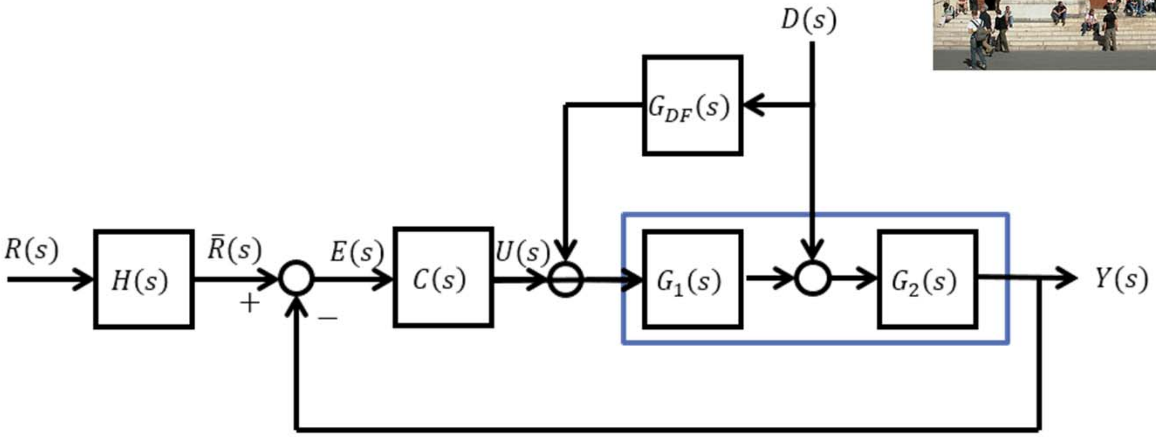
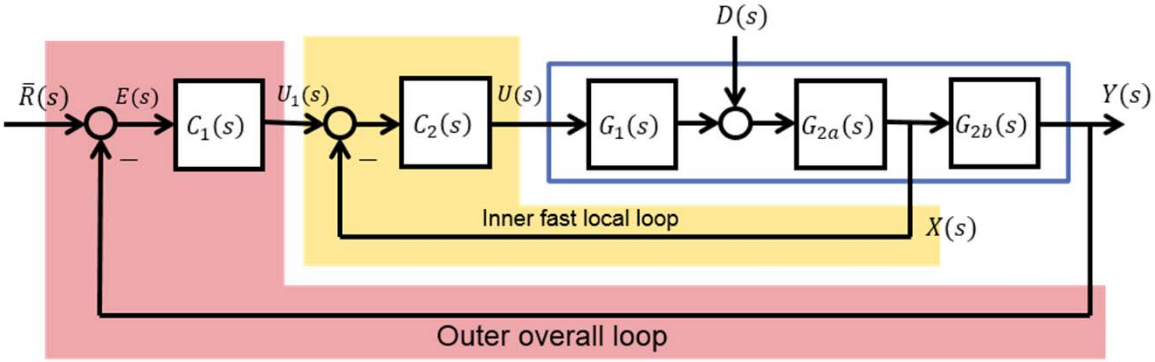
FROM LAST TIME...

Architectural Issues

- Internal Model Principle
- Feedforward
- Cascade Control





INTRO TO STATE-SPACE MODELS

Topics

- Review of State Space
- Transfer Functions and State Space
- Canonical Forms

At the end of this sections, students should be able to:

- Convert an ODE to state space form.
- Identify components of a state space model.
- Describe observable and controllable canonical forms.
- Transform

RECALL OUR PREVIOUS DISCUSSION ON CONTROL THEORY

Classical Control Theory

- Single-Input Single-Output (SISO) systems
- Linear Time-Invariant (LTI) systems
- Developed without computers
- Graphical tools, algebraic manipulation

Modern Control Theory

- Multiple-Input Multiple-Output (MIMO) systems
- Nonlinear systems
- Easy access to computers
- Numeric solutions, matrix manipulation

DEFINITIONS

State

- The smallest set of n variables (state variables) such that knowledge of these n variables at $t = t_0$, together with knowledge of the input for $t \geq t_0$, completely and uniquely determines system behavior for $t \geq t_0$.

State vector

- n[th] order vector whose components are the state variables

State space

- n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, etc.

State trajectory

- Path produced in the state space by the state vector as it changes over time

FOR LTI SYSTEMS WE CAN SIMPLIFY THE GENERAL STATE SPACE FORM

General state space equations:

$$\dot{x}(t) = f(x, u, t)$$
$$y(t) = g(x, u, t)$$

Assuming system is linear:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

Assuming system is time-invariant:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

WE WILL RESTRICT OURSELVES TO LINEAR STATE-SPACE MODELS

State Matrix

Input Matrix

$$\dot{x} = Ax(t) + Bu(t)$$

State Vector

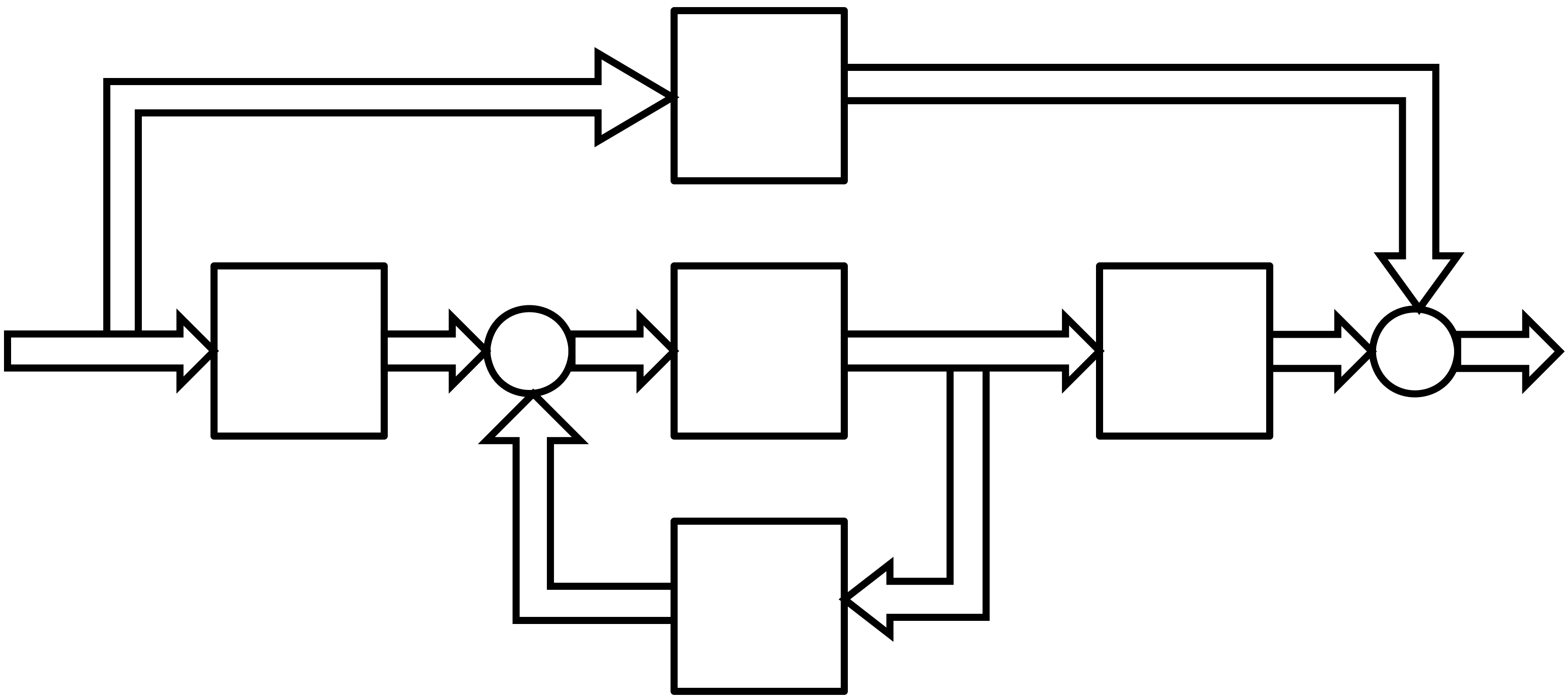
Input Vector

Output Vector

$$y = Cx(t) + Du(t)$$

Output Matrix

Feedthrough Matrix

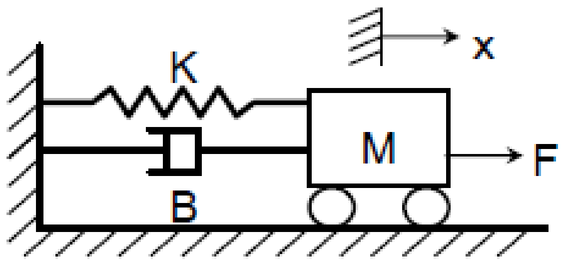


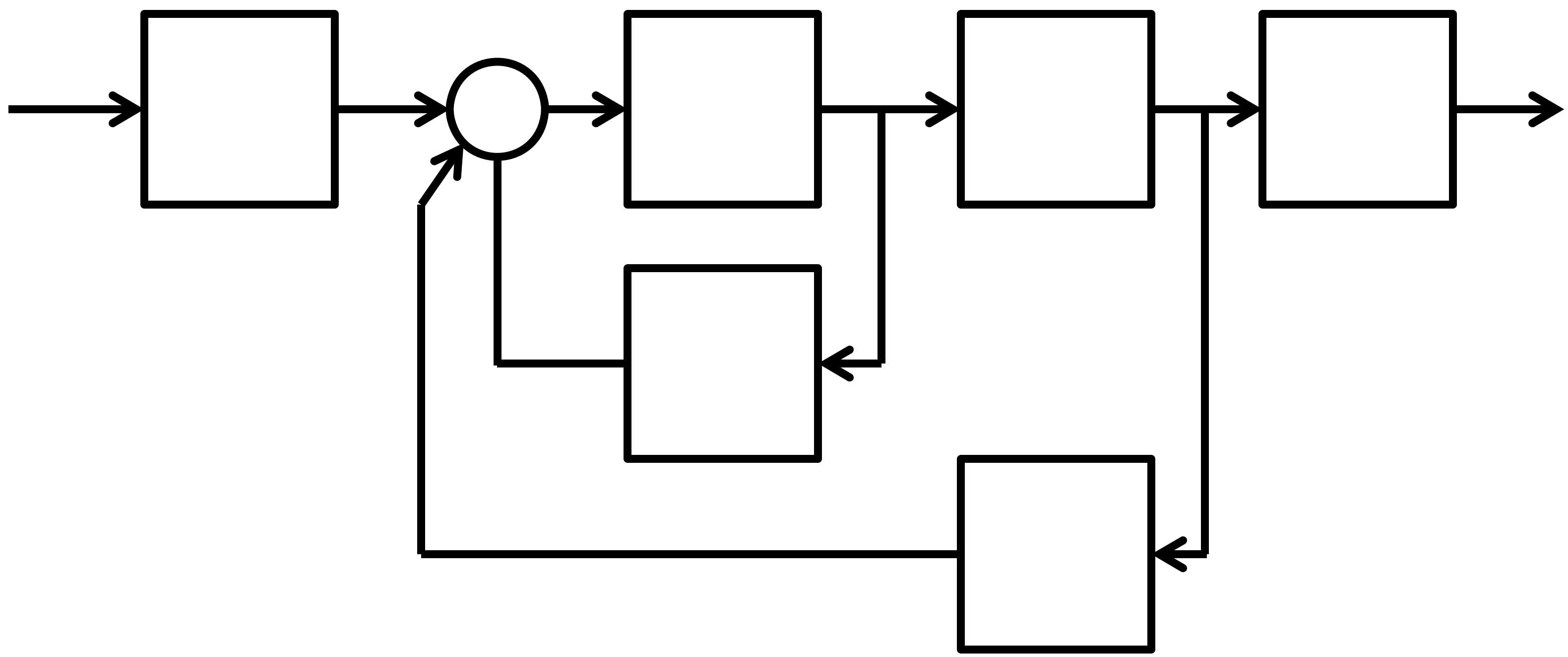
WE CAN CONVERT STATE-SPACE TO TRANSFER FUNCTION FORM

$$\dot{x} = Ax(t) + Bu(t)$$
$$y = Cx(t) + Du(t)$$

$$Y(s) = [C(sI - A)^{-1}B + D]U(s)$$

CONSIDER A SECOND-ORDER SYSTEM: THE MASS-SPRING-DAMPER





CONSIDER A GENERAL n^{th} ORDER ODE

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1\dot{y} + a_0 y = u$$

$$\Downarrow$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

WE CAN WRITE THIS IN MATRIX FORM

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \\ \end{bmatrix} u$$

$$y = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} x$$

WHAT IF THE GENERAL ODE INCLUDES INPUT DERIVATIVES?

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1\dot{y} + a_0 y = b_m u^{(m)} + b_{m-1}u^{(m-1)} + \cdots + b_1\dot{u} + b_0 u$$

$$\Downarrow$$

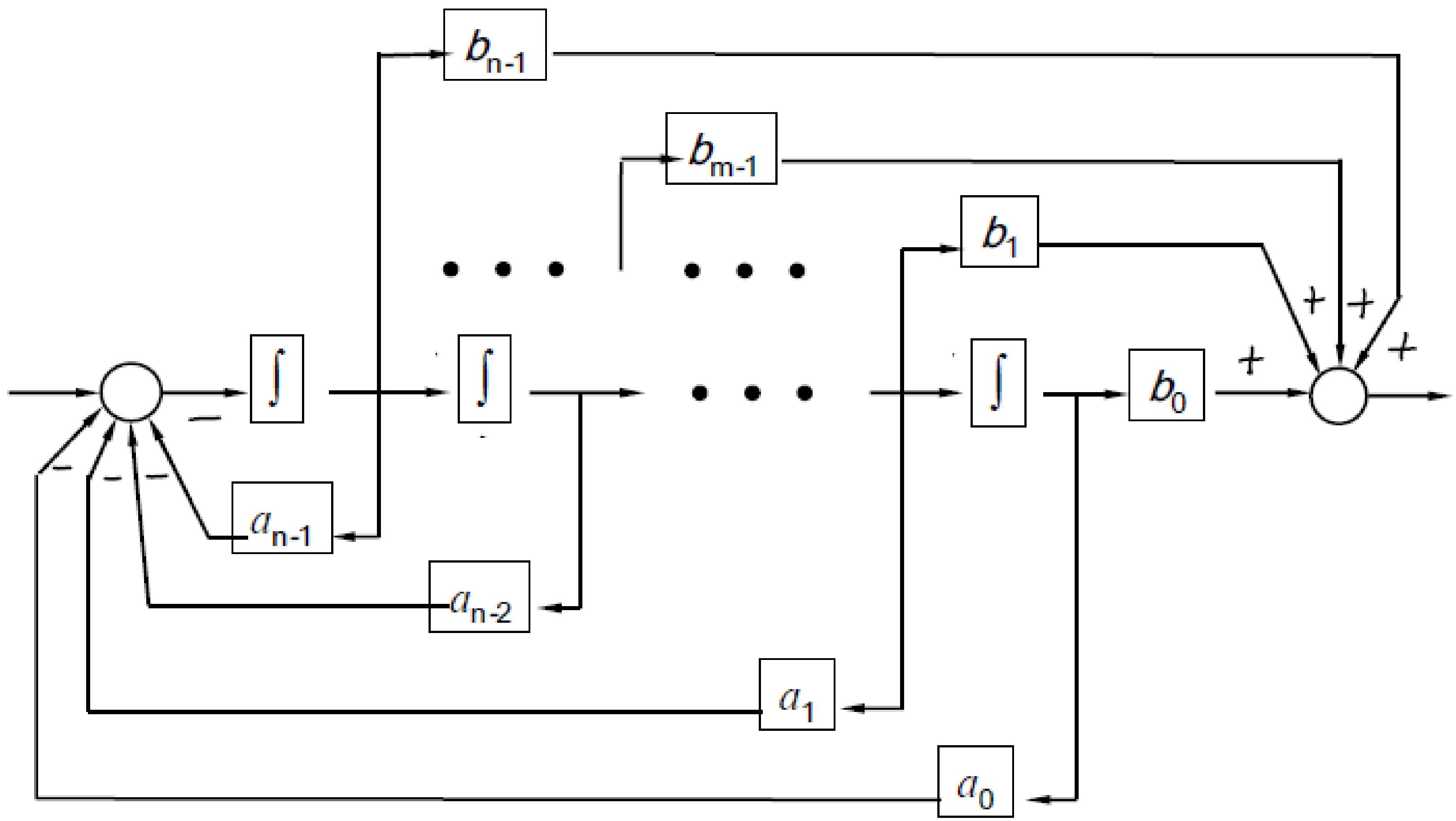
$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

Define a new dummy variable $Z(s)$

$$\frac{Y(s)}{Z(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{1}$$

$$\frac{Z(s)}{U(s)} = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

CONTROLLABLE CANONICAL FORM

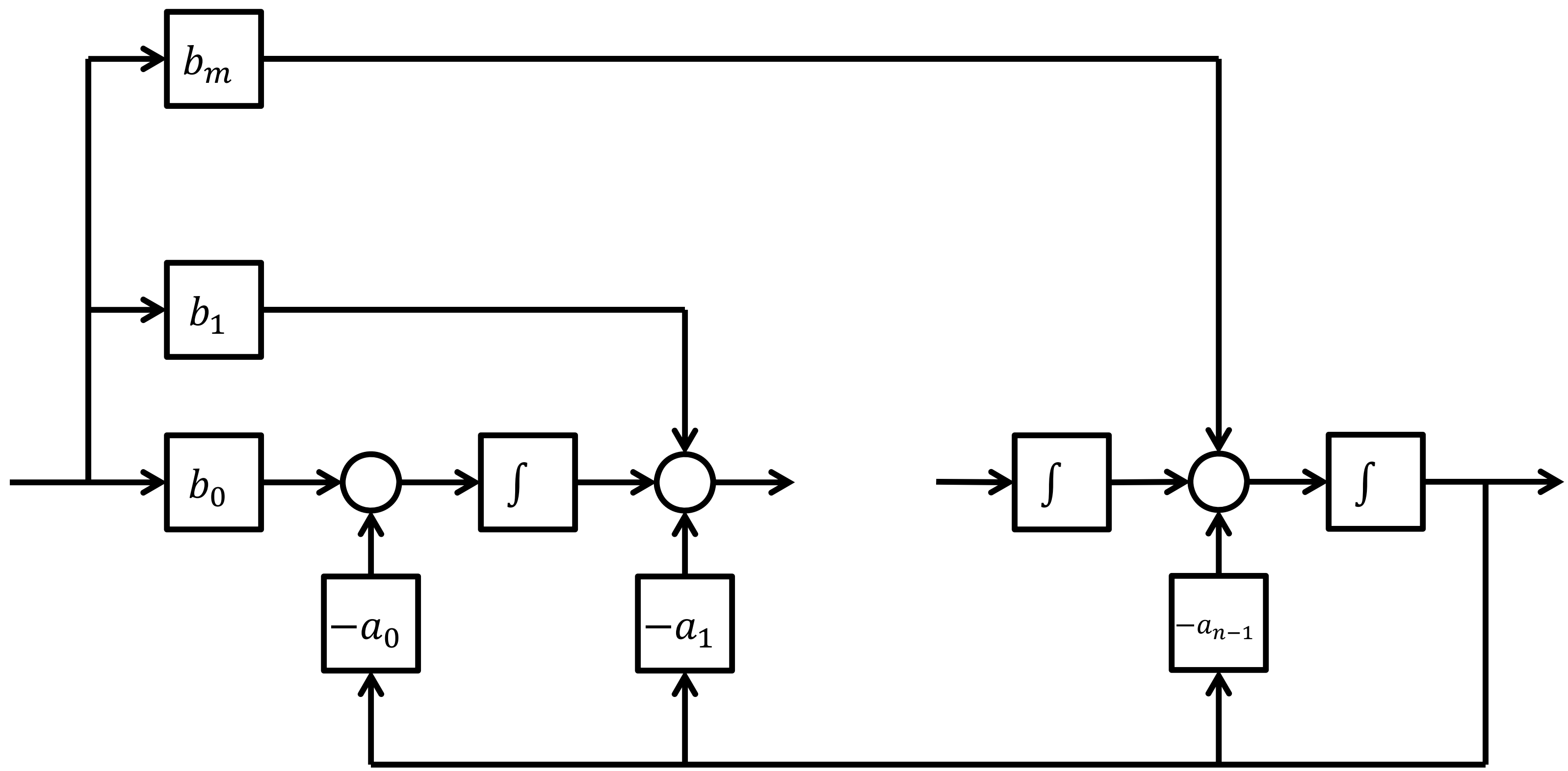


CONTROLLABLE CANONICAL FORM

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} u$$

$$y = [\qquad \qquad] \, x$$

OBSERVABLE CANONICAL FORM



OBSERVABLE CANONICAL FORM

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \left[\qquad\qquad\qquad\qquad \right] \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} + \left[\qquad \right] u$$

$$y = [\qquad\qquad\qquad\qquad]\, x$$

OBSERVABLE AND CONTROLLABLE CANONICAL FORMS ARE RELATED BY A TRANSFORMATION.

STATE TRANSFORMATIONS

Two sets of states can be related by a transformation matrix T:

Original system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

New System:

$$\dot{z} = \hat{A}z + \hat{B}u$$
$$y = \hat{C}z + \hat{D}u$$

RELATE THE TWO FORMULATIONS

TF IS INDEPENDENT OF STATE

Start with the TF:

$$Y(s) = [C(sI - A)^{-1}B + D]U(s)$$

Apply state transformation:

$$\hat{A} = T^{-1}AT, \qquad \hat{B} = T^{-1}B$$
$$\hat{C} = CT, \qquad \hat{D} = D$$

COMING UP…

Linear Algebra Review

- Matrix Inverses
- Eigenvalues and Eigenvectors
- Jordan Canonical Form
- Solution of LTI State Equations

Solution of LTI State Equations

- State Transition Matrix
- Free Response
- Forced Response