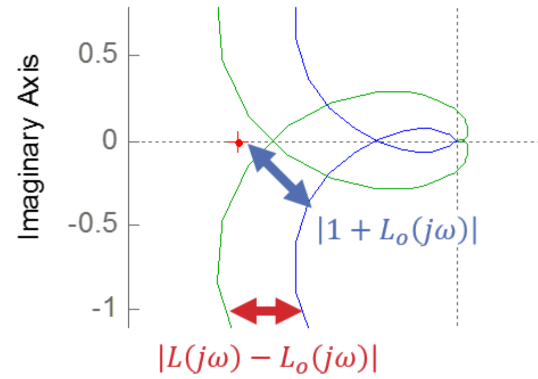
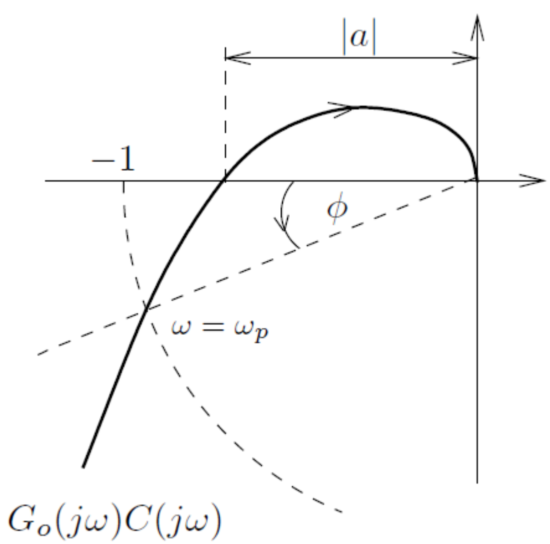
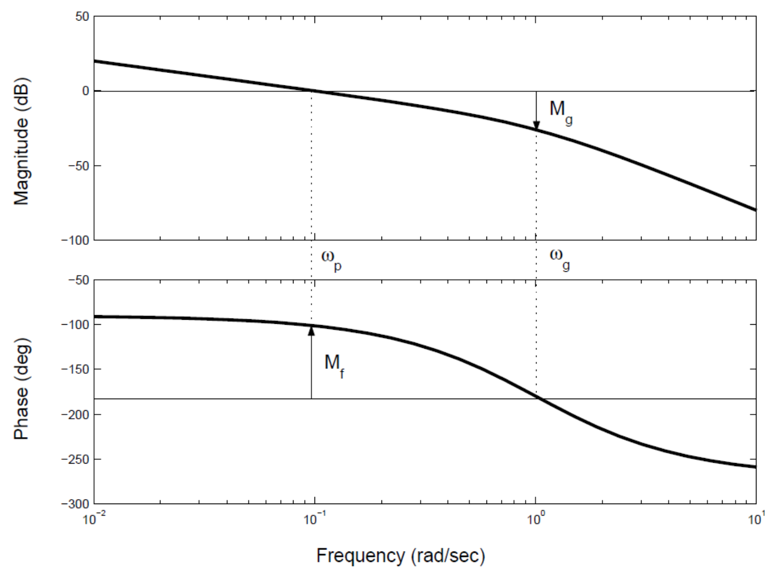
FROM LAST TIME

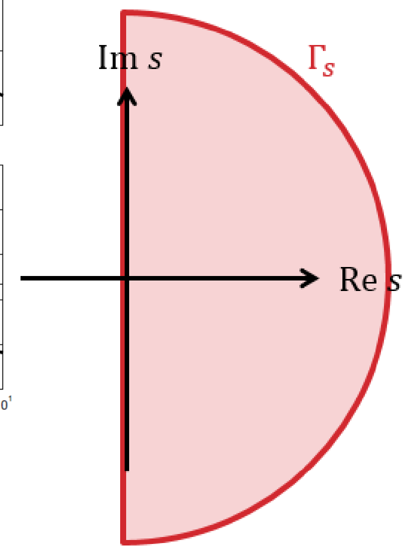
More Stability

- Nyquist test for stability
- Relative stability
- Robust stability









POLE PLACEMENT DESIGN

Topics

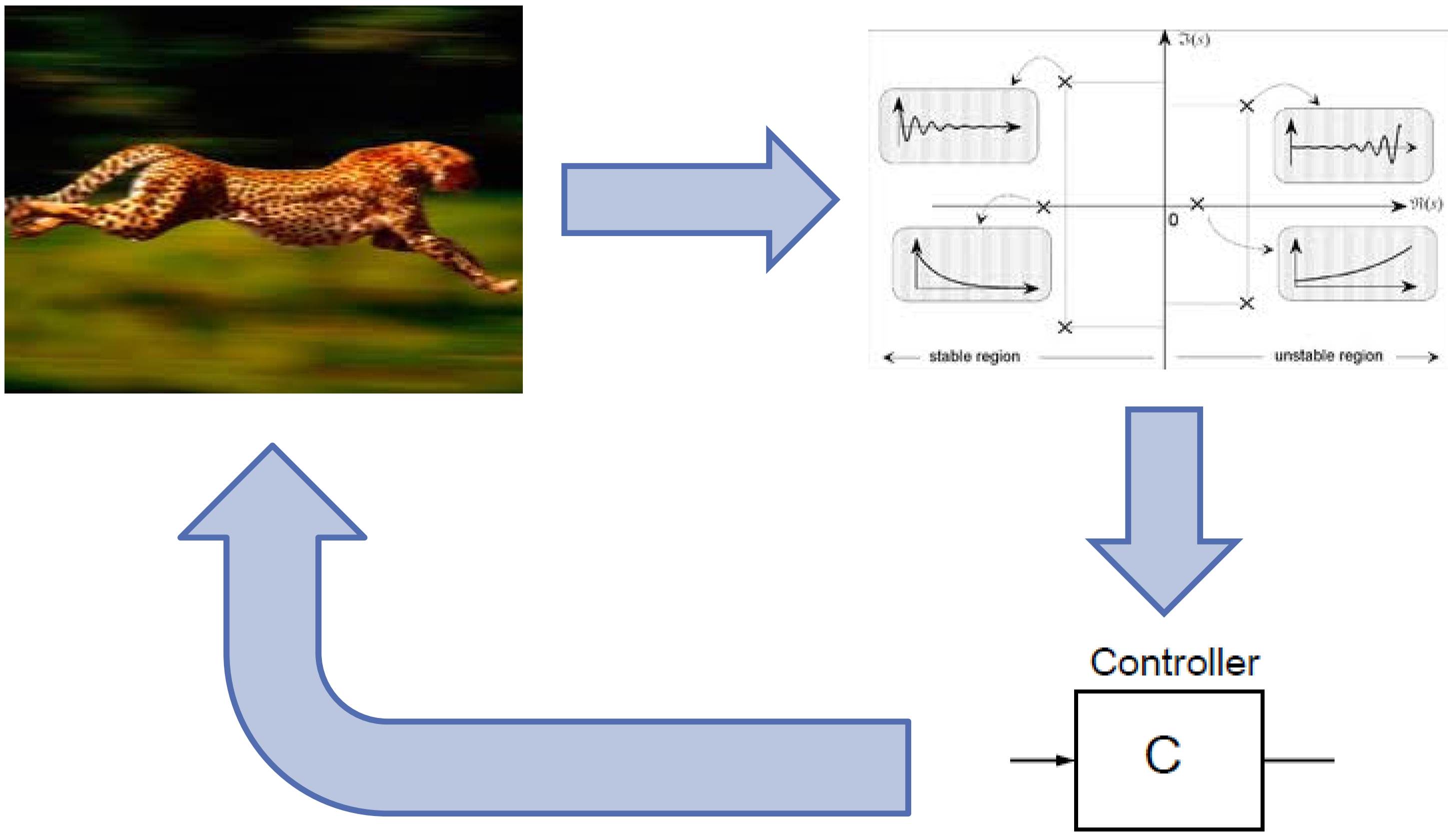
- Pole placement design
- Controller with integration
- PID via pole placement
- Smith predictor

At the end of this section, students should be able to:

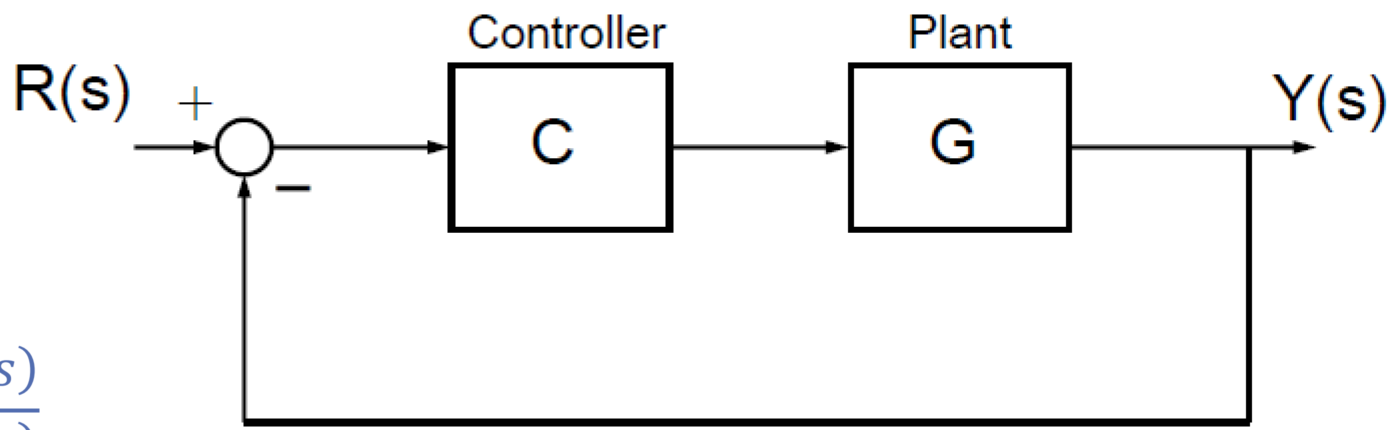
- Design a controller using pole placement method.
- Describe effects of P, I, and D terms.
- Design PID controllers using pole placement.
- Describe the operation and benefits of a Smith predictor.

WHAT CONTROLLER DESIGN TECHNIQUES DO YOU KNOW FROM PREVIOUS COURSES?

WE WANT A SYSTEMATIC PROCEDURE TO SYNTHESIZE A CONTROLLER FOR SISO LTI SYSTEMS



RECALL THE CLOSED-LOOP CHARACTERISTIC EQUATION



Let

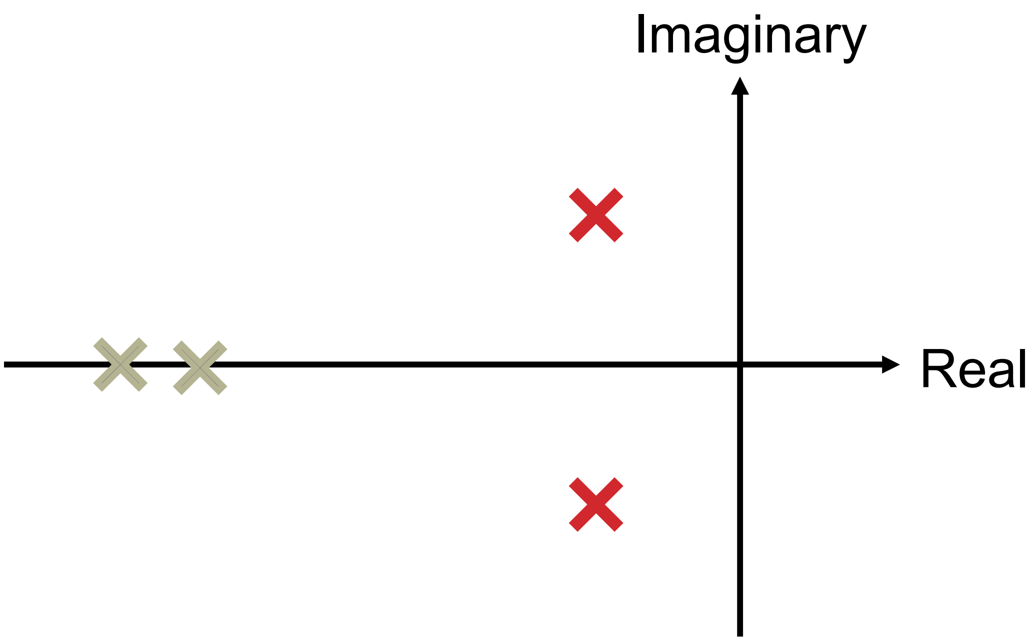
$$C(s) = \frac{N_C(s)}{D_C(s)}$$

$$G_o(s) = \frac{N_{G_o}(s)}{D_{G_o}(s)}$$

$$T_o(s) = \frac{G_o(s)C(s)}{1 + G_o(s)C(s)}$$

CL characteristic eq:

BASED ON DESIRED PERFORMANCE, SELECT DESIRED CLOSED-LOOP POLES



- Choose dominant poles first
- Place remaining poles far to the left of dominant poles
- Combine the desired closed-loop poles into a desired closed-loop characteristic polynomial

$$D_{CL}(s) = a^c_{n_{CL}}(s - p_1)(s - p_2)\cdots\left(s - p_{n_{CL}}\right)$$

POLE PLACEMENT EQUATES THE DESIRED AND ACTUAL CHARACTERISTIC POLYNOMIALS

$$D_{CL}(s) = a^c_{n_{CL}}(s-p_1)(s-p_2)\cdots(s-p_{n_{CL}})$$
$$= a^c_{n_{CL}}s^{n_{CL}} + a^c_{n_{CL}-1}s^{n_{CL}-1} + \cdots + a^c_1 s^1 + a^c_0$$

$$D_{CL}(s) = D_{G_o}(s)D_C(s) + N_{G_o}(s)N_C(s)$$

- This gives unknown coefficients due to controller $N_C(s)$ and $D_C(s)$
- Match coefficients and solve

EXAMPLE OF POLE PLACEMENT

Let

$$G_o(s) = \frac{1}{s^2 + 3s + 1} \qquad C(s) = \frac{b_{C1}s + b_{C0}}{a_{C1}s + a_{C0}}$$

Characteristic Equation:

$$(s^2 + 3s + \ 1)(a_{C1}s + a_{C0}) + (1)(b_{C1}s + b_{C0}) = 0$$

Choose poles such that the characteristic polynomial is:

$$(s + 10)(s^2 + 6s + 25)$$

EXAMPLE OF POLE PLACEMENT

Solve for controller coefficients:

WE CAN WRITE THE GENERALIZED PROBLEM AS:

Given:

$$D_{G_o}(s)D_C(s) + N_{G_o}(s)N_C(s) = 0$$

where

$$\begin{aligned}
N_{G_o}(s) &= b_{Gm}s^m + b_{G(m-1)}s^{m-1} + \cdots + b_{G1}s + b_{G0} \\
D_{G_o}(s) &= a_{Gn}s^n + a_{G(n-1)}s^{n-1} + \cdots + a_{G1}s + a_{G0} \\
N_C(s) &= b_{Cm_C}s^{m_C} + b_{C(m_C-1)}s^{m_C-1} + \cdots + b_{C1}s + b_{C0} \\
D_C(s) &= a_{Cn_C}s^{n_C} + a_{C(n_C-1)}s^{n_C-1} + \cdots + a_{C1}s + a_{C0}
\end{aligned}$$

and

$$\begin{aligned}
D_{CL}(s) &= a_{n_{CL}}^c s^{n_{CL}} + a_{n_{CL}-1}^c s^{n_{CL}-1} + \cdots + a_1^c s + a_0^c \\
&= a_{n_{CL}}^c (s-p_1)(s-p_2)\cdots\left(s-p_{n_{CL}}\right)
\end{aligned}$$

Find:

$$N_C(s), \qquad D_C(s)$$

DOES A SOLUTION EXIST?

Given $G_o(s)$ and any desired CL pole locations, i.e., known n_{CL} and $D_{CL}(s)$ but with coefficients being arbitrarily specified, does there exist a proper $C(s)$ that can achieve the desired poles?

Lemma:

Assume that $N_{Go}(s)$ and $D_{Go}(s)$ are coprime (no common factor). Then, as long as order of desired CL polynomial $D_{CL}(s)$ is no less than $2n-1$, there always exists a proper controller $C(s)$ that solves the pole placement problem:

$$D_{Go}(s)D_C(s) + N_{Go}(s)N_C(s) = D_{CL}(s)$$

In fact, when $n_{CL} = 2n-1$, the solution is unique with $C(s)$ of order $n_C = n-1$.

THE GENERAL SOLUTION FOR THE CONTROLLER COEFFICIENTS IS GIVEN BY:

$$\begin{bmatrix} a_{C(n-1)} \\ a_{C(n-2)} \\ \vdots \\ a_{C0} \\ b_{C(n-1)} \\ \vdots \\ b_{C0} \end{bmatrix} = \mathcal{S}^{-1} \begin{bmatrix} a_{2n-1}^{c} \\ a_{2n-2}^{c} \\ \vdots \\ a_{n}^{c} \\ a_{n-1}^{c} \\ \vdots \\ a_{0}^{c} \end{bmatrix}$$

$$\mathcal{S} = \begin{bmatrix} a_{Gn} & & & & b_{Gn} & & & \\ a_{G(n-1)} & \ddots & & & b_{G(n-1)} & \ddots & & \\ \vdots & & \ddots & & \vdots & & \ddots & \\ a_{G0} & & & a_{Gn} & b_{G0} & & & b_{Gn} \\ & \ddots & & a_{G(n-1)} & & \ddots & & b_{G(n-1)} \\ & & \ddots & \vdots & & & \ddots & \vdots \\ & & & a_{G0} & & & & b_{G0} \end{bmatrix}$$

- $\mathcal{S}$ is called the eliminant or Sylvester matrix

WHAT IF THE CONTROLLER MUST INCLUDE AN INTEGRATOR?

CONTROLLER WITH INTEGRATION

Want $D_C(s) = s\bar{D}_C(s)$

- Pole placement problem

$$D_{Go}(s)s\bar{D}_C(s) + N_{Go}(s)N_C(s) = D_{CL}(s)$$

- Equivalent pole placement problem

$$\bar{D}_{Go}(s)\bar{D}_C(s) + N_{Go}(s)N_C(s) = D_{CL}(s)$$

- Can be solved as before by assuming an equivalent fictitious plant of order $n + 1$ with a new denominator of

$$\bar{D}_{Go}(s) = sD_{Go}(s)$$

Solution always exists if n_{CL} is no less than $2n$. When $n_{CL} = 2n$, the solution is unique with order of $\bar{D}_C(s)$ being $n - 1$ and order $N_C(s)$ of being n!

WHAT IF WE WANT TO CANCEL SOME STABLE PLANT POLES OR ZEROS?

- Example

$$D_{Go}(s) = (s - p_C)\bar{D}_G(s)$$
$$N_{Go}(s) = (s - z_C)\bar{N}_G(s)$$

- Pole Placement Problem

$$(s - p_C)\bar{D}_G(s)(s - z_C)\bar{D}_C(s) + (s - z_C)\bar{N}_G(s)(s - p_C)\bar{N}_C(s) = D_{CL}(s)$$

- which has a solution only if $D_{CL}(s)$ contains the cancelled poles and zeros:

$$D_{CL}(s) = (s - p_C)(s - z_C)\bar{D}_{CL}(s)$$

- Equivalent Pole Placement Problem

$$\bar{D}_G(s)\bar{D}_C(s) + \bar{N}_G(s)\bar{N}_C(s) = \bar{D}_{CL}(s)$$

Cancelled poles/zeros remain as CL poles!

EXAMPLE

We want to add an integrator

Let

$$\bar{G}_o(s) = \frac{1}{s(s^2 + 3s + 1)} \qquad \bar{C}(s) = \frac{b_{C2}s^2 + b_{C1}s + b_{C0}}{a_{C1}s + a_{C0}}$$

Characteristic Equation:

$$s(s^2 + 3s + 1)(a_{C1}s + a_{C0}) + (1)(b_{C2}s^2 + b_{C1}s + b_{C0}) = 0$$

Let desired characteristic polynomial be:

$$(s + 10)^2(s^2 + 6s + 25)$$

EXAMPLE

Solve for controller coefficients:

PID CONTROL

PID CONTROLLER VIA POLE PLACEMENT

Proper PID Controller Structure

$$C(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_D s + 1} \quad = \frac{(K_D + K_P \tau_D)s^2 + (K_P + K_I \tau_D)s + K_I}{\tau_D s^2 + s}$$

Equivalent Controller Form

$$C(s) = \frac{b_{C2}s^2 + b_{C1}s + b_{C0}}{s^2 + a_{C1}s}$$

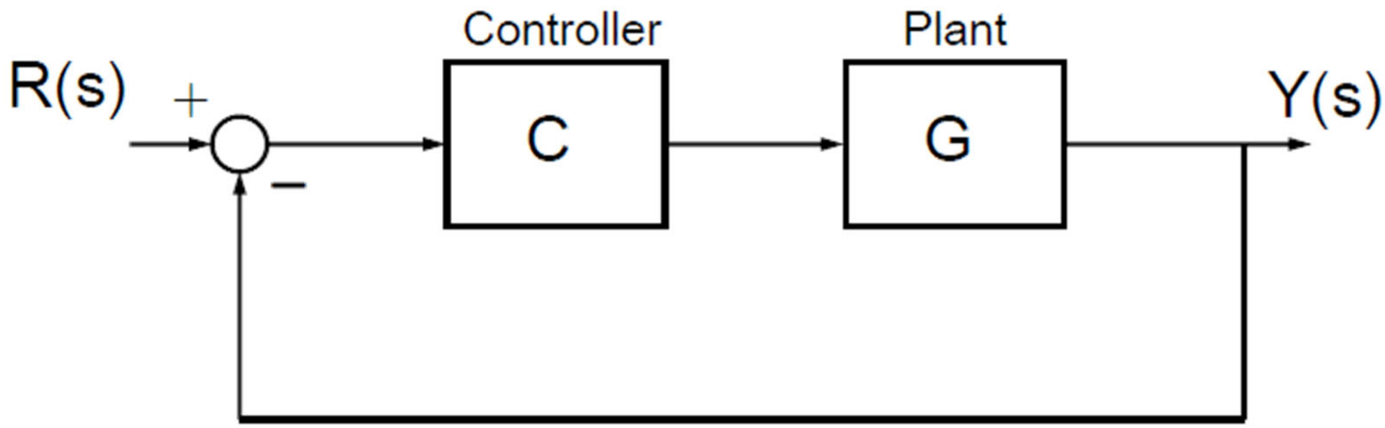
where

$$b_{C2} = \frac{K_D + K_P \tau_D}{\tau_D}$$

$$b_{C1} = \frac{(K_P + K_I \tau_D)}{\tau_D} \qquad a_{C1} = \frac{1}{\tau_D}$$

$$b_{C0} = \frac{K_I}{\tau_D}$$

PID CONTROL



$$u(t) = K_P e(t) + K_I \int_0^t e(t)dt + K_D \dot{e}(t)$$

RECALL THE EFFECTS OF P, I, AND D

Proportional (P)

- Improves rise time
- Reduces steady-state error
- Reduces effect of modeling error
- May introduce oscillation

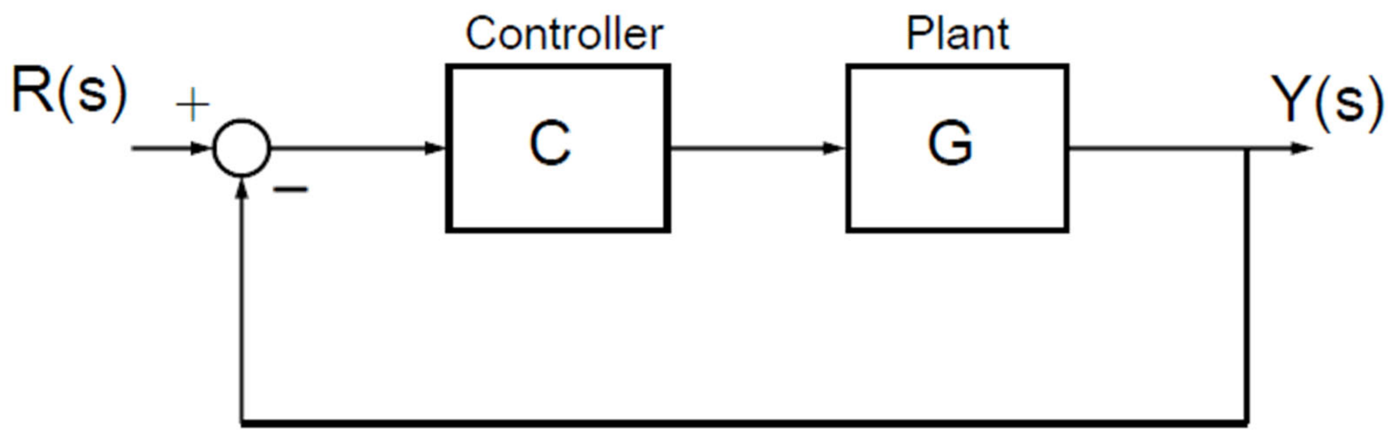
Integral (I)

- Eliminates steady-state error
- Increases system order
- May decrease stability margins

Derivative (D)

- Increases damping, may decrease settling time
- May increase overshoot

CONSIDER AN EXAMPLE



$$G_o(s) = \frac{4}{s(s+4)}$$

$$C(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_D s + 1} = \frac{b_{C2}s^2 + b_{C1}s + b_{C0}}{a_{C2}s^2 + a_{C1}s}$$

DESIGN A PID CONTROLLER FOR THE GIVEN POLE LOCATIONS

Case 1:

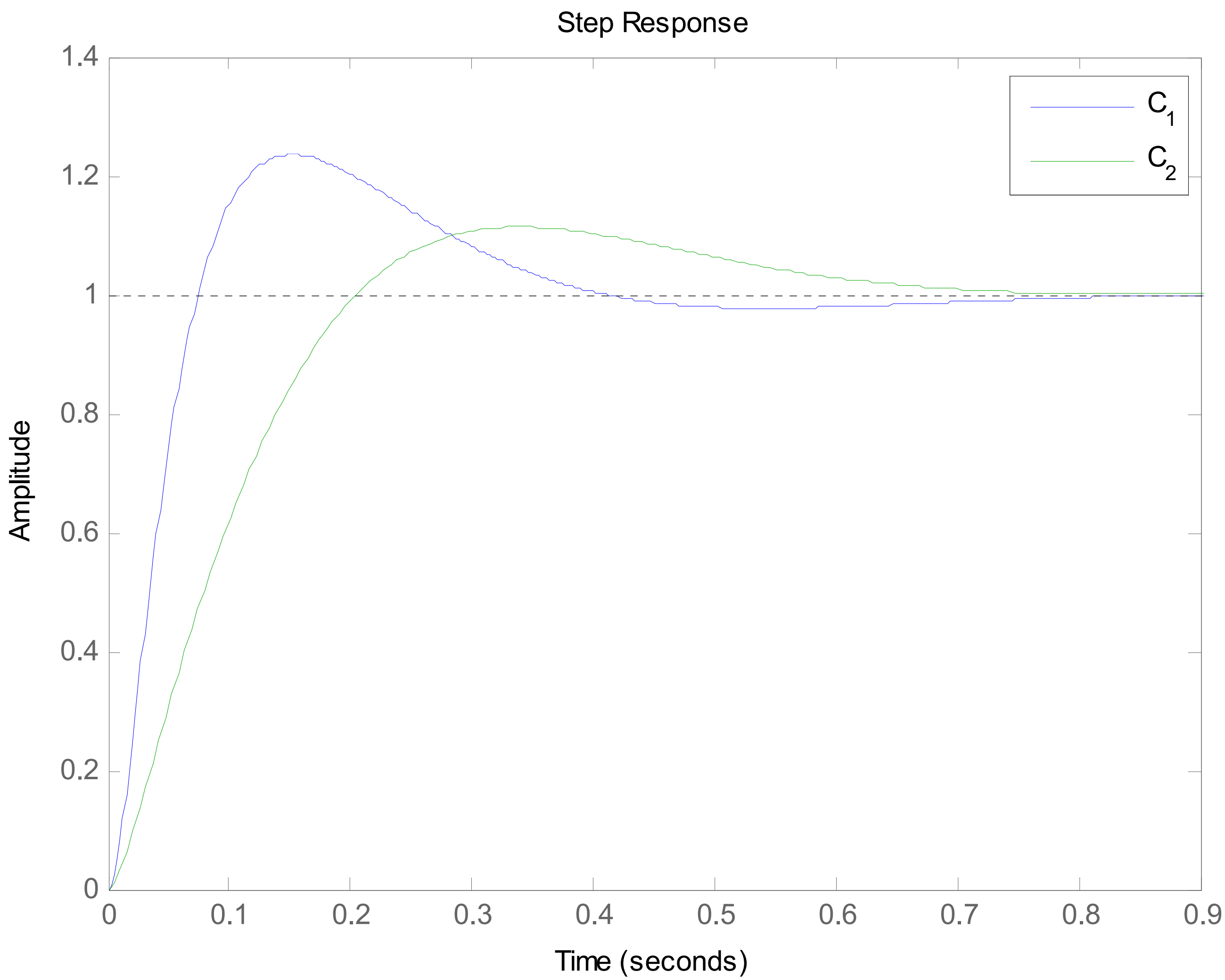
$$s_{1,2} = -6 \pm j6$$
$$s_3 = -20$$
$$s_4 = -70$$

DESIGN A PID CONTROLLER FOR THE GIVEN POLE LOCATIONS

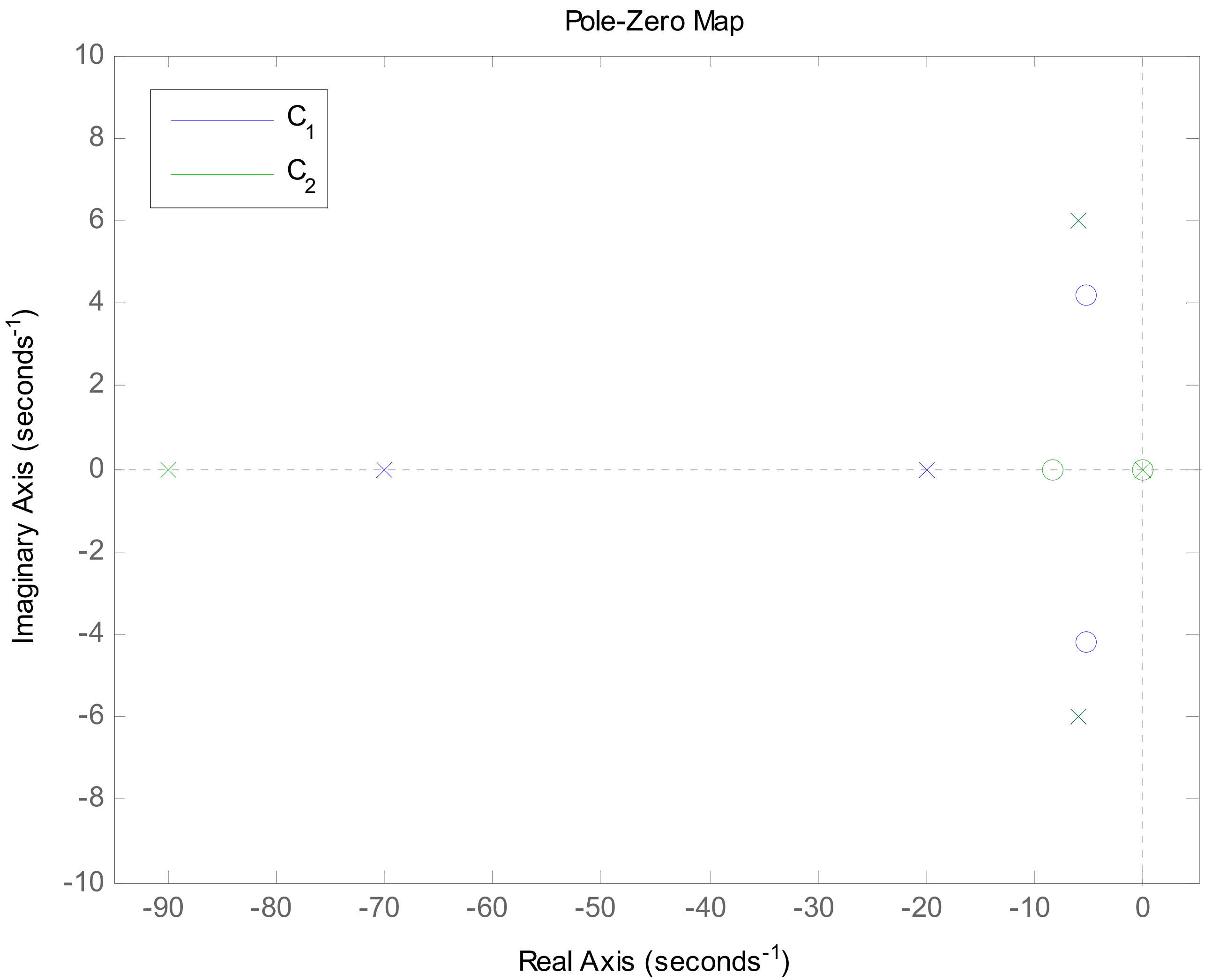
Case 2:

$$s_{1,2} = -6 \pm j6$$
$$s_3 = -0.1$$
$$s_4 = -90$$

USE MATLAB TO PLOT THE CLOSED-LOOP STEP RESPONSES

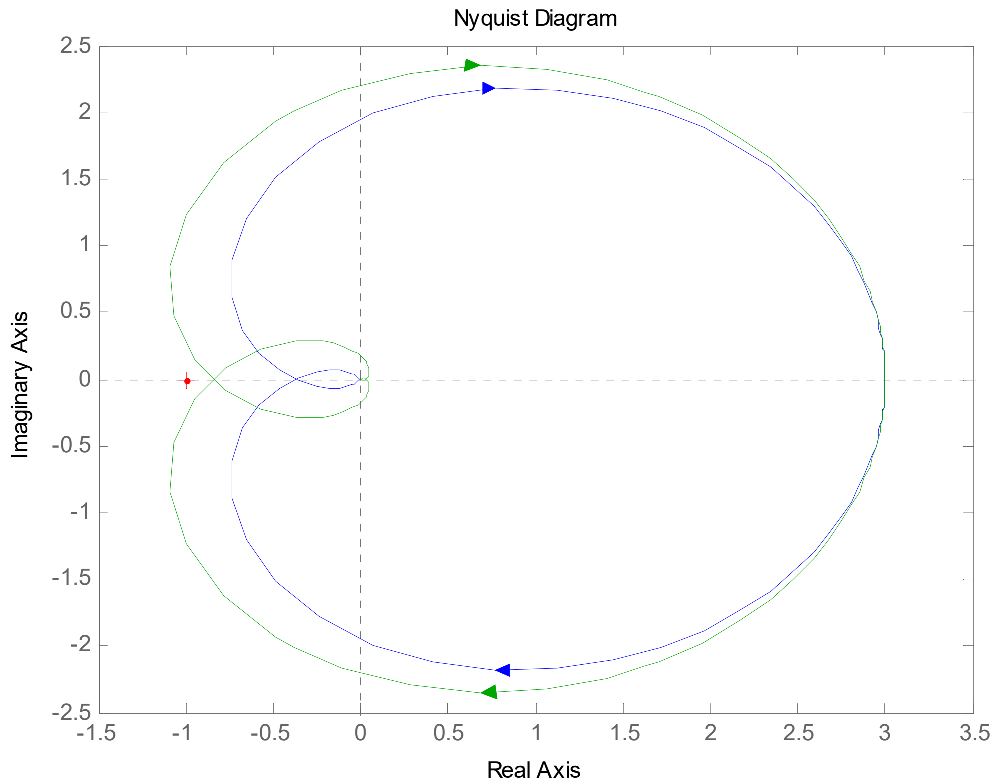


USE MATLAB TO PLOT THE CLOSED-LOOP POLES AND ZEROS



SMITH PREDICTOR

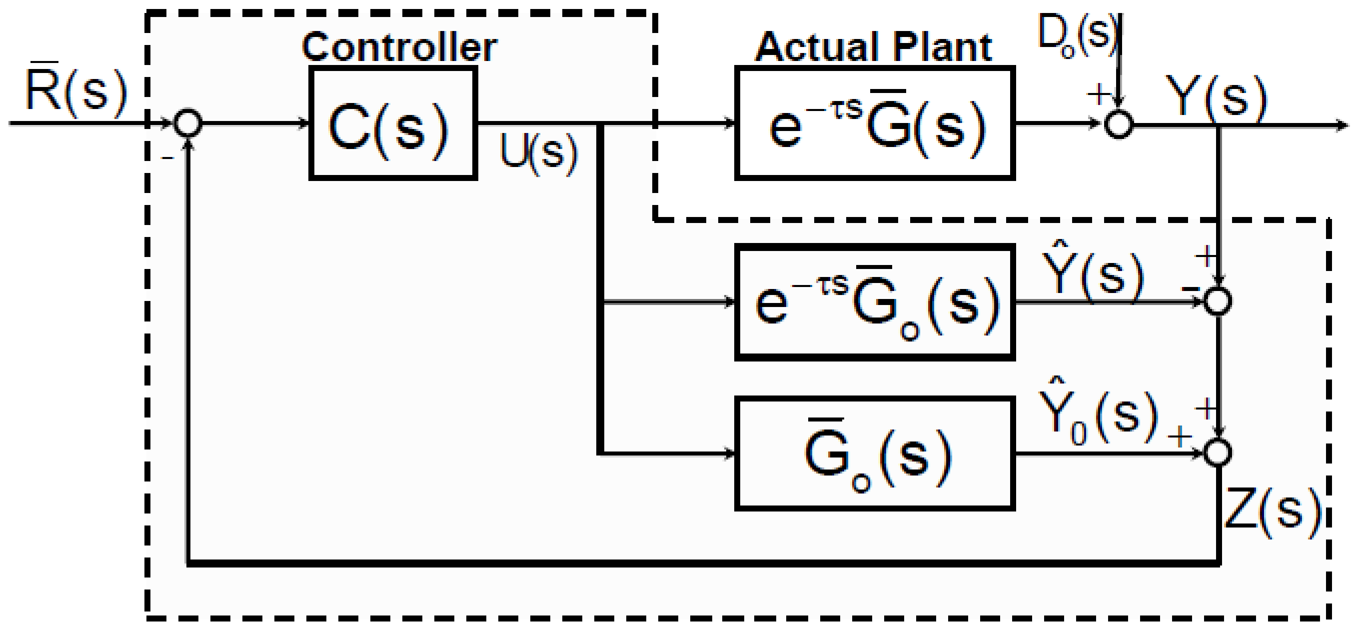
RECALL THE EFFECT OF A TIME DELAY ON THE NYQUIST STABILITY TEST



$$L_o(s) = \frac{3}{(s+1)^3}$$

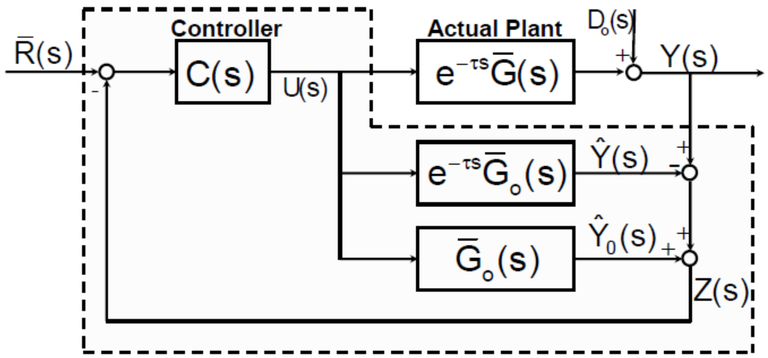
$$L(s) = \frac{3}{(s+1)^3} e^{-0.5s}$$

THE SMITH PREDICTOR USES A PARALLEL MODEL TO CANCEL THE DELAY EFFECT



- Requires stable open-loop system with known time-delay
- Controller can be designed for undelayed plant

TO VERIFY, DETERMINE THE TRANSFER FUNCTION FROM R TO Z



$$Z(s) = Y(s) - \hat{Y}(s) + \hat{Y}_o(s)$$

$$= D_o(s) + e^{-\tau s}\bar{G}(s)U(s) - e^{-\tau s}\bar{G}_o(s)U(s) + \bar{G}_o(s)U(s)$$

$$\approx D_o(s) + \bar{G}_o(s)U(s)$$

WHAT ARE THE LIMITATIONS OF THE SMITH PREDICTOR?

- Only works with stable plant!

- Significant robustness issues associated with the architecture

COMING UP...

SISO Design Limitations

- Free integrators
- Poles/Zeros

Frequency Domain Limitations

- Bode's Integral Constraints on Sensitivity
- Integral Constraints on Complementary Sensitivity
- Poisson Integral Constraint on Sensitivity
- Poisson Integral Constraint on Complementary Sensitivity