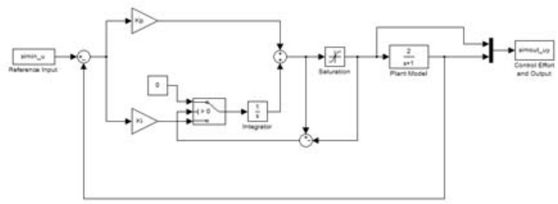
FROM LAST TIME…

Limitations and Remedies

- Fundamental Design Trade-offs
- Limitations
- Remedies

$$Y_D(s) = S(s)[D_o(s) + G(s)D_i(s)]$$
$$Y_N(s) = -T(s)N(s)$$
$$U(s) = S_u(s)[\bar{R}(s) - D_o(s) - N(s)]$$



WHAT DIFFERENCES DO YOU SEE?

ARCHITECTURAL ISSUES

Topics

- Internal Model Principle
- Feedforward
- Cascade Control

At the end of this section, students should be able to:

- Determine a disturbance generating polynomial.
- Use the Internal Model Principle in controller design.
- Determine a feedforward controller using inversion.
- Apply cascade control to a system with multiple measurable outputs.

WE WILL CONSIDER THESE STRUCTURAL CHANGES…

1. Modification of controller denominator by IMP
2. Addition of prefilter (two DOF controller)
3. High-gain inner control loop

There are other variations that we will not cover in class

ARCHITECTURAL ISSUES

INTERNAL MODEL PRINCIPLE

HOW DO WE MODEL A DISTURBANCE (OR REFERENCE)?

	$d(t)$	**Free response of an LTI system given by:**	$G_{dis}(s)$ **(assuming** $U(s) = 0$**)**
Step	A_{d0}	$\dot{d}(t) = 0$	$\frac{1}{s}$
Sinusoidal	$A_d \sin(\omega_d t + \psi_d)$	$\ddot{d}(t) + \omega_d d(t) = 0$	$\frac{1}{s^2 + \omega_d^2}$
Step + Sinusoidal	$A_{d0} + A_d \sin(\omega_d t + \psi_d)$		

WHAT ABOUT A GENERAL BOUNDED DISTURBANCE (OR REFERENCE)?

Free response of an LTI system given by:

$$d^{(q)}(t) + \gamma_{q-1} d^{(q-1)}(t) + \cdots + \gamma_0 d(t) = 0$$

⇩

$$G_{dis}(s) = \frac{1}{s^q + \gamma_{q-1} s^{q-1} + \cdots + \gamma_0}, \qquad U(s) = 0$$

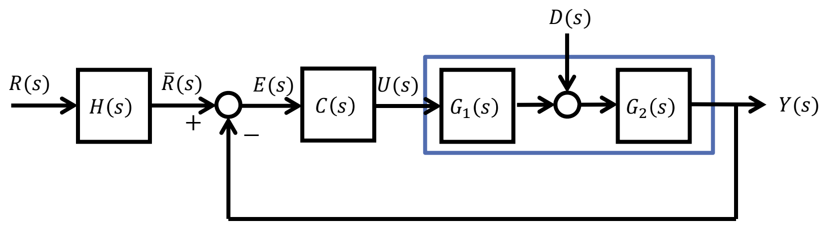
⇩

$$D(s) = \frac{N_{dis}(s)}{\Gamma_{dis}(s)}$$

where $\Gamma_{dis}(s)$ is the *disturbance generating polynomial*

WHAT IS THE DISTURBANCE GENERATING POLYNOMIAL FOR A DISTURBANCE WITH COMPONENT FREQUENCIES AT $\omega_{d1}, \ldots, \omega_{dm}$?

CONSIDER A CLOSED-LOOP SYSTEM WITH DISTURBANCE



$$Y(s) = S(s)G_2(s)D(s) = \frac{D_{G1}(s)D_{G2}(s)D_C(s)}{D_{CL}(s)} \cdot \frac{N_{G2}(s)}{D_{G2}(s)} \cdot \frac{N_{dis}(s)}{\Gamma_{dis}(s)}$$

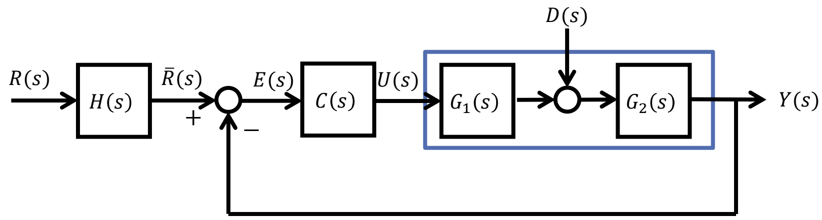
Condition for *Zero Steady-State* Output due to Disturbance:

STEADY-STATE DISTURBANCE COMPENSATION REQUIRES THAT GENERATING POLYNOMIAL OF DISTURBANCES BE INCLUDED AS PART OF THE CONTROLLER DENOMINATOR.

Internal Model Principle

Note that the roots of the generating polynomial, in particular the ones on the imaginary axis, impose the same performance trade-offs on the closed-loop as if those poles were part of the plant!

INTERNAL MODEL PRINCIPLE APPLIES TO REFERENCE TRACKING TOO!



$$Y(s) = T(s)H(s)R(s)$$

$$E(s) = \bar{R}(s) - Y(s) = S(s)H(s)R(s) = \frac{D_G(s)D_C(s)}{D_{CL}(s)} \cdot \frac{N_H(s)}{D_H(s)} \cdot \frac{N_R(s)}{\Gamma_R(s)}$$

Condition for *Zero Steady-State Error* due to Reference

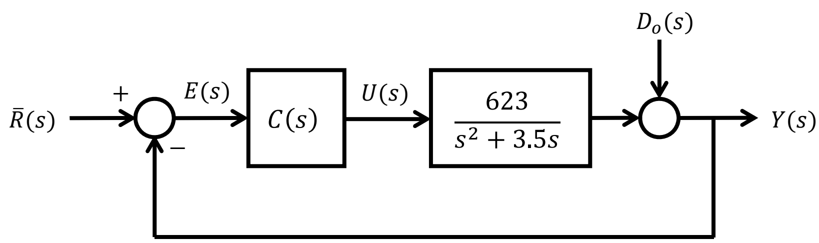
TO ELIMINATE TRACKING ERRORS, THE DENOMINATOR OF $L(s)$ MUST CONTAIN AN INTERNAL MODEL OF R OR D.

Design $C(s)$ in two parts:

$$C(s) = C_1(s)C_2(s)$$

- C_1 is based on steady-state performance
- C_2 is based on transients and stability

PID CONTROL OF DC MOTOR POSITION



- Assume that $D_o(s)$ is a sinusoidal at 5 rad/s
- Assume that $\bar{R}(s)$ is a step input
- Design $C(s)$ for zero steady-state error and good transient performance

AUGMENT THE PLANT, THEN DO POLE PLACEMENT

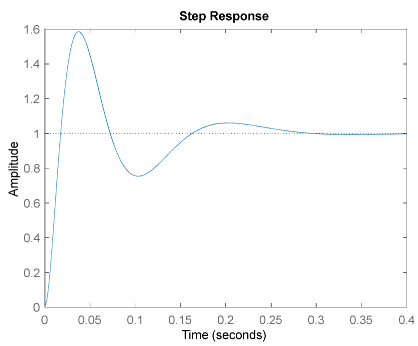
$$\bar{G}(s) = G_o(s)\left(\frac{1}{s^2 + 5^2}\right)\left(\frac{1}{s}\right)$$

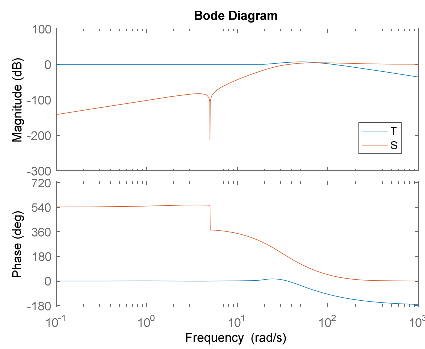
$$D_{\bar{G}}(s)\Gamma_d(s)\Gamma_r(s)D_C(s) + N_{\bar{G}}(s)N_C(s) = D_{CL}(s)$$

$$n = order\ of\ D_{\bar{G}}$$
$$q = order\ of\ \Gamma_d\Gamma_r$$
$$n_C = order\ of\ D_C$$
$$n_{CL} = 2n - 1 + q$$

$$C(s) = \frac{25.822(s^2 + 29.87s + 262.5)(s^2 + 17.56s + 485)}{(s + 196.5)s(s^2 + 25)}$$

CLOSED-LOOP ACHIEVES STEADY STATE ERROR SPECIFICATIONS

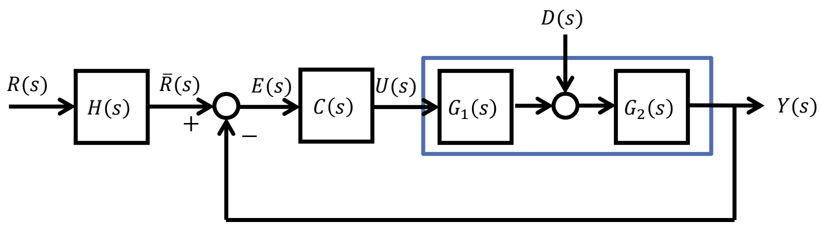




ARCHITECTURAL ISSUES

FEEDFORWARD CONTROL

FEEDFORWARD CAN IMPROVE REFERENCE TRACKING!



Closed-loop system output due to reference input

$$Y(s) = T(s)H(s)R(s)$$

Ideally, perfect tracking can be achieved even during the transient if one can choose feedforward transfer function H(s) such that

IS IDEAL FEEDFORWARD FOR REFERENCE TRACKING FEASIBLE?

1. The CLTF $T(s)$ has unstable CL zeros

2. The inversion 1/To(s) may not be proper and future reference input trajectory is not known, which means that

$$\bar{R}(s) = \frac{1}{T_o(s)} R(s)$$

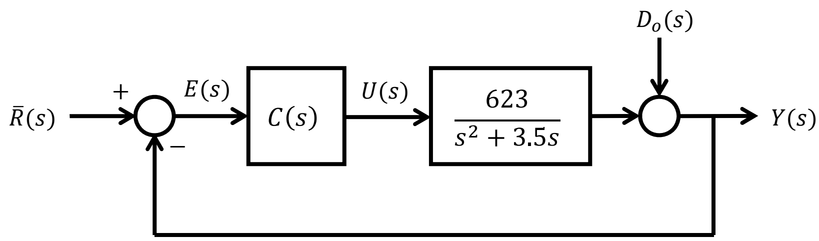
is not calculable or implementable.

IF IDEAL INVERSION DOES NOT WORK, CAN WE FIND AN ACCEPTABLE APPROXIMATION?

$$H(s) = \frac{1}{T_o(s)} \cdot \frac{1}{(\tau_h s + 1)^k}, \qquad \tau_h \ll 1$$

Insert fast poles to make the TF proper!

PID CONTROL OF DC MOTOR POSITION



- A proper PID controller has been previously synthesized to place CL poles at $p^c_{1,2d} = -20 \pm j20 \qquad p^c_{3,4d} = -40, -40$

$$C(s) = \frac{8.33s^2 + 205s + 2054}{s(s + 117)}$$

$$T_o(s) = \frac{623(8.33s^2 + 205s + 2054)}{\left((s + 20)^2 + 20^2\right)(s + 40)^2} = \frac{623(8.33)(s^2 + 24.6s + 246)}{D_{CL}(s)}$$

DESIGN $H(s)$ TO CANCEL THE STABLE ZEROS OF $T_o(s)$

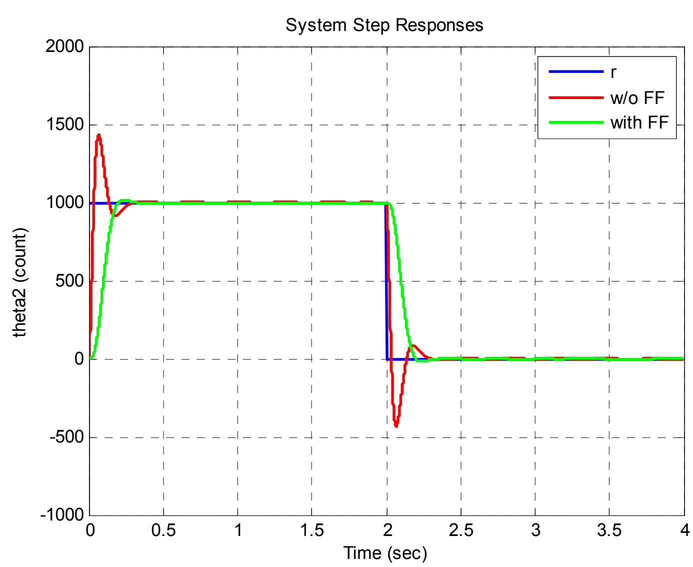
$$T_o(s) = \frac{623(8.33)(s^2 + 24.6s + 246)}{D_{CL}(s)}$$

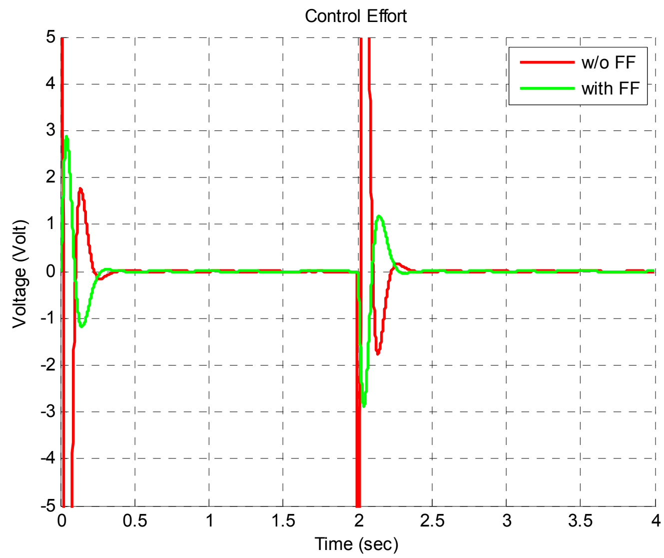
$$\Downarrow$$

$$H(s) = \frac{246}{s^2 + 24.6s + 246}$$

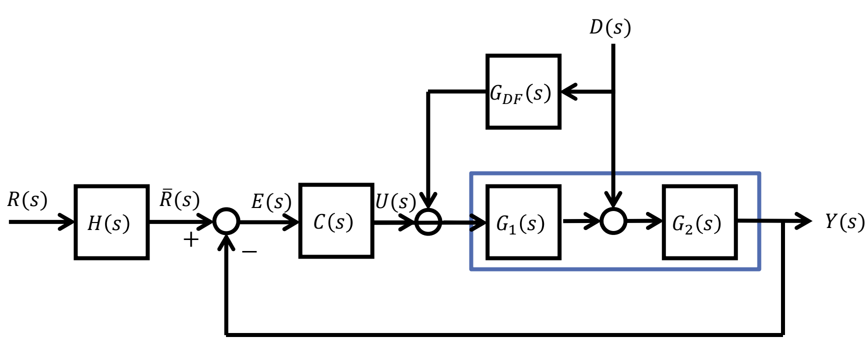
$$\Downarrow$$

$$Y(s) = H(s)T(s)R(s) = \frac{623(8.33)}{D_{CL}(s)}R(s)$$

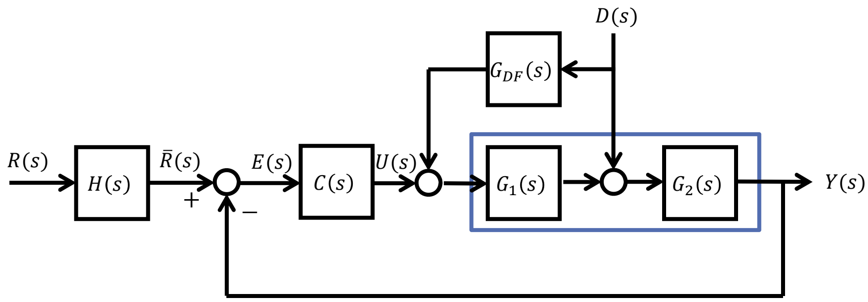




CAN WE APPLY FEEDFORWARD TO IMPROVE DISTURBANCE REJECTION?



Disturbance must be *measurable*.



Output due to Disturbance

$$Y_D(s) = S(s)G_2(s)[1 + G_1(s)G_{DF}(s)]D(s)$$

Ideal Disturbance Feedforward TF ($Y_D(s) = 0$)

$$G_{DF}(s) = \frac{-1}{G_1(s)}$$

ARCHITECTURAL ISSUES

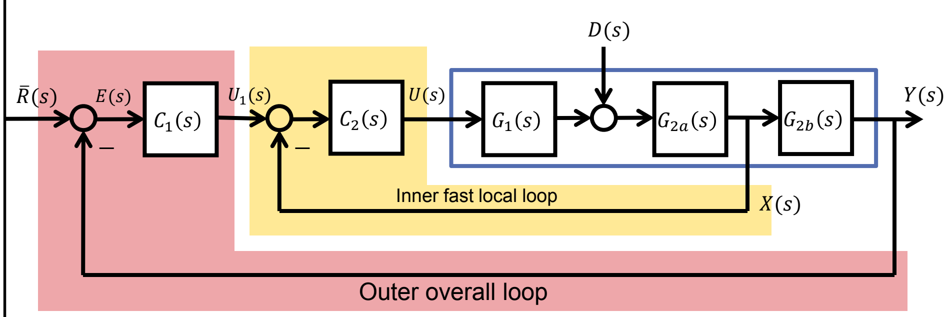
CASCADE CONTROL

UNTIL NOW, WE'VE USED ONLY OUTPUT FEEDBACK. IS THERE ADDITIONAL INFORMATION WE COULD USE?

Conflicting design requirements:

- High open-loop gain for small sensitivity values for good disturbance rejection capability
- Low open-loop gain for small complementary values for good noise attenuation capability and robust stability in the presence of modeling errors

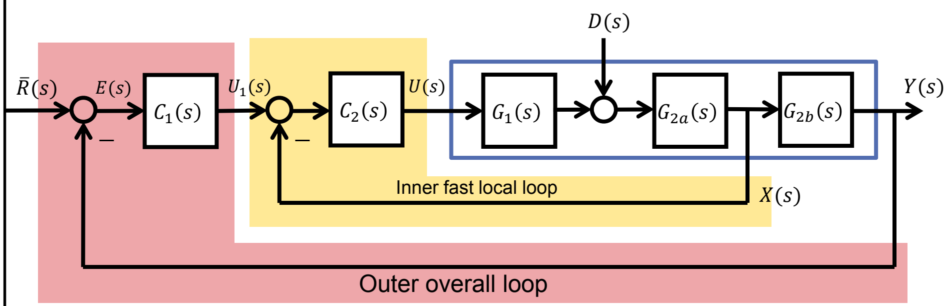
CASCADE CONTROL INCORPORATES A FAST (HIGH-GAIN) INNER LOOP FOR DISTURBANCE ATTENUATION



Design $C_2(s)$ to achieve a high CL bandwidth from $U_1(s)$ to $X(s)$

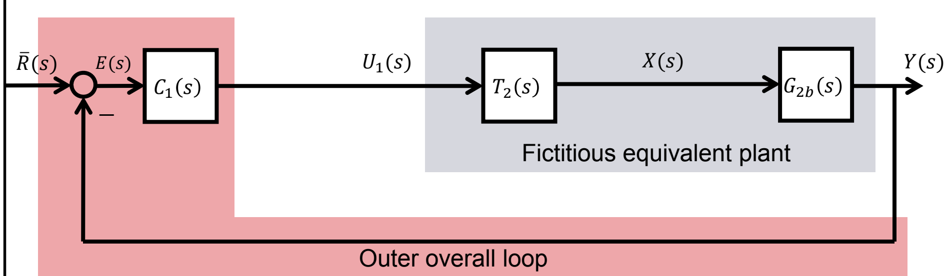
$$X(s) = S_2(s)G_{2a}(s)D(s) + T_2(s)U_1(s)$$

WITH INNER LOOP DESIGNED, OUTER LOOP DESIGN BECOMES EASIER



Replace inner loop with fictitious equivalent plant.

WITH INNER LOOP DESIGNED, OUTER LOOP DESIGN BECOMES EASIER



Apply previous design rules from to fictitious equivalent plant.

COMING UP…

Midterm Exam!

Intro to State-Space Models

- Review of State Space
- Transfer Functions and State Space
- Canonical Forms