CSE 321 Homework 5

1-) A small business – lets say, a photocopying service with a single large machine - faces the following scheduling problem. Each morning they get a set of jobs from customers. They want to do the jobs on their single machine in an order that keeps their customers happiest. Job of customer i will take t_i time to complete. Given a schedule (i.e., an ordering of the jobs), let C_i denote the finishing time of job i. For example, if job j is the first to be done, we would have $C_j = t_j$; and if job j is done right after job i, we would have $C_j = C_i + t_j$. Each customer i also has a given weight w_i that represents his or her importance to the business. The happiness of customer i is expected to be dependent on the finishing time of i's job. So the company decides that they want to order the jobs to minimize the weighted sum of the completion times, $\sum_{i=1}^{n} Wi.Ci$.

Propose a greedy algorithm to solve this problem. That is, you are given a set of n jobs with a processing time t_i and a weight w_i for each job. You want to order the jobs to minimize the weighted sum of the completion times, $\sum_{i=1}^{n} Wi.Ci$. Implement the algorithm with Python.

For an example, suppose there are two jobs: the first takes time $t_1 = 1$ and has weight $w_1 = 10$, while the second job takes time $t_2 = 3$ and has weight $w_2 = 2$. Then doing job 1 first would yield a weighted completion time of 10.1 + 2.4 = 18, while doing the second job first would yield the larger weighted completion time of 10.4 + 2.3 = 46.

2-) Suppose you're running a lightweight consulting business - just you, two associates, and some rented equipment. Your clients are distributed between the East Coast and the West Coast, and this leads to the following question. Each month, you can either run your business from an office in New York (NY) or from an office in San Francisco (SF). In month i, you'll incur an operating cost of N_i if you run the business out of NY; you'll incur an operating cost of S_i if you run the business out of SF. (It depends on the distribution of client demands for that month.) However, if you run the business out of one city in month i, and then out of the other city in month i + 1, then you incur a fixed moving cost of M to switch base offices.

Given a sequence of n months, a plan is a sequence of n locations - each one equal to either NY or SF - such that the ith location indicates the city in which you will be based in the ith month. The cost of a plan is the sum of the operating costs for each of the n months, plus a moving cost of M for each time you switch cities. The plan can begin in either city.

The problem: Given a value for the moving cost M, and sequences of operating costs

N_1 ..... N_n and S_1 ..... S_n, find a plan of minimum cost (Such a plan will be called optimal.).

For an example, let's suppose n=4, M=10 and the operating costs are given by the following table.

#	Month 1	Month 2	Month 3	Month 4
NY	1	3	20	30
SF	50	20	2	4

Then the plan of minimum cost would be the sequence of locations:

[NY, NY, SF, SF]

with a total cost of $1 + 3 + 2 + 4 + 10 = 20$, where the final term of 10 arises because you change locations once.

a) Show that the following algorithm does not correctly solve this problem by giving an instance which it does not return the correct answer.

for i= 1 to n

  if $N_i < S_i$ then

    Output "NY in Month i"

  else

    Output "SF in Month i"

end

b) Propose a dynamic programming algorithm that takes values for n, M, and sequences of operating costs N_1 ..... N_n and S_1.....S_n, and returns the cost of an optimal plan. Implement the algorithm with Python.

Explain the algorithms clearly, and analyze time complexity of them in a <u>single</u> PDF file. Add your .py file /s and the PDF file in a folder which is named as your number. Don't forget to <u>zip</u> your homework before upload it to the Moodle.

Note:

- Submit your homework on Moodle.
- You can send an email to <u>b.koca@gtu.edu.tr</u> to ask any question about the homework
- Do your homework personally, group studies will be considered as cheating.