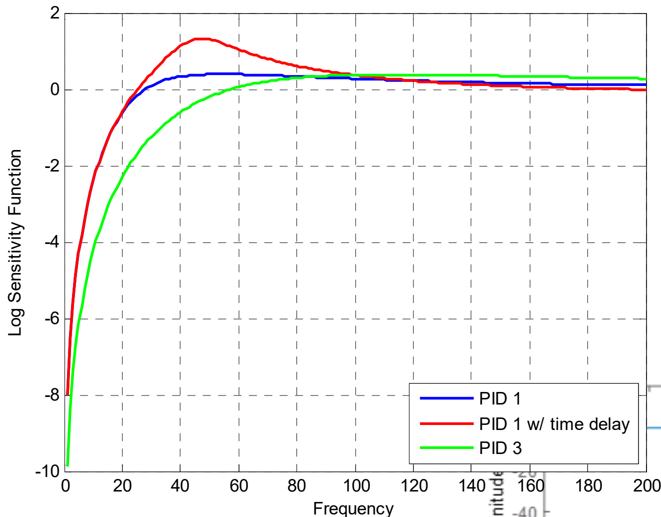
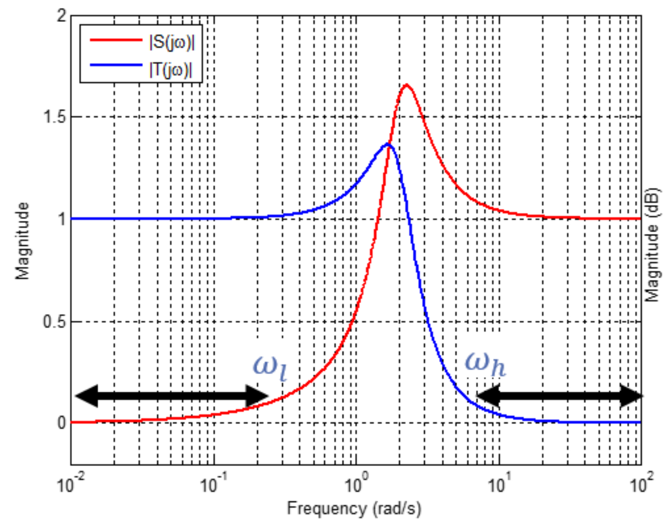
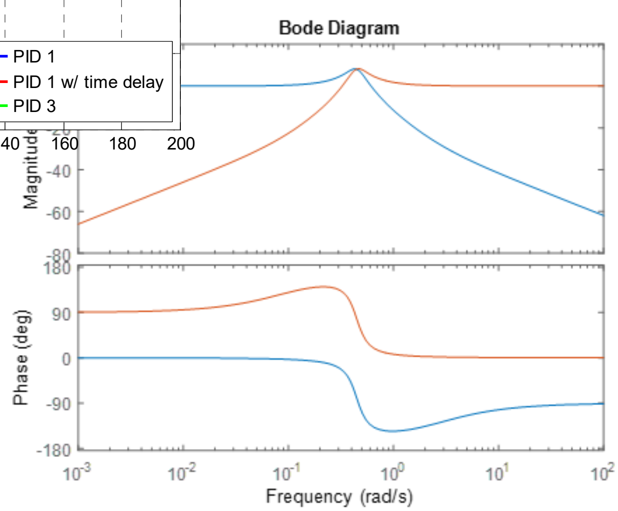
FROM LAST TIME…

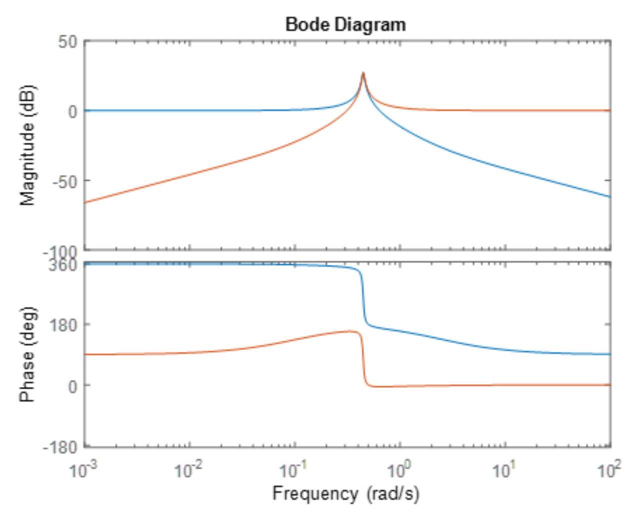
Frequency-Domain Design Limitations

- Bode’s Integral Constraints
- Poisson Integral Constraints









SUMMARY OF DESIGN LIMITATIONS

Topics

- Fundamental Design Trade-offs
- Limitations
- Remedies

At the end of this section, students should be able to:

- Identify design trade-offs.
- Identify limitations due to physical components.

HOW DO WE QUANTIFY CLOSED-LOOP PERFORMANCE?

- Various aspects of performance cannot be addressed independently via SISO design
- Must have a thorough understanding of fundamental SISO design limitations to come up with realistic performance specifications

DISTURBANCE REJECTION PROPERTY

$$Y_D(s) = S(s)[D_o(s) + G(s)D_i(s)] \qquad S(s) = 1 - T(s)$$

Disturbances are rejected only at frequencies where

$$|S(j\omega)| \approx 0 \quad \text{or} \quad |T(j\omega)| \approx 1$$

To achieve acceptable performance in the presence of disturbances, it will generally be necessary to place a lower bound on the closed- loop bandwidth!

MEASUREMENT NOISE EFFECT

$$Y_M(s) = -T(s)N(s)$$

Measurement noise is rejected only at frequencies where

$$|T(j\omega)| \approx 0$$

Given the fact that noise is typically dominated by high frequencies, there will be an upper limit on the bandwidth of closed-loop set by measurement noise!

CONTROL EFFORT

$$U(s) = S_u(s)[\bar{R}(s) - D_o(s) - N(s)] \qquad S_u(s) = T(s)/G(s)$$

Large control signals arise at frequencies where $|T(j\omega)| \approx 1$ but $|G(j\omega)| \ll 1$ which occurs when closed-loop is forced to be much more responsive than open-loop process

To avoid actuator saturation or slew rate problems, it will generally be necessary to place an upper limit on the closed-loop bandwidth!

MODELING ERROR EFFECT

Robust Stability Theorem: $|T_o(j\omega)||G_\Delta(j\omega)| < 1, \quad \forall\omega$

Robust Performance: $|S_\Delta(j\omega)| = \left|\dfrac{1}{1 + T_o(j\omega)G_\Delta(j\omega)}\right| \approx 1, \quad \forall\omega$

Design Constraint: $S_o(j\omega) + T_o(j\omega) = 1, \quad \forall\omega$

For frequencies with significant modeling errors:

Being responsive to reference changes and against disturbances at frequencies with significant modeling errors jeopardize stability; note also that MME $|G_\Delta(\mathrm{j}\omega)|$ is normally significant at high frequencies

> To achieve robust stability and acceptable performance in the presence of modeling errors, it will generally be desirable to place an upper limit on closed-loop bandwidth!

STRUCTURAL LIMITATIONS VIA TIME DELAYS

Best Attainable Ideal Sensitivity Functions for a delay of τ

$$T_o^*(s) = e^{-s\tau}$$
$$S_o^*(s) = 1 - e^{-s\tau}$$

- To achieve the above ideal result requires the use of Smith Predictor plus an ideal controller!

Practically Achievable Sensitivity Functions

- For $\eta\tau$ error of time delay, magnitude of MME $|G_\Delta(j\omega)|$ approaches 1 at approximately a bandwidth of $1/\eta\tau$, and thus limits the achievable CL bandwidth to this order for robust stability.

1. Delays limit disturbance rejection by requiring that a delay occurs before the disturbance can be cancelled.
2. Delays further limit the achievable CL bandwidth through the impact of modeling errors

SYSTEM TYPE

Want zero steady-state error to a step reference?

- Controller or plant must have at least one free integrator

Want zero steady-state error to a step input disturbance?

- Controller must have at least one free integrator

Requirements on steady-state error to reference or input disturbance place requirements on controller structure

OPEN LOOP POLES/ZEROS

Observations

- A real minimum-phase (LHP) zero to the right of all CL poles produces overshoot in the step response
- A real nonminimum-phase (RHP) zero always produces undershoot in the step response. The amount of undershoot grows as the zero approaches the origin
- Any real open-loop pole to the right of all CL poles will produce overshoot in a one-DOF control architecture
- Imaginary zeros smaller than CL poles will produce overshoot

1. The CL bandwidth should in practice be set less than the smallest RHP zero to avoid large undershoot in step response
2. It is advisable to set CL bandwidth greater than real part of any unstable OL pole to avoid large overshoot
3. Maximum error will be very large if one tries to make CL bandwidth greater than the position of the imaginary zeros

WATER BED EFFECT (BODE)

Independent of controller design

- Low sensitivity in certain prescribed frequency bands will result in a sensitivity larger than one in other frequency bands
- Unstable (RHP) open-loop poles make sensitivity minimization more difficult

- Low complementary sensitivity in certain prescribed frequency bands will result in a complementary sensitivity larger than one in other frequency bands
- RHP open-loop zeros makes the allocation of complementary sensitivity in the frequency domain more difficult.

POISSON

Observations

- When one RHP zero approaches a RHP open-loop pole, sensitivity allocation in frequency domain becomes almost impossible.
- When CL bandwidth is large when compared to the speed of NMP (RHP) zero, there will be a large sensitivity peak
- When CL bandwidth is small compared to unstable RHP poles, there will be a large complementary sensitivity peak.
- Sharp transitions in the sensitivity frequency response, i.e., ω_l close to ω_h, will contribute to large sensitivity peaks.

The design problem becomes more difficult when the system has *fast RHP open-loop poles* and *slow RHP zeros*, relative to CL bandwidth

SISO DESIGN LIMITATIONS (PART 2)

REMEDIES

GIVEN THE LIMITATIONS IN SISO DESIGN, WHAT CAN WE ACTUALLY DO WITH ALL OF THEM?

1. Identify unachievable specifications

2. Identify where additional effort would be fruitful or wasted

For example, each of the following impose an upper limit on usable bandwidth:

- Actuator slew-rate and amplitude limits
- Model error
- Delays
- Right-half-plane or imaginary-axis zeros

GIVEN THESE TRADE-OFFS AND LIMITATIONS, WHAT REMEDIES ARE AVAILABLE?

WHAT CHANGES TO SENSOR SELECTION WILL HELP?

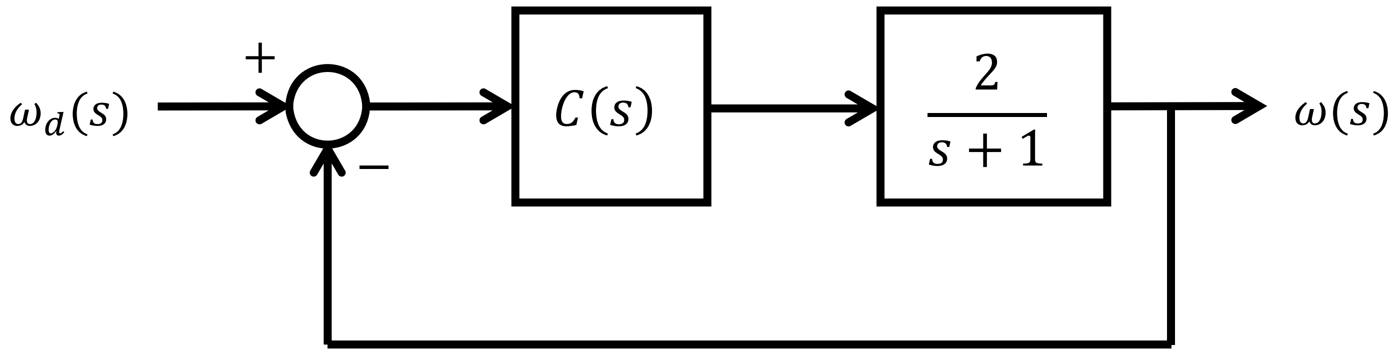
- More accurate or faster sensor
- Virtual sensor

WHAT CHANGES TO ACTUATOR SELECTION WILL HELP?

More powerful or faster actuators

EXAMPLE: CHANGING PID STRUCTURE

SPEED CONTROL OF MOTOR

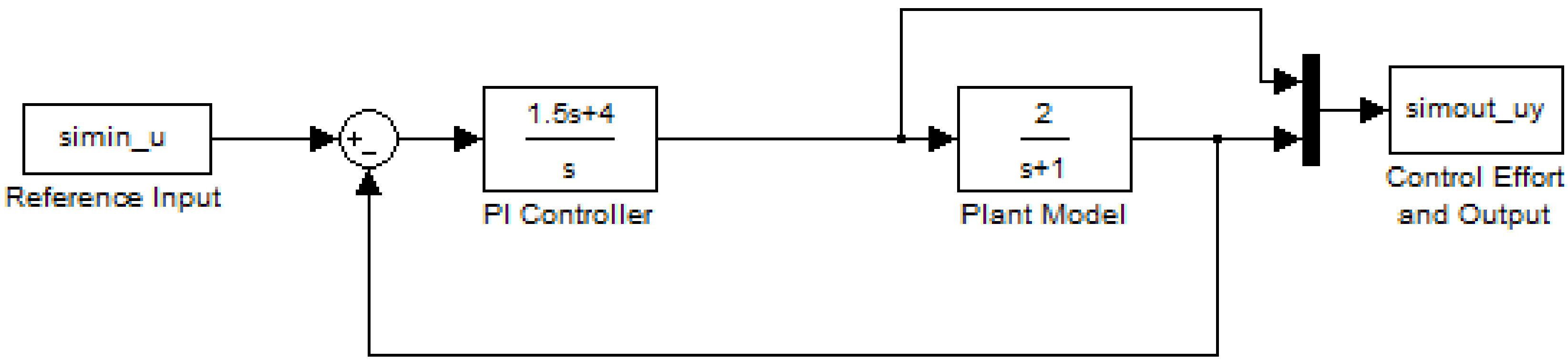


Performance Specifications:

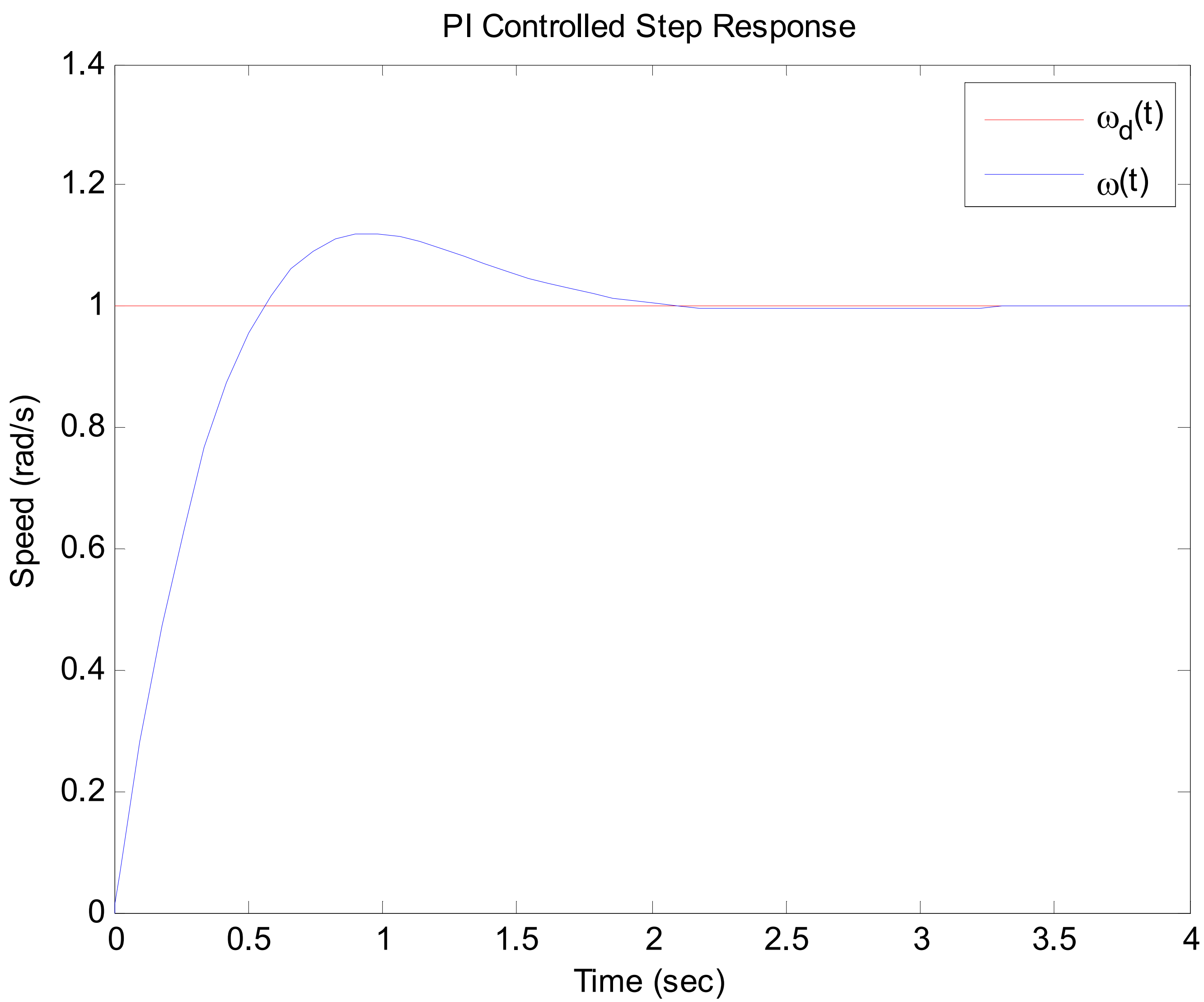
- Damping ratio = 0.707
- 2% Settling time = 2 sec.
- S.S. error to step inputs = 0

CHOOSE A PI CONTROLLER

CREATE CONTROL LOOP IN SIMULINK

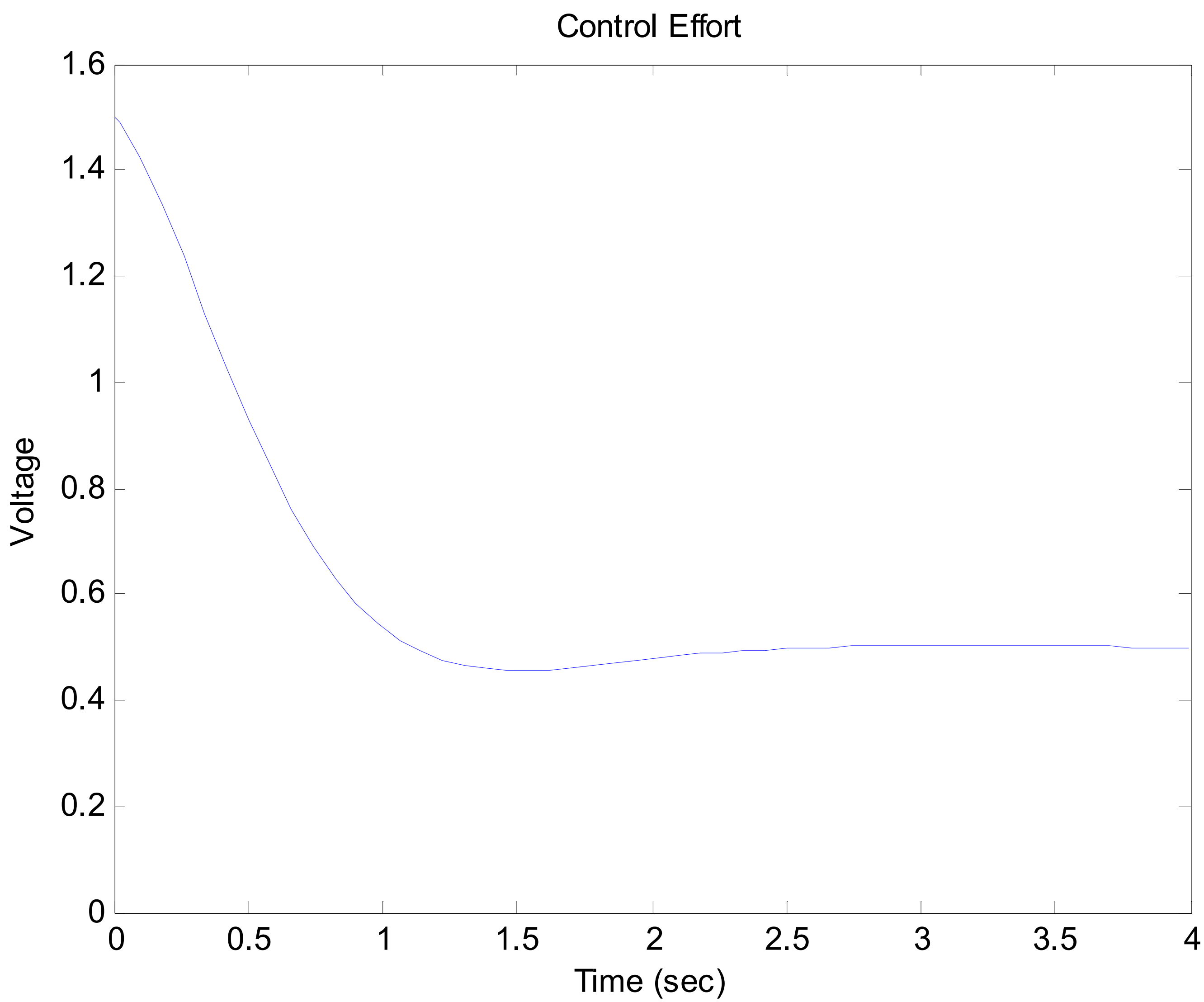


PI CONTROL

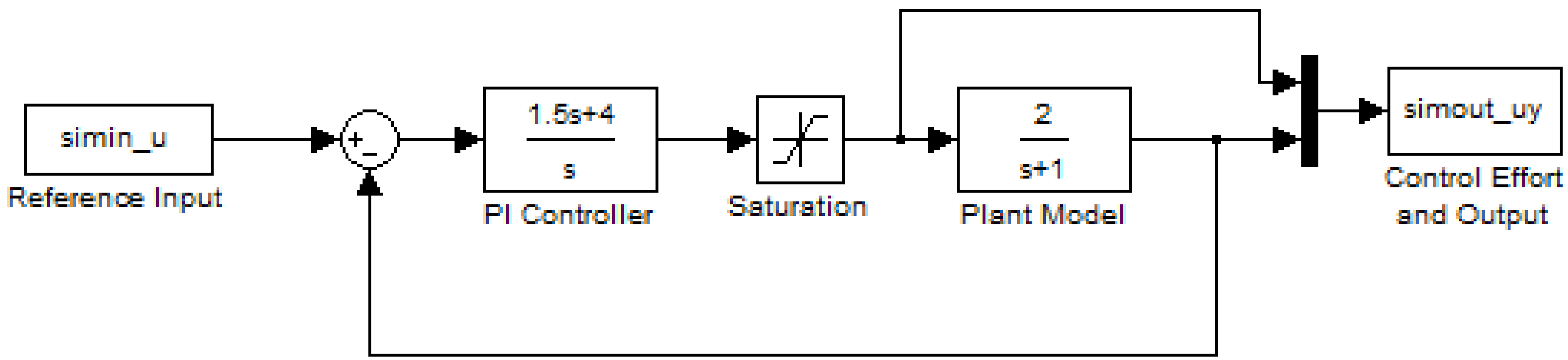


PI Controlled Step Response
Speed (rad/s)
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.5
1
1.5
2
2.5
3
3.5
4
$\omega_d(t)$
$\omega(t)$

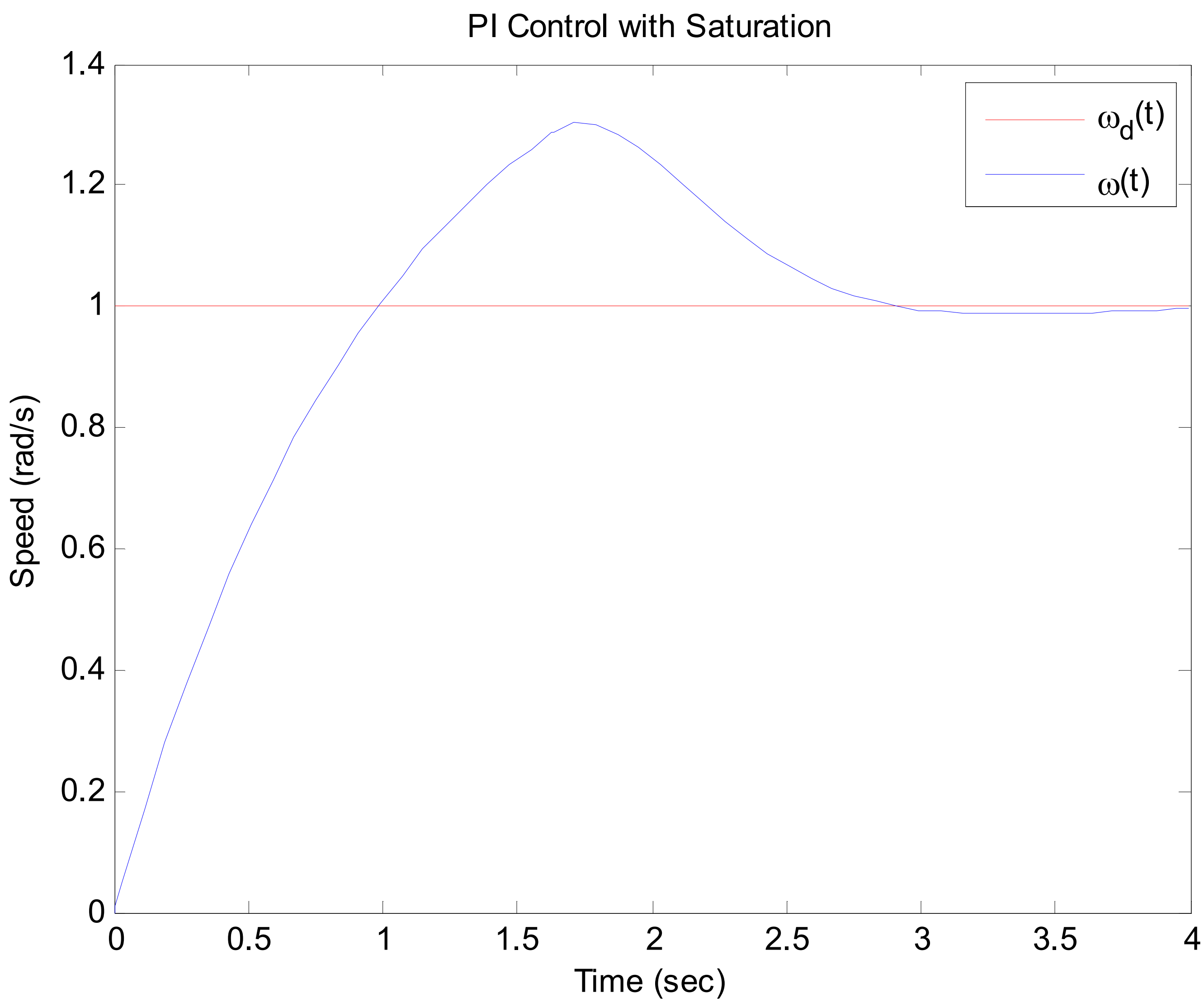
PI CONTROL


Control Effort
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Voltage
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec)

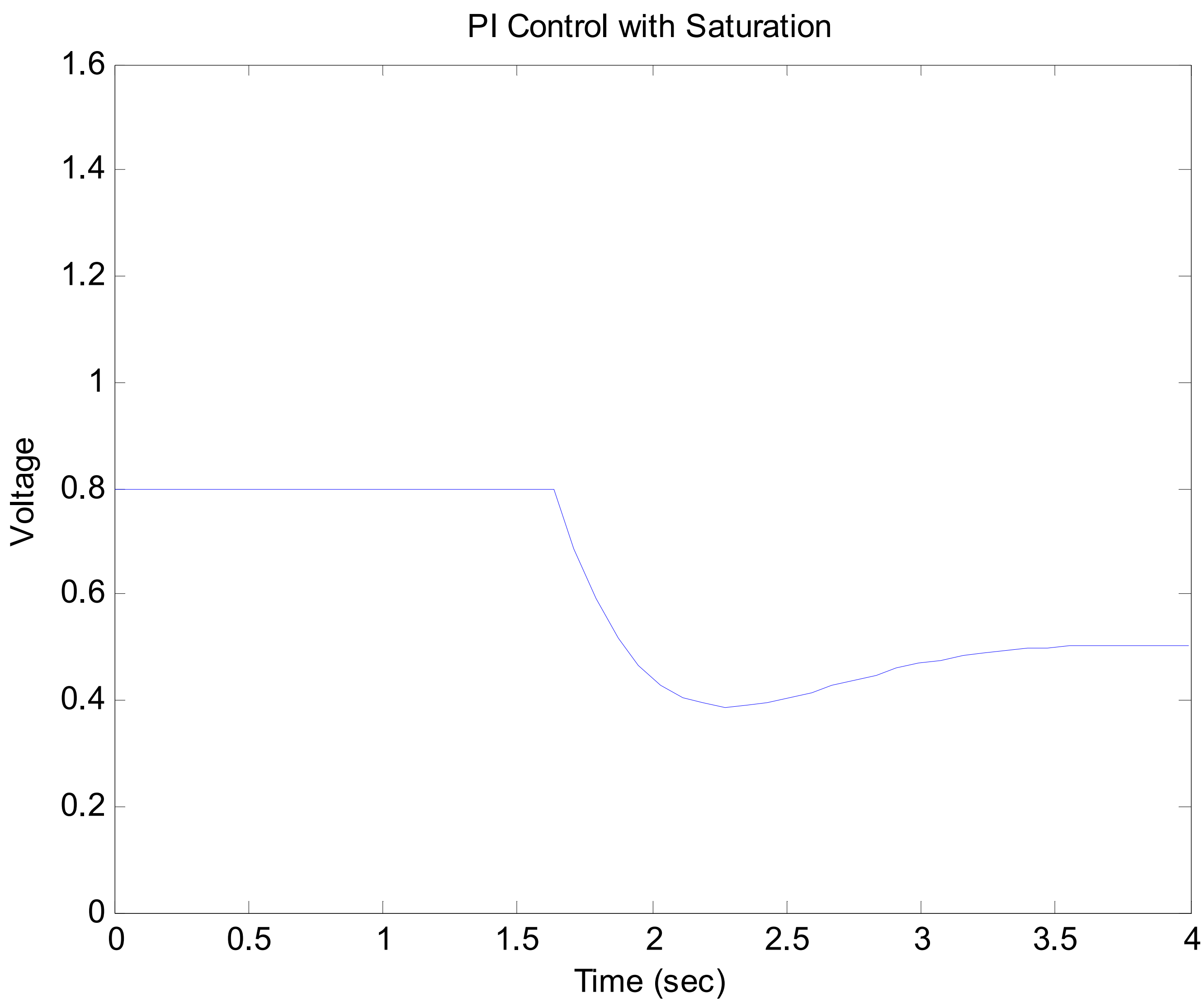
ADD ACTUATOR SATURATION TO CONTROL LOOP IN SIMULINK



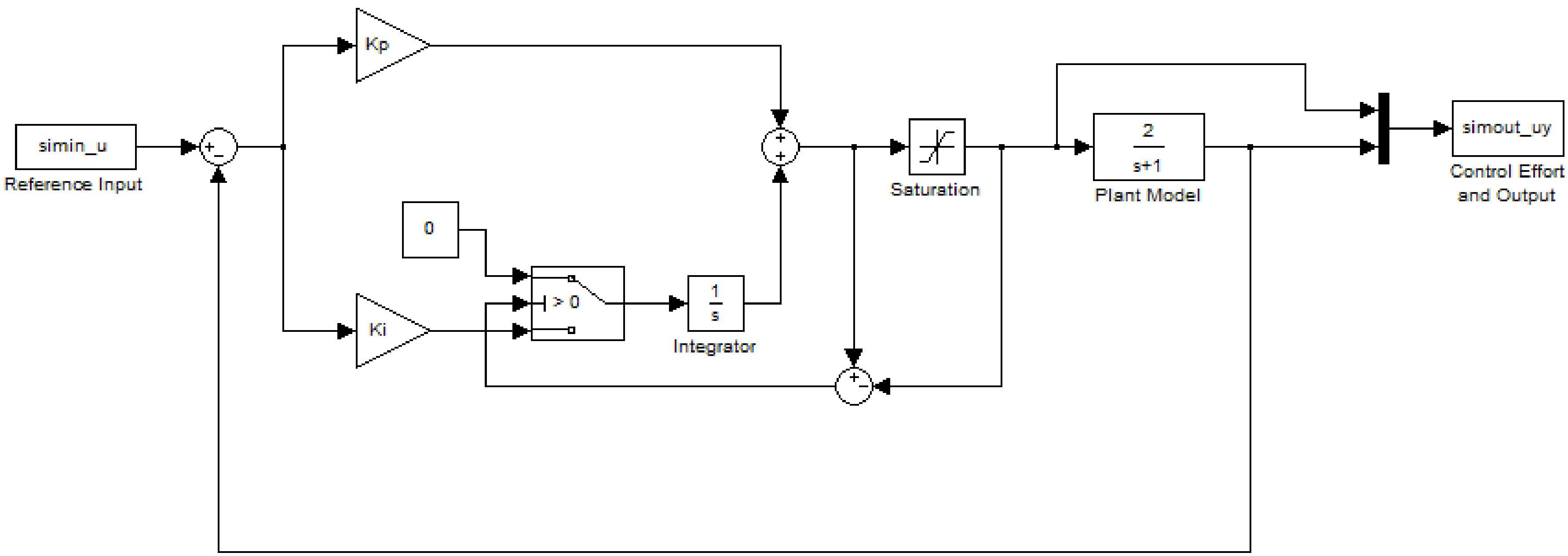
PI CONTROL WITH SATURATION



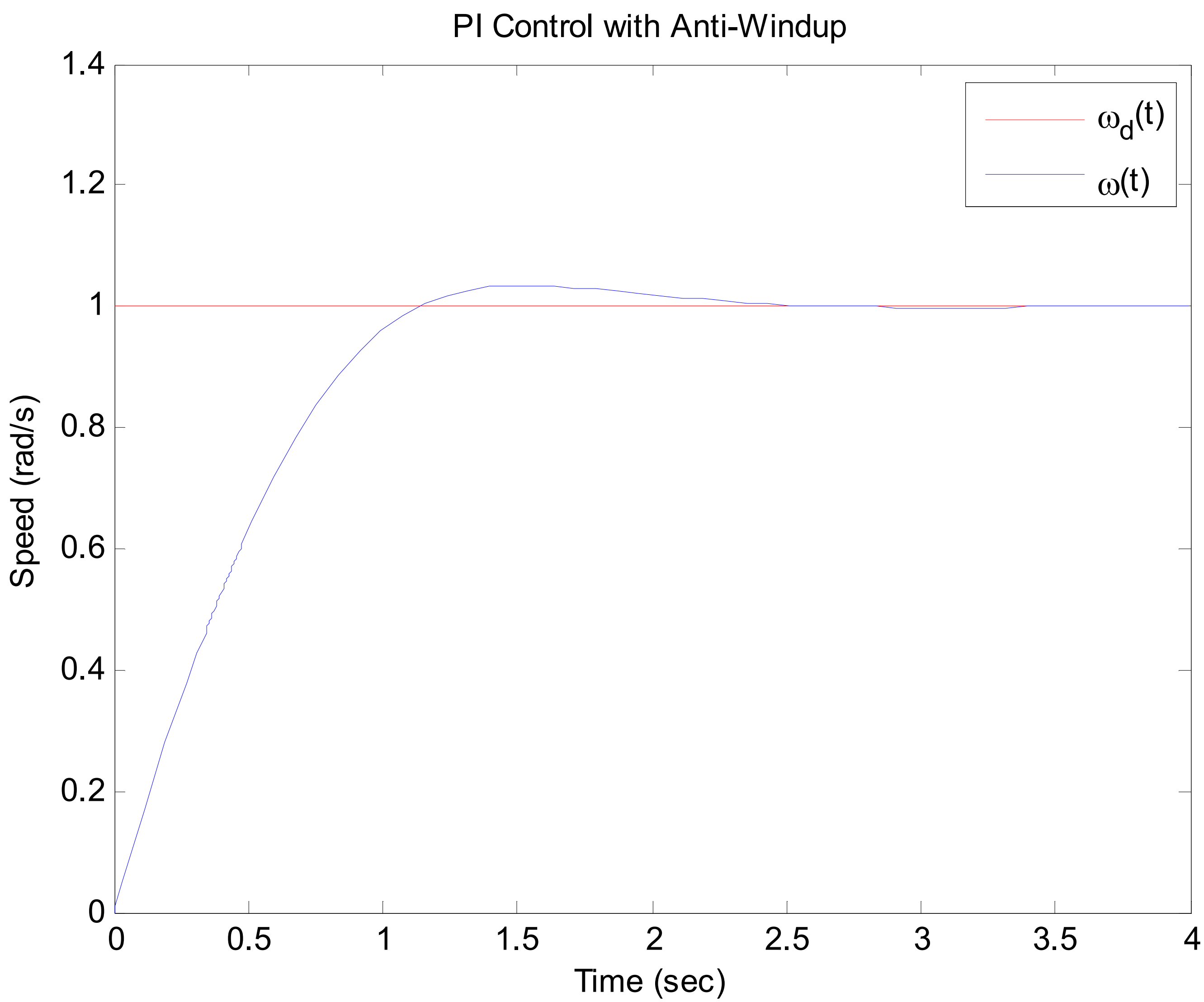
PI CONTROL WITH SATURATION



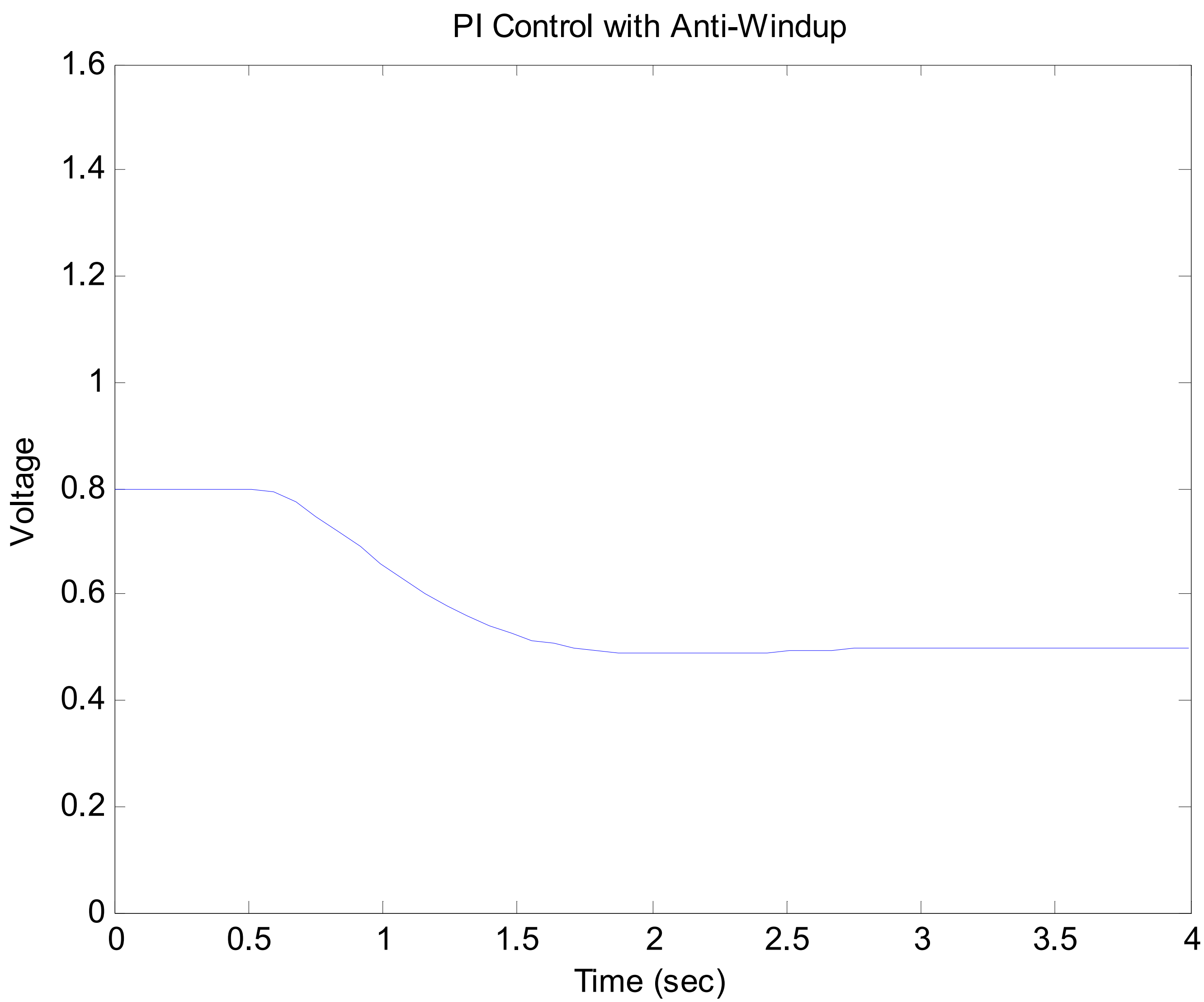
ADD ANTI-WINDUP MECHANISM TO CONTROL LOOP IN SIMULINK



PI CONTROL WITH ANTI-WINDUP



PI CONTROL WITH ANTI-WINDUP



COMING UP…

Architectural Issues

- Internal Model Principle
- Feedforward
- Cascade Control

Midterm Exam!