ☆ unclustered on sid



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## B+-tree cost example

- ☆ Relation R(A,B,C,D,E,F)
- ☆ A and B are int (each 6 Bytes), C-F is char[40] (160 Bytes)
  - Size of tuple: 172 Bytes
- ☆ 200,000 tuples
- ☆ Each data page has 4 K and is around 80% full
  - $200,000 / ((0.8 * 4000) / 172) = 10750$  pages
  - $200,000 * 172 / (0.8 * 4000) = 10750$  pages
- ☆ Values of B are within [0:19999] uniform distribution
- ☆ Non-clustered B-tree for attribute B, indirect indexing (2)
- ☆ An index page has 4K and intermediate pages are filled between 50% - 100%
- ☆ The size of an rid = 10 Bytes
- ☆ The size of a pointer in intermediate pages: 8 Bytes
- ☆ Index entry size in root and intermediate pages:
  - $\text{size}(\text{key}) + \text{size}(\text{pointer}) = 6 \text{ Bytes} + 8 \text{ Bytes} = 14 \text{ Bytes}$

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## Size of B+tree

- ☆ The average number of rids per data entry
  - Number of tuples / different values (if uniform) (Example  $200,000 / 20,000 = 10$ )
- ☆ The average length per data entry:
  - Key value + #rids \* size of rid (Example:  $6 + 10 * 10 = 106$ )
- ☆ The average number of data entries per leaf page:
  - Fill-rate \* page-size / length of data entry
  - Example:  $0.75 * 4000 / 106 = 28$  entries per page
- ☆ The estimated number of leaf pages:
  - Number of entries = number of different values / #entries per page
  - Example  $20000 / 28 = 715$
- ☆ Number of entries intermediate page:
  - Fill-rate \* page-size / length of index entry
  - Min fill-rate: 0.5, max fill rate: 1
  - Example:  $0.5 * 4000 / 14 = 143$  entries ;  $1 * 4000 / 14 = 285$  entries
- ☆ Height is 3: the root has between three and four children
  - Three children: each child has around  $715 / 3 = 238$  entries
  - Four children: each child has around  $715 / 4 = 179$  entries

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## B+ Trees in Practice

- Typical order d of inner nodes: 100 (I.e., an inner node has between 100 and 200 index entries)
  - ☆ Typical fill-factor: 67%.
  - ☆ average fanout = 133
- Leaf nodes have often less entries since data entries larger (rids)
- Typical capacities
  - ☆ Height 4:  $133^4 = 312,900,721$  records
  - ☆ Height 3:  $133^3 = 2,352,637$  records
  - ☆ Height 2:  $133^2 = 17,689$  records
- Can often hold top levels in buffer pool:
  - ☆ Level 1 (root) = 1 page = 4 Kbytes
  - ☆ Level 2 = 133 pages = 0.5 Mbyte
  - ☆ Level 3 = 17,689 pages = 70 Mbytes

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## Index in DB2

- Simple
  - ☆ `CREATE INDEX ind1 ON Skaters(sid) ;`
  - ☆ `DROP INDEX ind1;`
- Index also good for referential integrity (uniqueness)
  - ☆ `CREATE UNIQUE INDEX indname ON Skaters(name)`
- Additional attributes
  - ☆ `CREATE UNIQUE INDEX ind1 ON Skaters(sid) INCLUDE (name)`
  - ☆ Index only on sid
  - ☆ Data entry contains key value (sid) + name + rid
  - ☆ `SELECT name FROM Skaters WHERE sid = 100`
    - Can be answered without accessing the real data pages of Skaters relation!
- Clustered index
  - ☆ `CREATE INDEX ind1 on Skaters(sid) CLUSTER`

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## Index in DB2

- Index on multiple attributes:
  - ☆ `CREATE INDEX ind1 ON Skaters(age,rating) ;`
  - ☆ Order is important:
    - Here data entries are first ordered by age
    - Skaters with the same age are then ordered by rating
  - ☆ Supports:
    - `SELECT * FROM Skaters WHERE age = 20 ;`
    - `SELECT * FROM Skaters WHERE age = 20 AND rating < 5 ;`
  - ☆ Does not support
    - `SELECT * FROM Skaters WHERE rating < 5 ;`

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