

Relationship Prediction in Dynamic Heterogeneous Information Networks

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Abstract. Most real-world information networks, such as social networks, are heterogeneous and relations between different entities have different semantic meanings. Therefore techniques for link prediction in homogeneous networks can not be directly applied on heterogeneous ones. On the other hand, works that investigate link prediction in heterogeneous networks, do not consider the dynamics of networks in sequential time intervals. In this work we propose a technique that leverages a combination of latent and topological meta path-based features to predict a target relationship between two nodes of given types in a dynamic heterogeneous information network. Our results indicates that combining these features helps in building a more accurate predictive model compared to current techniques that use either of these features.

Keywords: Link prediction · Relationship prediction · Dynamic heterogeneous networks · Social networks · Network topology · Meta path.

1 Introduction

The goal of link prediction in a network graph [7] is to estimate the likelihood of the relationship between two nodes in future, based on the observed graph. Predicting such connections in a network have multiple applications such as friend/item/ad recommending, network completion, or biological applications such as predicting protein-protein interactions. Traditional link prediction techniques, such as [7], consider networks to be homogeneous, i.e., graphs with only one type of edges and nodes. However, most real-world networks, such as social networks, scholar networks, patient networks [4] and knowledge graphs [19] are heterogeneous information networks (HINs) [14] and have multiple node and relation types. For example, in a bibliographic network there are nodes of types authors, papers, and venues, and edges of types write, cite and publish.

In a HIN relations between different entities have different semantic meanings. Thus techniques for homogeneous networks can not be directly applied on heterogeneous ones. A few works such as [17, 15] investigated the problem of link/relationship prediction in HINs, however, they do not consider the dynamics of social networks and ignore analysis of sequence of network snapshots. On the other hand, it has been shown that for link prediction in homogenous networks incorporating temporal changes helps in a more accurate prediction [23]. Previous work on temporal link prediction scarcely studied HINs and to

the best of our knowledge, the problem of relationship prediction for dynamic heterogeneous networks was not studied before.

In this work we study the problem of predicting relationships in a dynamic heterogeneous information network (DHIN) i.e., a network with different types of nodes and links associated with timestamps, which is stated as follows: *Given a DHIN graph G , how can we predict the future structure of G ?*

Amin ► *This section is done up to here! Contributions TBA.* ◀

The main contributions of our work include:

- We present a technique, called **RelationPredict**, that predicts a target relationships between two nodes of given types;
- An evaluation of the accuracy and performance of the proposed algorithm on real social network data.

[23] [17] [16] [6] [20] [18] [15] [21] [7]

2 Problem Statement

The following definition is an extended version of HIN [17] for the dynamic network case.

Definition 1 (Dynamic heterogeneous information network). *A dynamic heterogeneous information network (DHIN) is a directed graph $G = (V, E)$ with a node type mapping function $\phi : V \rightarrow \mathcal{A}$ and a link type mapping function $\psi : E \rightarrow \mathcal{R}$, where V , E , \mathcal{A} , and \mathcal{R} denote sets of nodes, links, node types, and relation types. Each node $v \in V$ belongs to a node type $\phi(v) \in \mathcal{A}$, each link $e \in E$ belongs to a relation $\psi(e) \in \mathcal{R}$, and $|\mathcal{A}| > 1$ and $|\mathcal{R}| > 1$. Also each edge $e = (u, v, t)$ is a temporal edge from a vertex u to a vertex v at time t . □*

The DBLP bibliographic network¹ is an example of a DHIN, containing different types of nodes such as papers, authors, topics, and publication venues, with publication links associated with date. Another example is the Twitter social network with nodes of types posted tweets, users, topics, and hashtags and time window associated with these tweets.

In the context of a heterogenous network, a *relation* can be in the form of a *direct link* or an *indirect link*, where an indirect link is a sequence of direct links in the network. Thus, two nodes might not be directly connected, however they might be connected considering the semantic of a sequence of links of different types. In this work, we use the terms *relationship prediction* and *link prediction* interchangeably referring to predicting whether two nodes will be connected in future via a *sequence of relations* in the graph, where the *length* of a sequence is greater than or equal to one. For instance in a bibliographic network a direct link exist between an author and a paper he wrote, and an indirect link exist between him and his co-authors through the paper, which they wrote together.

¹ <http://dblp.uni-trier.de/db/>



Fig. 1. Network schema for DBLP network.

In order to better understand different types of nodes and their relation in a network, the concept of *network schema* [17] is used. A network schema is a meta structure graph that summarizes a HIN and is formally defined as bellow.

Definition 2 (Network schema). For a heterogeneous network $G = (V, E)$, the network schema $S_G = (\mathcal{A}, \mathcal{R})$ is a directed meta graph where \mathcal{A} is the set of node types in V and \mathcal{R} is the set of relation types in E . \square

Figure 1 shows the network schema for the DBLP bibliographic network with $\mathcal{A} = \{Author, Paper, Venue, Topic\}$. In this paper, we refer to different types of nodes in the DBLP bibliographic network with abbreviations P for paper, A for author, T for topic, and V for venue.

Similar to the notion of network schema that provides a meta structure for the network, a *meta path* [17] provides a meta structure for paths between different nodes in the network.

Definition 3 (Meta path). A meta path \mathcal{P} is a path in the network schema graph $S_G = (\mathcal{A}, \mathcal{R})$, denoted in the form of $\mathcal{P} = A_1 \xrightarrow{R_1} A_2 \dots \xrightarrow{R_n} A_{n+1}$, as a sequence of links between node types, which defines a composite relationship between a node of type A_1 and one of type A_{n+1} , where $A_i \subseteq \mathcal{A}$ and $R_i \subseteq \mathcal{R}$. \square

The length of \mathcal{P} is the number of relations in \mathcal{P} . Note that given two node types A_i and A_j , there may exist multiple meta paths of different lengths between them. The co-author relationship in DBLP can be described with the meta path $A \xrightarrow{write} P \xrightarrow{write^{-1}} A$ or in short $A-P-A$. Paths in thick solid lines in Figure 2 correspond to $A-P-V-P-A$ meta paths between *Max* and *Ada*, indicating they published in the same venue, such as *Max-P1-KDD-P3-Ada*. Each meta path indicates a different semantic and defines a unique topology representing a special relation. The relationship between two authors are different in meaning when they are co-authors ($A-P-A$) versus one citing another's paper ($A-P-P-A$).

Definition 4 (Relationship prediction problem). Given a DHIN graph G at time t , and a target relation meta path $P(A_i, A_j)$ between nodes of type A_i and A_j , we aim to predict the existence of a meta path P between two given nodes of types A_i and A_j at time $t + 1$. \square

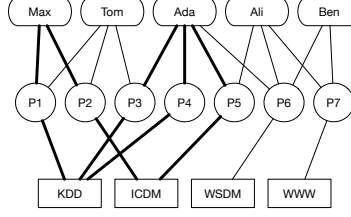


Fig. 2. An example of $A-P-V-P-A$ meta paths between two authors Max and Ada.

3 Relationship Prediction Approach

Given a DHIN graph $G = (V, E)$, and the number of graph snapshots t , we first decompose G to a sequence of t HIN graphs G_1, \dots, G_t based on links with associated timestamps. We then apply our techniques to predict G_{t+1} . As mentioned in Definition 4, in this work we intend to predict *existence of a given type of relationship* (target meta path) between two given nodes. Therefore we define a new type of graph, called *augmented reduced graph*, that is generated based on a given heterogeneous graph and a target relation meta path.

Definition 5 (Augmented reduced graph). *Given a HIN graph $G = (V, E)$ and a target meta path $P(A_i, A_j)$ between nodes of type A_i and A_j , an augmented reduced graph $G^P = (V^P, E^P)$ is a weighted graph, where $V^P \subseteq V$ and nodes in V^P are of type A_i and A_j , edges in E^P indicates relationships of type P in G , and each edge $e^P = (u, v, w)$ is a weighted edge from a vertex u to a vertex v with a weight $w = \text{Sim}(u, v)$ indicating a similarity measure between u and v . \square*

Examples of relation similarity measure between two nodes in a HIN are path count [17, 15], PathSim [17] or normalized path count [15], random walk and symmetric random walk [15], and HeteSim [13]. Path count measures the number of path instances between the source and target nodes of a given meta path, and can be calculated by multiplying adjacency matrices of relations in the meta path [15]. Random walk measures the probability of the random walk from source to target nodes. HeteSim evaluates relevance of heterogeneous nodes given an arbitrary path, while PathSim[5] evaluate similarity of same-typed nodes based on a symmetric path.

An augmented reduced graph for the network in Figure 2 and target meta path $P(A, A)=A-P-V-P-A$ is a graph with nodes of type *Author* and edges that represent relationship of *publishing in the same venue*. For example (Max, Ada) is an edge in the corresponding augmented reduced graph because they both published at KDD and ICDM. If we consider meta path $P(A, A)=A-P-A$, the augmented reduced graph represents a co-authorship graph, where nodes are of type *Author* and edges, such as (Max, Tom) , represent *co-authorship*.

3.1 Homogenize link prediction

Zhu et al. [23] studied the problem of temporal link prediction in the context of homogeneous networks, where the input is a sequence of graphs G_1, \dots, G_t and the output is the estimated G_{t+1} . They present a matrix factorization (MF) with block-coordinate gradient descent technique that for each adjacency matrix G_τ at time τ infers a low rank k -dimensional latent space representation matrix Z_τ that minimizes

$$\begin{aligned} & \underset{Z_1, \dots, Z_t}{\operatorname{argmin}} \sum_{\tau=1}^t \|G_\tau - Z_\tau Z_\tau^T\|_F^2 + \lambda \sum_{\tau=1}^t \sum_u (1 - Z_\tau(u) Z_{\tau-1}(u)^T) \\ & \text{subject to } \forall u, \tau, Z_\tau \geq 0, Z_\tau(u) Z_\tau(u)^T = 1 \end{aligned} \quad (1)$$

where λ is a regularization parameter, and $(1 - Z_\tau(u) Z_{\tau-1}(u)^T)$ penalizes node u for suddenly changing its latent position. $Z_\tau(u)$ is a row vector denoting u 's temporal latent space representation at time τ , and $Z_\tau(u, i)$ indicates the position of u in the i -th dimension at Z . The intuition behind their prediction model is that 1) nodes move smoothly in the latent space over time and it is less likely to have large moves [12, 22], and 2) user interactions are more likely to occur between similar users in a latent space representation. To predict adjacency matrix G_{t+1} they used $Z_t Z_t^T$, however, they mentioned that G_{t+1} can be formulated as $\Phi(f(Z_1, \dots, Z_t))$, where Φ and f are the link and the temporal functions that one may apply techniques such as nonparametric approaches [11] to learn them.

Algorithm 1 is an adaptation of the above MF technique applied on a sequence of augmented reduced graphs G_i^P (Definition 5) given a target meta path P , which changes equation (1) by replacing G_τ with G_τ^P . The algorithm gets as an input a DHIN graph G , the number of graph snapshots t , a target relation meta path $P(a, b)$, and the latent space dimension k . The algorithm first decomposes G into a sequence of t graphs $\{G_1, \dots, G_t\}$ (line 1) by considering the associated timestamps on edges. Next from each graph G_i , a corresponding augmented reduced graph G_i^P is generated (lines 2-8) for which nodes are of type a and b (beginning and end of target relation meta path P). Finally the MF technique in [23] is applied (lines 9-10) to infer latent spaces Z_1, \dots, Z_t and estimate G_{t+1}^P by $Z_t Z_t^T$. Note that Z_τ depends on $Z_{\tau-1}$ as used in the temporal regularization term in equation (1).

3.2 Meta path-based relationship prediction

The above homogenize approach does not consider different semantics of meta paths between the source and destination nodes. In fact, Zhu et al. [23] assume that the probability of a link between nodes depends only on their latent positions. However, we also include meta path-based features in our prediction model. Our intuition is that along with latent space features leveraging meta path-based features, as in [15], helps to boost the prediction accuracy. In other word, we combine latent space features with topological meta path-based features. Sun et al. [15] proposed a supervised learning framework, called *PathPredict*, that uses

Algorithm. 1 Homogenize Link Prediction

Input: A DHIN graph G , the number of snapshots t , a target meta path $P(a, b)$, the latent space dimension k

Output: The predicted graph G^P at time $t + 1$ based on the target relation P

- 1: $\{G_1, \dots, G_t\} \leftarrow \text{DecomposeGraph}(G, t)$
- 2: **for** each graph $G_i = (V_i, E_i)$ **do**
- 3: Let a and b be the node types of beginning and end of P
- 4: **for** each node $x \in V_i$ that $\phi(x) = a$ **do**
- 5: Follow P to reach a node $y \in V_i$ that $\phi(y) = b$
- 6: Add nodes x and y , and edge (x, y) to the augmented reduced graph G_i^P
- 7: **end for**
- 8: **end for**
- 9: $\{Z_1, \dots, Z_t\} \leftarrow \text{MatrixFactorization}(G_1^P, \dots, G_t^P, k)$
- 10: $G_{t+1}^P \leftarrow Z_t Z_t^T$
- 11: Return G_{t+1}^P

Algorithm. 2 Meta path-based Relationship Prediction

Input: A DHIN graph G , the number of snapshots t , a network schema S , a target meta path $P(a, b)$, the maximum length of a meta path l , the latent space dimension k

Output: The predicted graph G^P at time $t + 1$ based on the target relation P

- 1: $\{G_1, \dots, G_t\} \leftarrow \text{DecomposeGraph}(G, t)$
- 2: Generate target augmented reduced graphs G_1^P, \dots, G_t^P following Algorithm 1 lines 2-8
- 3: $\{P_1, \dots, P_n\} \leftarrow \text{GenerateMetaPaths}(S, P(a, b), l)$
- 4: **for** each graph $G_i = (V_i, E_i)$ **do**
- 5: **for** each node $x \in V_i$ that $\phi(x) = a$ **do**
- 6: **for** each meta path P_j **do**
- 7: Follow P_j to reach a node y that $\phi(y) = b$
- 8: $w_{xy} \leftarrow \text{Similarity}(x, y, G_i)$
- 9: Add nodes x and y , and edge (x, y) with weight w_{xy} to augmented reduced graph $G_i^{P_j}$
- 10: **end for**
- 11: **end for**
- 12: $\{Z_1, \dots, Z_t\} \leftarrow \text{MatrixFactorization}(G_1^{P_j}, \dots, G_t^{P_j}, k)$
- 13: $G_{t+1}^{P_j} \leftarrow Z_t Z_t^T$
- 14: **end for**
- 15: $\forall (a, b \in N(a)) \in G_t^P$, add a feature vector to the training set with w_{ab} in $G_{t-1}^{P_j}$ for each meta paths P_j , and label=1 if $(a, b) \in E_t^P$ otherwise label=0.
- 16: Learn the model and apply it to the feature vector of $G_{t+1}^{P_j}$ with different meta path P_j .
- 17: Build G_{t+1}^P based on the cut-off values for the output of prediction model.
- 18: Return G_{t+1}^P

the meta path-based features in a past time interval to predict the relationship building in a future time interval. Their model learns coefficients associated with each feature by maximizing the likelihood of new relationship formation.

We define a set of meta paths [17] on a given network schema and a target relation meta path P to generate an augmented reduced graph (Definition 5) G_i^P from G_i based on P . We then leverage the technique in [23] to predict G_{t+1}^P given G_1^P, \dots, G_t^P , by inferring the temporal latent space representation for nodes at time $t + 1$. Amin ► *this para seems wrong!!!* ◀

Algorithm 2 gets as an input a DHIN graph G , the number of graph snapshots t , a network schema S , a target relation meta path $P(a, b)$ between node types a and b , the maximum length of a meta path l , and the latent space dimension k . Same as Algorithm 1, it decomposes G into a sequence of graphs (line 1). It then produces the set of all meta paths between nodes of type a and type b defined in

the given target relation $P(a, b)$ (line 2). This is done by traversing the network schema S (for instance through a BFS traversal) and generating meta paths with the maximum length of l . Next from each graph snapshot G_i , a corresponding augmented reduced graph G_i^P is generated (lines 3-9) for which nodes are of type a and b (beginning and end of meta path P) and edges have weight based on a relation similarity measure, such as path count, between a and b . Once the sequence of augmented reduced graphs $\{G_1^P, \dots, G_t^P\}$ are generated, we apply the matrix factorization technique in [23] to find a low rank k -dimensional latent space representation matrix Z_τ for nodes at time τ .

The predictive model. Given the training pairs of nodes and their corresponding meta path-based features (similarity weights w), we build a prediction model to learn the weights associated with these features by apply logistic regression. We define the probability of forming a *new link in the future* from node a to b as $Pr(label = 1|a, b; \theta) = \frac{1}{e^{-z} + 1}$, where $z = \sum_{i=1}^n \theta_i \cdot f_i(a, b)$ and θ_i is the associated weight for n topological meta path-based features f_i at current time between a and b . We use logistic regression with L_2 regularization to estimate the optimal θ , given a training dataset with l instance-label pairs, where $\alpha \sum_{j=1}^n \theta_j^2$ is the regularization term, and $C > 0$ is a penalty parameter.

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} C \sum_{i=1}^l -\log Pr(label|a_i, b_i; \theta) + \alpha \sum_{j=1}^n \theta_j^2 \quad (2)$$

In the testing phase, we predict a data point x as positive if $\theta^T x > 0$, and negative otherwise.

In the training phase, for each pair of nodes (a, b) in G_t^P , where $b \in N(a)$, we add a feature vector $\mathbf{f}_t(a, b)$ to the training set with corresponding w_{ab} in $G_{t-1}^{P_j}$ for each meta paths P_j , and with label=1 if $(a, b) \in E_t^P$ otherwise label=0. We then perform logistic regression to learn the model. Finally we apply the model to the feature vector of predicted graphs $G_{t+1}^{P_j}$ with different meta path P_j . Finally it builds G_{t+1}^P based on the cut-off values for the output of prediction model.

3.3 Combining latent and meta path-based features

Although the gradient descent algorithm used in the MF technique to infer latent space matrices in [23] is fast, we cannot apply it to meta paths efficiently since computing meta path values for all possible pairs of nodes is not scalable. Thus we restrict it only to those links that make new connections in the next time interval or negative samples and use logistic regression.

$$\begin{aligned} & [23] \text{ approximated } G_{ij} \text{ with } \sum_{l=1}^k Z_{il} Z_{jl} \text{ and [17] with } \sum_{l=1}^n \theta_l f_l(i, j) \\ & - G_{ij} \approx \sum_{l=1}^n \theta_l f_l(i, j) + \sum_{l=1}^k \beta_l Z_{il} Z_{jl} \\ & \underset{Z}{\operatorname{argmin}} \sum_{(i,j) \in O} \|G_\tau(i, j) - \Phi(z_i^T z_j + \theta^T f(i, j))\|_F^2 \end{aligned}$$

, where Φ and ϕ is the link function.

- Augmenting latent with explicit features helps to combine latent features with the results of any other link prediction model. Let $f(i, j)$ be the meta path feature between nodes i and j . Then, we can learn latent features that fit the residual of f . In general then, the latent feature approach has a natural mechanism by which any predictive signal can be incorporated, whether it is an explicit feature vector or model predictions. Our hypothesis is that combining latent features with meta path-based features will improve performance on the test data. However, if the latent features learn similar structure to the meta path features, then combining the two may not be beneficial. We only consider the linear combination of latent and meta path features, however, these can be combined in different ways that is beyond the scope of this work.

3.4 Implementation

Amin ► *add system and OS spec* ◀ We use the implementation of temporal latent space inference for a sequence of dynamic graph snapshots ²[23]. For the classification part, we use the efficient LIBLINEAR [5] package³ and set the type of solver to L2-regularized logistic regression (primal). We performed 5-fold cross validation for the training phase.

4 Experiments

4.1 Experiment Setting

Dataset. We conduct our experiments on two real-world datasets that have different characteristics and evolution behaviour.

- *Publications dataset:* The *aminer* citation dataset⁴ V8 (2016-07-14) is extracted from DBLP, ACM, and other sources. It contains 3,272,991 papers and 8,466,859 citation relationships for 1,752,443 authors, who published in 10,436 venues, from 1930 to 2016. Each paper is associated with abstract, authors, year, venue, and title. **Amin** ► *We consider only those papers published since 1996, which includes X papers and Y authors.* ◀ Authors in [15] used a similar dataset but considered only authors with more than 5 publications. We generate two datasets: one that contains all publications, and one that considers authors with at least 5 papers. We consider $k = 3, 5, \text{and } 10$ different time intervals for the dynamic analysis. In our evaluation, we execute the learned model on the last interval to measure the prediction accuracy.
- *Movies dataset:* The RecSys HetRec 2011 movie data set [1] is an extension of MovieLens10M dataset, published by GroupLens research group ⁵ that

² <https://github.com/linhongseba/Temporal-Network-Embedding>

³ <https://github.com/cjlin1/liblinear>

⁴ <https://aminer.org/citation>

⁵ <http://www.grouplens.org>

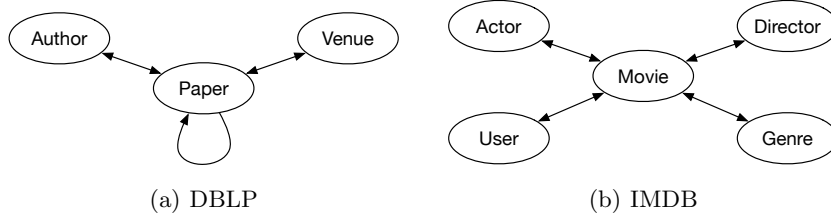


Fig. 3. The simplified network schema used for our experiments.

links the movies of MovieLens dataset with their corresponding web pages at Internet Movie Database (IMDb⁶) and Rotten Tomatoes⁷ movie review systems. It contains information of 2,113 users, 10,197 movies, 20 movie genres (avg. 2.04 genres per movie), 4,060 directors, 95,321 actors (avg. 22.78 actors per movie), 72 countries, 855,598 ratings (avg. 404.92 ratings per user, and avg. 84.64 ratings per movie), and 13,222 tags (avg. 22.69 tags per user, avg. 8.12 tags per movie).

Movies once release, users can rate them but a paper is published once and new co-authorship is made only at that time.

In co-authorship all connections are new in MovieDB new connections to an existing movie. This is a common problem with all rating datasets.

We also conduct our experiments on two variations of the DBLP, one with min 5 paper as in ... and one with all.

Meta paths and target relationships. We consider different type of target relationship.

Network schema for the two datasets are shown in Figure X. Note that we consider a simplified version and ignore nodes such as topic for papers or tag for movies.

We consider ... based on the work in ... we con

Authors in [15] conducted Wald test in a case study and found that the p-value for the feature associated with each meta path and their significance level. From the results, we can see that the shared co-authors, shared venues, shared topics and co-cited papers for two authors all play very significant roles in determining their future collaboration(s). For...

Similarly we only calculated the PC for these meta paths. Note that the goal of our paper is not to select the best features but to show the strength of using...

Table 1 shows meta paths between authors under length 4 for the publications dataset.

Unlike the $A-P-A$ target relation for the publication dataset for which both ends of the relation is of the same kind, we consider $U-M$ as the target meta

⁶ <http://www.imdb.com>

⁷ <http://www.rottentomatoes.com>

Table 1. Publications Dataset Meta Paths (A = author, P = paper, V = venue).

Meta path	Meaning
$A-P-A$	<i>[The target relation]</i> Authors are coauthors
$A-P-V-P-A$	Authors publish in the same venue
$A-P-A-P-A$	Authors have the same co-author
$A-P-P-P-A$	Authors cite the same papers

path for the movie dataset to show the effectiveness of our proposed methods in predicting such relationships. **Amin** ► *The issue with matrix factorization is that originally $G_{n \times n}$ is for homogenous network with the same type of nodes. In our case ZZ^T vs. VU^T* ◀

Table 2. Movies Dataset Meta Paths (U = user, M = movie, A = actor, D = director, G = genre).

Meta path	Meaning
$U-M$	<i>[The target relation]</i> A user watches a movie
$U-M-A-M$	A user watches a movie with the same actor
$U-M-D-M$	A user watches a movie with the same director
$U-M-G-M$	A user watches a movie of the same genre
$U-M-U-M$	A user watches a movie that another user

Baseline methods. Considering the effect of time-wise data decomposition. What if we shorten timespans of each G_t ? The extreme is having only one graph or having it for each year. Can we find a trade-off?

- Heterogeneous non-temporal (PathCount, PathSim, NormalPathCount, RandomWalk, SymmetricRandomWalk)
- Homogeneous non-temporal (Katz, Jaccard)
- Homogeneous temporal (BCGD)

Evaluation Metrics. We use prediction error to evaluate the inference accuracy. Given the training graph G_1, \dots, G_t , prediction error is defined as... Therefore, a smaller prediction error indicates better inference accuracy. For link prediction accuracy, we use Area Under Curves (both Receiver Operating Characteristic (ROC) and Precision-Recall (PR) curves), termed as AUCROC and AUCPR [3]. Also in order to decide which classifier has a lower error rate, we perform McNemar’s test that assess the significance of the difference between two correlated proportions.

4.2 Results and Findings

Amin ► *One reason that $A-P-V-P-A$ is better with intervals is that one may publish in KDD but there are so many publishing there....* ◀

4.3 Discussion

Our proposed technique can also be used in other applications. For example link recommendation

predicting missing edges in graphs.

Vertex Recommendation similar to [10]

In this work we modelled the predicted graph $\hat{G}_\tau(i, j)$ as a combination of meta path features and latent features $\Phi(z_i^T z_j + f_D(z_{i,j}; w))$. As explained in [8], one may also augment the model by incorporating some information regarding node affinities using implicit/explicit attributes and define node features x_i , which makes the model $\hat{G}_\tau(i, j) = \Phi(z_i^T z_j + f_D(z_{i,j}; w))$

As shown in [8] and [23], latent features are more predictive of linking behaviour compared to unsupervised scoring techniques such as Katz, Preferential Attachment, and Adamic.

Experiments in [8] shows combining the latent structure and side-information increases the prediction accuracy.

Connection to link privacy research such as [9]

5 Related Work

[23] [17] [16] [6] [20] [18] [15] [21] [7]

Sun et al. proposed PathSelClus [18] that uses limited guidance from users in the form of seeds in some of the clusters and automatically learn the best weights for each meta-path in the clustering process.

The concept of temporal smoothness has been used in evolutionary clustering [36] and link prediction in a dynamic network [23].

There exist many embedding methods for static networks, however very few considered dynamic networks. Zhu et al. [23] attempt dynamic link prediction by adding a temporal-smoothing regularization term to a non-negative matrix factorization objective. Their goal is to reconstruct the adjacency matrix of different time-stamps of a graph. They use a Block-Coordinate Gradient Descent (BCGD) algorithm to perform non-negative factorization. Their formulation is almost identical to the algorithm of Chi et al. [2], who perform evolutionary spectral clustering that captures temporal smoothness. Because matrix factorization provides embedding vectors of the nodes for each time-stamp, the factorization by-product from this work can be considered as dynamic network embeddings.

6 Conclusions and Future Work

TBA.

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