From PCP to ZK-STARK

Eli Ben-Sasson|Chief Scientist (East) | February 2019

Overview

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Cryptoproofs

2

PCP, IOP
STIK, STARK
FRI

3

Concrete
Questions

Proofs of Computational Integrity (CI)

INTEGRITY

The quality of being honest

(Dictionary)

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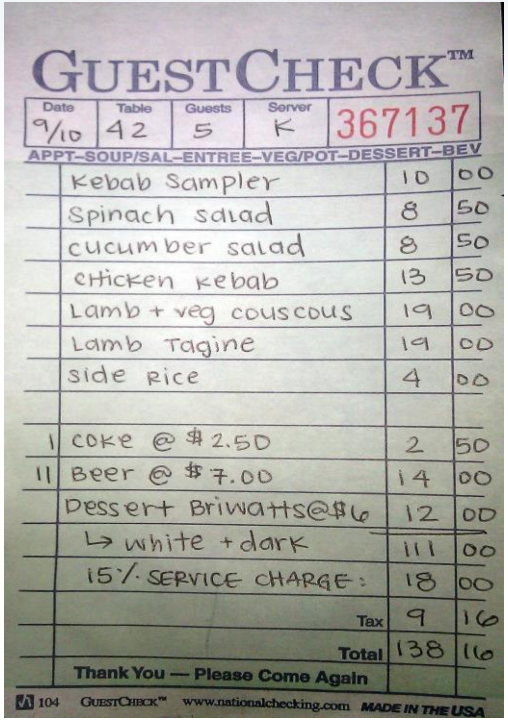
CI Statement: total=$138.16

- Prover: Party producing proof (here: Grocer)
- Verifier: Party checking proof (here: Customer)



- Generic CI statement: Computation *C*, with public input *x* and auxiliary private input *w*, reached output *y* in *T* steps

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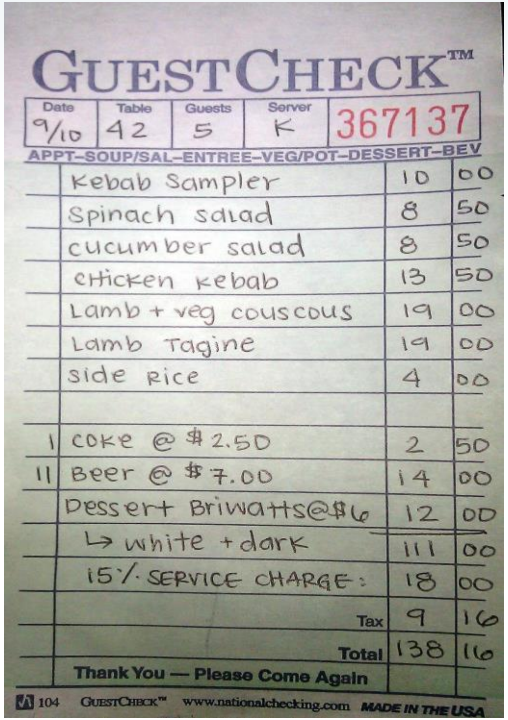
Grocery receipts are proofs of computational integrity

- Verification via naive re-execution of computation
- Proof is (i) deterministic, (ii) error free, (iii) one-shot (non-interactive)



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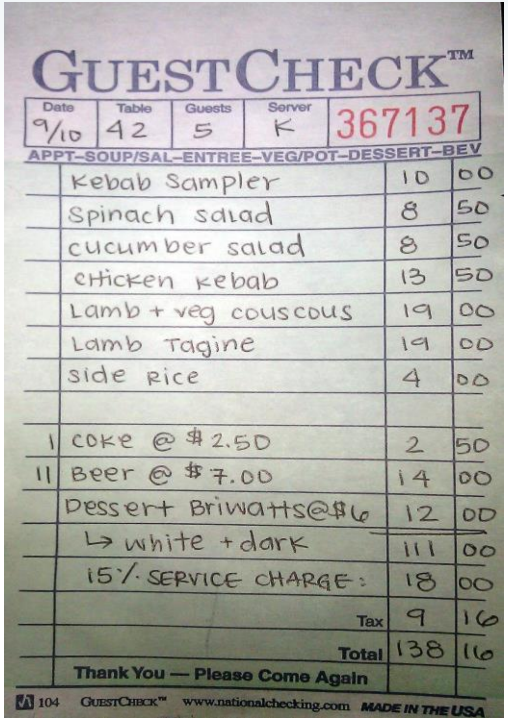
- Modern CI proofs have (i) randomness, (ii) small error, (iii) interaction; in return, offer many benefits…



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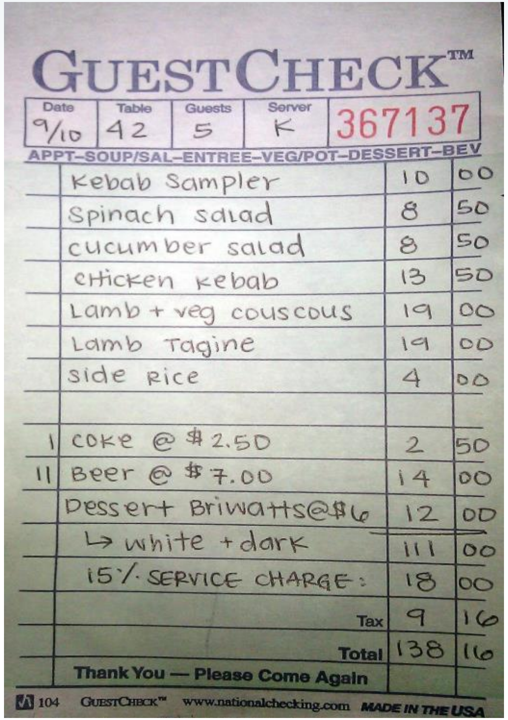
Modern Computational Integrity proofs [GMR85]

IP, ZK, CS, PCP, MIP, IPCP, LPCP, PCIP, IOP, …



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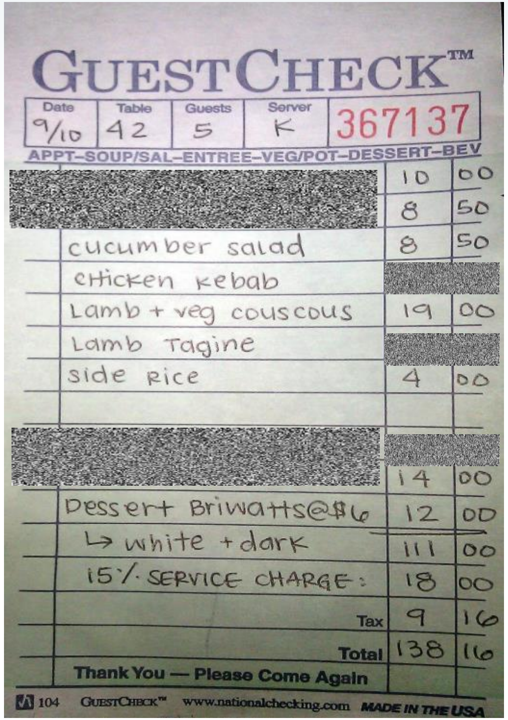

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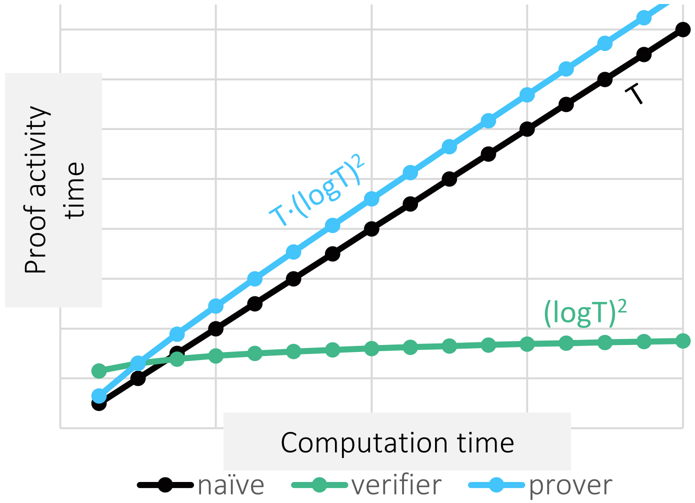

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Privacy examples

- **Zcash shielded transaction: shield payer, payee and payment amount**
- **Paid taxes on all my 2018 transactions, without revealing them**
- **My crypto exchange is in the black, without showing my positions**
- …

Modern Computational Integrity proofs [GMR85]

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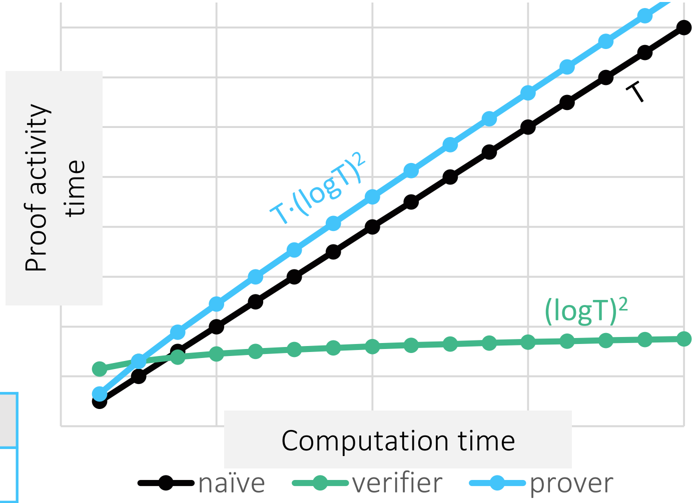

Scalability examples

Naïve computation time	Verifier time	Prover time
Mega (2^{20})	400	400·Mega
Giga (2^{30})	900	900·Giga
Tera (2^{40})	1600	1600·Tera
Peta (2^{50})	2500	2500·Peta

Modern Computational Integrity proofs [GMR85]

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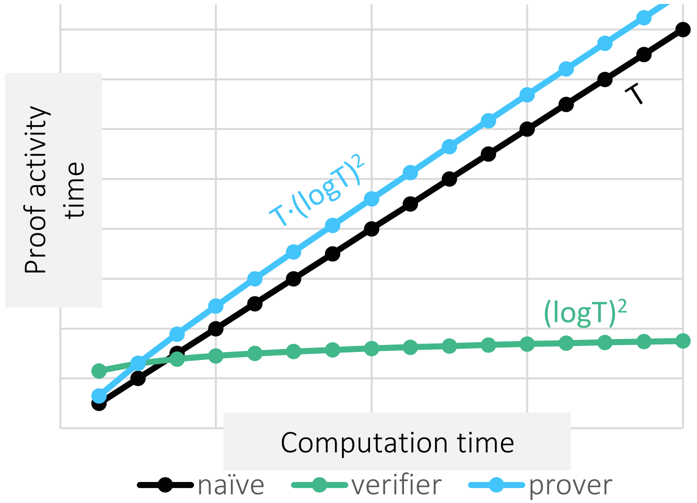

Scalability examples

Proof scalability can solve blockchain scalability problems

- Suppose computing latest Bitcoin state takes 1Peta (2^{50}) steps
- A single prover spends $2500 \cdot 2^{50}$ steps, posts proof
- All other nodes verify exponentially faster, in 2500 steps

Modern Computational Integrity proofs [GMR85]

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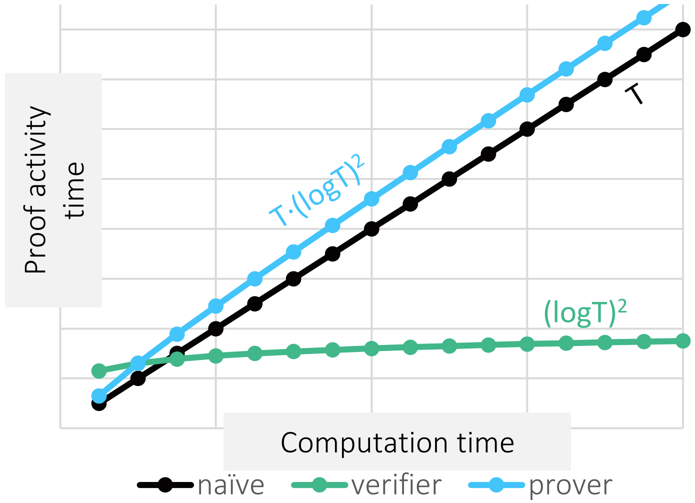

tl;dr my research: PCP-based proofs, concrete efficiency

- 1995: ZK w/ scalable verifier was "galactic algorithm"
- 2018: scalable ZK realized in code for meaningful computation
- using *scalable PCPs* and *Interactive Oracle Proofs (IOPs)*

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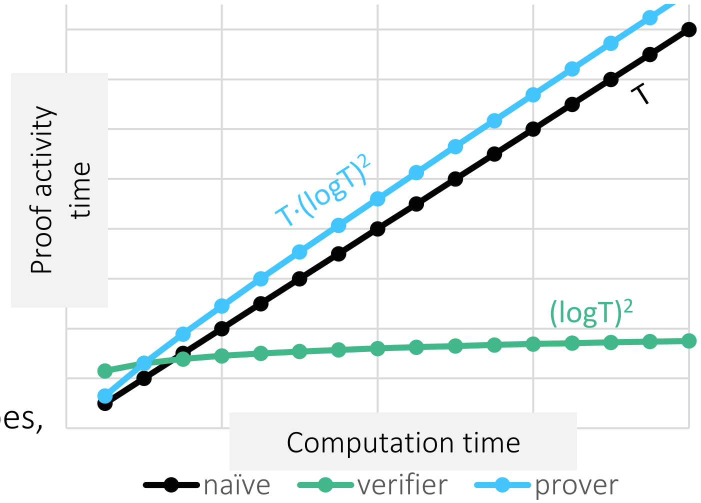
Co-authors [2001-2019]: Iddo Bentov, Alessandro Chiesa, Michael Forbes, Ariel Gabizon, Daniel Genkin, Oded Goldreich, Lior Goldberg, Tom Gur, Matan Hamilis, Prahladh Harsha, Yinon Horesh, Yohay Kaplan, Swastik Kopparty, Or Meir, Evgenya Pergament, Michael Riabzev, Shubhangi Saraf, Mark Silberstein, Hennig Stichtenoth, Nicholas Spooner, Madhu Sudan, Eran Tromer, Salil Vadhan, Madars Virza, Avi Wigderson.



Many flavors of proof systems

Variety of theoretical constructions

(past 30 yrs)

PCP based, linear PCPs, elliptic curve+pairing based succinct NIZKs, proofs for muggles, quadratic span/arithmetic programs (QAP/QSP), interactive oracle proofs (IOP), …

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PCP based, linear PCPs, elliptic curve+pairing based succinct NIZKs, proofs for muggles, quadratic span/arithmetic programs (QAP/QSP), interactive oracle proofs (IOP), …

…and implementations

(past 5 yrs)

Pinocchio, libsnark, Zcash, Pepper, Buffet, ZKboo, Ligero, Bulletproofs, Hyrax, libstark, Aurora, ...

See zkp.science

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zk-STARK definition [BBHR18]

An argument system is a zk-STARK if it satisfies:

zk — **zero knowledge:** private inputs are shielded

S — **Scalable:** proofs for CI of computation lasting T cycles are
- generated in roughly T cycles (quasi-linear in T), and
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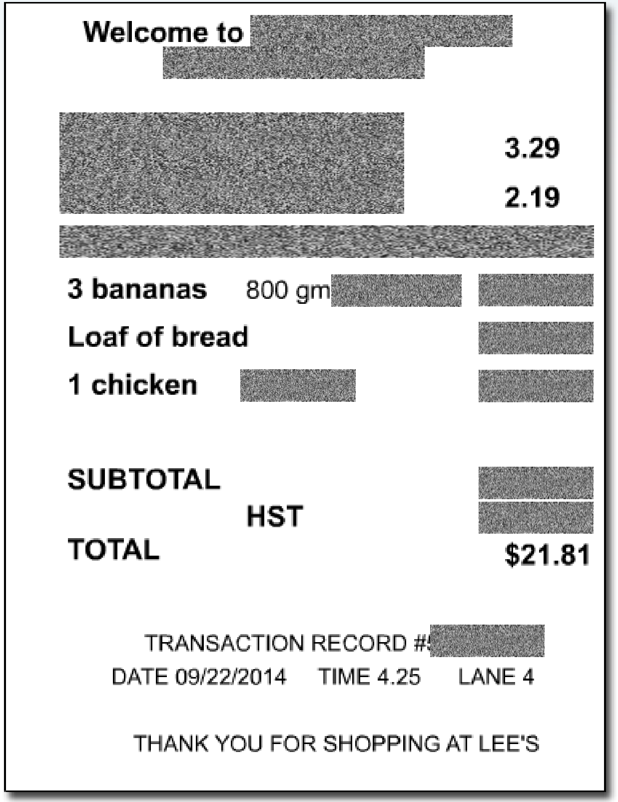
T — **Transparent:** verifier messages are random coins; no trusted setup

AR K — **ARgument of Knowledge:** proof can be generated only by party knowing private input (formally: an efficient procedure can extract the secrets from a prover)


Welcome to [redacted]

[redacted] 3.29
2.19
[redacted]

3 bananas 800 gm [redacted]
Loaf of bread [redacted]
1 chicken [redacted]

SUBTOTAL [redacted]
HST
TOTAL $21.81

TRANSACTION RECORD #[redacted]
DATE 09/22/2014 TIME 4.25 LANE 4

THANK YOU FOR SHOPPING AT LEE'S

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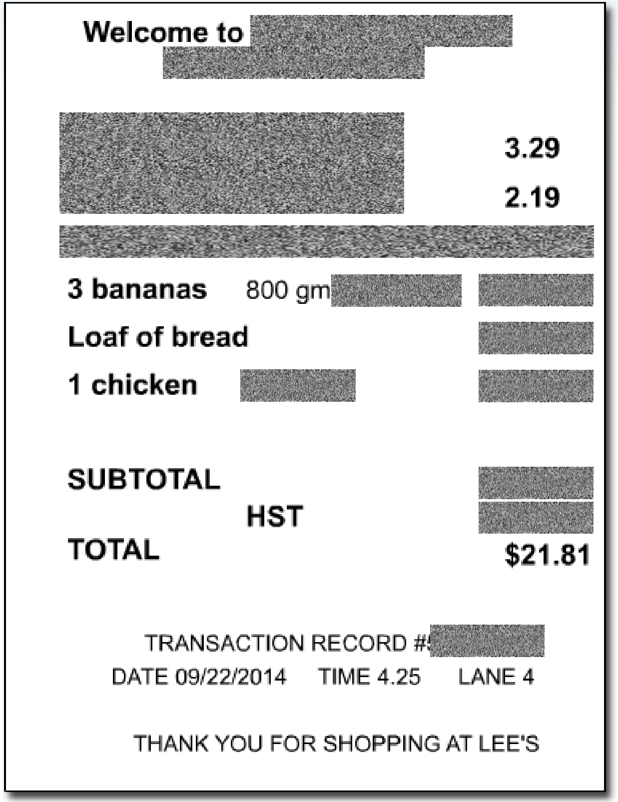
- **STARKs may be interactive** (use blockchain as source of transparent randomness), gives shorter & safer proofs
- 1st STARK implementation: SCI-POC [BCG+16]; 1st zk-STARK: libstark [BBHR18]



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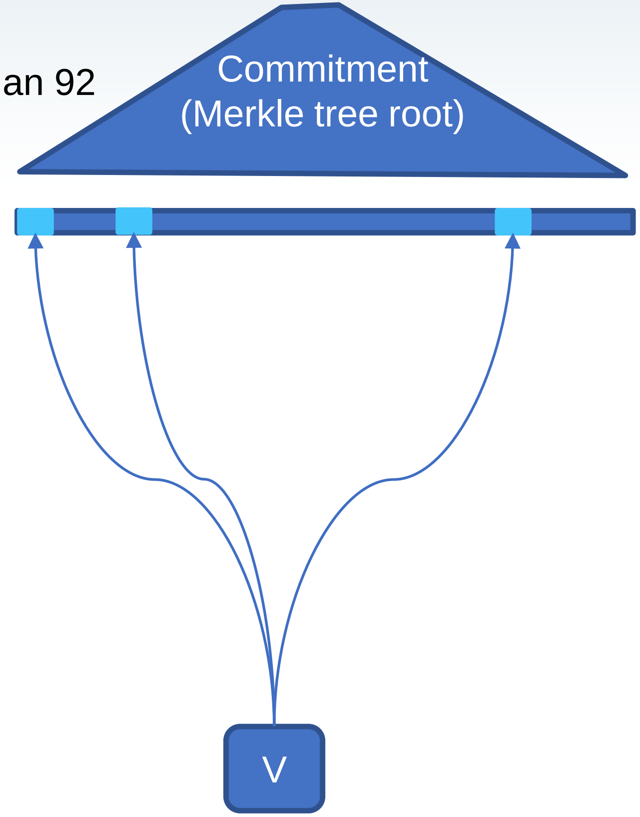
PCPs and polylogarithmic verification

Kilian 92



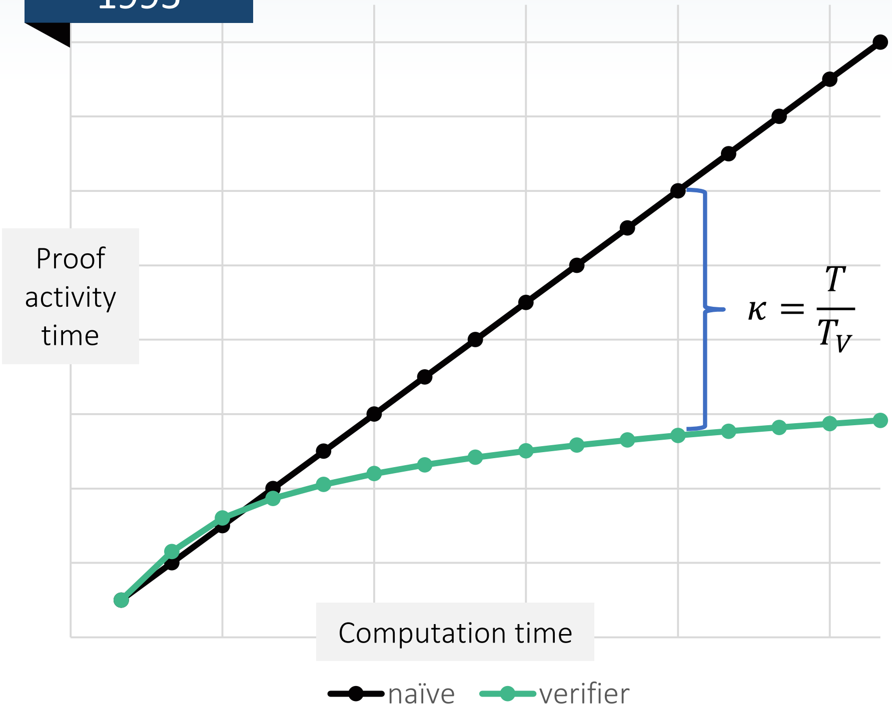
"In <u>*this setup*</u>, a single reliable PC can monitor the operation of a herd of supercomputers working with possibly extremely powerful but unreliable software and untested hardware" [Babai, Fortnow, Levin, 1991]

Setup: to prove $x \in L$ for some $L \in NTIME(T(n))$

- Verifier has oracle access to PCP π,
- Verifier runs in time poly($n + \log(T(n))$)
- If $x \in L$ then exists π accepted w.p. 1
- If $x \in L$ then all π rejected w.p. > ½

PCP and scalability [BFL, BFLS, AS, AL, K, M 1991-4]

1995



Naive:

Verification = proving

PCP:

Poly-logarithmic verification

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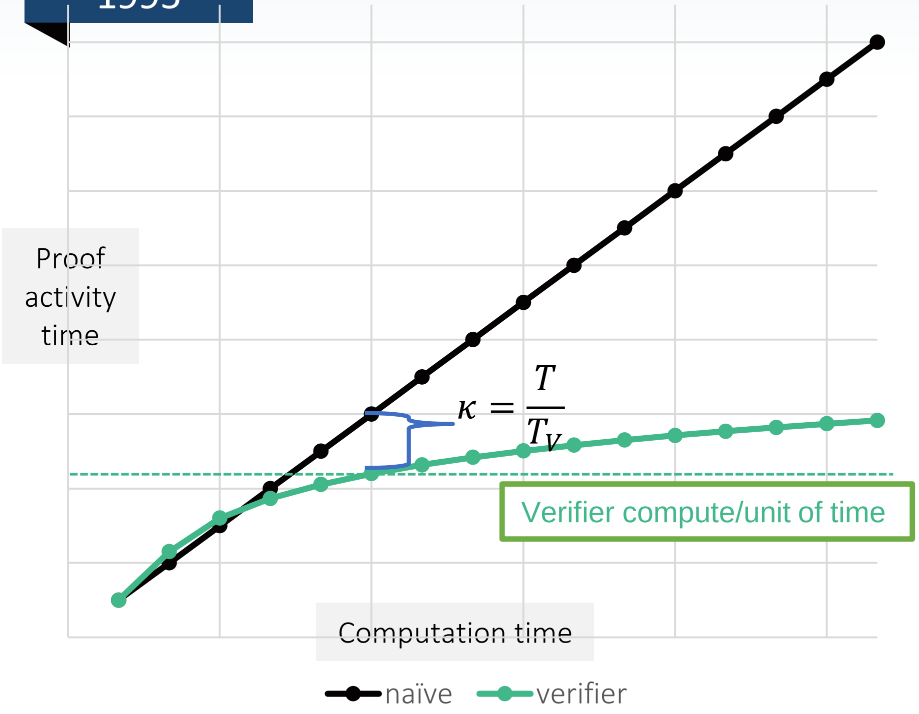

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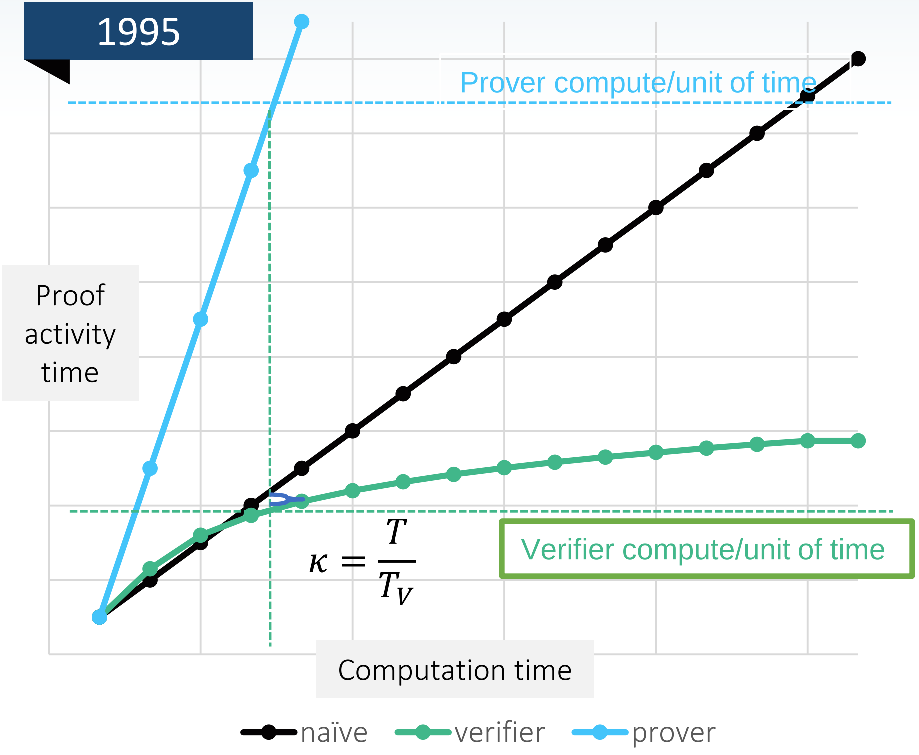

Naive:

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PCP:

Polynomial proving time

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PCP and scalability [BS, BGHSV, D, M, 2003-8]

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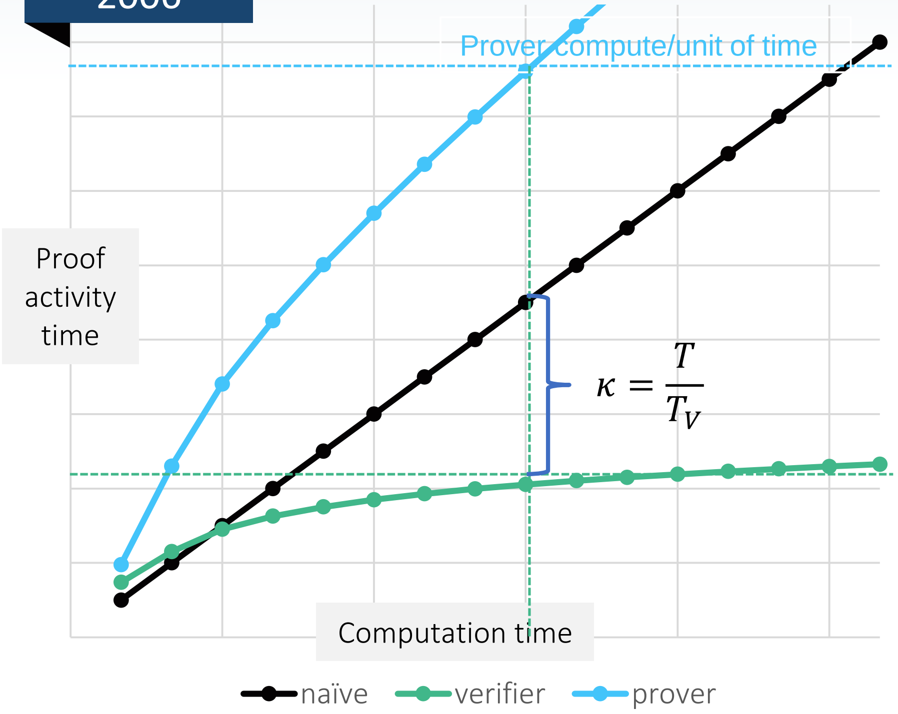

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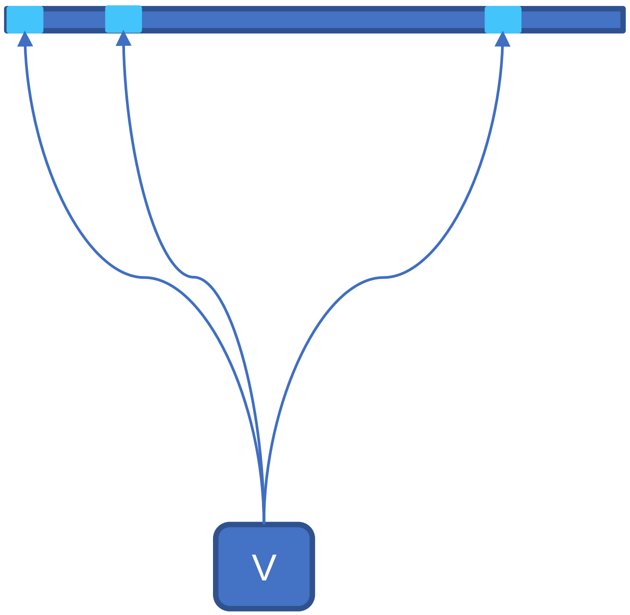

Interactive Oracle Proofs (IOP) [RRR16, BCS16]

IOP setup: to prove $x \in L$ for some $L \in NTIME(T(n))$

- Verifier has oracle access to 1st oracle π_0,
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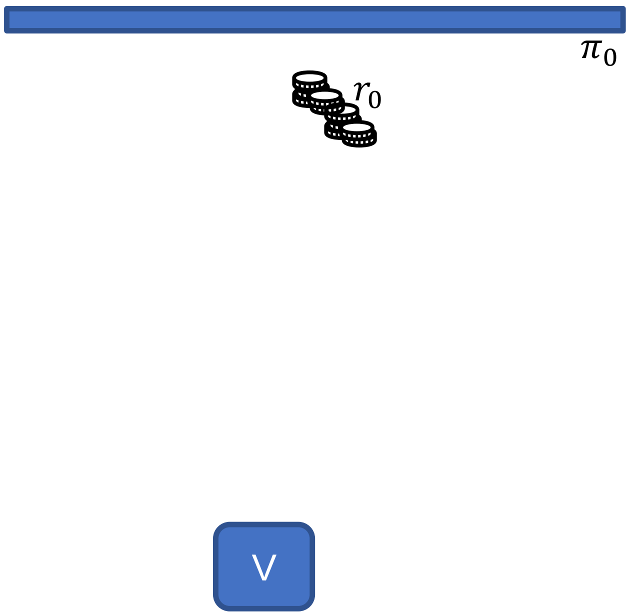

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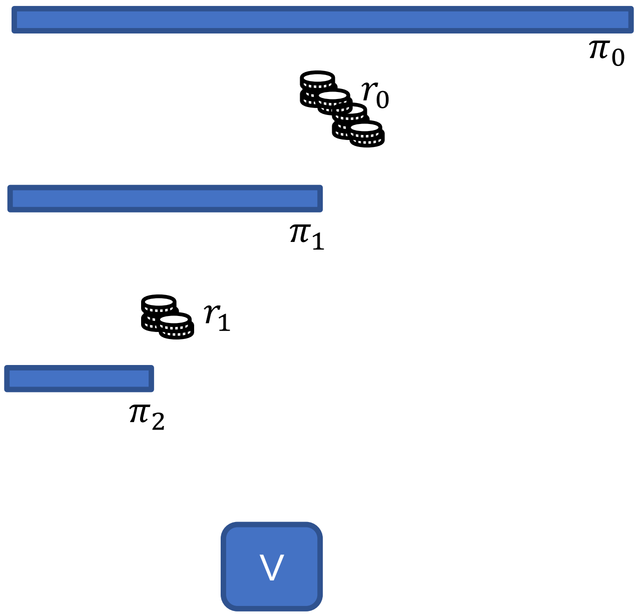

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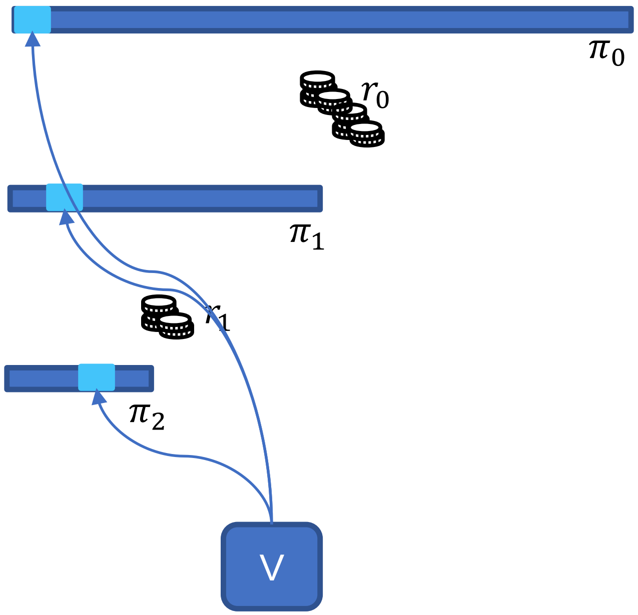

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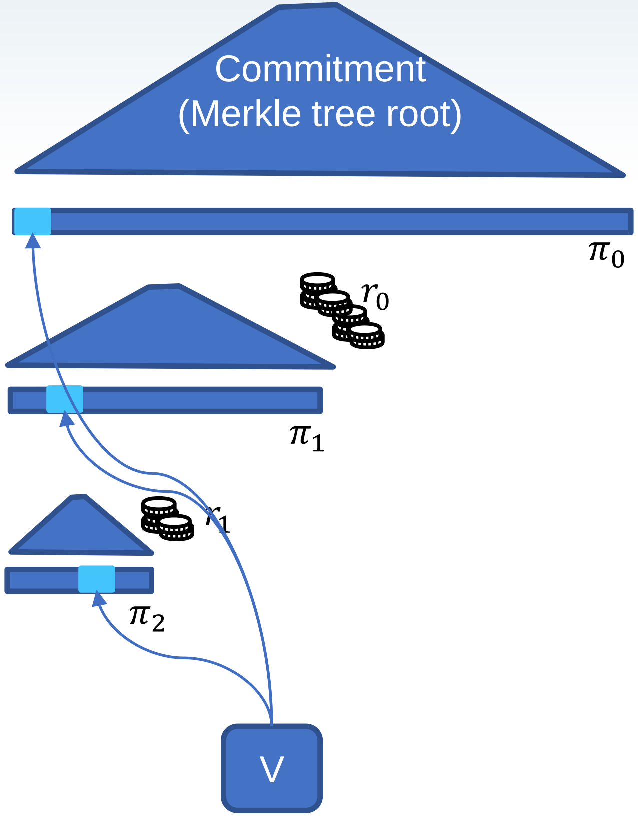

IOP tl;dr

With IOPs, can prove results that are yet unknown in PCP model

- 2-round IOP with perfect ZK, non-adaptive verifier for NP [BCGV16]
- Doubly-efficient, constant round, Interactive Proofs [RRR16]
- O(1)-round IOP with linear bit-length proofs and constant query comp [BCGRS17]
- Proximity protocols for Reed-Solomon with linear arithmetic complexity and logarithmic query complexity [BBHR18]
- …
- Concretely efficient ZK-STARKs [BBC+16, BBHR18, …]

PCP and scalability [BS, BGHSV, D, M, 2003-8]

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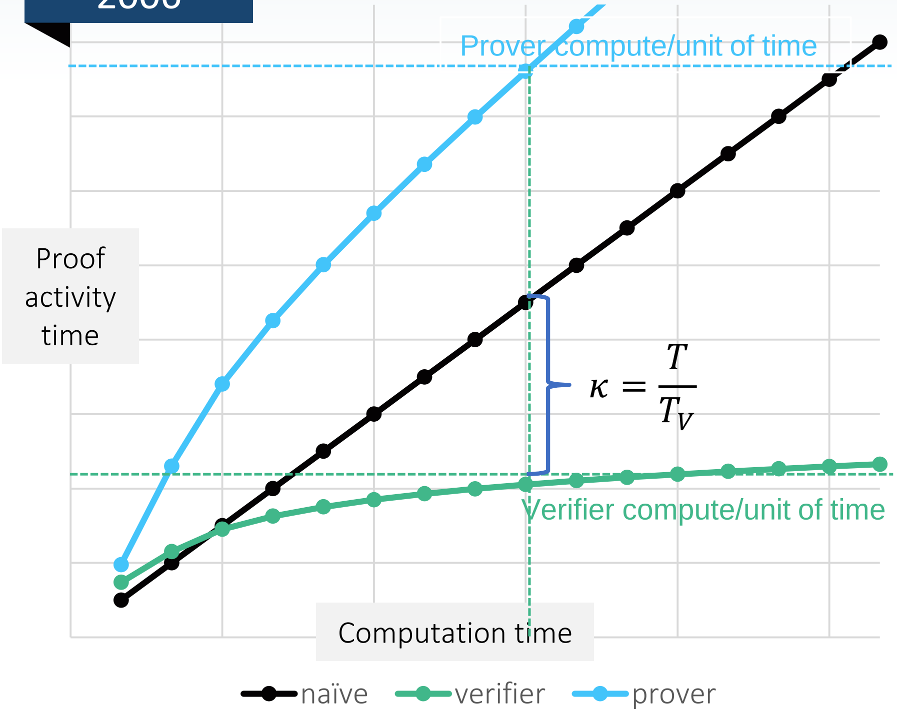
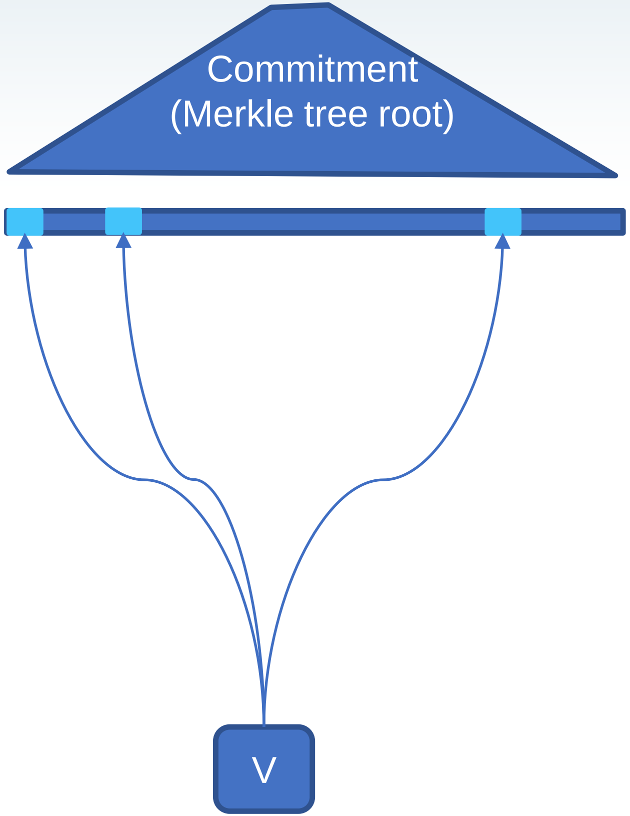
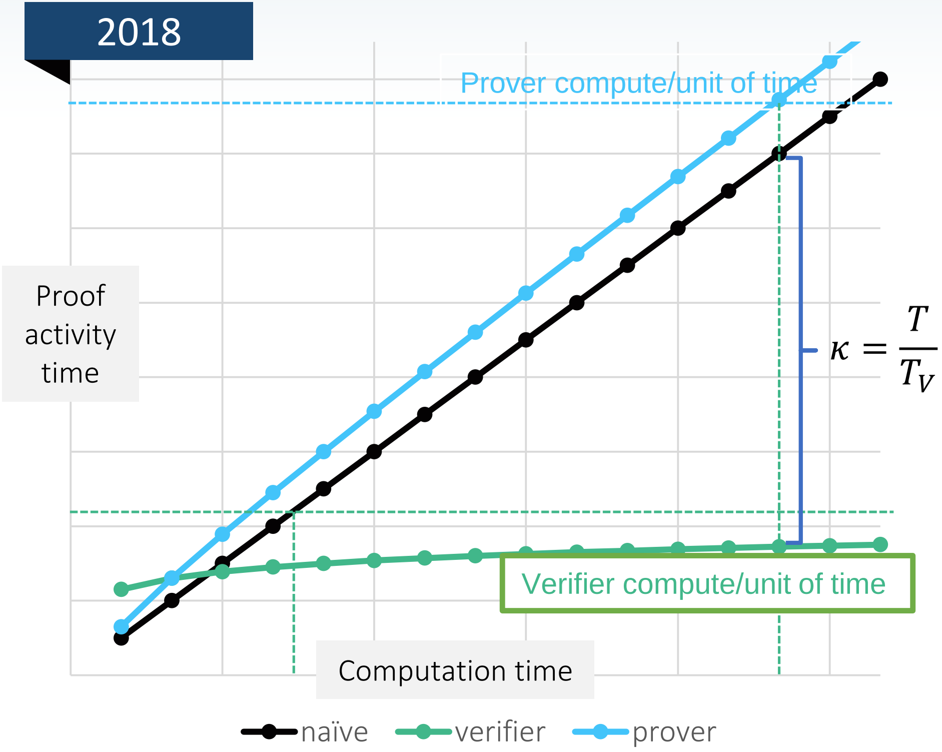
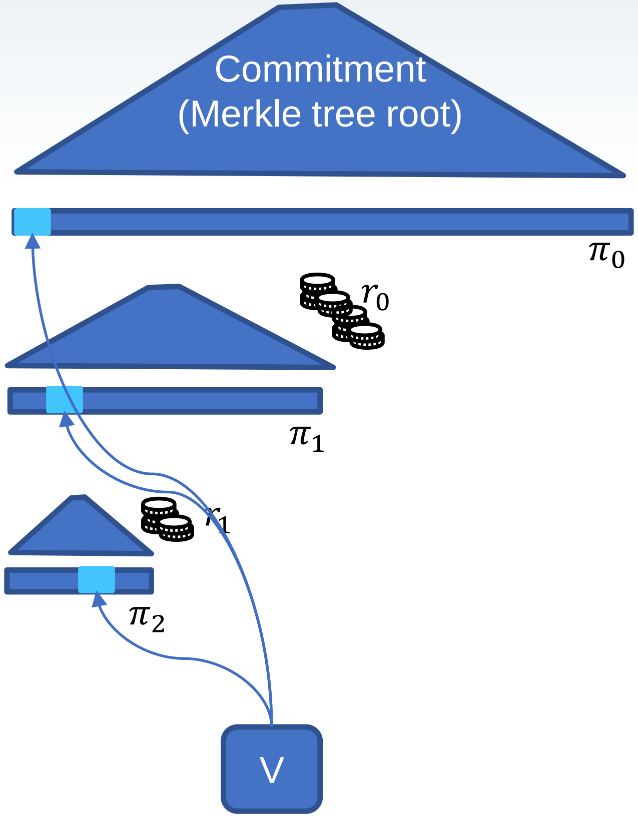




IOP and Scalability [BBC+16, BBHR18]

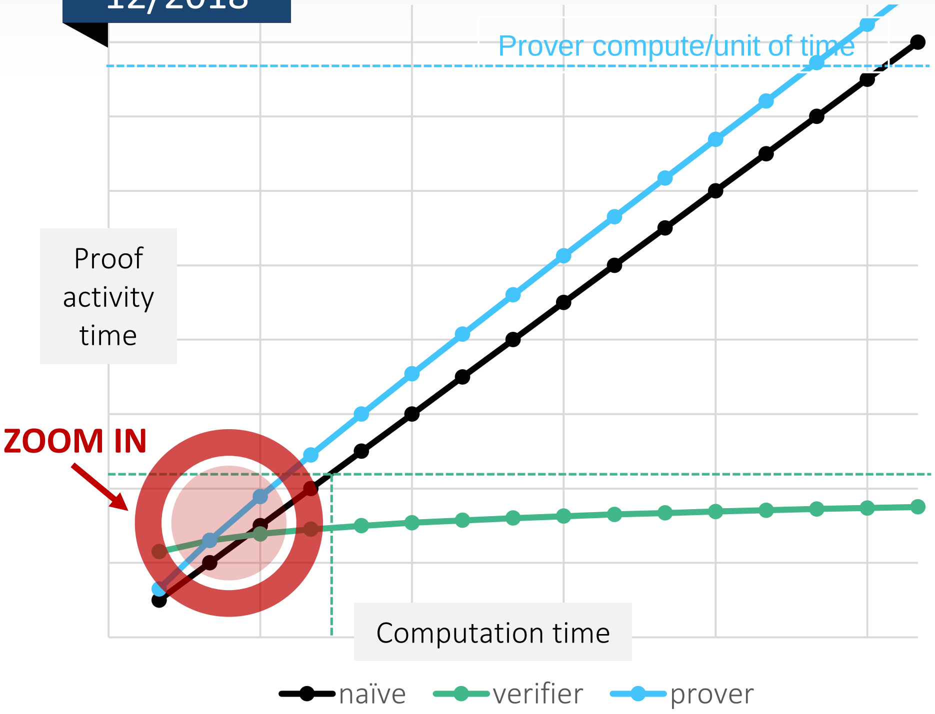


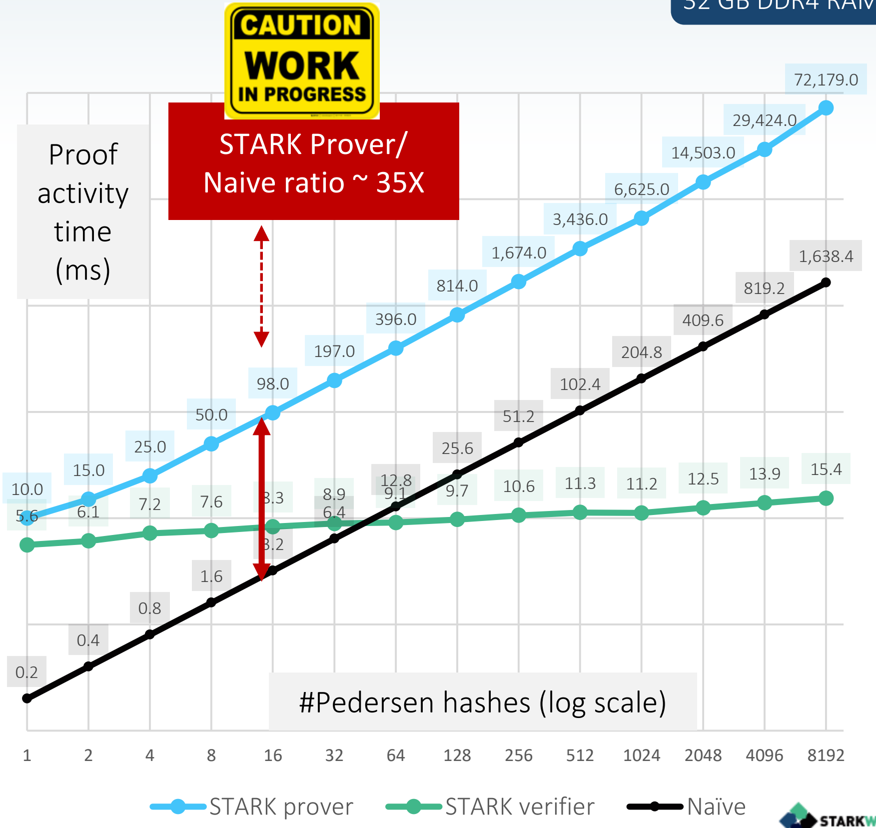


IOP and Scalability

T480 laptop
i7-8550U CPU
@ 1.80GHz
Quad-core
32 GB DDR4 RAM

12/2018





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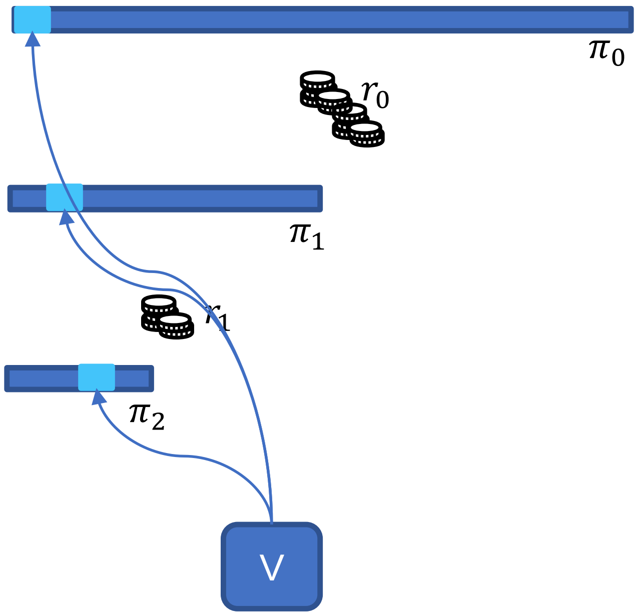
Scalable Transparent IOP of Knowledge (STIK) [BBHR18]

Definition:

An IOP for $L \in NTIME(T(n))$ is said to be:

- **Scalable** if both of following hold:
  - Proving time $T_P = \tilde{O}(T(n)) + poly(n) = T(n) \cdot \log^{O(1)} T(n) + poly(n)$
  - Verifying time $T_V = \text{poly} \log T(n)$
- **Transparent** if all verifier messages are public random coins (Arthur-Merlin protocols)
- **IOP of Knowledge** if exists Extractor E that extracts in time $poly\ T(n)$ a witness w for membership of $x \in L$, from ``good'' prover P_x



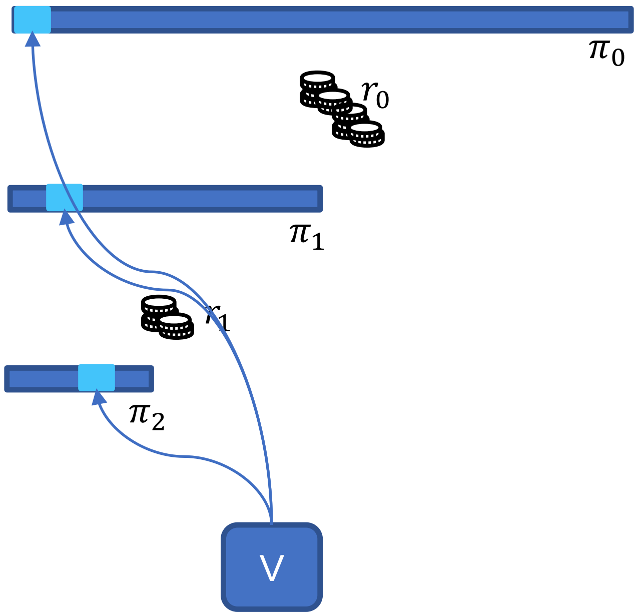
STARKWARE

Strict STIK (arithmetic complexity)

Definition:

An STIK for $L \in NTIME(T(n))$ is

- **Strictly Scalable** if both of following hold:
  - Proving time $T_P = O(T(n) \log T(n))$
  - Verifying time $T_V = O(\log T(n)) + \tilde{O}(n)$



Thm [B, Chiesa, Goldberg, Gur, Riabzev, Spooner, 2019]:

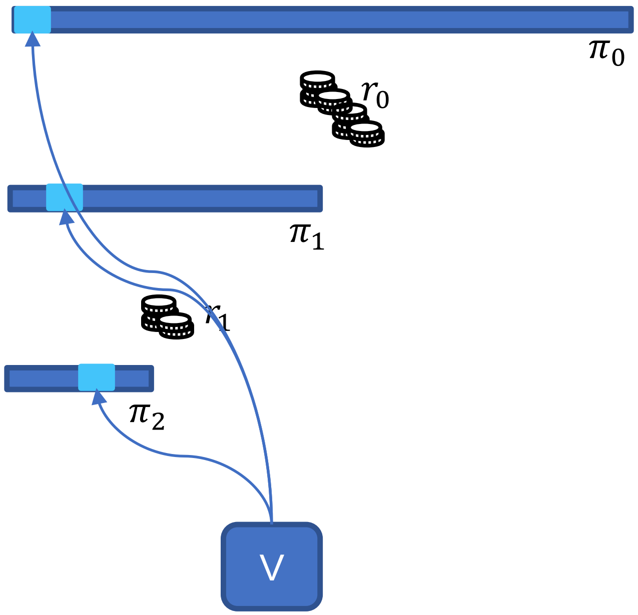
Every $L \in NTIME(T(n))$ has a strict (ZK)-STIK, where T_P, T_V are measured using arithmetic complexity over field of size $O(T(n))$

Question: Strict STIK, Boolean complexity?

Interactive Oracle Proofs of Proximity (IOPP) [RRR16, BCS16]

IOPP: to prove oracle f close to code $C \subset F^n$

- Verifier sends public randomness r_0
- Verifier has oracle access to 1st oracle π_1
- …
- Verifier runs in time poly($n + \log(T(n))$)
- If $f \in C$ then $\exists \pi_0, \pi_1, \ldots, \pi_t$ accepted w.p. 1
- Otherwise, $\forall \vec{\pi}$ rejected w.p. $> s(\Delta(f, C))$
- s is soundness function, want to maximize it



Fast Reed-Solomon IOPP (FRI) [B, Bentov, Horesh, Riabzev 2018]

Definition: Reed-Solomon code (low deg polys)

$$RS[F,S,\rho] = \{f: S \to F \mid \deg(f) < \rho|S|\}$$

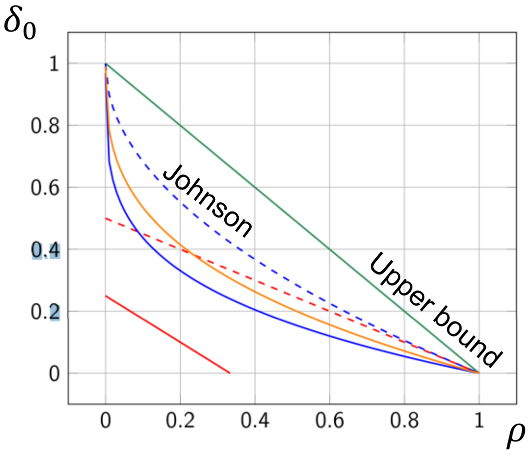
Thm [BBHR 2018]:

$\forall S \subseteq F, S$ is a group of size $N = 2^n$, the code $RS[F,S,\rho]$ has a (fast) IOPP with

- $T_P \leq 6 \cdot N$
- $T_V \leq 21 \cdot \log N$
- $s(\delta) \geq \min\{\delta_0, \delta\}$ for $\delta_0 \approx \frac{1-\rho}{4}$

$\delta_0 \approx 1 - \rho^{\frac{1}{4}}$ [BKS18]

New: $\delta_0 \approx 1 - \rho^{\frac{1}{3}}$ [BGKS19]



Question: Is $s(\delta) \geq \delta - \left(\frac{|S|}{|F|}\right)^{O(1)}$ for δ as large as $1 - \rho$?

Overview

1

Cryptoproofs

2

PCP, IOP
STIK, STARK
FRI

3

Concrete
Questions

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Theory questions with practical impact

1. Strict STIK, Boolean complexity
   - $T_P = O(T(n)\log T(n))$ and $T_P = O(\log T(n))$
   - Approach: use AG codes over constant alphabets
   - Requires quasi-linear encoding time for AG codes
2. Better soundness analysis for FRI
   - Is $s(\delta) \geq \delta - \left(\frac{|S|}{|F|}\right)^{O(1)}$ for δ greater than $1 - \rho^{\frac{1}{3}}$?
   - Reaching Johnson bound $(1 - \sqrt{\rho})$? Beyond it?
3. Sliding scale conjecture for IOP and STIK?
   - Currently soundness error greater than rate (ρ)
   - Want soundness error closer to $\mathrm{poly}(\frac{1}{|\mathrm{F}|})$
   - Perhaps simpler to solve for IOPs than for PCPs?

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Crypto-Security Questions

1. STARK-friendly crypto primitives
   - SHA2/3 STARK "cost" $\approx 10^4$
   - Pedersen STARK "cost" $\approx 10^3$
   - MiMC/Jarvis/Friday "cost" $\approx 10^2$ [AGRRT16, AD18]
   - How low can you get? Algebraic security analysis?
2. STARK/STIK security analysis
   - Best efficient attack on FRI? on other PCPs/PCPPs? [BBGR16]
3. STARK-friendly commitments and accumulators
   - Replace Merkle trees with more efficient data structures? [LM18, BBF18]
   - With Merkle trees in RO model, do you need λ or 2λ bits RO to reach $2^{-\lambda}$ soundness error?
4. How to formally verify a STIK/STARK constraint system?

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Questions?

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