MATH 545 - Assignment 1

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Q1) The following results will be used frequently:
For any 2 rv's X and Y, if X and Y are independent, then $Cov(X,Y)=0 \Rightarrow E(X.Y)=E(X).E(Y)$, and $Cov(X,X)=Var(X)$,
$E(X^2)-[E(X)]^2=Var(X)$, $\Rightarrow E(X^2)=Var(X)+[E(X)]^2$, in general.
The mean and autocovariance functions are denoted μ_X and γ_X respectively.

(a) $\mu_X(t)=E[a+bZ_t+cZ_{t-2}]=a+0+0=0$, $E(X_{t+h})=a$ also.

$\gamma_X(t+h,t)=E(X_{t+h}.X_t)-\underbrace{E(X_{t+h})}_{a}.\underbrace{E(X_t)}_{a}=E(X_{t+h}.X_t)-a^2$,

$=E[(a+bZ_t+cZ_{t-2})(a+bZ_{t+h}+cZ_{t+h-2})]$

$=E[a^2+abZ_{t+h}+acZ_{t+h-2}+abZ_t+b^2Z_{t+h}.Z_t+bcZ_{t+h-2}.Z_t$
$+acZ_{t-2}+bcZ_{t-2}.Z_{t+h}+c^2Z_{t-2}.Z_{t+h-2}]$.

If $h=0$, this equals $a^2+0+0+0+b^2\sigma^2+0+0+0+c^2\sigma^2$
$=a^2+b^2\sigma^2+c^2\sigma^2-a^2=b^2\sigma^2+c^2\sigma^2$..

If $h=2$, then this is $a^2+0+0+0+0+bc\sigma^2+0+0=a^2+bc\sigma^2-a^2$
$=bc\sigma^2$

else, this equals $a^2+0+0+0+0+0+0+0+0-a^2=0$//

So the autocov function is $\begin{cases}\sigma^2(b^2+c^2) & \text{if } h=0,\\ bc\sigma^2 & \text{if } h=2,\\ 0 & \text{else}\end{cases}$..

$\mu_X(t)$ and $\gamma_X(t+h,t)$ don't depend on t, so X_t is stationary.

(b) let $\alpha_1 = \cos(ct)$, $\beta_1 = \cos(c(t+h))$,
$\alpha_2 = \sin(ct)$, $\beta_2 = \sin(c(t+h))$

Then $M_X(t) = E(Z_1)\alpha_1 + E(Z_2)\alpha_2 = 0$.

Auto cov: $\gamma_X(t+h, t) = E[(Z_1 B_1 + Z_2 B_2)(Z_1\alpha_1 + Z_2\alpha_2)] - \overset{0}{\widetilde{E(Z_t).E(Z_{t+h})}}$

$= E[Z_1^2 \alpha_1 B_1 + Z_1 Z_2 B_2 \alpha_2 + Z_1 Z_2 B_1 \alpha_2 + Z_2^2 \alpha_2 B_2]$.

If $h=0$, this equals $\sigma^2 \alpha_1 B_1 + \sigma^2 \alpha_2 B_2 = \sigma^2 \alpha_1^2 + \sigma^2 \alpha_2^2$

$= \sigma^2(\cos^2(ct) + \sin^2(ct)) = \sigma^2$ as $E(Z_1 Z_2) = E(Z_1)E(Z_2) = 0$ by indep.

If $h \neq 0$, then this equals $\sigma^2 \alpha_1 B_1 + \sigma^2 \alpha_2 B_2$

$= \sigma^2 \cos(ct)\cos(c(t+h)) + \sigma^2 \sin(ct)\sin(c(t+h)) = \sigma^2(\cos(ct+ch-ct))$

$= \sigma^2 \cos(ch)$ //. No t dependence on mean and auto cov, thus this is stationary. Autocov. function: $\gamma = \begin{cases} \sigma^2 & \text{if } h=0, \\ \sigma^2\cos(ch) & \text{if } h\neq 0 \end{cases}$

(c) Mean: $M_X(t) = E(Z_t)\cos(ct) + E(Z_{t-1})\sin(ct) = 0$.

using the previous substitutions: Auto cov $= E(X_{t+h} \cdot X_t) - \overset{0}{\widetilde{E(X_t)}}.\overset{0}{\widetilde{E(X_{t+h})}}$

$= E[(Z_{t+h} \cdot B_1 + Z_{t+h-1} \cdot B_2)(Z_t \alpha_1 + Z_{t-1}\alpha_2)]$

$= E[\alpha_1 B_1 Z_t Z_{t+h} + Z_{t+h}.Z_{t-1} B_2 \alpha_2 + Z_{t+h-1}.Z_t B_2 \alpha_1$
$+ Z_{t+h-1}.Z_{t-1}.B_2\alpha_2]$

if $h=0$, $\Rightarrow \alpha_1^2\sigma^2 + 0 + 0 + \alpha_2^2\sigma^2 = \sigma^2(\cos^2 ct + \sin^2(ct))$
$= \sigma^2$.

if $h=1$, $= 0 + 0 + \sigma^2 B_2\alpha_1 + 0 = \sigma^2 \sin(c(t+1)).\cos(ct)$

else, $= 0 + 0 + 0 + 0 = 0$.

Autocov: $\gamma_X : \begin{cases} \sigma^2 & \text{if } h=0, \\ \sigma^2 \sin(c(t+1))\cos(ct) & \text{if } h=1 \\ 0 & \text{else} \end{cases}$. t dependence $\Rightarrow$ <u>not</u> stationary.

d) mean: $\mu_X(t) = a + b\,E(Z_0) = a$,

$E(X_{t+h}) = a$

$\mathrm{cov}(X_{t+h}, X_t) = E(X_{t+h} \cdot X_t) - a^2$,

$= E[(a + bZ_0)^2] = E[a^2 + 2abZ_0 + bZ_0^2]$

$= a^2 + b\sigma^2 - a^2 = b\sigma^2$ Thus autocov $\gamma_X(h) = \{b\sigma^2\}$,

mean or autocov. don't depend on $t \Rightarrow$ stationary.

e) mean $= \mu_X(t) = \cos(ct)\,E(Z_0) = 0$,

$\mathrm{Cov}(X_{t+h}, X_t) = E[Z_0 \cos(c(t+h)) \cdot Z_0 \cos(ct)] - 0^2$

if $h = 0$, then $E[Z_0^2 \cos^2(ct)] = \cos^2(ct)\,E(Z_0^2) = \sigma^2 \cos^2(ct)$

if $h \neq 0$, $= \cos(ct)\cos(c(t+h)) \cdot \sigma^2$, so autocov $\gamma_X(h) = \begin{cases} \sigma^2 \cos^2(ct) & \text{if } h = 0, \\ \cos(ct)\cos(c(t+h))\sigma^2 & \text{if } h \neq 0. \end{cases}$

depends on $t \Rightarrow$ not stationary.

f) mean $\mu_X(t) = E(Z_t \cdot Z_{t-1}) = 0$,

$\mathrm{Cov}(X_{t+h}, X_t) = E(X_{t+h} \cdot X_t) - 0^2 = E(Z_{t+h} \cdot Z_{t+h-1} \cdot Z_t \cdot Z_{t-1})$

if $h = 0$, then is $E(Z_t^2 \cdot Z_{t+h-1}^2) = E(Z_t^2) \cdot E(Z_{t+h-1}^2)$ by indep

$= \sigma^2 \sigma^2$

if $h = 1$, $= E(Z_{t+1} \cdot Z_t \cdot Z_t \cdot Z_{t-1}) = E(Z_t^2) \cdot E(Z_{t+1} \cdot Z_{t-1})$

$= 0$

else, $= 0$.

Autocov: $\gamma_X(t) = \begin{cases} \sigma^4 & \text{if } h = 0 \\ 0 & \text{else} \end{cases}$

no t dependence $\Rightarrow$ stationary.

Q2) a) For WN, we need 0 mean, and no correlation.

let t = odd, then $E(X_t) = E\left[\frac{Z_{t-1}^2 - 1}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}} E(Z_{t-1}^2) - 1$

$= \frac{1}{\sqrt{2}}(1-1) = 0$ as $Var = 1$.

$Var(Z_{t-1}) = E(Z_{t-1}^2) - \underbrace{E(Z_{t-1})}_{0}$

$\Rightarrow E(Z_{t-1}^2) = Var(Z_{t-1})$

t even $\Rightarrow E(X_t) = E(Z_t) = 0$

Suppose t is odd, and $t+h$ is odd.

Then $Cov(X_{t+h}, X_t) = E(X_{t+h} \cdot X_t) - 0^2 = E\left(\frac{Z_{t+h-1}^2 - 1}{\sqrt{2}} \cdot \frac{Z_{t-1}^2 - 1}{\sqrt{2}}\right)$

$= \frac{1}{2} E\left[Z_{t+h-1}^2 \cdot Z_{t-1}^2 - Z_{t+h-1}^2 - Z_{t-1}^2 + 1\right]$

$= \frac{1}{2}\left[1 \cdot 1 - 1 - 1 + 1\right] = \frac{1}{2}\left[1 - 2 + 1\right] = 0$

due to the definition of variance, $Var(Z_t) = 1$, and independence of Z's.

if $t+h$ is even and t is even, $Cov(X_{t+h} \cdot X_t) = E(Z_{t+h} \cdot Z_t) = 0$

if $t+h$ is even, t is odd, $= E\left(Z_{t+h} \cdot \frac{Z_{t-1}^2 - 1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} E\left(Z_{t+h} Z_{t-1}^2 - E(Z(t+h))\right)$

$= \frac{1}{\sqrt{2}} \cdot \underbrace{E(Z_{t+h})}_{0} E(Z_{t-1}^2) - \underbrace{E(Z_{t+h})}_{0}$

$= 0$

if $t+h$ is odd, t is even, $= E\left(\frac{Z_{t+h-1}^2 - 1}{\sqrt{2}} \cdot Z_t\right) = \frac{E(Z_t) \cdot E(Z_{t+h-1}^2) - E(Z_t)}{\sqrt{2}}$

$= 0$.

So we have AutoCov $= \gamma_X(h) = \begin{cases} 0 & \text{if } h \neq 0, \\ \sigma^2 = 1 & \text{if } h = 1 \end{cases}$ no correlation.

as $Cov(X_t, X_t) = E(Z_t^2) = Var(Z_t) = 1$ if t even,

$E\left[\frac{(Z_{t-1}^2 - 1)^2}{2}\right] = \frac{1}{2} E(Z_{t-1}^4 - 2Z_{t-1}^2 + 1) = \frac{1}{2}(3 - 2 + 1) = 1$.

Thus X_t is white noise, as $E(Z_{t-1}^4)$ = fourth moment of SND $= 3\sigma^4 = 3$. a const!

b) if n is odd, $X_{n+1} = Z_{n+1}$

$$X_n = \frac{Z_{n-1}^2 - 1}{\sqrt{2}}, \quad X_{n-1} = Z_{n-1}, \quad X_{n-2} = \frac{Z_{n-3}^2 - 1}{\sqrt{2}}, \text{ etc...}$$

Each odd observation depends on the previous even one, and the even cases show no dependence; we will never get another Z_{n+1} term within $X_1, \ldots, X_n$. Thus $E(X_{n+1} | X_1 \ldots X_n) = E(X_{n+1}) = E(Z_{n+1}) = 0$.

if n is even, $X_{n+1} = \frac{X_n^2 - 1}{\sqrt{2}}$, and X_{n+1} depends only on this.

let $X_n = k_n$, then $E(X_{n+1}) = \frac{k_n^2 - 1}{\sqrt{2}}$.

a) cont: This shows dependence, so for part a), we conclude that X_t is not IID.

Q3) Claim: ∇ is a linear operator.

proof: let $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, $m(t)$ and $n(t)$ some functions of t.

Then, $\nabla(\alpha m(t) + \beta n(t)) = 1 - B(\alpha m(t) + \beta n(t)) = \alpha m(t) + \beta n(t) - \alpha m(t-1) - \beta m(t-1) = \alpha m(t) - \alpha m(t-1) + \beta n(t) - \beta n(t-1)$

$= \alpha \nabla m(t) + \beta \nabla n(t)$ $\square$.

Induction on p: base case: $p=1$.

$$m_t = \sum_{k=0}^{1} C_k t^k = C_0 + C_1 t,$$

$$\nabla m_t = C_0 + C_1 t - C_0 - C_1(t-1)$$

$$= C_1 t - C_1 t + C_1 = C_1, \text{ power } 0,$$

holds.

Suppose $\nabla \sum_{k=0}^{n} C_k t^k$ has power $n-1$. We need to show

$\nabla \sum_{k=0}^{n+1} C_k t^k$ has power n,

$$\sum_{k=0}^{n+1} C_k t^k = \sum_{k=0}^{n} C_k t^k + C_{(n+1)} t^{(n+1)}, \text{ Apply } \nabla;$$

$$= \nabla \sum_{k=0}^{n} C_k t^k + \nabla C_{(n+1)} t^{(n+1)} \text{ by linearity,}$$

$$\nabla C_{(n+1)} t^{(n+1)} = C_{(n+1)} t^{n+1} - C_{(n+1)} t^{n+1} = \cancel{C_{n+1} t^{n+1}} - C_{n+1} t^{n+1} + \underbrace{\ldots\ldots}_{\text{lower order } t \text{ terms of power at most } n}$$

And the left term has power $(n-1)$ by the induction hypothesis, thus the whole polynomial has power n at most. This concludes the proof.

$\nabla^{p+1}(m_t)$ means $\underbrace{\nabla(\nabla(\ldots(m_t)}_{p+1 \text{ times}}$. Initially power p, each ∇ reduces power by 1,

So applying p times gives some constant c, and $\nabla c = c - c = 0$ being the $p+1$'th application.

4) a) $\nabla \nabla_{12} X_t = (1-B)(1-B^{12})X_t = (1-B)(X_t - X_{t-12})$

$= X_t - X_{t-12} - X_{t-1} + X_{t-13}$

$= a + bt + S_t + Y_t - a - bt + 12b - S_{t-12} - Y_{t-12} - a - bt + b$

$- S_{t-1} - Y_{t-1} + a + bt - 13b + S_{t-13} + Y_{t-13}$

Since $S_{t-12} = S_t$, $S_{t-13} = S_{t-1}$,

$= Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}$. Let this $= Z_t$.

$E(Z_t) = 0$, $\gamma_Z(t) = Cov(Z_{t+h}, Z_t) = E(Z_{t+h}, Z_t) - 0^2$

$= E[Y_{t+h}, Y_t] - E(Y_{t+h} \cdot Y_{t-1}) - E(Y_{t+h} \cdot Y_{t-12}) + E(Y_{t+h} \cdot Y_{t-13})$

$- E(Y_{t+h-1}, Y_t) + E(Y_{t+h-1}, Y_{t-1}) + E(Y_{t+h-1}, Y_{t-12}) - E(Y_{t+h-1}, Y_{t-13})$

$- E(Y_{t+h-12}, Y_t) + E(Y_{t+h-12}, Y_{t-1}) + E(Y_{t+h-12}, Y_{t-12}) - E(Y_{t+h-12}, Y_{t-13})$

$+ E(Y_{t+h-13}, Y_t) + E(Y_{t+h-13}, Y_{t-1}) + E(Y_{t+h-13}, Y_{t-12}) + E(Y_{t+h-13}, Y_{t-13})$

$= \underline{\gamma_Y(h)} - \gamma_Y(h+1) - \gamma_Y(h+12) + \gamma_Y(h+13) + \gamma_Y(h-1) + \underline{\gamma_Y(h)}$

$+ \gamma_Y(h+11) - \gamma_Y(h+12) - \gamma_Y(h-12) + \gamma_Y(h-11) + \underline{\gamma_Y(h)} - \gamma_Y(h+1)$

$+ \gamma_Y(h-13) - \gamma_Y(h-12) - \gamma_Y(h-1) + \underline{\gamma_Y(h)}$

$= 4\gamma_Y(h) - 2\gamma_Y(h+1) - 2\gamma_Y(h-1) + \gamma_Y(h+11) + \gamma_Y(h-11) - 2\gamma_Y(h+12)$

$- 2\gamma_Y(h-12) + \gamma_Y(h+13) + \gamma_Y(h-13)$,

as in general $E(Z_{t+h}, Z_{t+k}) = \gamma_y(h-k)$. There is no t dependence on mean or autocov, so $\underline{\text{stationary}}$.

b) $\nabla_{12}^2\{X_t\} = (1-B^{12})(1-B^{12})X_t = (1-B^{12})X - X_{t-12}$

$= X_t - 2X_{t-12} + X_{t-24}$, $X_t = aS_t + btS_t + Y_t$

$= aS_t + btS_t + Y_t - 2[aS_{t-12} + b(t-12)S_{t-12} + Y_{t-12}]$

$+ aS_{t-24} + b(t-24)S_{t-24} + Y_{t-24}$

$= aS_t + btS_t + Y_t - 2[aS_t + (bt-12b)S_t + Y_{t-12}]$

$+ aS_t + (bt-24b)S_{t-24} + Y_{t-24}$

$= aS_t + btS_t + Y_t - 2[aS_t + btS_t - 12bS_t + Y_{t-12}]$

$+ aS_t + S_t bt - 24bS_t + Y_{t-24}$

$= Y_t - 2Y_{t-12} + Y_{t-24}$ using $S_t = S_{t-12} = S_{t-24}$.

call this Z_t. $E(Z_t) = 0$.

$\gamma_Z(t+h,t) = Cov(Z_{t+h}, Z_t) = E(Z_{t+h} Z_t) =$

$= E(Y_{t+h} Y_t) - 2E(Y_{t+h} Y_{t-12}) + E(Y_{t+h} Y_{t-24})$

$+ 2E(Y_{t+h-12} Y_t) + 4E(Y_{t+h-12}, Y_{t-12}) - 2E(Y_{t+h-12}, Y_{t-24})$

$+ E(Y_{t+h-24}, Y_t) + 2E(Y_{t+h-24}, Y_{t-12}) + E(Y_{t+h-24}, Y_{t-24})$

$= \gamma_Y(h) - 2\gamma_Y(h+12) + \gamma_Y(h+24) - 2\gamma_Y(h-12) + 4\gamma_Y(h)$

$-2\gamma_Y(h+12) + \gamma_Y(h-24) - 2\gamma_Y(h-12) + \gamma_Y(h)$

$= 6\gamma_Y(h) - 4\gamma_Y(h+12) - 4\gamma_Y(h-12) + \gamma_Y(h+24) + \gamma_Y(h-24)$.

No t dependance on mean or autocov $\Rightarrow$ stationary.

Question 5, Assignment 1

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```
library(tidyverse)
library(itsmr)
library(forecast)
library(tibbletime)
library(tsbox)
library(gridExtra)
library(TTR)
```

In order to make our life easier later down the line, we first create a tbl from our original data.

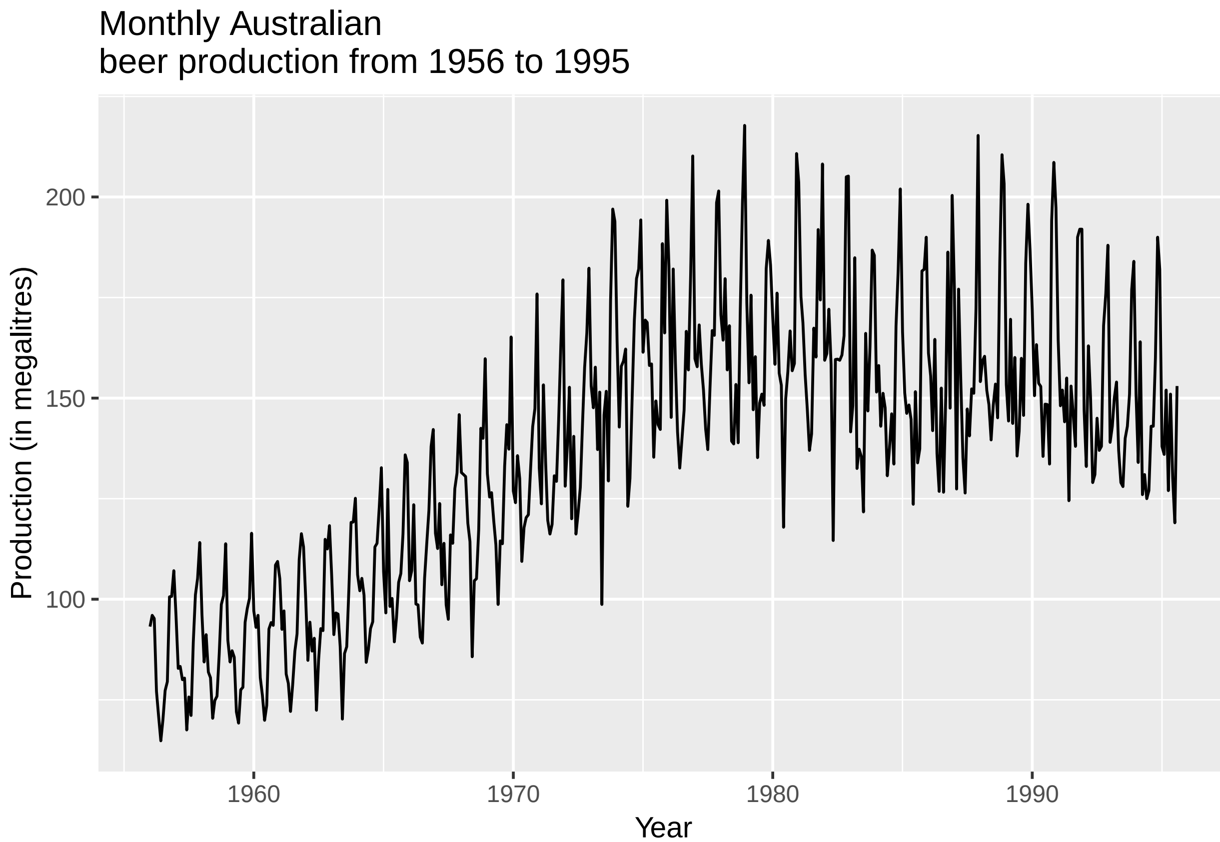
```
australian_beer = read_csv("AusBeer.csv")
head(australian_beer)
```

```
## # A tibble: 6 x 2
##   Date       Production
##   <date>          <dbl>
## 1 1956-01-01       93.2
## 2 1956-02-01       96
## 3 1956-03-01       95.2
## 4 1956-04-01       77.1
## 5 1956-05-01       70.9
## 6 1956-06-01       64.8
```

```
australian_beer_tbl = as_tbl_time(ts_df(australian_beer),index=Date)
```

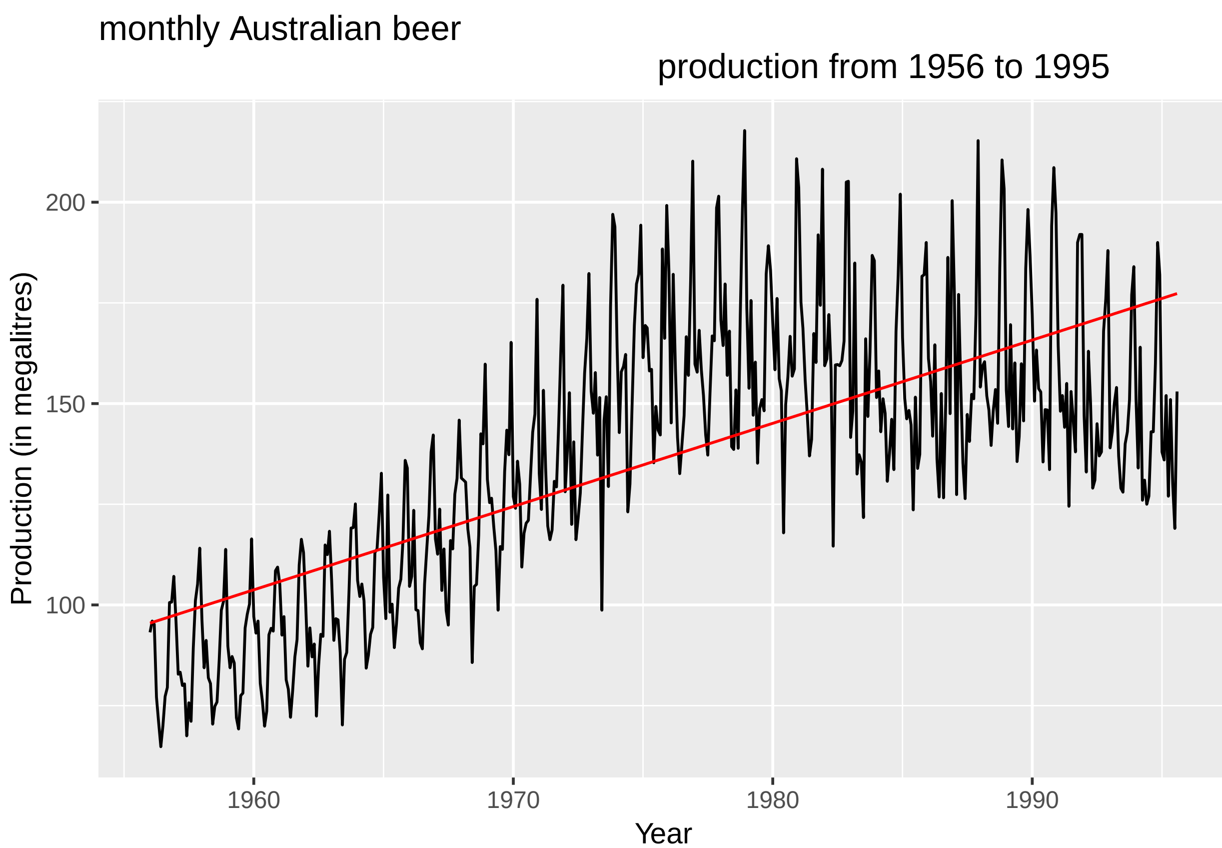
First let's take a look at our data.

```
ggplot(australian_beer,aes(x=Date,y=Production)) + geom_line() + ylab("Production (in megalitres)") +
  ggtitle("Monthly Australian
beer production from 1956 to 1995") + xlab("Year")
```



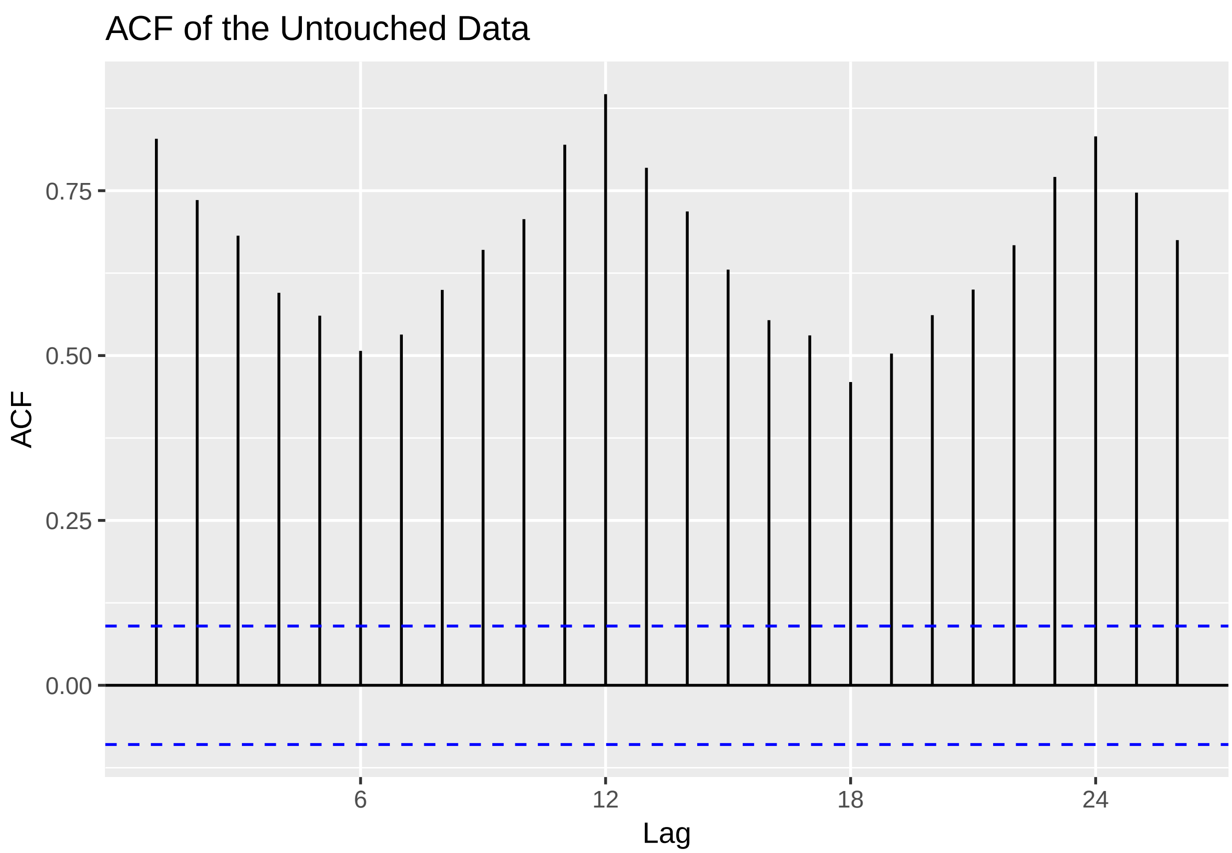
There does appear to be some sort of increasing trend. Let's assume an upward (increasing) linear trend, and see what this could look like.

```
australian_beer_tbl = australian_beer %>% mutate(lin_trend=trend(Production,p=1))

ggplot(australian_beer_tbl,
       aes(x=Date,y=Production))+geom_line()+geom_line(aes(y=lin_trend),
  color="red")+ylab("Production (in megalitres)")+ggtitle("monthly Australian beer
                                                          production from 1956 to 1995") + xlab("Year")
```



Seems somewhat reasonable, but not a very good fit as the data fluctuates a lot. Let's take a look at the Autocovariance Function (ACF):

```
ggAcf(ts(australian_beer_tbl %>% pull(Production),
         start=c(1956,1),freq=12))+ggtitle("ACF of the Untouched Data")
```



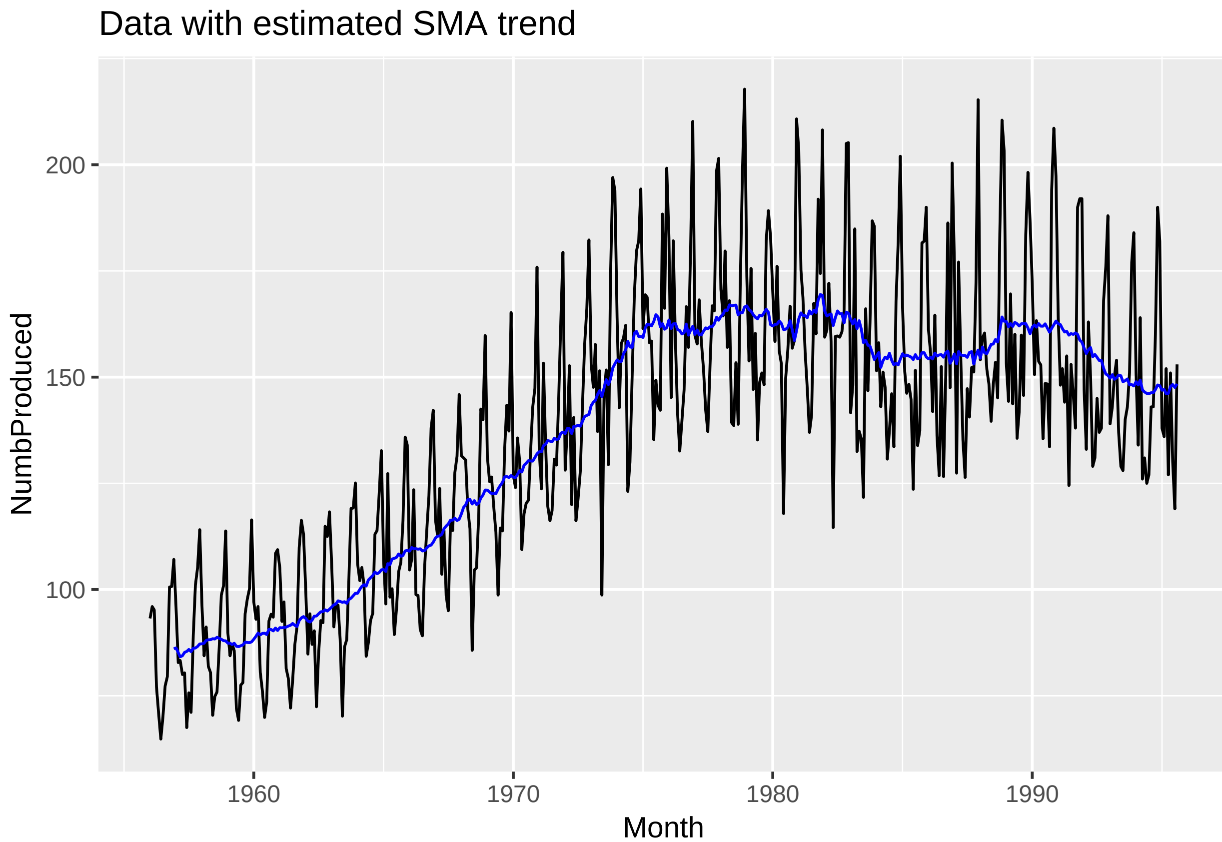
Indeed high correlation across the board, and some consistence between lags 0,12 and 24 which could imply some sort of period 12 seasonality.

Now we set it up for further analysis by creating a time series (ts) and then adjust our dataframe and table to be properly indexed.

```
australian_beer_ts = ts(australian_beer_tbl %>% pull(Production), start=c(1956,1),freq=12)
australian_beer_df = ts_df(australian_beer_ts) %>% rename(Month = time,
                                                           NumbProduced=value)
australian_beer_tbl = as_tbl_time(australian_beer_df,index=Month)
```

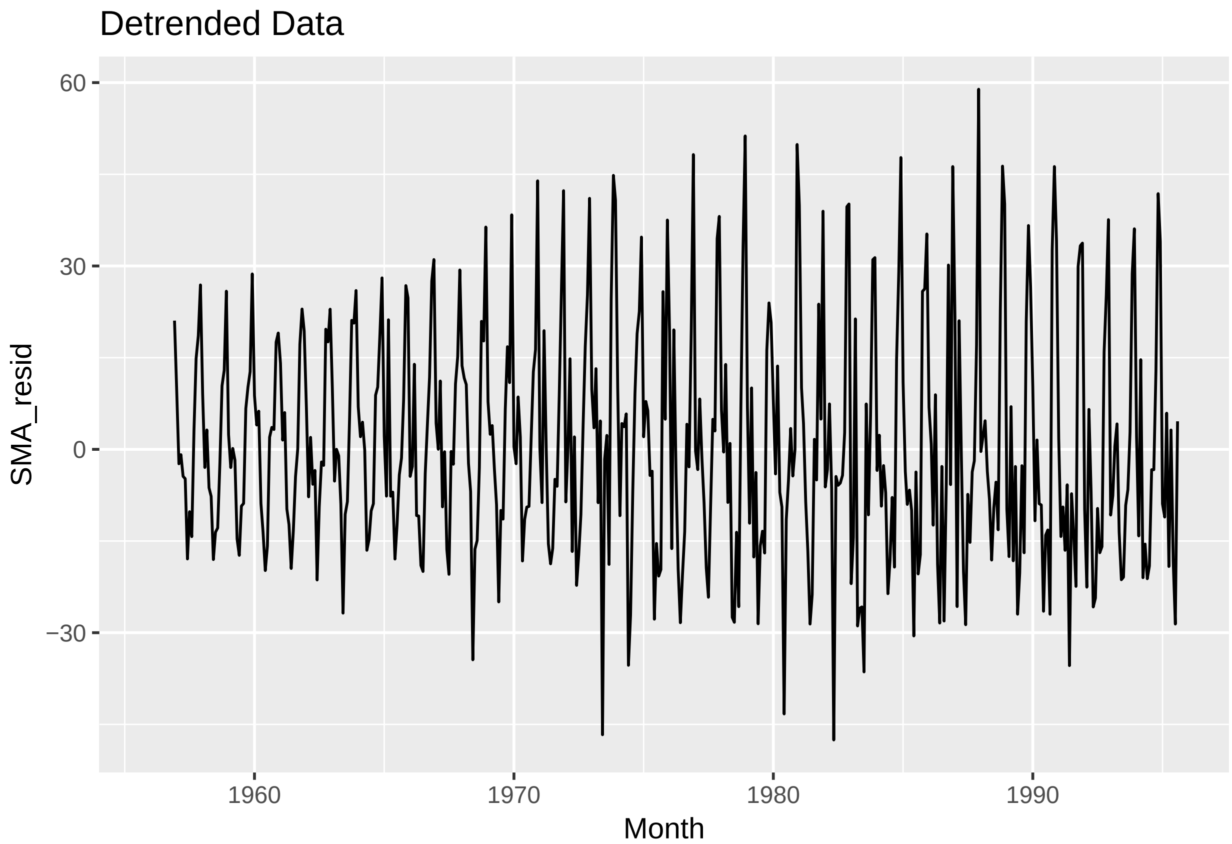
We are now ready to find the trend component and plot it together with our data. Using an SMA of order 12 yields:

```
trend_comp = ts_df(SMA(australian_beer_ts,n=12)) %>% rename(Month=time,SMA_12=value)

australian_beer_tbl  =  full_join(australian_beer_tbl,trend_comp) %>%
  mutate(SMA_resid=NumbProduced-SMA_12)

ggplot(australian_beer_tbl,aes(x=Month,y=NumbProduced)) + geom_line() +
  geom_line(aes(y=SMA_12),color="blue") + ggtitle("Data with estimated SMA trend")
```



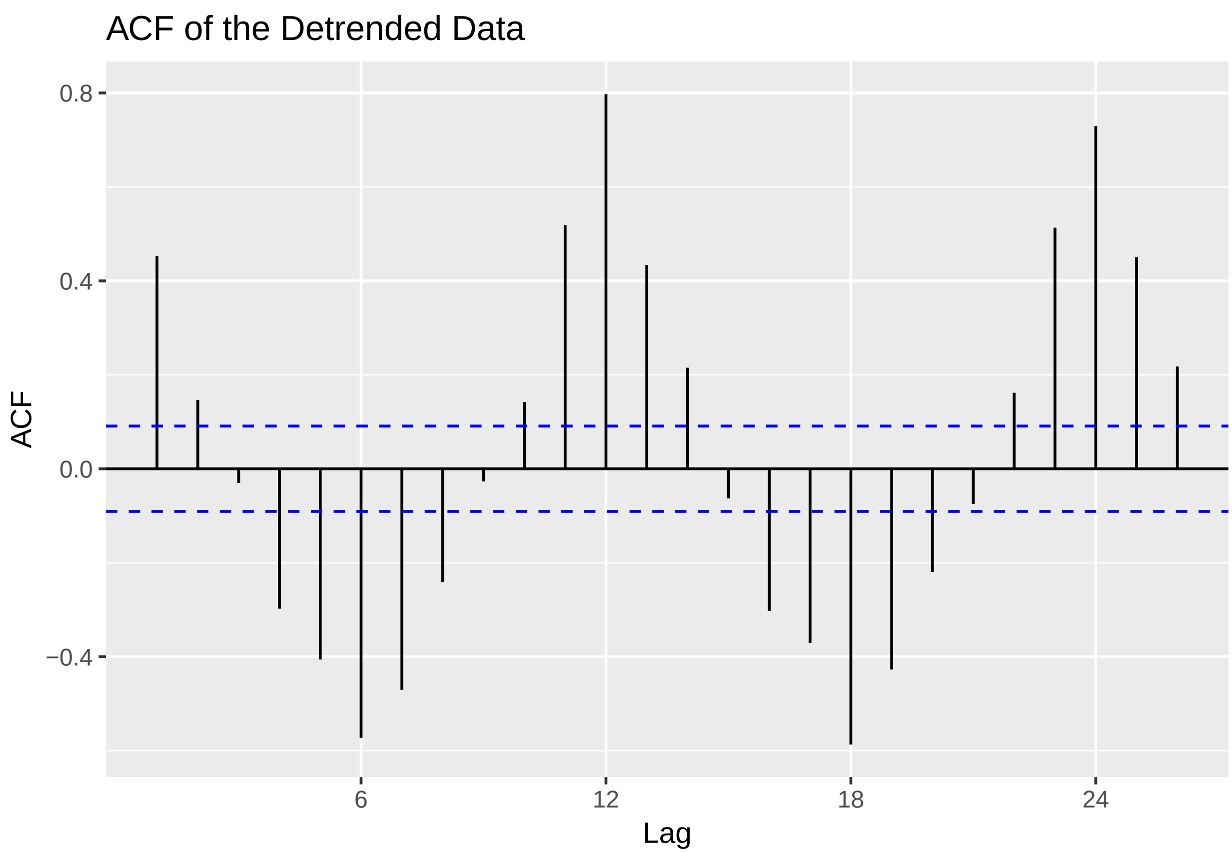
A very well fit, so we caught the trend. And here is the residue, that is our data with the trend component removed:

```
ggplot(australian_beer_tbl,aes(x=Month,y=SMA_resid)) + geom_line(col="black")+ggtitle("Detrended Data")
```



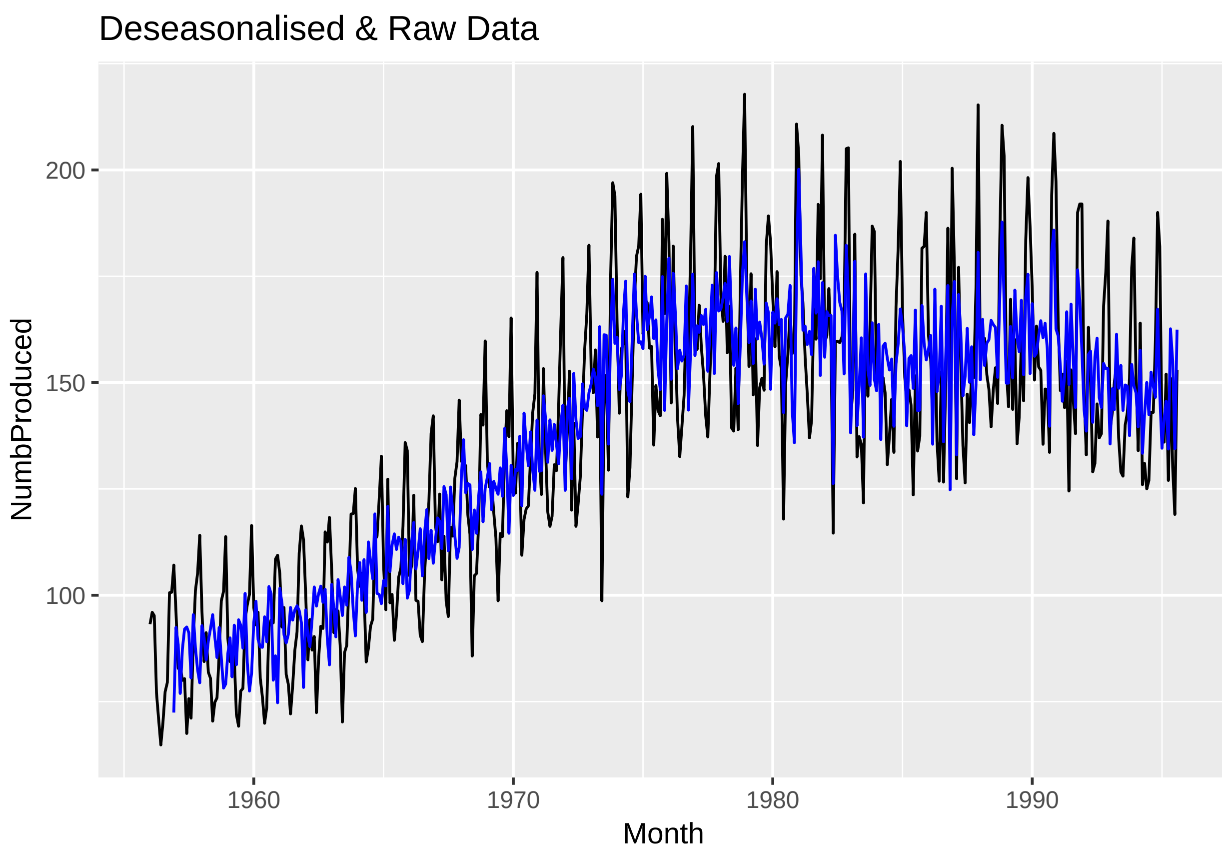
Let's see if and how the ACF got affected:

```
ggAcf(ts(australian_beer_tbl %>% pull(SMA_resid),
         start=c(1956,1),freq=12))+ggtitle("ACF of the Detrended Data")
```



Indeed a lot of it seems to be trimmed out, and we're left with a very patterned, seasonal looking correlation. Now to deseasonalise it by using the season & residue functions and plot the raw & deseasonalised data sets on the same graph:

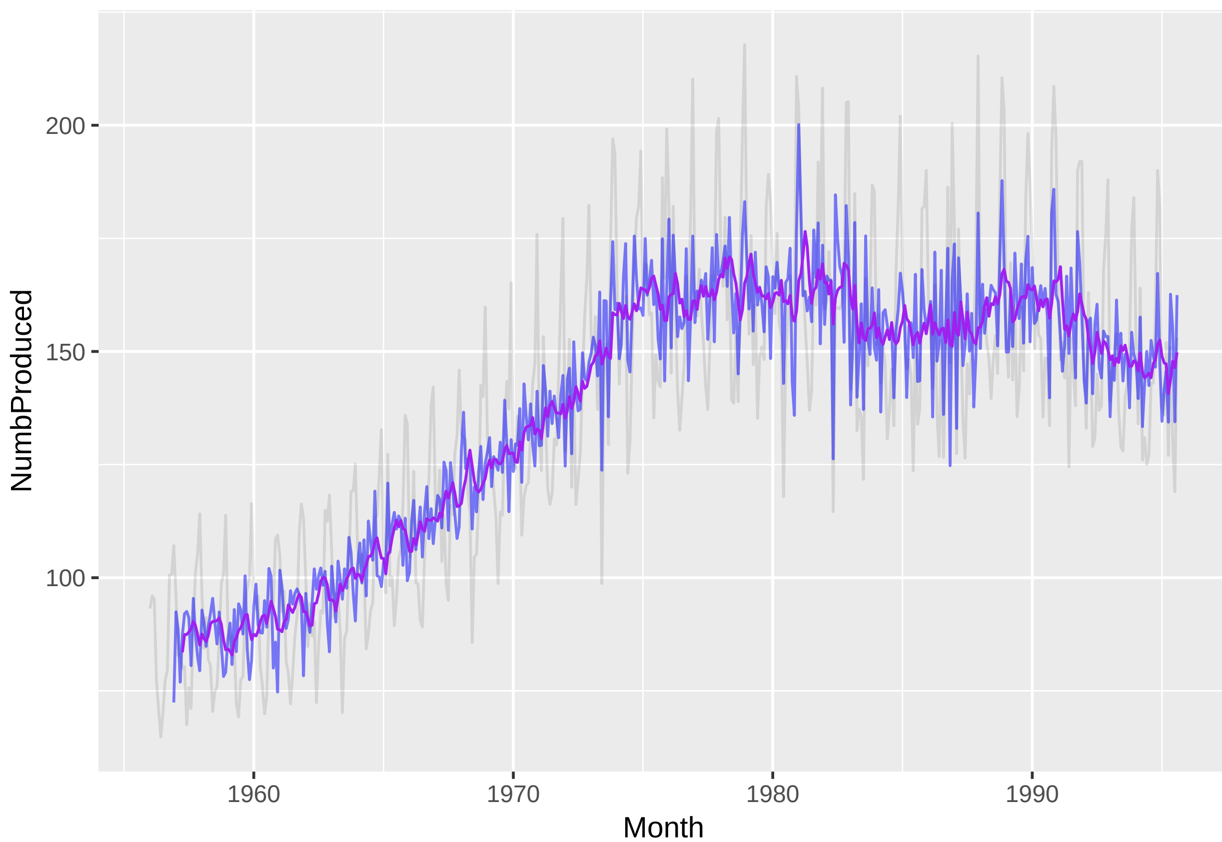
```
SMA_resid_ts = ts(australian_beer_tbl%>% filter(!is.na(SMA_resid)) %>% pull(SMA_resid),
                  start=c(1956,12),frequency=12)

season_comp = season(SMA_resid_ts,d=12)


SMA_resid_tbl = ts_df(SMA_resid_ts) %>% rename(Month=time,
                                                SMA_resid=value) %>%
  mutate(seasonal = season_comp) %>% as_tbl_time(index=Month)



australian_beer_tbl = full_join(australian_beer_tbl,SMA_resid_tbl)


australian_beer_tbl = australian_beer_tbl %>% mutate(deseason=NumbProduced-seasonal)


ggplot(australian_beer_tbl,aes(x=Month,y=NumbProduced)) + geom_line() +
  geom_line(aes(y=deseason),color="blue")+ggtitle("Deseasonalised & Raw Data")
```



Where the blue line is the deseasonalised data, and the black is our original data.

Indeed this seems to fit the data very well. Now we re-estimate trend (that is estimate the trend of the deseasonalised component) by applying SMA of order 5 and join it up to the data once again.

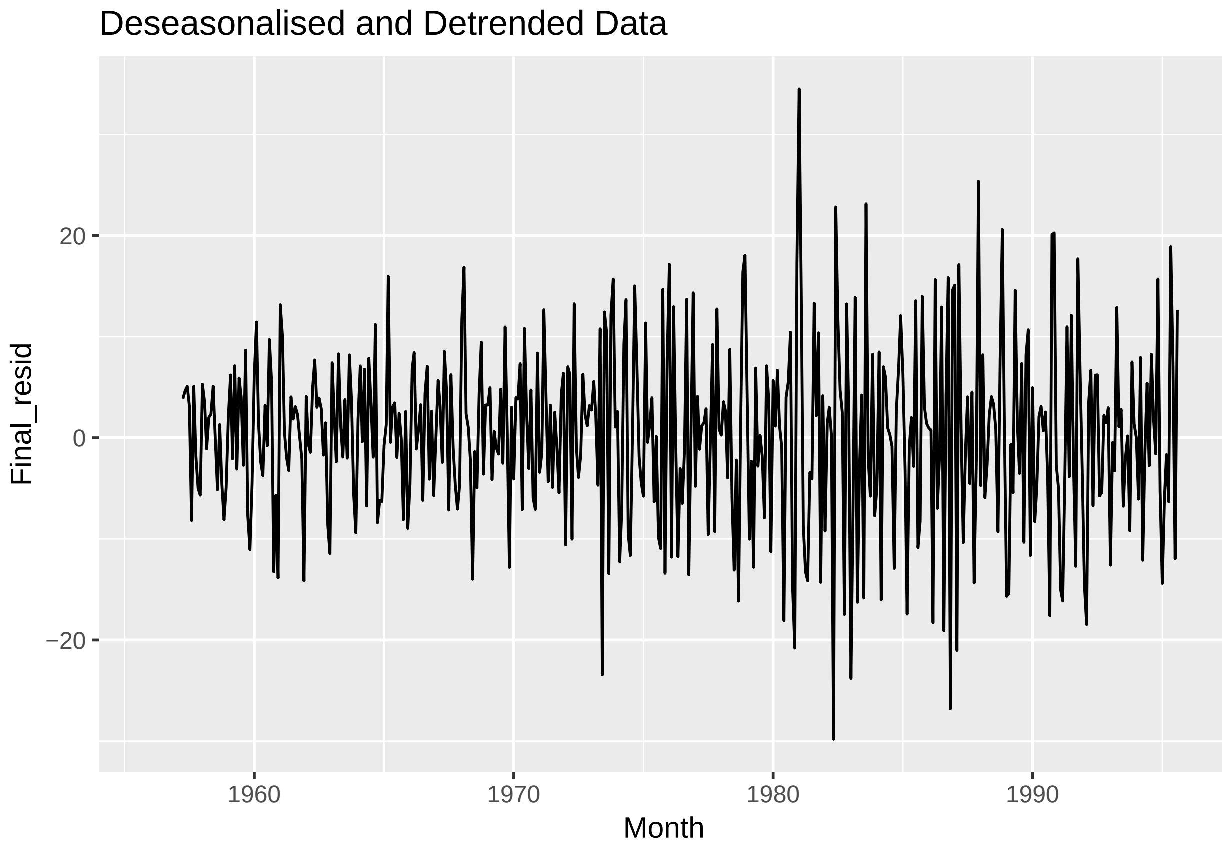
```
deseason_ts = ts(australian_beer_tbl %>% filter(!is.na(deseason)) %>% pull(deseason),
                 start=c(1956,12),frequency=12)

deseason_trend = ts_df(SMA(deseason_ts,n=5)) %>% rename(Month=time,de_SMA_5 = value)

australian_beer_tbl = full_join(australian_beer_tbl,deseason_trend)





australian_beer_tbl = australian_beer_tbl %>% mutate(Final_resid=NumbProduced-de_SMA_5-seasonal)

ggplot(australian_beer_tbl,aes(x=Month,y=NumbProduced))+geom_line(alpha=0.1)+geom_line(aes(y=deseason)
,color="blue",alpha=0.5)+geom_line(aes(y=de_SMA_5),color="purple")
```



The purple line represents the trend of the deseasonalised data, and now we shall remove both the season and the trend.

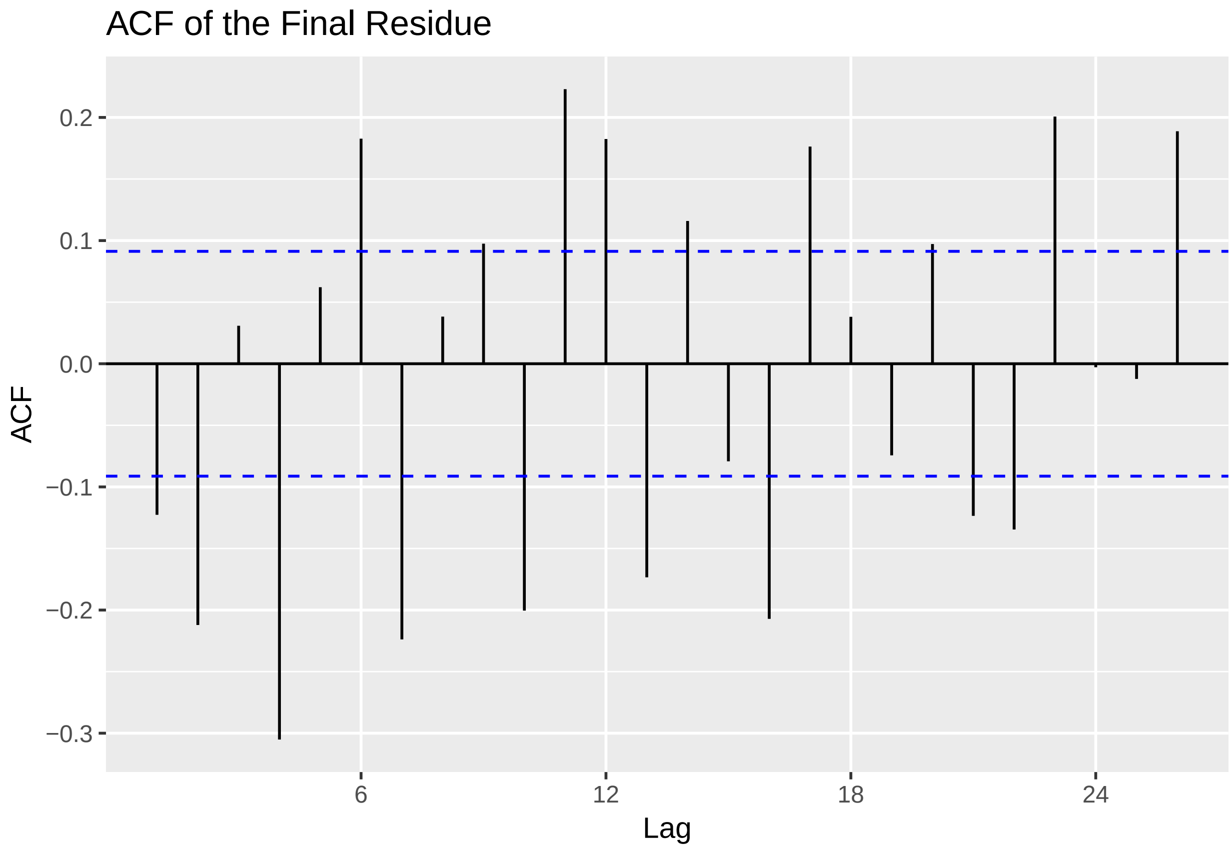
And thus, the final deseasonalised and detrended data looks like this:

```
ggplot(australian_beer_tbl,
       aes(x=Month,y=Final_resid)) + geom_line()+ggtitle("Deseasonalised and Detrended Data")
```



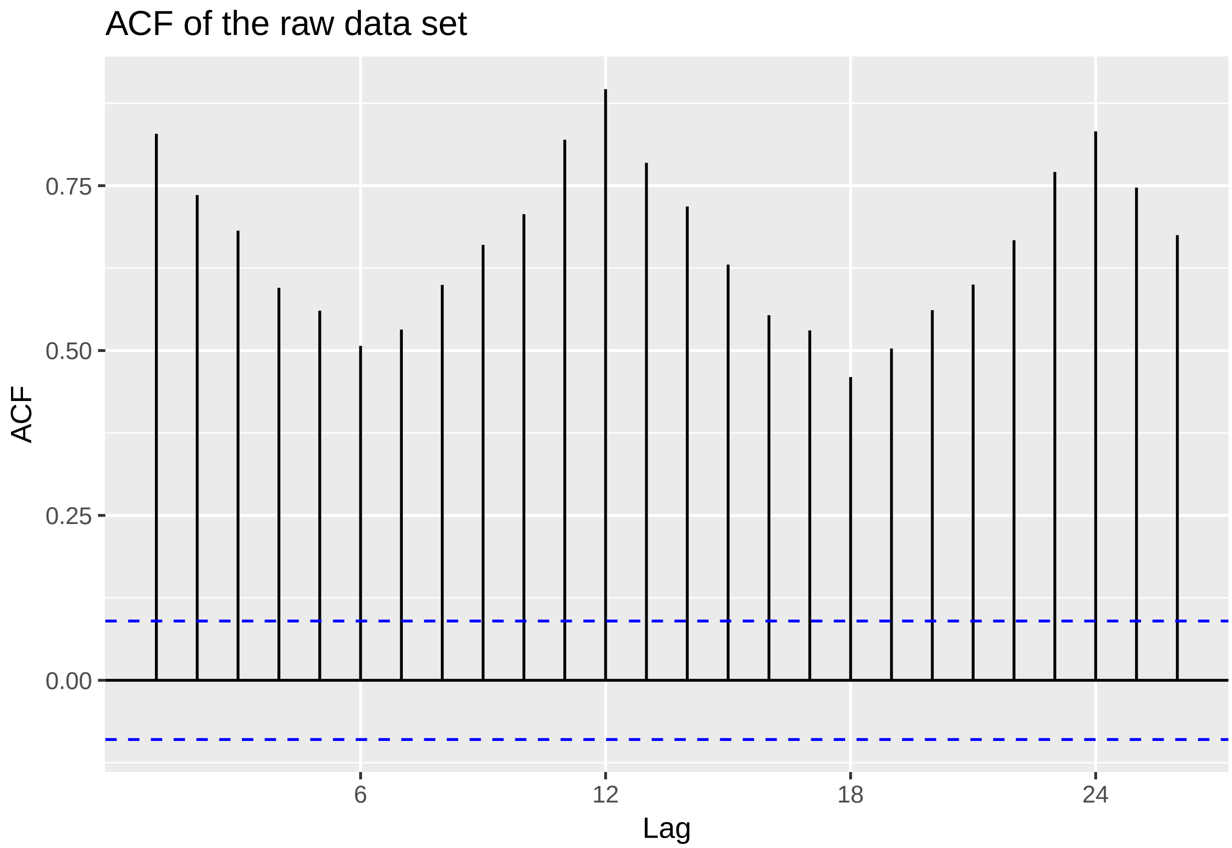
We presume that what's left is noise alone. Isolating this "final residue" we get the following autocovariance:

```
ggAcf(ts(australian_beer_tbl %>% pull(Final_resid),
         start=c(1956,12),freq=12))+ggtitle("ACF of the Final Residue")
```



There are still spikes of correlaiton; perhaps not as little as we'd hoped, however it's much less patterned and the seasonality/trend seems to have dissipated significantly. Here is the initial, raw ACF for comparison:

```
ggAcf(ts(australian_beer_tbl %>% pull(NumbProduced),
         start=c(1956,12), freq=12))+ggtitle("ACF of the raw data set")
```



As one can see, this is was much more structured.