BLG354-HW-2-

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1 Q1

For some DT signals, equations representing one of their periods are given below. Determine the DTFS coefficients for these signals.

In this problem I first plot the signals using python for the purpose to compare the reobtained visuals in the second question. The signals can be seen here in their fundamental period:

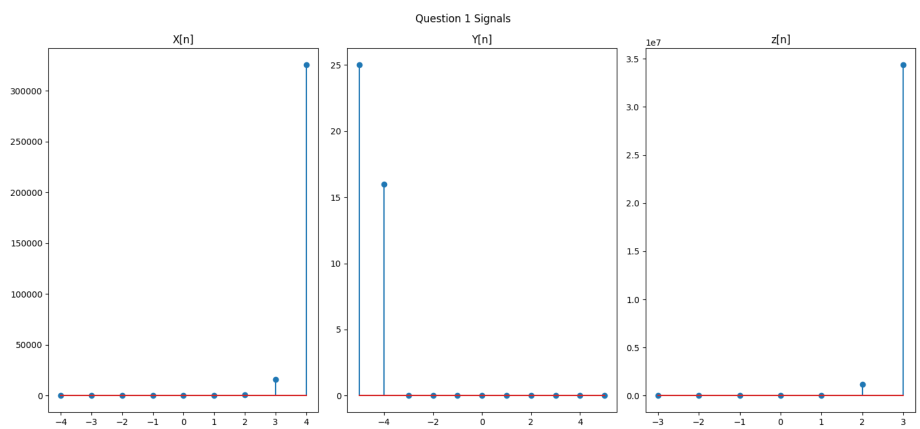
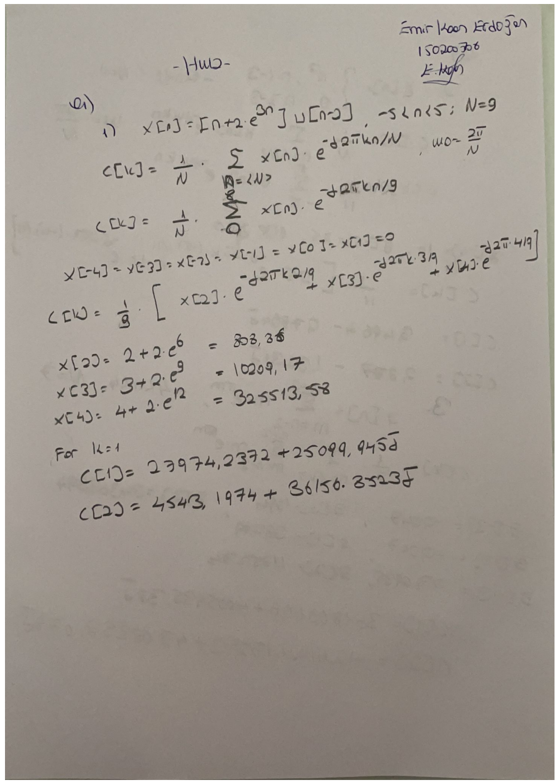


Figure 1: Signals

Then, as it is asked in the question, I calculated the DTFS coefficients using the formula.Then the complicated equations are calculated using the function I implemented in python ,and print the results in terminal.
Below, you can find the handwritten mathematical calculations:



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-HW2-

Q1)

1) $x[n] = [n + 2 \cdot e^{3n}]\, u[n-2]$, $-5 < n < 5$; $N = 9$

$c[k] = \frac{1}{N} \cdot \sum_{n=\langle N \rangle} x[n] \cdot e^{-j2\pi kn/N}$, $w_0 = \frac{2\pi}{N}$

$c[k] = \frac{1}{N} \cdot \sum x[n] \cdot e^{-j2\pi kn/9}$

$x[-4] = x[-3] = x[-2] = x[-1] = x[0] = x[1] = 0$

$c[k] = \frac{1}{9} \cdot \left[x[2] \cdot e^{-j2\pi k2/9} + x[3] \cdot e^{-j2\pi k \cdot 3/9} + x[4] \cdot e^{-j2\pi \cdot 4/9} \right]$

$x[2] = 2 + 2 \cdot e^{6} = 808{,}36$

$x[3] = 3 + 2 \cdot e^{9} = 10209{,}17$

$x[4] = 4 + 2 \cdot e^{12} = 325513{,}58$

For $k = 1$

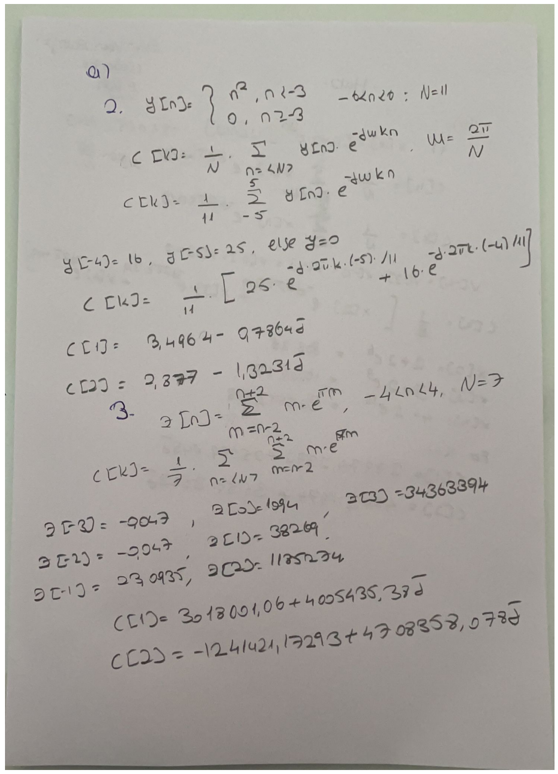
$c[1] = 23974{,}2372 + 25099{,}9458j$

$c[2] = 4543{,}1974 + 36150.3523j$

Figure 2: Calculations

The calculations for the b and c part of the first part are as follows:

Below you can see the coefficients, in the a range I randomly pick, it can be extended by rearranging the parameters.



Q1)

2. $y[n] = \begin{cases} n^2, & n<-3 \\ 0, & n \geq -3 \end{cases} \quad -6<n<0 : N=11$

$C[k] = \frac{1}{N} \cdot \sum_{n=\langle N \rangle} y[n] \cdot e^{-jwkn}, \quad w = \frac{2\pi}{N}$

$C[k] = \frac{1}{11} \cdot \sum_{-5}^{5} y[n] \cdot e^{-jwkn}$

$y[-4] = 16, \; y[-5] = 25, \; \text{else } y=0$

$C[k] = \frac{1}{11} \cdot \left[25 \cdot e^{-j \cdot 2\pi k \cdot (-5)/11} + 16 \cdot e^{-j \cdot 2\pi k \cdot (-4)/11} \right]$

$C[1] = 3,4964 - 0,7864j$

$C[2] = 2,877 - 1,3231j$

3. $z[n] = \sum_{m=n-2}^{n+2} m \cdot e^{\pi m}, \quad -4<n<4, \; N=7$

$C[k] = \frac{1}{7} \cdot \sum_{n=\langle N \rangle} \sum_{m=n-2}^{n+2} m \cdot e^{\pi m}$

$z[-3] = -2047, \; z[0] = 1094, \; z[3] = 34363394$

$z[-2] = -2047, \; z[1] = 38269,$

$z[-1] = 23,0935, \; z[2] = 1135234$

$C[1] = 3018001,06 + 4005435,38j$

$C[2] = -1241421,17293 + 4708358,078j$

Figure 3: Calculations Q1-B

```
Print out the dtfs coefficients for the function x1 for the k values: [1,2]
C[1]:  (27974.237217650734+25099.944802771806j)
C[2]:  (4543.197361066154+36156.852309365226j)
Print out the dtfs coefficients for the function y1 for the k values: [1,2]
C[1]:  (3.4963687750271726-0.7863866435717782j)
C[2]:  (2.8769672916391076-1.3231010841520268j)
Print out the dtfs coefficients for the function z1 for the k values: [1,2]
C[1]:  (3018001.0601161798+4005435.380979954j)
C[2]:  (-1241421.1729278238+4708358.0778113445j)
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Figure 4: Coefficients

2 Q2

In the second question, I am requested to reobtain the signals given in the first question from their DFTS coefficients. I use python inline function. The resulting plots can be seen in the following:

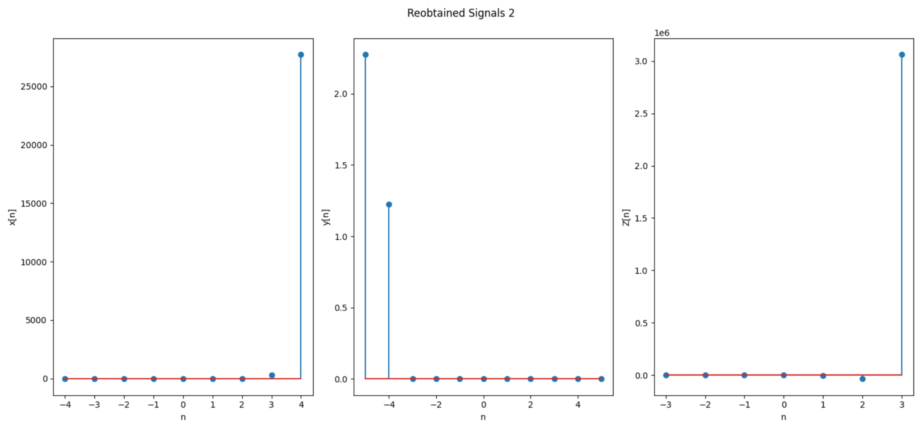


Figure 5: Plots

3 Q3

In the third question, we meet a continous time signal. In the question, we are requested to find the CTFS coefficient and reobtain the signal as in the first parts using python.
The signal equation is: $\mathbf{x(t) = 2sin(2t) + 4cos(3t)}$ First I share a visual of the main signal:

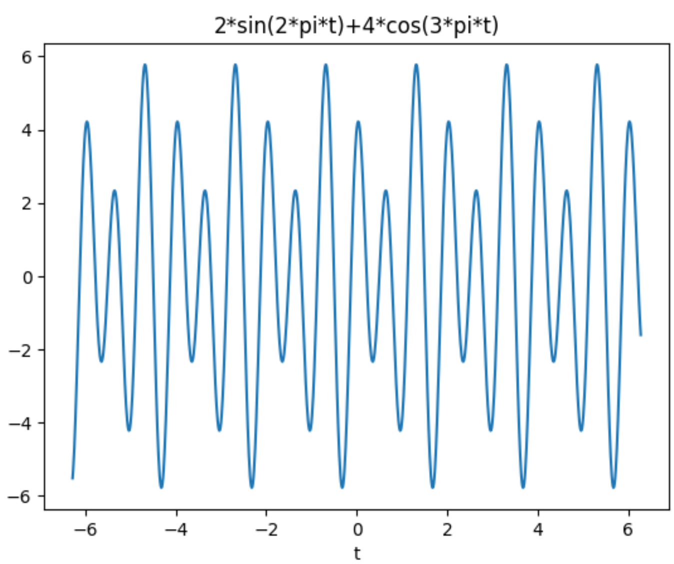
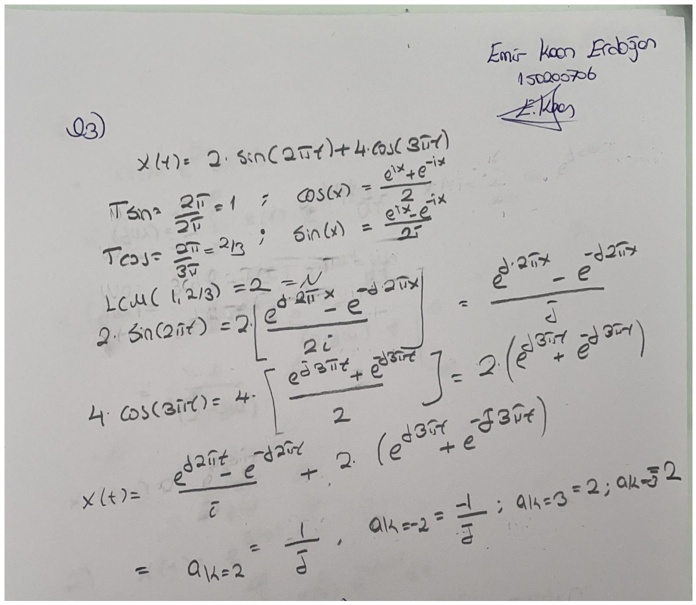


Figure 6: Signals

The hand-written calculations are as follows: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Q3)

$x(t) = 2 \cdot \sin(2\pi t) + 4 \cdot \cos(3\pi t)$

$T_{\sin} = \frac{2\pi}{2\pi} = 1$; $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

$T_{\cos} = \frac{2\pi}{3\pi} = 2/3$; $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$

$LCM(1, 2/3) = 2 = N$

$2 \cdot \sin(2\pi t) = 2 \cdot \left[\frac{e^{j \cdot 2\pi x} - e^{-j 2\pi x}}{2i} \right] = \frac{e^{j \cdot 2\pi x} - e^{-j 2\pi x}}{j}$

$4 \cdot \cos(3\pi t) = 4 \cdot \left[\frac{e^{j 3\pi t} + e^{-j 3\pi t}}{2} \right] = 2 \cdot \left(e^{j 3\pi t} + e^{-j 3\pi t} \right)$

$x(t) = \frac{e^{j 2\pi t} - e^{-j 2\pi t}}{i} + 2 \cdot \left(e^{j 3\pi t} + e^{-j 3\pi t} \right)$

$= a_{k=2} = \frac{1}{j}$, $a_{k=-2} = \frac{-1}{j}$; $a_{k=3} = 2$; $a_{k=-3} = 2$

Figure 7: Signals

The reconstructed signal is below:

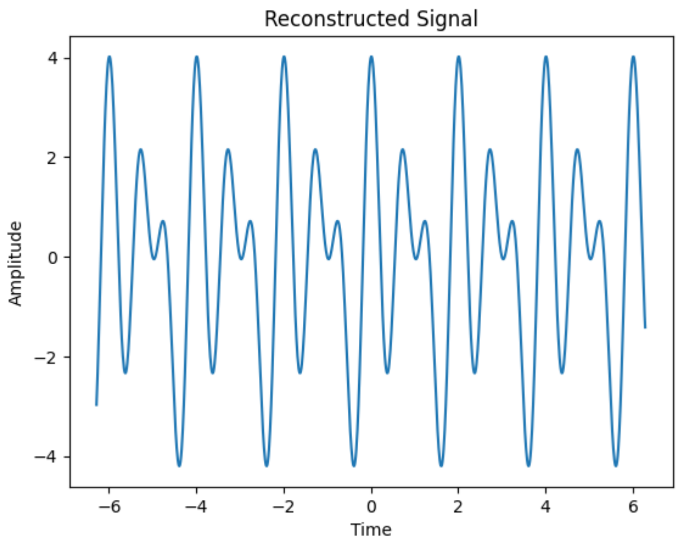
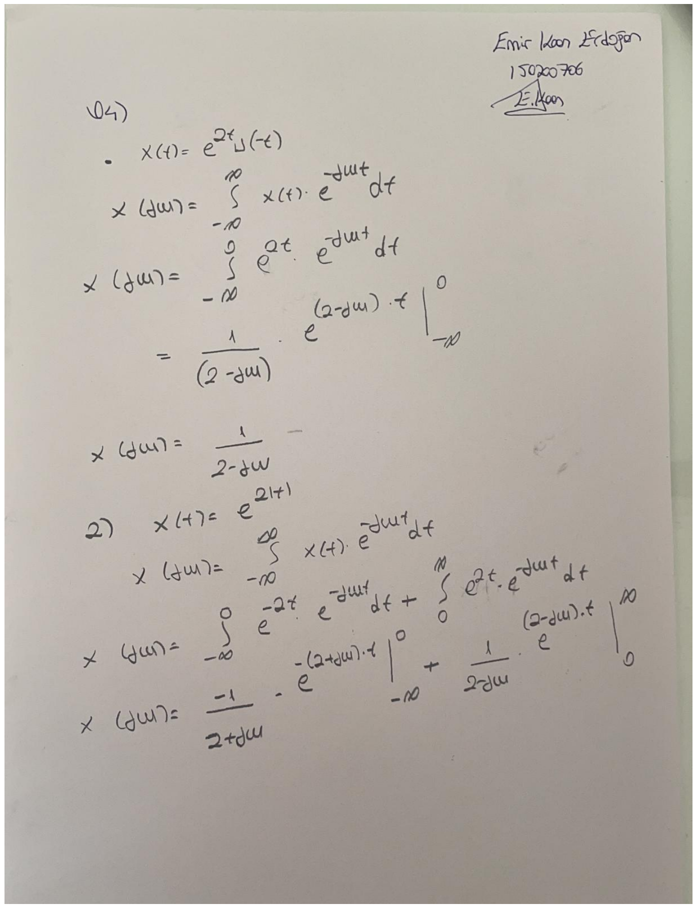


Figure 8: Signals

4 Q4

Here, we need to find the Fourier Transform for the given continuous time signals The calculations are shared below:



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Q4)

• $x(t) = e^{2t}u(-t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$X(j\omega) = \int_{-\infty}^{0} e^{2t}\cdot e^{-j\omega t}\,dt$$

$$= \frac{1}{(2-j\omega)}\cdot e^{(2-j\omega)\cdot t}\Big|_{-\infty}^{0}$$

$$X(j\omega) = \frac{1}{2-j\omega}$$

2) $x(t) = e^{2|t|}$

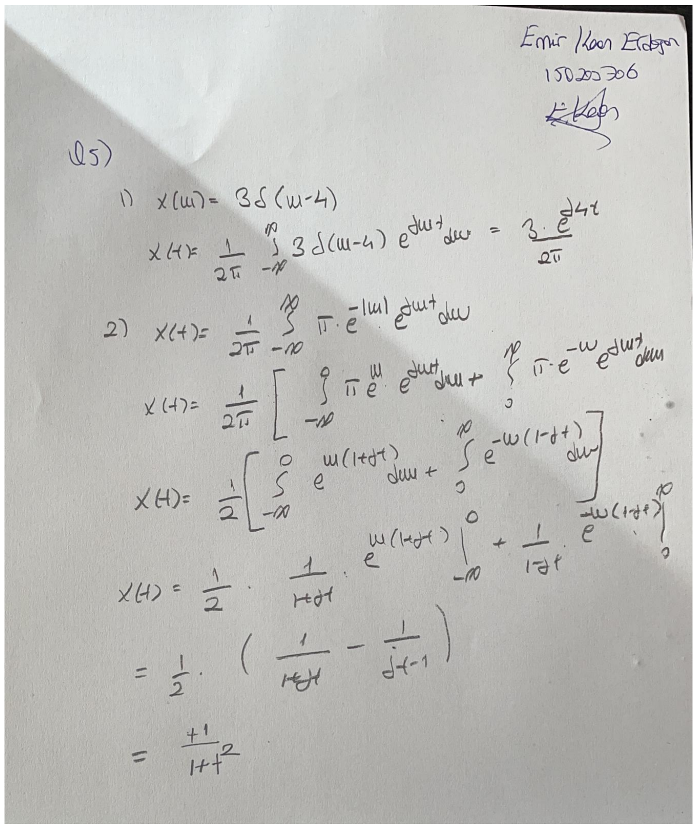
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$X(j\omega) = \int_{-\infty}^{0} e^{-2t}\cdot e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{2t}\cdot e^{-j\omega t}\,dt$$

$$X(j\omega) = \frac{-1}{2+j\omega}\cdot e^{-(2+j\omega)\cdot t}\Big|_{-\infty}^{0} + \frac{1}{2-j\omega}\cdot e^{(2-j\omega)\cdot t}\Big|_{0}^{\infty}$$

Figure 9: Q4

5 Q5



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Q5)

1) $X(\omega) = 3\delta(\omega-4)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 3\delta(\omega-4)\, e^{j\omega t}\, d\omega = \frac{3\cdot e^{j4t}}{2\pi}$$

2) $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\cdot e^{-|\omega|}\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\left[\int_{-\infty}^{0} \pi e^{\omega}\cdot e^{j\omega t}\, d\omega + \int_{0}^{\infty} \pi\cdot e^{-\omega} e^{j\omega t}\, d\omega\right]$$

$$x(t) = \frac{1}{2}\left[\int_{-\infty}^{0} e^{\omega(1+jt)}\, d\omega + \int_{0}^{\infty} e^{-\omega(1-jt)}\, d\omega\right]$$

$$x(t) = \frac{1}{2}\cdot \frac{1}{1+jt}\cdot e^{\omega(1+jt)}\Big|_{-\infty}^{0} + \frac{1}{1-jt}\cdot e^{-\omega(1+jt)}\Big|_{0}^{\infty}$$

$$= \frac{1}{2}\cdot\left(\frac{1}{1+jt} - \frac{1}{jt-1}\right)$$

$$= \frac{1}{1+t^2}$$

Figure 10: Q5