VIDEOS

Quadcopter Acrobatics

https://youtu.be/XxFZ-VStApo

Inverted Pendulum

https://youtu.be/a4c7AwHFkT8

Questions to consider

- How are these similar?
- How are these different?
- How are these tasks accomplished?
- How will you accomplish these tasks?

MODELING PHYSICAL SYSTEMS

Topics

- Why we model dynamic systems
- How we model dynamic systems

At the end of this section, students should be able to:

- Model dynamic systems with ODEs.
- Model dynamic systems with transfer functions.
- Model dynamic systems with state-space representation.

HOW DO WE MODEL A PHYSICAL SYSTEM FOR CONTROL?

Physical Description

↓

Schematic

↓

Differential Equations

↙ ↘

Transfer Function   State Space Model

WE LIKE TO MAKE SEVERAL MODELING ASSUMPTIONS

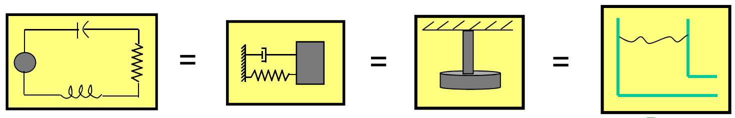
Linear – obeys linear superposition

Time Invariant – constant parameters

Lumped Parameters – characteristics are lumped into discrete elements

DYNAMIC SYSTEMS

Models are the same regardless of the physical domain:



$y = \text{Charge}$, $u = e$

$y = \text{Transl. displ.}$, $u = f$

$y = \text{Angular displ.}$, $u = \tau$

$y = \text{Pressure}$, $u = \frac{dq}{dt}$

We only have to understand one model, but we have four different systems!

$$\ddot{y} + \dot{y} + y = u(t)$$

IDEALIZED MODELING ELEMENTS

Basic Components

- Inertia
- Energy Storage
- Energy Dissipation

Mechanical – Force driven

- Mass
- Linear Spring
- Linear Damper

Electrical – Voltage driven

- Inductor
- Capacitor
- Resistor

Thermal

- Thermal capacitance (specific heat)
- Thermal resistance from conduction or convection (actually energy transfer, not dissipation)

Hydraulic

- Usually distributed (i.e. along a length of pipe)
- Lumped approximations may be used

TERMINOLOGY

Nominal Model

- Simplified approximate model used for control design

Calibration (or Simulation) Model

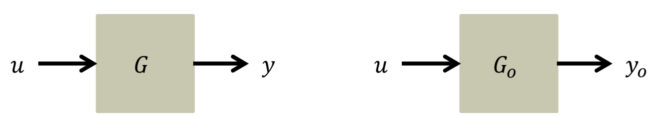
- Model that captures all pertinent aspects of plant behavior to be used for controller validation

Model Error

- The difference between the nominal and calibration model

MODEL ERRORS

Let y_o be the nominal system output and y be the actual system output



(True plant) (Nominal plant)

Additive Modeling Error (AME):

$$AME = y - y_o \quad = [G - G_o]U(s) \quad = \Delta G \cdot U(s)$$

Multiplicative Modeling Error (MME):

MME

$$Y(s) = G(s)U(s) \ = [G_o + \Delta G]U(s) \ = G_o\left[1 + \frac{\Delta G}{G_o}\right]U(s)$$

BUILDING MODELS

Three approaches:

- From physical principles (white-box model)
- From experimental data (black-box model)
- Combination of the above

Always use the simplest model that captures the essential aspects of the process for control design.

WHAT PROBLEMS EXIST WITH A WHITE-BOX MODEL?

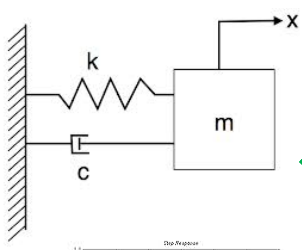
Damping coefficients (friction) can be difficult (or impossible) to estimate

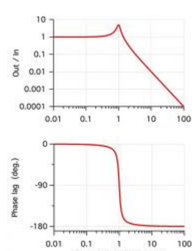
WHAT PROBLEMS EXIST WITH A BLACK-BOX MODEL?

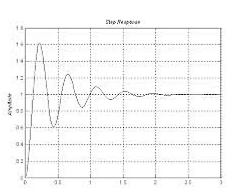
How do you select a model structure?

Can you gain any intuition from the model?

CAN WE REMEDY THESE PROBLEMS WITH A GRAY-BOX MODEL?







$$G(s) = \frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Use what we know (intuition) as a starting point

Fill in the gaps using data

DEFINITIONS

State

- The smallest set of n variables (state variables) such that knowledge of these n variables at $t = t_0$, together with knowledge of the input for $t \geq t_0$, completely and uniquely determines system behavior for $t \geq t_0$.

State vector

- n[th] order vector whose components are the state variables

State space

- n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, etc.

State trajectory

- Path produced in the state space by the state vector as it changes over time

FOR LTI SYSTEMS WE CAN SIMPLIFY THE GENERAL STATE SPACE FORM

General state space equations:

$$\dot{x}(t) = f(x, u, t)$$
$$y(t) = g(x, u, t)$$

Assuming system is linear:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

Assuming system is time-invariant:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

WHAT IF WE HAVE A NONLINEAR STATE SPACE MODEL?

$$\dot{x}(t) = f(x, u, t)$$
$$y(t) = g(x, u, t)$$

Choose operating point: x_Q, u_Q

$$\dot{x}(t) \approx f(x_Q, u_Q) + \frac{\partial f}{\partial x}\Big|_{\substack{x=x_Q \\ u=u_Q}} (x(t) - x_Q) + \frac{\partial f}{\partial u}\Big|_{\substack{x=x_Q \\ u=u_Q}} (u(t) - u_Q)$$

$$y(t) \approx g(x_Q, u_Q) + \frac{\partial g}{\partial x}\Big|_{\substack{x=x_Q \\ u=u_Q}} (x(t) - x_Q) + \frac{\partial g}{\partial u}\Big|_{\substack{x=x_Q \\ u=u_Q}} (u(t) - u_Q)$$

WHEN LINEARIZING ABOUT THE ORIGIN, THE LINEARIZATION IS CLEAR

$$x_Q = 0$$
$$u_Q = 0$$

$$\dot{x}(t) \approx Ax(t) + Bu(t) + f(0,0)$$

$$y(t) \approx Cx(t) + Du(t) + g(0,0)$$

- For other operating points we might consider an incremental model (see textbook)

CONTROL THEORY

Classical Control Theory

- Single-Input Single-Output (SISO) systems
- Linear Time-Invariant (LTI) systems
- Developed without computers
- Graphical tools, algebraic manipulation

Modern Control Theory

- Multiple-Input Multiple-Output (MIMO) systems
- Nonlinear systems
- Easy access to computers
- Numeric solutions, matrix manipulation

COMMENTS ON MODEL TYPES

Transfer Function Models:

- Are conceptually simple
- Are easily converted to frequency domain transfer functions that are more intuitive to practicing engineers
- Are difficult to solve in the time domain (solution: Laplace transformation)

State Space Models:

- Consider the internal behavior of a system
- Can easily incorporate complicated output variables
- Have significant computation advantage for computer simulation
- Can represent multi-input multi-output (MIMO) systems and nonlinear systems

COMING UP...

Case Study: Hard disk drive read/write head

Transfer Functions

- Poles and zeros
- First and second order systems

Dynamic Response

- Types of inputs
- Free and Force responses
- Transient and Steady-State responses