Spacecraft Dynamics & Missions 3

Solution 3.1

1. From Kepler's third law we have

$$\mu = n^2 a^3$$

.

As the probe is massless we can make the simplification $\mu \approx mG$. As a result we have

$$m = \mu/G = n^2 a^3/G$$

$$m = \frac{4\pi^2 \times 1200^3}{4.7^2 \times 86400^2 \times 6.672 \times 10^{-20}}$$

hence m = 6.2×10^{18} kg. Similarly, for the volume we have

$$V = \frac{4}{3}\pi R^3$$

where R is Eugenia's radius, equal to 107.5 km. The above equation then gives $V = 5.2 \times 10^{15}\text{m}^3$ so $\rho = m/V = 1200$ kg m^3 = 1.2 g cm^3.

2. Differentiating Kepler's third law with respect to the orbital period gives

$$|\Delta\mu| = 8\pi^2 \left(\frac{a}{T}\right)^3 \Delta T.$$

The percent relative error is then computed to be

$$\frac{\Delta\mu}{\mu} \times 100 = 8\pi^2 \left(\frac{1200}{4.7 \times 86400}\right)^3 \frac{0.1 \times 86400}{0.4137} \times 100 \simeq 4\%.$$

Solution 3.2

We compute the magnitude r and v from $\boldsymbol{r}$ and $\boldsymbol{v}$. This gives

$$r = 7.1386 \times 10^3\text{km}$$

$$v = 9.83 \text{ kms}^{-1}$$

Hence,

$$E = v^2/2 - \mu/r = -7.5286 \text{ km}^2\text{s}^{-2}$$

and

$$a = -\mu/2E = 26472.3 \text{ km}$$

Solution 3.3

$$H_x = y\dot{z} - z\dot{y} = 10892.045 \text{ km}^2\text{s}^{-1}$$
$$H_y = z\dot{x} - x\dot{z} = -61799.992 \text{ km}^2\text{s}^{-1}$$
$$H_z = x\dot{y} - y\dot{x} = 31285.44 \text{ km}^2\text{s}^{-1}$$

Hence total angular momentum is $H = 70118.86 \text{ km}^2\text{s}^{-1}$ and $\cos I = H_z/H \rightarrow I = 63.5^\circ$

Solution 3.4

$a = R_\oplus + 650 = 7028.14$ km. Hence $n = \sqrt{\mu/a^3} = 0.00107 \text{ s}^{-1}$. Hence period is 1.628 hours or $1^h37^m40.8^s$.
For exactly 16 orbits, we need a period of 24/16 hours, i.e. 90 minutes or 5400 secs.
$n = 2\pi/5400 = 0.00116\text{s}^{-1}$. Hence $a = 6652.55$km or an altitude of 274.413 km.