AIMS | African Institute for Mathematical Sciences
NEXT EINSTEIN INITIATIVE

GAUSSIAN DISCRIMINANT ANALYSIS (GDA)

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Introduction

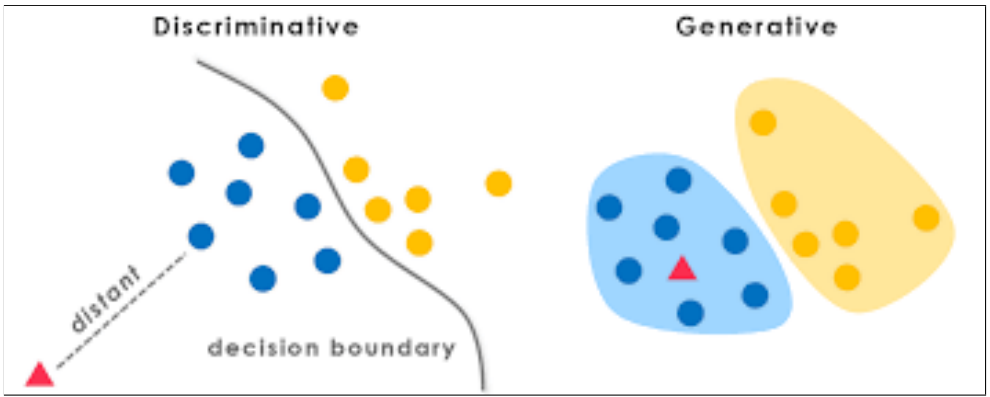
Introduction

The main aim of this project is to implement Gaussian Discriminative Analysis. Machine Learning models can be classify as discriminative or generative models.

- Discriminative Models
  We model probability distribution $\mathbb{P}(\boldsymbol{y}|\boldsymbol{x})$ with the training data.

- Generative Models
  We This is a part of unsupervised learning task in

Machine Learning Models



Driscriminative models Vs Generative models

Gaussian Discriminant Analysis

Gaussian Discriminant Analysis

GDA is a generative learning algorithm.

- Input features are continuous random variables
- Assume $\mathbb{P}(\boldsymbol{x}|\boldsymbol{y})$ and $\mathbb{P}(\boldsymbol{y})$ are distributed according to multivariate normal distribution and Bernuolli distribution respectively.

Gaussian Discriminant Analysis

Assumptions

$$\mathbb{P}[\boldsymbol{y}_i; \phi] = \phi^{\boldsymbol{y}_i}(1-\phi)^{1-\boldsymbol{y}_i},\ \boldsymbol{y}_i \in \{0, 1\}. \tag{1}$$

$$\mathbb{P}\big[\boldsymbol{x}_i | \boldsymbol{y}_i = 0; \boldsymbol{\mu_0}, \boldsymbol{\Sigma}\big] = \frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{\mu_0})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_i - \boldsymbol{\mu_0}) \right) \tag{2}$$

$$\mathbb{P}\big[\boldsymbol{x}_i | \boldsymbol{y}_i = 1; \boldsymbol{\mu_1}, \boldsymbol{\Sigma}\big] = \frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{\mu_1})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_i - \boldsymbol{\mu_1}) \right) \tag{3}$$

Gaussian Discriminant Analysis

From Bayes rule , we can compute $\mathbb{P}(\boldsymbol{y}|\boldsymbol{x})$.

$$\mathbb{P}(\boldsymbol{y}|\boldsymbol{x}) = \frac{\mathbb{P}(\boldsymbol{x}|\boldsymbol{y}) \times \mathbb{P}(\boldsymbol{y})}{\mathbb{P}(\boldsymbol{x})}$$

$$\text{argmax}_{\boldsymbol{y}}\mathbb{P}(\boldsymbol{y}|\boldsymbol{x}) = \text{argmax}_{\boldsymbol{y}}\frac{\mathbb{P}(\boldsymbol{x}|\boldsymbol{y}) \times \mathbb{P}(\boldsymbol{y})}{\mathbb{P}(\boldsymbol{x})}$$

$$\text{argmax}_{\boldsymbol{y}}\mathbb{P}(\boldsymbol{y}|\boldsymbol{x}) \propto \text{argmax}_{\boldsymbol{y}}\mathbb{P}(\boldsymbol{x}|\boldsymbol{y}) \times \mathbb{P}(\boldsymbol{y})$$

The likelihood function L is given by

$$L(\boldsymbol{y}|\boldsymbol{x}; \boldsymbol{\mu_k}, \boldsymbol{\Sigma}, \phi) = \mathbb{P}(\boldsymbol{x}|\boldsymbol{y}; \boldsymbol{\mu_k}, \boldsymbol{\Sigma})\mathbb{P}(\boldsymbol{y}; \phi) \text{ where } k = \mathbb{1}_{\{\boldsymbol{y}_i=1\}}$$

$$\phi = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{y}_i = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{\boldsymbol{y}_i=1\}}$$

$$\boldsymbol{\mu_0} = \frac{\sum_{i=1}^{n} \mathbb{1}_{\{\boldsymbol{y}_i=0\}}\boldsymbol{x}_i}{\mathbb{1}_{\{\boldsymbol{y}_i=0\}}}$$

$$\boldsymbol{\mu_1} = \frac{\sum_{i=1}^{n} \mathbb{1}_{\{\boldsymbol{y}_i=1\}}\boldsymbol{x}_i}{\mathbb{1}_{\{\boldsymbol{y}_i=1\}}}$$

$$\Sigma = \frac{1}{n}\sum_{i=1}^{n} \left(\boldsymbol{x}_i - \boldsymbol{\mu_k}\right)\left(\boldsymbol{x}_i - \boldsymbol{\mu_k}\right)^T \text{ where } k = \mathbb{1}_{\{\boldsymbol{y}_i=1\}}$$

Sentimental Analysis using GDA

Sentimental Analysis using GDA

Preprocessing

Steps

- lower case
- remove stopwords
- remove punctuations
- remove apostrophe
- tokenize
- lemmatization
- convert numbers

TF -IDF

- Term- Frequency

$$TF = \frac{\text{Nbr of repetitions of word in a doc}}{\text{\# of words in a doc}}$$

- Inverse Document Frequency

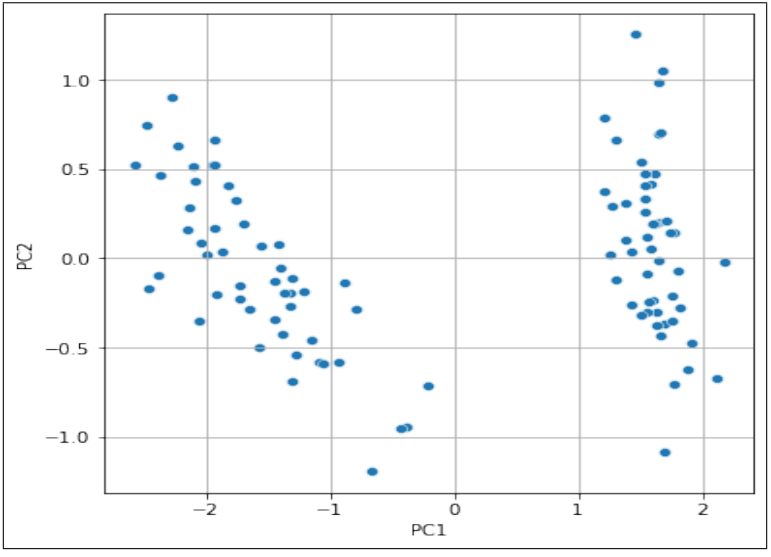
$$IDF = \log\left[\frac{\text{\# Number of docs}}{\text{Nbr of docs containing the word}}\right]$$

Sentimental Analysis using GDA

The estimated parameters from the training dataset

$$
\begin{aligned}
\phi^* &= 0.75 \\
\boldsymbol{\mu_0}^* &= (-0.0262559, -0.01121153)^T \\
\boldsymbol{\mu_1}^* &= (0.00595905, 0.0021744)^T \\
\boldsymbol{\Sigma}^* &= \begin{pmatrix} 4.62948277 \cdot 10^{-4} & -9.65664189 \cdot 10^{-6} \\ -9.65664189 \cdot 10^{-6} & 1.49865708 \cdot 10^{-3} \end{pmatrix}
\end{aligned}
$$

The accuracy after prediction is 50%.



Visualization of the Iris Dataset using PCA

Conclusion

In conclusion,

- GDA model does not need any hyperparameter.
- GDA models works perfectly with small dataset.
- Time complexity is expensive


THANK
YOU