MATH 3MB3 Group Project
Fall 2019

# Modelling Traffic During Rush Hour in the Greater Toronto Area


Patricia Loaiza
Dylan Melville
Emily Sanderson
Claire Styba
Brenden Suh


August 20, 2020


**Abstract**

Commuting to work is an integral part of a working Canadian's life. This model analyzes commute duration from Hamilton to Toronto at different times during the day. We include commuters entering from Burlington, Oakville and Mississauga. This paper takes into account how lane characteristics, weather conditions, population, and a redistribution of normal work hours affects one's daily commute time. We concur that a increase in population and a reduction of lanes can drastically increase commute time. We modelled how slow downs can affect multiple sections of road and lead to disruptions in the schedules of commuters. Concurrently, a redistribution of work hours has a positive impact on commute time. With these findings we can make actionable recommendations on improving traffic flow in the years to come.

# Contents

# 1 Introduction

Cities have a large migrant workforce and rely on major roads and highways to transport commuters. The commute between Hamilton and Toronto utilizes some of Canada's busiest highways. Up to 200,000 cars drive on them daily. As populations grow, Canadians will be spending more and more time traveling. Hoping to boost the economy and travel efficiency, cities spend considerably on highways and roads. Through our model, we analyze commute duration throughout a week day morning. We account for changes in population and physical/intangible changes of road conditions.

The QEW, the 403, and the Gardiner Expressway are the main three highways commuters use to get from Hamilton to Toronto. While the population of Toronto was 1.82 million in 1961, it has skyrocketed to 2.93 million as of 2017. Infrastructure has adapted to keep up with growing populations. "One of the fundamental principles of transportation economics is that infrastructure supply can reduce the cost of travel, inducing the potential for users to reduce costs, become more productive, and invest savings in other economically productive or valuable endeavors." [4]. Our model sidesteps the cost of real data simulations, to accurately look at how changes in the road environment effect commute time.

Traffic has an opportunity cost. While waiting in traffic, workers are not contributing to the economy. A 2009 Metrolink study found that road congestion costs the GTA $6 billion per year in lost productivity [1]. Considering the cost, one can understand the importance of having efficient road systems. However, road congestion has a set of unique problems. City's must balance the cost of increasing efficiency versus the opportunity cost of traffic. The model accounts for changes in population, road structure, and driver behavior. Understanding these three variables, city planners will be able to predict the impact their changes will make. For instance, construction increases long run transit efficiency, but has a short-term cost of materials, labor and traffic congestion. Utilizing our model, decision makers can understand the best times to close lanes to minimize cost of production, allowing for more efficient decision making.

# 2 Assumptions

The assumptions made helped the model stay as simple and realistic as possible.

1. The speed of a section of highway is based on the number of cars and lanes in that stretch of highway
2. The speed decreases linearly based on the density of cars
3. Only a percentage of the population of the city will enter the highway and they will do so with a normal distribution based how long they believe it will take them to get to their destination
4. Each city only has one highway exit/entrance
5. A kilometer of road can only take so many cars before it gets jammed

## 3 Results of Simulation

Based on the research done, we came up with multiple formulas to simulate the morning commute. The first of these being the flow of traffic. The flow is the rate at which cars move through a stretch of road and is based on the number of cars and the concentration of cars which is a function of lanes.

$$q = u_f(1 - \frac{k}{k_j}) \tag{1}$$

Where $q$ is the flow, or cars passing per minute, $u_f$ is the free flow speed, or the speed that cars drive without traffic, $k$ is the density of the cars, calculated as

$$k = \frac{V}{ld} \tag{2}$$

Where $V$ is the number of vehicles, $l$ is the number of lanes on the road and $d$ is the length of road. Finally. $k_j$ is the jam density, where the traffic is not moving. From there, we can calculate the average speed from the density and the flow as:

$$u = \frac{q}{k} \tag{3}$$

This leads us to a system that models the speed in each section of highway. Using the speed of each stretch we are able to simulate how long it takes a person to get to Toronto from Hamilton, Burlington, Oakville or Mississauga

Therefore, we can model the change in vehicle population as increasing from on-ramp traffic, and from the flow of the previous road section, and decreasing as its own traffic flows to the next section of road:

$$V_{t+1}^i = V_t^i - q_t^i + q_t^{i-1} + O_t^i \tag{4}$$

Where O is the on-ramp traffic, estimated as 1.1% of the population normally distributed.

With the assumptions made and the parameters being adjusted to the realistic values stated in Appendix A under constants, we get a graph representing the time an individual leaves against the time it takes them to arrive to Union Station (Figures 1 and 2).

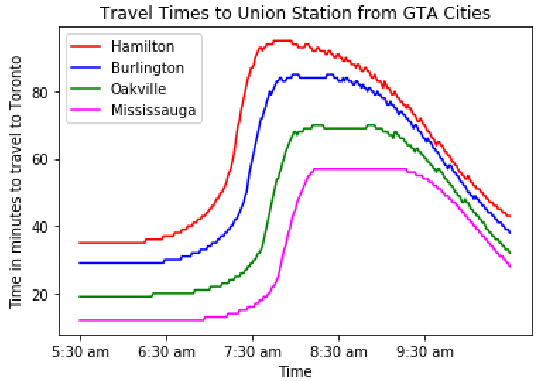


Figure 1: First Model Results

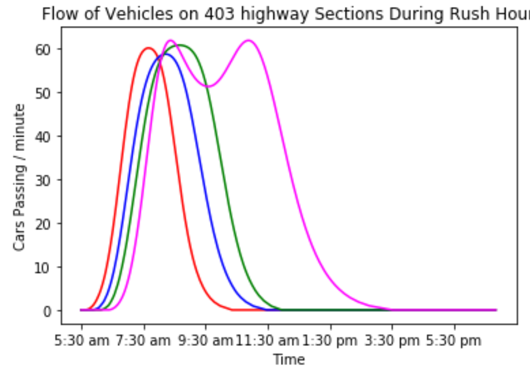


Figure 2: Flow of first model

From this graph, we see that the deeper we go into rush hour, the longer it takes for a commuter to get to Toronto from any city. Then traffic starts dying down and the commute takes less time. This model does pretty well when there are no jams to mess with the system. However, if we were to change the stretch from Mississauga to only have 2 lanes, the time of arrival decreases (Figures 3 and 4).

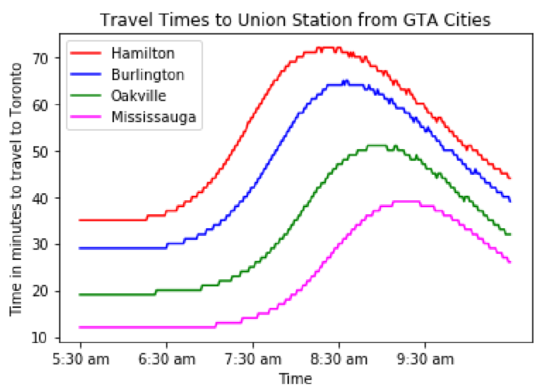


Figure 3: Reduced Flow in Mississauga has no effect

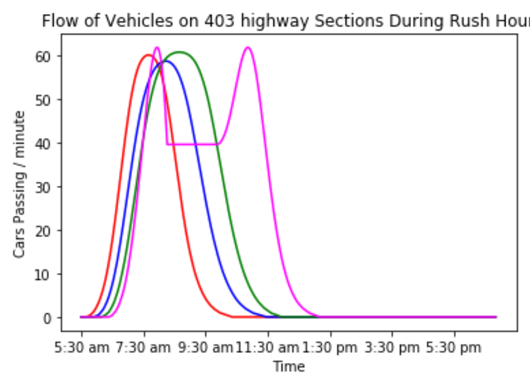


Figure 4: The dipped flow has little effect

This is obviously not correct. It is because this model assumes that if the next stretch of highway is at capacity, people will simply not enter it and get off the highway. This is unrealistic, as commuters will not give up on getting to work because of a little traffic! Therefore, we must adjust our model to take into account the next section's flow so that jams in the section ahead slow down the previous section.

This is done by having each section of road check the available room in the next section, before giving its outflow. If the outflow would be higher than the available space, the flow is decreased to only the number of cars that could fit in the next road. Refer to Appendix A for code updating vehicle populations and the Figures 5 and 6 for our new, more realistic models.

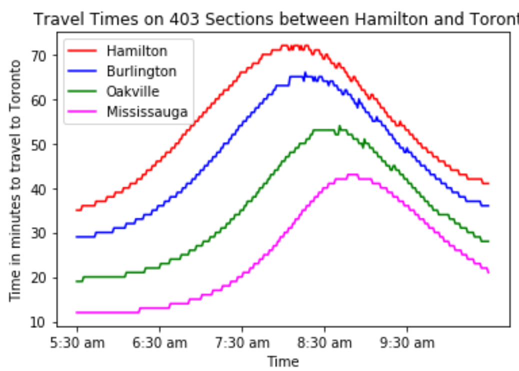


Figure 5: New time approximations

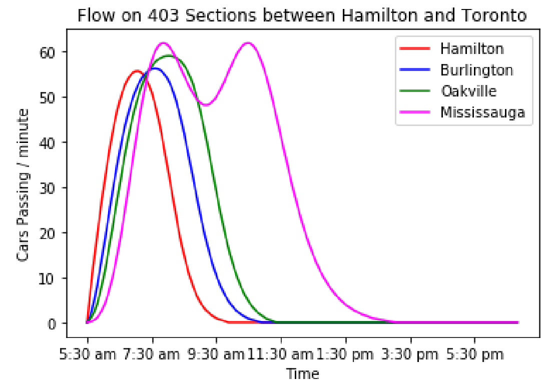


Figure 6: New flow approximations are more accurate

| Departure City | | Our times | Google's times |
|---|---|---|---|
| Hamilton | → | 59 min | 60 min |
| Burlington | → | 52 min | 55 min |
| Oakville | → | 40 min | 45 min |
| Mississauga | → | 31 min | 35 min |

Table 1: Here we can see our predictions were similar to those from Google, when leaving to arrive at work at 9:00 am.

Our travel times are fairly accurate to what Google Maps predicts the commute time will be. This gives us confidence that our model will be able to simulate real life condition changes when we adjust the parameters in the next sections.

# 4 Population Growth

Each road section has new commuters entering the system. The number of commuters entering a road section is a percentage of the corresponding city's population. This determines the rate at which cars move through a stretch of road. Thus with unchanging road parameters, an increase of a city's population injects a large amount of cars in one section causing a traffic jam. This increases commute time for all commuters prior to said city. As shown in Fig 7 all commuters suffer from longer commute times when Mississauga's population increases. Traffic from Oakville, Burlington, and Hamilton are congested at Mississauga's road section. More cars equates to a lower flow rate, and slower commute times.
This brings us to our first application of the model. Mississauga is expected to grow to 812,000 people by 2031. If a city is expecting a certain increase in population, they can then adjust the roads prior to this so that they are suitable for the future.

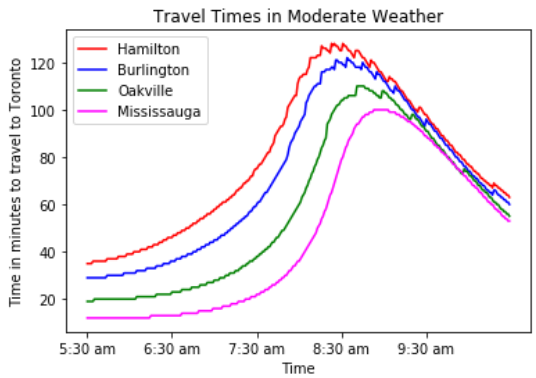


Figure 7: Growing Mississauga Time

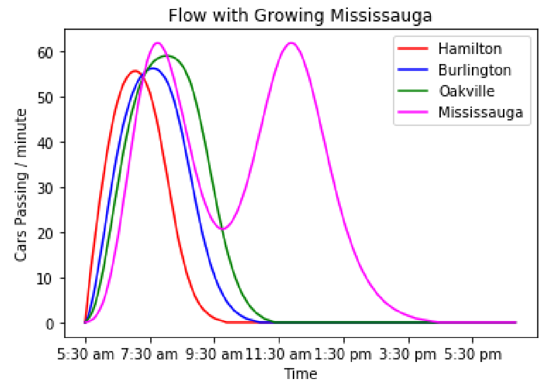


Figure 8: Growing Mississauga Flow

# 5 Structural Changes of the Road

There are many reasons for lane closures but they all have large affects on the flow of traffic especially during rush hour. This model takes into account the number of lanes for each stretch of highway. When the number of lanes increases, traffic flows better. This causes a decrease in the travel time. When the number of lanes decreases, there is a build up of traffic causing jams and that increases travel time. If one lane were to close in Hamilton (the first stretch of highway), this would only increase travel time for commuters leaving from Hamilton and decrease travel time for commuters leaving from the later cities due to the reduced outflow of traffic from Hamilton. Whereas, if one lane were to close in Mississauga (the fourth stretch of highway), it would increase travel time for all commuters since they must travel through Mississauga to reach Toronto. The model can replicate this situation. The following two graphs depict the outcome of a lane closure in Oakville. A change in the number of vehicles, the speed and the overall travel time to Toronto is demonstrated. It can be seen that commuters who leave at 8am in Hamilton will experience the worst travel time increase. Commuters from Mississauga see no change in travel time around 8:00 am, because the lane closure does not change the density of their route, until later in the morning when the continued traffic from Oakville's jam increases the late morning density. Travellers from Hamilton see a 30 minute increase at peak traffic.

## 5.1 Graphs produced from the simulations with one closed lane in Oakville

This model shows that as the number of lanes are reduced, traffic becomes jammed very quickly and these jams can make travel times exponentially larger. The main cause for lane closures are accidents and construction. Accidents are more avoidable yet unpredictable but construction is the opposite. Construction must happen to make roads better and improve traffic flow but it needs to be done in the proper manner to keep travel times realistic. If lane closures could happen in the middle of the day and not during rush hour times, or if better transit could be made available during times of increased construction this could help greatly decrease the effects of lane closures on travel time. Below we see the effect of closing one lane in Oakville, which reduces the speed in Oakville, and increases average commute time of Oakville, Burlington, and Hamilton.

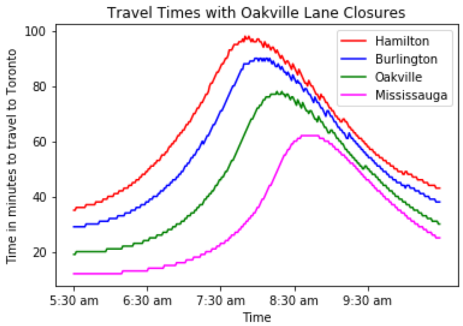


Figure 9: Travel time to Toronto with one lane closure in Oakville

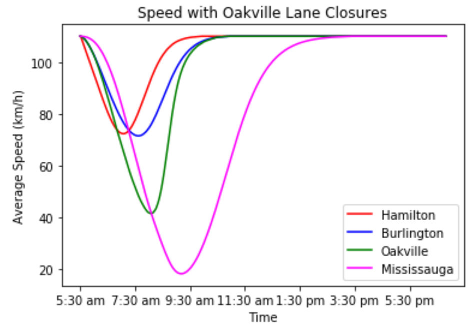


Figure 10: Average vehicle speed with one lane closure in Oakville

# 6 Changes in Behaviour

## 6.1 How weather impacts the flow of traffic

Weather has a large impact on the flow of traffic. In the winter, roads can get very icy and visibility may be severely decreased due to blizzards. In months where there is no snow, there could be severe rainfall which also affects visibility. Heavy snow or rain causes drivers to be more cautious and thus drive slower. The sun can also affect visibility, since this model looks at morning traffic to Toronto there are stretches of road that face the sunrise which again may reduce the flow of traffic.

The model uses a parameter for weather, which reduces the free-flow speed linearly with 0 being perfect conditions, and 1 representing the average driver driving at less than 1 $\frac{km}{h}$. The following equation shows how the parameter $w$ changes the free flow speed of the model.

$$q = u_f(1 - w)(1 - \frac{k}{k_j}) \tag{5}$$

For example, if there is rain that reduces visibility over the entire stretch of road, this may be considered a 0.3 on our weather condition parameter. If there is snowfall that is accumulating on the road and reducing visibility and tire friction, this may be considered a 0.5 on our weather condition parameter. It can be seen in the following two figures how both these situations would impact travel time. The usual hour trip from Hamilton to Toronto for a 9am arrival increases to an hour and a half with rain. The same trip, under snowfall conditions would take more than two hours for a 9am arrival.
Weather had a significant effect on the derivative of the commute time graph. With slow moving traffic in poor conditions, towards rush hour the commute time increases very quickly. In the worse case, it increases by 80 minutes in less than an hour. This is extremely frustrating for commuters, since a small delay in the morning leads to a large increase in commute time.

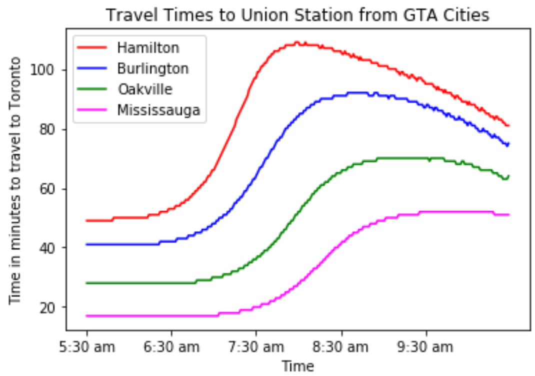


Figure 11: Travel time with rain (weather parameter set to 0.3)

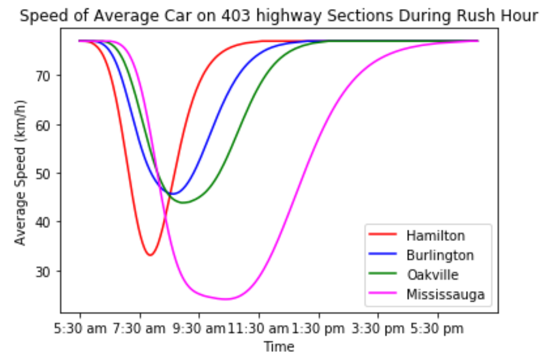


Figure 12: Speed with with rain (weather parameter set to 0.3)

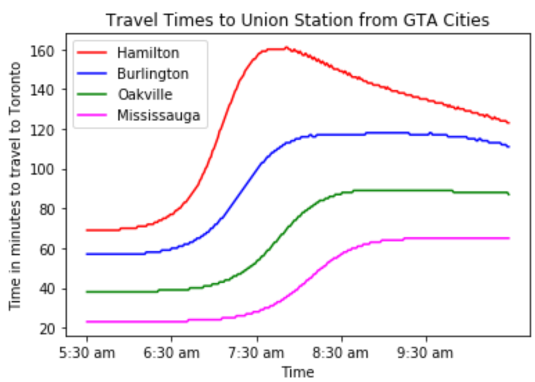


Figure 13: Travel time with snow (weather parameter set to 0.5)

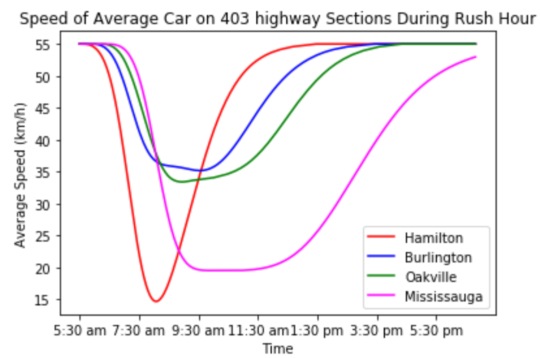


Figure 14: Speed with snow (weather parameter set to 0.5)

### 6.2 How the addition of public transit alleviates traffic

Reducing the amount of vehicles on the highway has a great impact on travel times. If a portion of the population starts carpooling or getting on public transit, the number of vehicles on the road decreases while having the same number of people commuting.

It is observed that when there is a reduction of 20% of vehicles entering the highway, the travel times greatly decrease for everyone. The slight decrease in vehicles causes the decrease in vehicle flow around 8:30 am in Mississauga to disappear. This is equivalent to removing the traffic jam. This reduces peak travel time travelling from Hamilton to Toronto by 20 minutes.

Encouraging the usage of public transit and car pooling is an easy way for policy makers to decrease traffic without spending on new infrastructure.

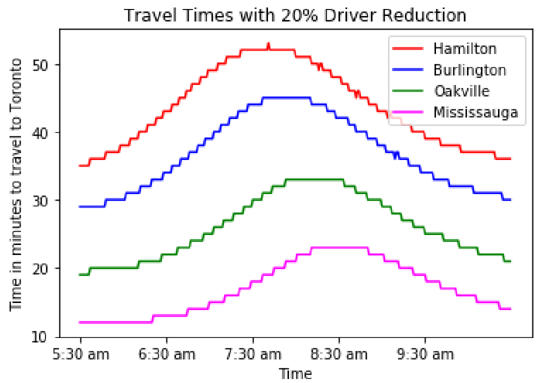


Figure 15: Travel time with 20% vehicle reduction

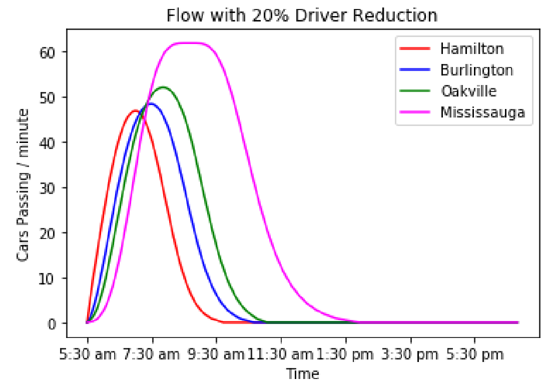


Figure 16: On-ramp flow with varied work hours

### 6.3 How different/multiple work hours impact the flow of traffic

Companies are becoming more comfortable with having their employees work from home or work different hours (7am-3pm, 8am-4pm, 10am-6pm) instead of the typical 9-5. This causes changes in the distribution of cars entering the highway. The original assumption was that leave time is a normal distribution, with one peak. With a change/movement in normal work hours this may lead to a multi-peak distribution.

If half of the commuters from Mississauga and Oakville started working 11-7 and the other half continued to work 9-5, this would help with travel times for the entire commuter population. For example, suppose the two average leave times for Oakville are now 7:45 am and 9:45am to get there at 9:00 am and 11:00 am respectively. Drivers looking to arrive at 11 would change their leave times equivalently. By spreading out the inflow of cars more evenly, the density never slows traffic to the same level as in the original model. This removes the exponential increase in travel time that is seen in the original model. Instead the travel time increases more linearly, punishing commuters less for leaving later in the morning. The range of commute times decreases as well, with the highest commute time being only around 15 minutes longer than the early morning commutes.

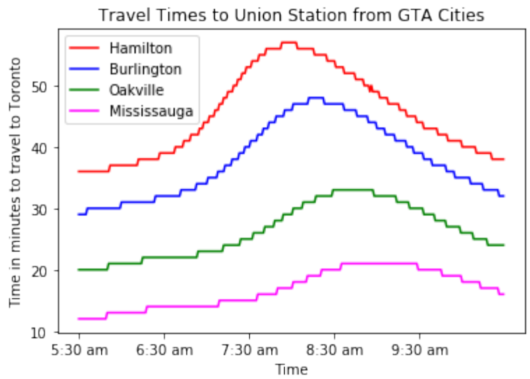


Figure 17: Travel time with 2 different leave times from Oakville and Mississauga

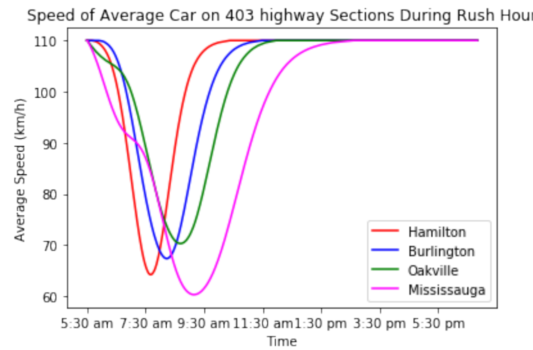


Figure 18: Speed with 2 different leave times from Oakville and Mississauga

Having 2 different sets of work hours (7-3, 9-5) decreases travel time thus employers should consider making work hours more flexible. This benefits their employees by making their commute time shorter allowing them to dedicate this found time to family or leisure activities, which could lead to higher productivity in the workplace.

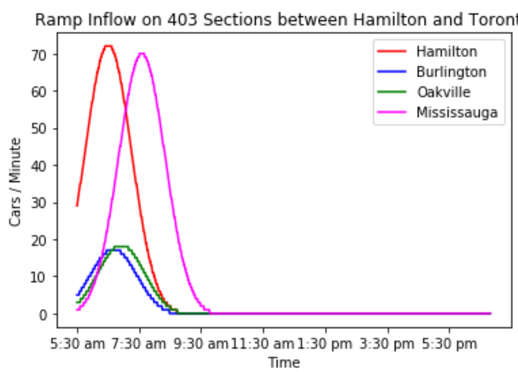


Figure 19: On-ramp flow with normal work hours

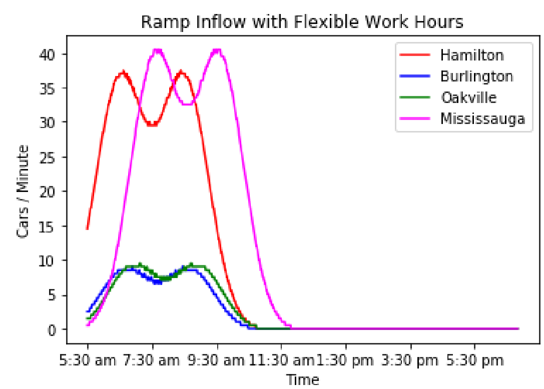


Figure 20: On-ramp flow with varied work hours

## 7 Concluding Remarks

In conclusion, there are many factors that affect travel time around rush hour. After accounting for the traffic flow ahead, we see that changes in conditions on one section may have an effect on the others. For example, lane closures in Mississauga will slow down the entire system. It can also be seen that population changes of a single city have a massive effect on the average commute time. Then, when we factor in changes to weather, that is to say changes in driver behaviour, we can more accurately predict a day's travel time. Finally, the inclusion of multiple work start times to simulate flexible work hours, decreases the time it takes to get to Toronto and therefore showing a possible insight into how traffic can be better managed. Traffic is a part of most Canadian's lives and it is important to be able to understand how it works and what affects it. Municipalities can use these kinds of simulations to better determine the most efficient use of resources. For example, they can figure out if the addition of a lane has the intended effect and whether or not it is worth it for their city. They can also be aware of the effects that a growing population will have on commute time and plan accordingly.

# Appendix A

Listing 1: Functions from the Road class that update the car populations

```
def outflow(self):
    self.speed = free_flow_speed*(1 - (self.density/jam_density))
    outflow = self.density*self.speed
    #if there is a next road, there's a possibility it is jammed, and therefore
    # cars cannot flow at the rate they want to.
    # in this case we check the capacity and only allow the number of cars that
    # can fit.
    if (self.next_road != None):
        #if there is 0 space, allow no car movement
        if (self.next_road.max_vehicles == self.next_road.vehicle_count):
            outflow = 0
            self.speed = 0
        # if there is limited space, less than the outflow allow only those cars
        # to move forwards
        elif (self.next_road.max_vehicles - self.next_road.vehicle_count <
        outflow):
            outflow = self.next_road.max_vehicles - self.next_road.vehicle_count
            self.speed = outflow/self.density
    return outflow

def update(self,time,road_inflow):
    onramp_inflow = self.city.onramp(time)
    outflow = self.outflow()
    #Assumption: Once the highway is jammed, all cars that would come on exit
    the highway
    self.vehicle_count = min(math.floor(self.vehicle_count + onramp_inflow +
        road_inflow - outflow),self.max_vehicles)
    self.density = self.vehicle_count / (self.lanes * self.length)
    return self.vehicle_count
```

# Appendix B - Constants

Populations [2]

1. Hamilton - 747,545
2. Burlington - 183,314
3. Oakville - 193,832
4. Mississauga - 721,599

Distances [5]

1. Hamilton to Burlington - 11.1 km
2. Burlington to Oakville - 17.0 km
3. Oakville to Mississauga - 13.8 km
4. Mississauga to Toronto - 20.9 km

Number of Lanes [5]

1. Hamilton to Burlington - 4 lanes
2. Burlington to Oakville - 3 lanes
3. Oakville to Mississauga - 3 lanes
4. Mississauga to Toronto - 4 lanes

Average Speed without Traffic - 110 km/h [5]
Percentage of Commuters - 1/100 [5]
Jam Density - 135 Cars per km per lane [3]
(Note, this is the number of cars at which point traffic is jammed)

## References

[1] Star Editorial Board, *Toronto needs action on road congestion*, Tech. report, The Toronto Star, 2018.

[2] Statistics Canada, *Municipalities in canada with the largest and fastest-growing populations between 2011 and 2016*, Available from this website: https://www12.statcan.gc.ca/census-recensement/2016/as-sa/98-200-x/2016001/98-200-x2016001-eng.cfm, 2017.

[3] V.L. Knoop and W. Daamen, *Automatic fitting procedure for the fundamental diagram*, Transportmetrica B: Transport Dynamics, 2017.

[4] Pavlos Kanaroglou Matthias Sweet, Carly Harrison, *Congestion trends in the city of toronto*, Tech. report, City of Toronto, 2015.

[5] Queen's Printer of Ontario, *Ontario traffic volume*, Tech. report, Government of Ontario, 2015.