Prj04_Fourier_S025235

March 11, 2024

0.0.1 EE 421/521 Image Processing

0.1 Project 4 - Frequency Domain Processing

In this project, you will implement the following filters in the **Fourier** domain:

1. Ideal low-pass filter
2. Ideal high-pass filter
3. Ideal horizontal low-pass filter
4. Ideal horizontal high-pass filter

This project will be graded for both EE 421 (HW3) and EE 521 (HW3) students.

[225]:

```python
# STEP 0 Import the necessary packages

# reading/writing image files
from skimage import io
from skimage import color

# displaying images and plots
import matplotlib.pyplot as plt

# array operations
import numpy as np

# signal processing operations
from scipy import signal
from scipy.linalg import circulant

# DFT calculations
from scipy import fftpack as ft

# mathematical calculations
import math
import cmath


# Mount Google Drive folder to Colab
```

```
from google.colab import drive
drive.mount('/content/drive',force_remount = True)
```

```
Mounted at /content/drive
```

[226]:
```
# my function to set P to 0 if M infinitesmall
# (e.g., for setting phase to 0 when magnitude is infinitesmall)

def my_epsilon2zero(P, M):

    epsilon = 1.0e-6

    shapeP = P.shape
    shapeM = M.shape
    assert shapeP == shapeM

    P = P.reshape(-1)
    M = M.reshape(-1)

    for i in range(P.size):
        if M[i] < epsilon:
            P[i] = 0.0

    return P.reshape(shapeP)

# end of function
```

[227]:
```
# my function to obtain a display-friendly version of 2-D DFT of an image
# (used for displaying DFT magnitude, DFT real part, and DFT imaginary part)

def my_log_display(X):

    shapeX = X.shape
    X = X.reshape(-1)

    for i in range(X.size):
        if X[i] < 0:
            # this is needed for real and imaginary parts
            X[i] = - np.log(1-X[i])
        else:
            # magnitude is always non-negative
            X[i] = np.log(1+X[i])

    return X.reshape(shapeX)

# end of function
```

[228]:
```python
# my function to multiply an image with (-1)^(i+j)
# so that the origin of its DFT is displayed at the center

def my_img_shift(img):

    height, width = img.shape
    assert height%2 == 0 and width%2 == 0

    img_shift = img.copy() + 0.0

    for i in range(height):
        for j in range(width):
            if (i+j)%2 == 1:
                img_shift[i,j] *= -1.0

    return img_shift

# end of function
```

[229]:
```python
# my function to round image data to nearest integer, then
# truncate to range [0, 255], and then set data type to uint8

def my_imgTruncate(img):

    img = np.round(img, 0)
    img = np.minimum(img, 255)
    img = np.maximum(img, 0)
    img = img.astype('uint8')

    return img

# end of function
```

[230]:
```python
# my function to construct a low/high-pass rectangular uniform mask

def my_mask_Rectangle(img_shape, mask_shape, isLowPass = True):

    assert len(img_shape) == 2 and len(mask_shape) == 2
    assert mask_shape < img_shape

    # height & width of image
    hi, wi = img_shape

    # height & width of mask
    hm, wm = mask_shape

    if isLowPass:
```

```
        # zero except at the center
        img_mask = np.zeros(img_shape)
        val = 1
    else:
        # one except at the center
        img_mask = np.ones(img_shape)
        val = 0

    # set the center value (1 for low pass, 0 for high pass)
    img_mask[(hi-hm)//2:(hi+hm)//2, (wi-wm)//2:(wi+wm)//2] = val

    return img_mask

# end of function
```

[231]:
```
# my function to convert to lumincance, round to nearest integer,
# truncate to range [0, 255], and then set data

def my_imgLuminance(imgRGB):

    # make sure it is a color image
    dim_img = imgRGB.shape[2]
    assert dim_img >= 3

    # get the luminance data
    if dim_img == 3:
        imgLum = color.rgb2gray(imgRGB)
    else:
        # ignore the alpha channel
        imgLum = color.rgb2gray(imgRGB[:,:,0:3])

    imgLum = np.round(imgLum * 255, 0)
    imgLum = np.minimum(imgLum, 255)
    imgLum = np.maximum(imgLum, 0)
    imgLum = imgLum.astype('uint8')

    return imgLum

# end of function
```

[232]:
```
# STEP 1 Pick an image for filtering

# set image folder
image_folder = r'/content/drive/MyDrive/Colab Notebooks/421_Images'

# read input image
image_file = r'/cameraman.tiff'
```

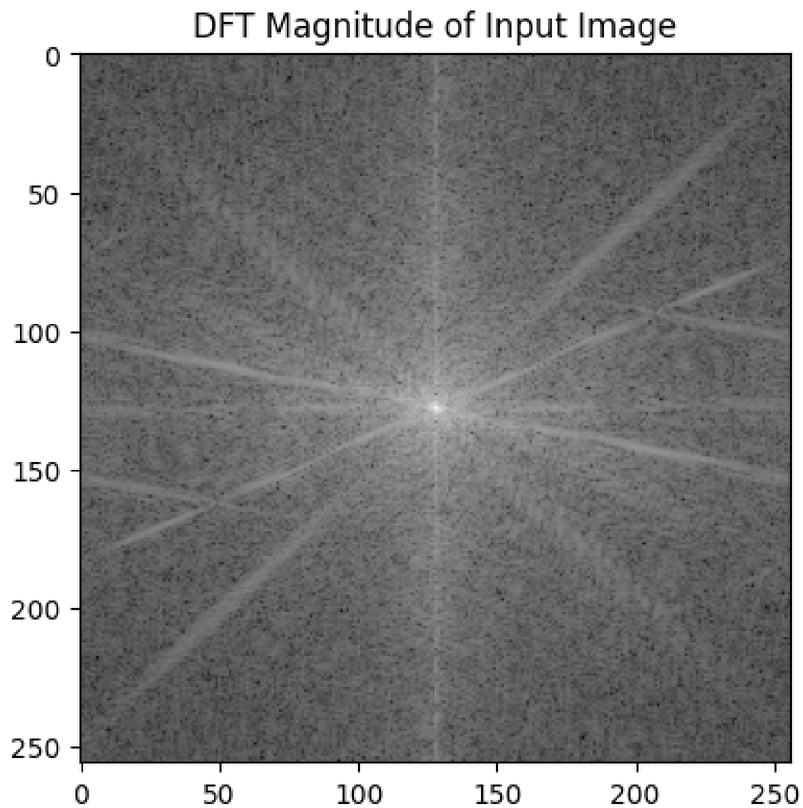
```python
image_path = image_folder + image_file
imgRGB = io.imread(image_path)

if imgRGB.ndim >= 3:
    # color image
    # calculate the luminance image
    x_img = my_imgLuminance(imgRGB)
else:
    # monochrome image
    x_img = imgRGB

height = x_img.shape[0]
width = x_img.shape[1]
datatype = x_img.dtype

print("Image width is {} and image height is {}.".format(width, height))
print("Image data type is {}.\n".format(datatype))

# display luminance image
plt.imshow(x_img, cmap='gray', vmin=0, vmax=255)
plt.title('Input Image')
plt.xticks([]), plt.yticks([])
plt.show()
```

```
Image width is 256 and image height is 256.
Image data type is uint8.
```

Input Image

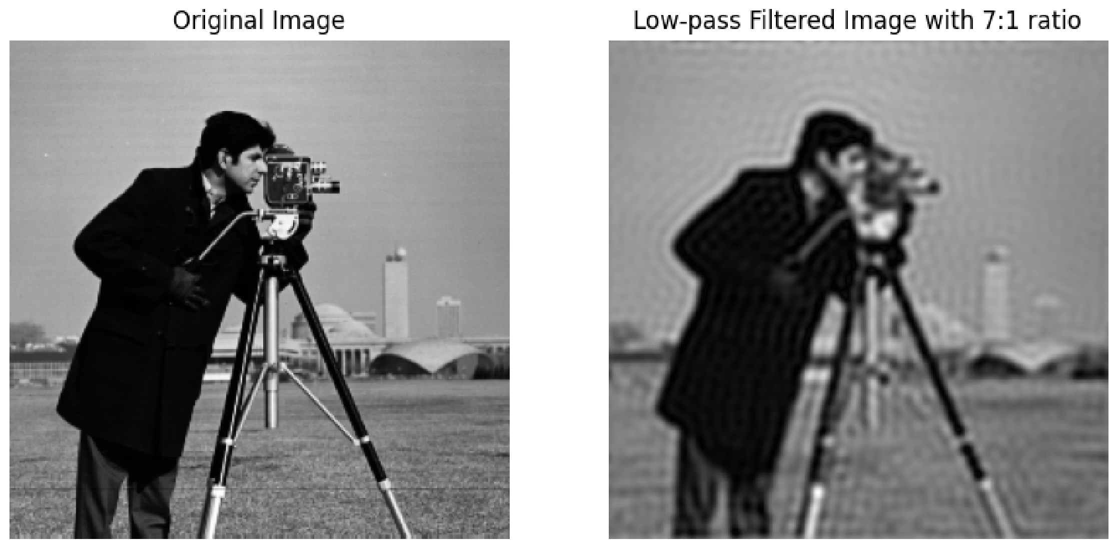
[233]:
```
# STEP 2 Take the DFT of the image

# multiply the image with (-1)^(i+j) before DFT so that DFT origin is displayed
↪at the center
# note that for this to work, both image width and height must be even
x_img_shift = my_img_shift(x_img)

# calculate the 2-D DFT via SciPy's 2-D DFT function
X_img_shift = ft.fft2(x_img_shift)

# get DFT size
height, width = X_img_shift.shape

# get a display friendly version for DFT magnitude
X_img_mag_pr = my_log_display(np.abs(X_img_shift))

# get min and max values for scaling during display
mag_max = X_img_mag_pr.max()
mag_min = X_img_mag_pr.min()

# display the DFT magnitude of the image
plt.imshow(X_img_mag_pr, cmap='gray')
```

```
plt.title('DFT Magnitude of Input Image')
plt.show()
```



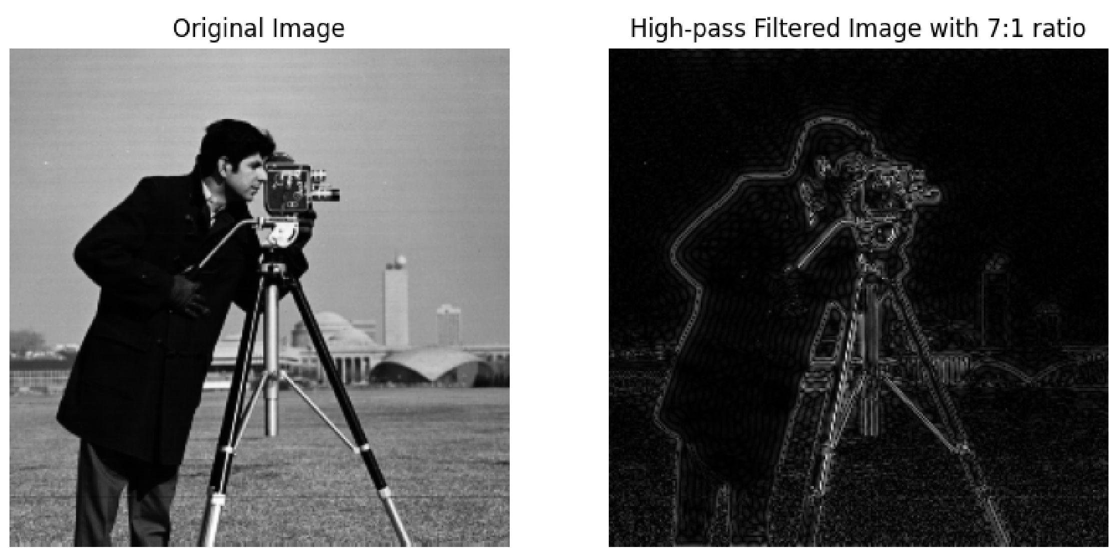
[234]:
```
# STEP 3 Apply an ideal low-pass filter in the Fourier domain

#----------------------------------------------------------------
# 7:1 ideal low-pass filter implementation in the Fourier domain
#----------------------------------------------------------------

def low_pass_filter(img, cutoff_ratio):
    # Fourier Transform
    f_img = ft.fft2(img)
    f_shift = ft.fftshift(f_img)

    # Mask for low-pass filter
    rows, cols = img.shape
    crow, ccol = rows//2, cols//2

    # Cutoff frequency
    cutoff_frequency = min(rows, cols) / cutoff_ratio
```

```
    # Mask
    mask = np.zeros((rows, cols), dtype=np.float64)
    mask[np.sqrt((np.arange(rows) - crow)**2+(np.arange(cols)[:,np.
↪newaxis]-ccol)**2) <=cutoff_frequency] = 1
    f_shift_filtered = f_shift * mask

    # Inverse Fourier Transform
    img_filtered = ft.ifftshift(f_shift_filtered)
    img_filtered = ft.ifft2(img_filtered)
    img_filtered = np.abs(img_filtered)

    return img_filtered

# Read the image
img = io.imread(image_path, as_gray=True)

# Low-pass filter (cutoff ratio = 7:1)
cutoff_ratio = 7
filtered_img = low_pass_filter(img, cutoff_ratio)

#----------------------------------------------------------------
# Display original and filtered image
#----------------------------------------------------------------
plt.figure(figsize=(10, 5))

plt.subplot(1, 2, 1)
plt.imshow(img,cmap='gray')
plt.title('Original Image')
plt.axis('off')

plt.subplot(1, 2, 2)
plt.imshow(filtered_img,cmap='gray')
plt.title('Low-pass Filtered Image with 7:1 ratio')
plt.axis('off')
plt.show()
```



[235]:
```
# STEP 4 Apply an ideal high-pass filter in the Fourier domain

#-----------------------------------------------------------------
# 7:1 ideal high-pass filter implementation in the Fourier domain
#-----------------------------------------------------------------

def high_pass_filter(img, cutoff_ratio):
    # Fourier Transform
    f_img = ft.fft2(img)
    f_shift = ft.fftshift(f_img)

    # Mask for the high-pass filter
    rows, cols = img.shape
    crow, ccol = rows//2, cols//2

    # Cutoff frequency
    cutoff_frequency = min(rows,  cols) / cutoff_ratio

    # Mask
    mask = np.ones((rows, cols), dtype=np.float64)
    mask[np.sqrt((np.arange(rows) - crow)**2 + (np.arange(cols)[:,np.newaxis] - ccol)**2) <= cutoff_frequency] = 0
    f_shift_filtered = f_shift * mask

    # Inverse Fourier Transform
    img_filtered = ft.ifftshift(f_shift_filtered)
    img_filtered = ft.ifft2(img_filtered)
    img_filtered = np.abs(img_filtered)
```

```python
    return img_filtered

# Read the image
img = io.imread(image_path, as_gray=True)

# High-pass filter (cutoff ratio = 7:1)
cutoff_ratio = 7
filtered_img = high_pass_filter(img, cutoff_ratio)

#----------------------------------------------------------------
# Display original and filtered image
#----------------------------------------------------------------

plt.figure(figsize=(10, 5))

plt.subplot(1,2,1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(filtered_img, cmap='gray')
plt.title('High-pass Filtered Image with 7:1 ratio')
plt.axis('off')

plt.show()
```

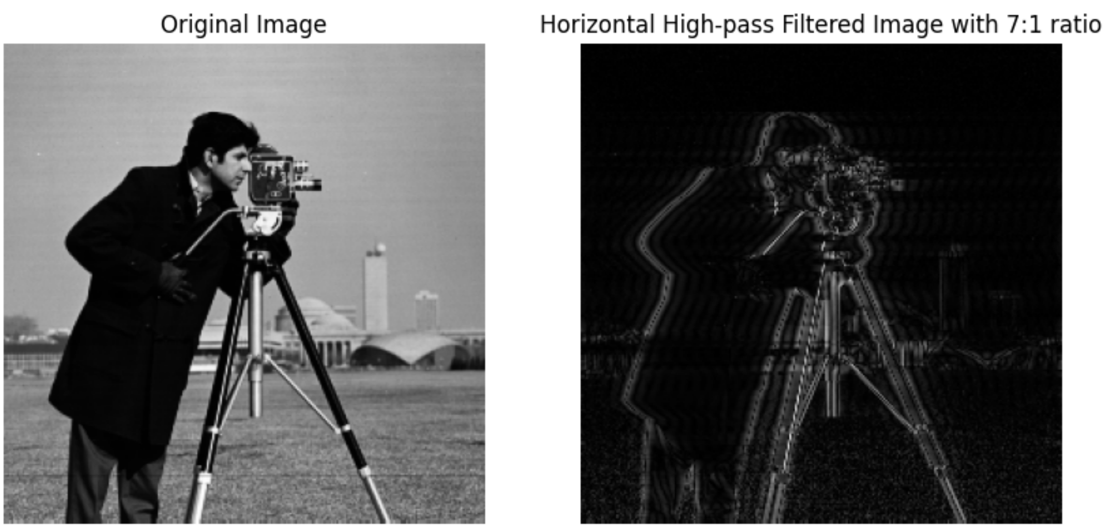


[236]:
```python
# STEP 5 Apply an ideal horizontal low-pass filter in the Fourier domain

#----------------------------------------------------------------
# 7:1 ideal horizontal low-pass filter implementation in the Fourier domain
#----------------------------------------------------------------

def horizontal_low_pass_filter(img, cutoff_ratio):
    # Fourier Transform
    f_img = ft.fft2(img)
    f_shift = ft.fftshift(f_img)

    # Mask for the horizontal low-pass filter
    rows, cols = img.shape
    crow, ccol = rows//2, cols//2

    # Cutoff frequency
    cutoff_frequency = cols/cutoff_ratio

    # Create mask
    mask = np.zeros((rows, cols), dtype=np.float64)
    mask[:, (ccol-int(cutoff_frequency/2)):(ccol+int(cutoff_frequency/2))] = 1

    # Apply the mask
    f_shift_filtered = f_shift * mask

    # Perform Inverse Fourier Transform
    img_filtered = ft.ifftshift(f_shift_filtered)
    img_filtered = ft.ifft2(img_filtered)
    img_filtered = np.abs(img_filtered)

    return img_filtered

# Read the image
img = io.imread(image_path, as_gray=True)

# Horizontal low-pass filter (cutoff ratio of 7:1)
cutoff_ratio = 7
filtered_img = horizontal_low_pass_filter(img,cutoff_ratio)

#----------------------------------------------------------------
# Display original and filtered images
#----------------------------------------------------------------
plt.figure(figsize=(10, 5))

plt.subplot(1, 2, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
```

```python
plt.axis('off')

plt.subplot(1, 2, 2)
plt.imshow(filtered_img, cmap='gray')
plt.title('Horizontal Low-pass Filtered Image with 7:1 ratio')
plt.axis('off')

plt.show()
```

[237]:
```python
# STEP 6 Apply an ideal horizontal high-pass filter in the Fourier domain

#----------------------------------------------------------------
# 7:1 ideal horizontal high-pass filter implementation in the Fourier domain
#----------------------------------------------------------------

def horizontal_high_pass_filter(img, cutoff_ratio):
    # Fourier Transform
    f_img = ft.fft2(img)
    f_shift = ft.fftshift(f_img)

    # Mask for horizontal high-pass filter
    rows, cols = img.shape
    crow, ccol = rows//2, cols//2

    # Cutoff frequency
    cutoff_frequency = cols/cutoff_ratio

    # Mask
    mask = np.ones((rows, cols), dtype=np.float64)
```

```
    mask[:, (ccol - int(cutoff_frequency / 2)):(ccol + int(cutoff_frequency /␣
↪2))] = 0
    f_shift_filtered = f_shift*mask

    # Inverse Fourier Transform
    img_filtered = ft.ifftshift(f_shift_filtered)
    img_filtered = ft.ifft2(img_filtered)
    img_filtered = np.abs(img_filtered)

    return img_filtered

# Read the image
img = io.imread(image_path, as_gray=True)

# Horizontal high-pass filter (cutoff ratio = 7:1)
cutoff_ratio = 7
filtered_img = horizontal_high_pass_filter(img, cutoff_ratio)

#-----------------------------------------------------------------
# Display original and filtered image
#-----------------------------------------------------------------

plt.figure(figsize=(10,5))

plt.subplot(1,2,1)
plt.imshow(img, cmap='gray')
plt.title('Original Image')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(filtered_img, cmap='gray')
plt.title('Horizontal High-pass Filtered Image with 7:1 ratio')
plt.axis('off')

plt.show()
```



STEP 7 Comments on the results

ADD YOUR COMMENTS HERE

Comment on the results in Steps 3-6. Do they look as expected? Are there any ringing artefacts observed?

Using lowpass filter for the imag, deleted high valued frequency components. By using low-pass filter, we obtained the overall image. However while losing sharp edges (high frequency component).

There's only sharp edges in high pass filtered images, while we can see the ringing in lowpass filtered ones.

[ ]:
```
!sudo apt-get install texlive-xetex texlive-fonts-recommended␣
↪texlive-plain-generic
```

[239]:
```
!jupyter nbconvert --to pdf '/content/drive/MyDrive/Colab Notebooks/
↪Prj04_Fourier_S025235.ipynb'
```

```
[NbConvertApp] WARNING | pattern '/content/drive/MyDrive/Colab
Notebooks/Prj04_Fourier_S025235.ipynb' matched no files
This application is used to convert notebook files (*.ipynb)
        to various other formats.

        WARNING: THE COMMANDLINE INTERFACE MAY CHANGE IN FUTURE RELEASES.

Options
=======
The options below are convenience aliases to configurable class-options,
as listed in the "Equivalent to" description-line of the aliases.
To see all configurable class-options for some <cmd>, use:
    <cmd> --help-all
```

```
--debug
    set log level to logging.DEBUG (maximize logging output)
    Equivalent to: [--Application.log_level=10]
--show-config
    Show the application's configuration (human-readable format)
    Equivalent to: [--Application.show_config=True]
--show-config-json
    Show the application's configuration (json format)
    Equivalent to: [--Application.show_config_json=True]
--generate-config
    generate default config file
    Equivalent to: [--JupyterApp.generate_config=True]
-y
    Answer yes to any questions instead of prompting.
    Equivalent to: [--JupyterApp.answer_yes=True]
--execute
    Execute the notebook prior to export.
    Equivalent to: [--ExecutePreprocessor.enabled=True]
--allow-errors
    Continue notebook execution even if one of the cells throws an error and
include the error message in the cell output (the default behaviour is to abort
conversion). This flag is only relevant if '--execute' was specified, too.
    Equivalent to: [--ExecutePreprocessor.allow_errors=True]
--stdin
    read a single notebook file from stdin. Write the resulting notebook with
default basename 'notebook.*'
    Equivalent to: [--NbConvertApp.from_stdin=True]
--stdout
    Write notebook output to stdout instead of files.
    Equivalent to: [--NbConvertApp.writer_class=StdoutWriter]
--inplace
    Run nbconvert in place, overwriting the existing notebook (only
            relevant when converting to notebook format)
    Equivalent to: [--NbConvertApp.use_output_suffix=False
--NbConvertApp.export_format=notebook --FilesWriter.build_directory=]
--clear-output
    Clear output of current file and save in place,
            overwriting the existing notebook.
    Equivalent to: [--NbConvertApp.use_output_suffix=False
--NbConvertApp.export_format=notebook --FilesWriter.build_directory=
--ClearOutputPreprocessor.enabled=True]
--no-prompt
    Exclude input and output prompts from converted document.
    Equivalent to: [--TemplateExporter.exclude_input_prompt=True
--TemplateExporter.exclude_output_prompt=True]
--no-input
    Exclude input cells and output prompts from converted document.
```

```
            This mode is ideal for generating code-free reports.
    Equivalent to: [--TemplateExporter.exclude_output_prompt=True
--TemplateExporter.exclude_input=True
--TemplateExporter.exclude_input_prompt=True]
--allow-chromium-download
    Whether to allow downloading chromium if no suitable version is found on the
system.
    Equivalent to: [--WebPDFExporter.allow_chromium_download=True]
--disable-chromium-sandbox
    Disable chromium security sandbox when converting to PDF..
    Equivalent to: [--WebPDFExporter.disable_sandbox=True]
--show-input
    Shows code input. This flag is only useful for dejavu users.
    Equivalent to: [--TemplateExporter.exclude_input=False]
--embed-images
    Embed the images as base64 dataurls in the output. This flag is only useful
for the HTML/WebPDF/Slides exports.
    Equivalent to: [--HTMLExporter.embed_images=True]
--sanitize-html
    Whether the HTML in Markdown cells and cell outputs should be sanitized..
    Equivalent to: [--HTMLExporter.sanitize_html=True]
--log-level=<Enum>
    Set the log level by value or name.
    Choices: any of [0, 10, 20, 30, 40, 50, 'DEBUG', 'INFO', 'WARN', 'ERROR',
'CRITICAL']
    Default: 30
    Equivalent to: [--Application.log_level]
--config=<Unicode>
    Full path of a config file.
    Default: ''
    Equivalent to: [--JupyterApp.config_file]
--to=<Unicode>
    The export format to be used, either one of the built-in formats
            ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook',
'pdf', 'python', 'rst', 'script', 'slides', 'webpdf']
            or a dotted object name that represents the import path for an
            ``Exporter`` class
    Default: ''
    Equivalent to: [--NbConvertApp.export_format]
--template=<Unicode>
    Name of the template to use
    Default: ''
    Equivalent to: [--TemplateExporter.template_name]
--template-file=<Unicode>
    Name of the template file to use
    Default: None
    Equivalent to: [--TemplateExporter.template_file]
--theme=<Unicode>
```

```
    Template specific theme(e.g. the name of a JupyterLab CSS theme distributed
    as prebuilt extension for the lab template)
    Default: 'light'
    Equivalent to: [--HTMLExporter.theme]
--sanitize_html=<Bool>
    Whether the HTML in Markdown cells and cell outputs should be sanitized.This
    should be set to True by nbviewer or similar tools.
    Default: False
    Equivalent to: [--HTMLExporter.sanitize_html]
--writer=<DottedObjectName>
    Writer class used to write the
                                    results of the conversion
    Default: 'FilesWriter'
    Equivalent to: [--NbConvertApp.writer_class]
--post=<DottedOrNone>
    PostProcessor class used to write the
                                    results of the conversion
    Default: ''
    Equivalent to: [--NbConvertApp.postprocessor_class]
--output=<Unicode>
    overwrite base name use for output files.
                can only be used when converting one notebook at a time.
    Default: ''
    Equivalent to: [--NbConvertApp.output_base]
--output-dir=<Unicode>
    Directory to write output(s) to. Defaults
                                  to output to the directory of each notebook.
To recover
                                  previous default behaviour (outputting to the
current
                                  working directory) use . as the flag value.
    Default: ''
    Equivalent to: [--FilesWriter.build_directory]
--reveal-prefix=<Unicode>
    The URL prefix for reveal.js (version 3.x).
            This defaults to the reveal CDN, but can be any url pointing to a
copy
            of reveal.js.
            For speaker notes to work, this must be a relative path to a local
            copy of reveal.js: e.g., "reveal.js".
            If a relative path is given, it must be a subdirectory of the
            current directory (from which the server is run).
            See the usage documentation
            (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-js-
html-slideshow)
            for more details.
    Default: ''
    Equivalent to: [--SlidesExporter.reveal_url_prefix]
```

```
--nbformat=<Enum>
    The nbformat version to write.
            Use this to downgrade notebooks.
    Choices: any of [1, 2, 3, 4]
    Default: 4
    Equivalent to: [--NotebookExporter.nbformat_version]

Examples
--------

    The simplest way to use nbconvert is

            > jupyter nbconvert mynotebook.ipynb --to html

            Options include ['asciidoc', 'custom', 'html', 'latex', 'markdown',
'notebook', 'pdf', 'python', 'rst', 'script', 'slides', 'webpdf'].

            > jupyter nbconvert --to latex mynotebook.ipynb

            Both HTML and LaTeX support multiple output templates. LaTeX
includes
            'base', 'article' and 'report'.  HTML includes 'basic', 'lab' and
            'classic'. You can specify the flavor of the format used.

            > jupyter nbconvert --to html --template lab mynotebook.ipynb

            You can also pipe the output to stdout, rather than a file

            > jupyter nbconvert mynotebook.ipynb --stdout

            PDF is generated via latex

            > jupyter nbconvert mynotebook.ipynb --to pdf

            You can get (and serve) a Reveal.js-powered slideshow

            > jupyter nbconvert myslides.ipynb --to slides --post serve

            Multiple notebooks can be given at the command line in a couple of
            different ways:

            > jupyter nbconvert notebook*.ipynb
            > jupyter nbconvert notebook1.ipynb notebook2.ipynb

            or you can specify the notebooks list in a config file, containing::

                c.NbConvertApp.notebooks = ["my_notebook.ipynb"]
```

```
            > jupyter nbconvert --config mycfg.py

To see all available configurables, use `--help-all`.
```

[ ]:

```

```