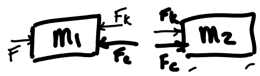
STATE FEEDBACK

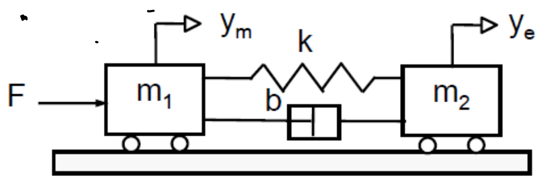
CASE STUDY: ROBOTIC WELDING

ROBOTIC WELDING

https://www.youtube.com/watch?v=ebX5hU_MDAY

MODEL THE WELDING TORCH AS A TWO-MASS SYSTEM WITH SOME STIFFNESS AND DAMPING





We can change input force F

We want to control end effector position y_e

Your intern has developed a transfer function for the system:

$$G(s) = \frac{Y_e(s)}{F(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

b k k²

b/m k k

DERIVE THE STATE-SPACE SYSTEM MODEL AND VERIFY YOUR INTERN'S WORK → NO IC's

$m_1\ddot{y}_m = -b(\dot{y}_m - \dot{y}_e) - K(y_m - y_e) + F$

$m_2\ddot{y}_e = b(\dot{y}_m - \dot{y}_e) + K(y_m - y_e)$

$G(s) = \frac{Y_e(s)}{F(s)}$

$m_1 s^2 y_m(s) = -b y_m(s)s - b y_e(s)s - K y_m(s) + K y_e(s) + F(s)$

$m_2 s^2 y_e(s) = b y_m(s)s - b y_e(s)s + K y_m(s) - K y_e(s)$

$y_m(s)(-bs - K - m_1s^2) + y_e(s)(-bs + K) + F(s) = 0$

$y_m(s) = \frac{-y_e(s)(-bs+K) + F(s)}{(-bs - K - m_1s^2)}$

$m_2 s^2 y_e(s) = y_e(s)(-bs - K) + \frac{-y_e(s)(-bs+K) + F(s)}{(-bs-K-m_1s^2)}(bs+K)$

$y_e(s)\left(\frac{-(-bs+K)(bs+K)}{(-bs-K-m_1s^2)} + (-bs-K) - m_2s^2\right) + \frac{F(s)\,b(s+K)}{(-bs-K-m_1s^2)} = 0$

$y_e(s)\left(\frac{-(-bs+K)(bs+K) + (-bs-K)(-bs-K-m_1s^2) + (-m_2s^2)(-bs-K-m_1s^2)}{(-bs-K-m_1s^2)}\right.$

$= \frac{-F(s)\,b(s+K)}{(-bs-K-m_1s^2)}$

I FUCKED UP SOMEWHERE

- Assume that the system has parameters
  - $\omega_n = 1$ rad/sec
  - $\zeta = 0$
- Assumed ζ value is highly unrealistic, but shows a special case

CHECK CONTROLLABILITY OF THE SYSTEM

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix}}_{A} x \qquad y = \underbrace{\begin{bmatrix} b_0 & b_1 & b_2 & b_3 \end{bmatrix}}_{b} x$$

$$CCF \quad \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f \qquad \zeta = 0 \quad \omega_n = 1$$

$$y_e = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

PUT THE SYSTEM INTO CONTROLLABLE CANONICAL FORM

LET'S DESIGN A CONTROLLER!

Let

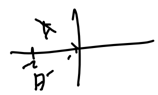
$$u = k_r r - Kx$$

Then

$$\dot{x} = (A - BK)x + Bk_r r$$

Choose design specifications:

- Closed-loop system should be 2nd-order dominant:
  - closed-loop damping ratio = 0.7 ~ ABOUT 45° FROM i AXIS
  - 2% settling time = 2 sec. $t_s = \frac{4}{|Re|} \rightarrow |Re| > 2$
- Plot plant poles and zeros and desired dominant closed-loop poles



$S = -2 \pm 2j$

PICK OTHERS 3 FAR AWAY

FIND K TO ACHIEVE DESIRED SPECIFICATIONS:

Let $|sI - A + BK| = \phi(s) = (s+2+2j)(s+2-2j)(s+10)(s+10)$

$$= s^4 + \underbrace{24}_{\alpha_3} s^3 + \underbrace{188}_{\alpha_2} s^2 + 560 s + 800$$

$$K =$$

BECAUSE OF OUR SPECIAL CASE, THE STATES ARE ALL DERIVATIVES OF THE OUTPUT

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \quad \Rightarrow \quad x = \begin{bmatrix} y_e \\ \dot{y}_e \\ \ddot{y}_e \\ \dddot{y}_e \end{bmatrix}$$

$$y_e = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

- With non-zero ζ, the C matrix would be more complicated
- Relationship between states and output would also be more complicated

ASIDE: WHAT IF WE HAD A MORE GENERAL STATE SPACE AND WANTED TO KNOW THE OUTPUT DERIVATIVES?

$$y = Cx$$

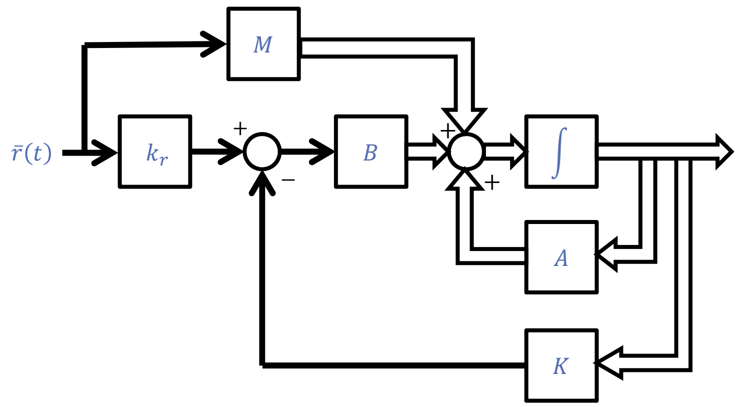
$$\dot{y} = C\dot{x} = C(Ax)$$

$$\ddot{y} = \frac{d}{dt}(CAx) = CA\dot{x} = CA(Ax)$$

$$\dddot{y} = \frac{d}{dt}(CA^2x) = CA^2\dot{x} = CA^2(Ax)$$

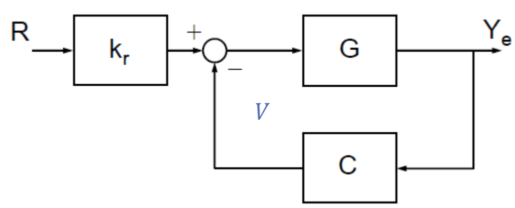
- We will use this fact when we talk about observability

RECALL THAT IN STATE FEEDBACK, THE CONTROLLER IS IN THE FEEDBACK PATH



USING STATE-OUTPUT RELATIONSHIP, DETERMINE EQUIVALENT CONTROLLER TRANSFER FUNCTION

$$x = \begin{bmatrix} y_e \\ \dot{y}_e \\ \ddot{y}_e \\ \dddot{y}_e \end{bmatrix}$$



$$C(s) = \frac{V(s)}{Y_e(s)} =$$

WE WANT ZERO STEADY-STATE TRACKING ERROR, USE THIS TO DESIGN k_r

$$\lim_{s \to 0} \frac{Y_e(s)}{R(s)} = 1$$

$$\lim_{s \to 0} \frac{Y_e(s)}{R(s)} =$$

$$k_r =$$

USE MATLAB TO VALIDATE THE CONTROLLER DESIGN

Closed-loop step response

- End effector displacement (m)
- Control effort (N)

Include 160 N maximum force output

- End effector displacement (m)
- Control effort (N)