Channel Mobility Incurred Source Routing in Cognitive Radio Networks

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Abstract

In Cognitive Radio Networks (CRN), Secondary users are allowed to use Primary Users' (PUs') idle channels for data transmission. However, PUs can reclaim their channels at any time and SUs must cease transmission immediately. Such a problem is known as the *spectrum mobility*, and may further cause break of pre-established routes in multi-hop CRNs where SUs act as relays. In this paper, we investigate a routing problem where channel switching and route re-selection are deployed to resolve SUs' conflicts with PUs and further improve spectrum mobility. In particular, we consider both protocol and physical interference models and form the routing game into a routing game which is proved to be a potential game with a pure Nash Equilibrium (NE). Finally, we present an efficient algorithm for finding the NE and analyze the *Price of Stability* of the proposed game.

1 Introduction

Wireless spectrum is currently regulated by government and is assigned to license holders and services on a long term basis. However, a large portion of the assigned spectrum is used sporadically, which leads to under utilization of frequency resources.The problem foresees the development of Cognitive Radio Networks (CRNs) to improve spectrum efficiency. The idea of CRN is that unlicensed users (Secondary Users, SUs) can flexibly access licensed users' (Primary Users, PUs) idle channels without interfering with other PUs' transmission. While such Dynamic Spectrum Access (DSA) improves spectrum utilization, new problems such as spectrum sensing, channel selection, MAC and routing protocols design occurred with regard to the architecture of CRNs, and one of the major problems is *spectrum mobility*.

In CRNs, due to higher priorities, PUs are allowed to reclaim their licensed channels at any time, while SUs must cease their transmissions immediately. Therefore, for multi-hop CRNs where SUs act as relays, spectrum mobility may break pre-established routes as it disables data transmission over certain links on those routes. To avoid conflicts with PUs and resume routing, each flow generator can either inform intermediate SUs to switch channels or select a new route. However, there is no clear answer as for which approach is better. While switching channels keeps the original spacial routes which effectively reduce routing costs, frequent channel switching may cause significant switching costs. On the other hand, while re-selecting a spacial route can reduce switching costs, it may result in additional routing costs in the mean time. Therefore, due to the trade off between channel switching and route re-selection, we combine the two approaches into a routing problem as a whole.

However, though the channel mobility problem in CRN is gaining increasing attention, most of the existing works focused only on either channel switching or route re-selection, but not both. For example, Zhao [3] proposed a reactive channel assignment scheme to minimize interference and maintain network connectivity, while Qinghai [2] presented a proactive one based on spectrum hole prediction. Yet recently, Qingkai [1] proposed a two-dimensional route switching game considering both channel switching and route re-selection. Nevertheless, he did not include the scenario where multiple flows sharing a same link, and considered only the protocol interference model without discussing the physical interference model. In this paper, we construct the routing problem into a routing game, where each flow generator acts as a player and selects its route selfishly to minimize its own cost, which results from both channel switching and route re-selection. In particular, we pay attention to the scenario where multiple flows can share a common link which further improves spectrum efficiency but introduces congestion costs. Furthermore, we consider both protocol and physical interference models, and construct the cost model and form the game accordingly.

The contributions of this paper include the following:

- We formulate the problem as the Route-Switching Game.
- We construct the game in regards to both protocol and physical interference models.
- We prove the existence of Nash Equilibria (NE).
- We simulate the *Fictitious Play Process* as the learning algorithm for the game, prove its feasibility and efficiency, and analyze the *Price of Stability* (PoS).

The rest of this paper is organized as following: We will first introduce some related works in section 2 and present the system model in section 3. Then in section 4, game formation for the Routing Game will be discusses. Next, we analyze the learning strategy for the game and the Price of Anarchy in section 5 and 6. At last, we present simulation results and conclusion in section 7 and 8 respectively.

2 Related Works

For routing problem in multi-hop CRN, the most relevant work would be the two dimensional routing game proposed by Qingkai [1], which also explored a joint scheme of channel switching and routing re-selection in the context of spectrum-mobility incurred route-switching problem in multi-hop CRN. However, his work failed to consider the scenario where multiple flows sharing a common link, and he considered only the protocol interference model but not the physical interference model. Most of the other works only considered either channel switching or spatial route selection. For instance, Qinghai [2] proposed a proactive channel selection scheme based on spectrum hole prediction. He introduced two opportunistic channel selection algorithms to optimize the throughput of the secondary user: minimum collision rate channel selection algorithm and minimum handoff rate channel selection algorithm. Besides, Zhao [3] provided a robust channel assignment scheme in the multi-hop CRN with both centralized and distributed algorithms, while Caleffi [4] proposed the criterion for an optimal routing metric in CRNs based on the diversity effects of spatial routes.

As for game formation, Roughgarden [5, p. 461- 487] discussed selfish routing game in general, and analyzed the existence and uniqueness of a Nash Equilibrium (NE) in the game. He then introduced the potential function method for finding NE, and brought up the concept of the Price of Stability (PoS) which compare the best NE to the socially optimal result to analyze the efficiency of self routing games. Furthermore, Ragavendran [6] discussed the 'hourglass' architecture for game-theoretic control, which allowed a diverse set of possible utility and learning designs, with an constrained interface connecting the two. In this paper, we construct players' utility function in terms of routing costs, and deploy the famous *Fictitious Play* (FP) process [7] as the learning rule for each player to update its strategy during the game. Additionally, the class *potential game* is chosen to be the constrained interface, which requires utility designs to guarantee that the resulting game is a potential game and required learning rules to guarantee to provide desirable behavior. In a potential game, a pure Nash Equilibrium will be reached where no player can further reduce its own cost under the specific utility and learning design.

3 System Model

What we consider is the channel-mobility-incurred source routing problem in the multi-flow multi-hop Cognitive Radio Networks (CRNs). In this section, we present the formal definitions of our system model.

3.1 Cognitive Radio Networks (CRNs)

A CRN can be characterized by a potential directed graph $G = (V, E)$, where V is the set of all active secondary users in the network and E denotes the set of all potential links. Here, potential links mean that not all the links in this graph can be established simultaneously. Similar to other recent works on CRNs, we only consider *unicast* in the communication, and assume that each node has already had a stable power level[1]. Therefore, with a known power level, we can get the wireless transmission range of each node and establish all the potential links. Formally, an edge $e^c_{i(u,v)} \in E$ from SU_u to SU_v is established if and only if SU_v is within the transmission range of SU_u, which is a potential link and therefore corresponds with a channel c. We assume that a node can be equipped with multiple antennae[2], so multiple wireless links can be established between two nodes. Two links are called opposite if they differ from each other only by the direction, and are denoted e and $\bar{e}$.

3.2 Channel

In CRNs, each secondary user (SU) is able to sense the licensed channel state to avoid colliding with any PU who has the priority to occupy channels. Each user[3] can establish a link with any SU within its own transmission range on any channel they have sensed. We denote $\mathcal{C} = \{1, 2, \ldots, C\}$ as the potential available channel set, and $\mathcal{C}_{\mathrm{SU}_i}$ is the available[4] channel set of the ith SU at a time. Since most channels in CRNs are occupied by PUs, C is a small number, and we can assume that the number of antennae of each wireless node is enough to support all its potential links working on different available channels simultaneously. Since different SU may collide with different PUs, $\mathcal{C}_{\mathrm{SU}_i}$ and $\mathcal{C}_{\mathrm{SU}_j}$ can be not equal given that $i \neq j$ at a time.

3.3 Flow

There are N flows indicated by $i \in \mathcal{N} = \{1, 2, ..., N\}$ being injected into the network concurrently, each of which can be considered being generated by an unique fictitious flow generator. That is to say, no two flows are from the same generator. Each flow corresponds to a source-destination pair (s, t) and has a number of information (packets) to transmit. For example, flow i has to route its α_i information from s_i to t_i in one period of time[5]. We assume that 1) the information can be divided into equal size to transmit no matter which flow they belong to; 2) each flow generator chooses only one path to route all its information; 3)the buffer at each node is large enough to sustain the whole process.

[1] Broadcast and power allocation is beyond our consideration here.

[2] i stands for link's (or edge's) identity; u, v stand for the nodes; and c stands for a channel. We will use e^c_i, e_i and e for denoting simplicity according to the context, all of which are literally equivalent to $e^c_{i(u,v)}$.

[3] We will use secondary user, user, SU and node interchangeably in this draft.

[4] "Available" means that the channel is sensed idle, and can be used by the user.

[5] In this paper, we consider the concurrent flows. Each generator will choose a route periodically. In each period of time, α_i information are expected to route.

3.4 Interference and Congestion

We deploy a linear interference measurement to generalize different kinds of interference model. We use a matrix $\mathcal{I}$ to characterize the influence of the transmission on one link on a transmission of another link. For links $e^c_{i(u,v)}$ and $e^{c'}_{j(u',v')}$, $\mathcal{I}(i,j)$ shows to what extent link $e^c_{i(u,v)}$ is interfered by link $e^{c'}_{j(u',v')}$ at a time. Note that only the link pairs assigned with the same channel ($c = c'$) can interfere with each other, for the ones assigned with different channels ($c \neq c'$), the corresponding entries are set to zero. After normalization, $0 \leq \mathcal{I}(i,j) \leq 1$ for any $e_i, e_j \in E$. In particular, we set $\mathcal{I}(i,j) = 1$ for any two links $e^c_{i(u,v)}$ and $e^c_{j(v,u)}$ between two nodes that are assigned with the same channel but have opposite directions; the special case, $\mathcal{I}(i,i) = 1$, is included, which denotes the congestion on this link (we will specify on it later). As for the other entries, the value will depend on the interference model we adopt. Two most frequently used models, the protocol model and the physical SINR model, are considered.

- Protocol interference model. Each entry of $\mathcal{I}$ is either 1 or 0. That is, for link pair $e_i, e_j \in E$, $\mathcal{I}(e_i, e_j) = 1$ if and only if e_i and e_j are assigned with the same channel and e_i is within the interference range of e_j[6], and $\mathcal{I}(e_i, e_j) = 0$ otherwise. The restriction in this model is that some nearby links may share a channel but only one link can support a successful transmission over this channel at one time. That is, for a successful transmission on link e_i, the constraint is

$$\sum_{e_j \in E \setminus \{e_i\}} \mathcal{I}(e_i, e_j) \cdot Z(e_j) = 0, \tag{1}$$

  where $Z(e_j) = 1$ if and only if e_j is activated, i.e., included in at least one path that is routed by at least one flow at a time, and $Z(e_j) = 0$ otherwise.

- Physical interference model. For two links $e^c_{i(u,v)}$ and $e^c_{j(p,q)}$, $\mathcal{I}(e^c_i, e^c_j)$ indicates how much a transmission on e^c_j interfere with a transmission on e^c_i, i.e., the ratio of the signal power at receiver SU_v from SU_p to that from SU_u. Specifically, we set $\mathcal{I}(e^c_i, e^c_j) = +\infty$ for any wireless link e^c_j which has the receiver of e^c_i as its sender in the same channel c, since one node cannot send and receive information simultaneously at one channel. Under the SINR model, the constraint for a successful transmission on link e^c_i is

$$\sum_{e_j \in E \setminus \{e_i\}} \mathcal{I}(e_i, e_j) \cdot Z(e_j) \leq \frac{1}{\beta}, \tag{2}$$

  where β is the threshold in the model, and we ignore the ambient noise.

Note that the above matrix can be established when the topology information and the interference model are known, with the assumption that the power level of each wireless device is stable.

[6]For example, the sender of e_j can interfere with the receiver of e_i.

3.5 Assumptions

We list following important assumptions that are related to our studies.

Assumption 1. *Each flow generator can only select one path between its own source-destination node pair to route all its information. Under the interference constraint of a interference model, for each flow i, there exists at least one feasible path from s_i to t_i, regardless of other flows' choices of strategies. And each secondary user will honestly follow the routing strategies developed by flow generators.*

Assumption 2. *At one (short) period of time, the channel states, the topology of CRNs and flows information including the source-destination pair and the expected number of information to be transmitted are known to each fictitious flow generator.*

Assumption 3. *The injection rate*[7] *of each flow is high enough to support the continuity of the routing process. That is, there will always be some information arriving at the source node ready for transmission, and the source node will not be idle unless the last packet of the related flow has been sent out.*

We find that a reasonable injection rate (i.e., the upper bound of it) has been issued by the researchers. And it can vary based on different assumptions of stochastic environment. However, this is beyond our concerns, so we just need to guarantee that it won't be too low.

Assumption 4. *Any channel is not allowed to be used by any PU and any SU simultaneously, even when interference is not high enough to break the links between PUs.*

Assumption 5. *("Backoff") Whenever a channel reclaimation occurs, all intermediate nodes drop the undelivered packets and the source nodes re-transmit the packets whose ACK is not received.*

4 Game Formulation

In this section, we formulate a routing and switching game in a cognitive radio network, which is asymmetric and non-identical-interest (selfish), considering that each flow comes from different application with selfish nature as stated above. Both the protocol interference model and the physical interference model are considered. We first introduce the cost of a link that a flow pays for when routing through it by specifying the components that constitute the link cost to illustrate the impacts on the performance of routing. Then, path cost is introduced, based on which a flow determines its path to route its packets. Finally, we formulate the game that corresponds to the routing problem.

[7] Here, injection rate corresponds to the receiving rate that the source node perceived.

4.1 Link Cost

We now present the cost of a link when a flow selects it to route its packets, which instructs each flow generator to choose its path. For the ith flow, set $\mathcal{P}_i$ as the set of its possible routing strategies. Only one path $p_i^k \in \mathcal{P}_i$ is chosen to route all its packets for any $i \in \{1, 2, ..., N\}$ and $k \in \{1, 2, ..., |\mathcal{P}_i|\}$. Let $\mathcal{P} = \bigcup_{i \in \mathcal{N}} \mathcal{P}_i$, which is the union of all the possible routing paths for the N flows. The strategies of all the N flows are within the space $\mathbf{P} = \mathbf{P}_1 \times \mathbf{P}_2 \times \cdots \times \mathbf{P}_N$, where a dimension $\mathbf{P}_i = \mathcal{P}_i$. According to Assumption 1, for each flow i, there exists at least one feasible path from s_i to t_i regardless of other flows' strategies.

We employ a two-dimensional matrix $\mathbf{X}_{\mathcal{P}}^{\mathcal{N}}$ to denote the path selection of each flow, where $X_p^i = 1$ if the ith flow chooses path p. Similarly, for any link $e \in E$, set $X_e^i = 1$ if this link is selected by the ith flow, which means $e \in p$ and $X_p^i = 1$ ($p \in \mathcal{P}_i$). Since self-impact can be reduced by some modern transmission technology, any self-impact (e.g. congestion and interference) is assumed to be ignored. We denote $Q_i(e)$ as the expected number of packets of all the other flows rather than the ith flow that will be transmitted on edge $e \in E$ in a period of time. Denote α_i as the expected number of packets that the ith flow will transmit over e during the same period of time. And we can have

$$Q_i(e) = \sum_{j \in \mathcal{N}_i} \sum_{p \in \mathcal{P} \ \& \ e \in p} X_p^j \cdot \alpha_j = \sum_{j \in \mathcal{N}_i} X_e^j \cdot \alpha_j, \tag{3}$$

where $\mathcal{N}_i := \mathcal{N} \setminus \{i\}$.

For link $e : X_e^i = 1$, the cost for flow i routing its packets on link e is characterized as follows, which includes *the congestion cost*, *the interference cost*, *the switching cost* and the *the energy cost*.

4.1.1 Congestion

Since we allow routing multiple flows on one link, there may exist congestion. Congestion that caused by different flows transmitting packets on the same link or on the links that have the opposite directions[8]. Recall that for flow i, $Q_i(e)$ and $Q_i(\bar{e})$ denote the expected number of packets of other flows transmitting on link e and its opposite link $\bar{e}$ in a period of time, respectively. We utilize a cost function to formulate the congestion cost of flow i on edge e, denoted as $D(i, e) = \lambda(\mathcal{I}(e, e)Q_i(e),\ \mathcal{I}(e, \bar{e})Q_i(\bar{e}))$. $\lambda(x, y)$ is a non-decreasing function of either x (with y fixed) or y (with x fixed). The cost function implies that the link will get more congested when the number of packets transmitted on it increases. We can see that the congestion link cost reflects the queuing delay in the cognitive radio networks.

[8]This has been discussed above (i.e., $\mathcal{I}(e, \bar{e}) = 1$). In this case, wireless links are assigned with the same channel but have opposite directions. We will use the term "opposite links" to denote this case.

4.1.2 Interference

Assigning a pair of links with the same channel incurs interference. Interference will lead to a fall in transmission rate no matter which interference model is taken into consideration. We have defined Matrix $\mathcal{I}$ to denote the interference relationship between any two of the potential communication links. So, we can formulate the interference cost under each of the two interference models with a cost function as follows.

- For the protocol interference model, we consider a CSMA-like contention cost. In this contention protocol, one link that is interfered by some other links will contend for transmission since only one of them can transmit successfully at one time. We model the interference link cost as the expected contention delay, as follows.

  $$F_1(i,e) = \mu_1(\mathcal{M}_i(e) - D(i,e)), \forall e \in E \tag{4}$$

  where $\mathcal{M}_i = \mathcal{I} \cdot Q_i$ is a vector. And $\mu_1(\cdot)$ is a non-decreasing function.

- For the physical interference model, we have

  $$F_2(i,e) = \begin{cases} \mu_2(\mathcal{M}_i(e) - D(i,e)), & \forall e \in E \& IF \leq \dfrac{1}{\beta} \\ 0, & otherwise \end{cases} \tag{5}$$

  where $IF = \sum_{e' \in E \setminus \{e\}} \mathcal{I}(e,e') \cdot Z_i(e')$. $Z_i(e)$ denotes whether some flows other than the ith one select the path including link e, since we ignore the impact of one's own transmission on itself. The inequality $IF \leq \frac{1}{\beta}$ corresponds to the condition for a successful transmission on a link (see Formula (2)). $\mu_2(\cdot)$ is a non-decreasing function. The expected amount of packets transmitting on the link that interferes with link e implies its active time.

4.1.3 Switching

Since a switching operation may cause delay and energy consumption, we split switching cost into two parts, one is the switching delay cost, and the other is the switching energy cost. Similar to the related works, we introduce a binary vector $\mathcal{H}$ to store the pre-established links. Formally, $H_e = 1$ implies that link e was pre-established (but may be broken now as the channel assigned on it is reclaimed by PUs or for other reasons) and $H_e = 0$ otherwise. Note that the switching cost of one link is shared by all the flows routed on it. The switching delay cost of flow i on link e is given by

$$G_a(i,e) = \frac{\alpha_i}{Q(e)} \cdot \nu \cdot (1 - H_e) \cdot X_e^i \tag{6}$$

where ν is a constant number denoting the unit switching delay cost.

The switching energy cost of flow i on link e is given by

$$G_b(i,e) = \frac{\alpha_i}{Q(e)} \cdot \xi \cdot (1 - H_e) \cdot X_e^i \tag{7}$$

where ξ is a constant number denoting the unit energy cost per switching operation.

We denote $G(i,e)$ as the total switching cost, which is a function of the combination of $G_a(i,e)$ and $G_b(i,e)$.

4.1.4 Energy

Power consumption when flows running on a link (note that almost all the power consumption occurs at the transmission end of a link) is significant especially when the transmission rate is relatively high. We assume that the energy consumption of a link is shared by all the flows running on it, and we use a general function to model the energy consumption of flow i on link e. It is

$$H(i,e) = \frac{\alpha_i}{Q(e)} \cdot \eta \cdot X_e^i \tag{8}$$

We denote the total cost of the ith flow routing on link e as $j_1(i,e)$ and $j_2(i,e)$ for the protocol and physical model respectively, which can be calculated as follows, where $\hat{f}$ is also a non-decreasing function:

- When considering the protocol interference model,

$$j_1(i,e) = \hat{f}(D(i,e), F_1(i,e), G(i,e), H(i,e)). \tag{9}$$

- When considering the physical interference model,

$$j_2(i,e) = \hat{f}(D(i,e), F_2(i,e), G(i,e), H(i,e)). \tag{10}$$

Note that, since the switching delay is only 150ms in practical, the congestion cost and interference cost dominates the impact of the delay cost in the total cost. Besides, there is a tradeoff between the communication quality and the energy consumption. The communication quality is considered as more important than energy consumption in CRNs since the PUs may reclaim the spectrum resource at any time. Based on this, one will always expect to consider only the congestion cost and the interference cost, which makes the total cost of the ith flow routing on link e become

$$j_k(i,e) = f(D(i,e), F_k(i,e)), \quad k = 1 \textit{ or } 2. \tag{11}$$

where f is also a non-decreasing function, for the protocol and physical model respectively.

4.2 Path (routing) Cost

Based on the above, we consider only the dominant costs, the congestion and the interference cost. Now, we use the aggregation function to characterize the path cost.

Denote the strategy profile of all players as $A = \{p_1, p_2, \cdots, p_N\}$. Let A_i denote the strategy of player i, which actually is the path p_i it chooses. Set the strategy profile of all players other than player i as

$$A_{-i} = \{p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_N\}.$$

For the ith flow that chooses strategy A_i containing link $e_1(A_i), e_2(A_i), \cdots, e_k(A_i)$, the path cost is

$$\begin{aligned} PC_i(A) &= PC_i(A_i, A_{-i}) \\ &= ||j(i, e_1(A_i)), j(i, e_2(A_i)), \cdots, j(i, e_k(A_i))||_m \end{aligned} \tag{12}$$

where $||\cdot||_m$ is a l_m-norm function with $m \in [1, \infty)$, and $j(i, e_i(A_i)), \forall i \in \mathcal{N}, e_i \in A_i$ is the link cost of the ith flow regardless of the interference model adopted. Note that the path cost is an expected measurement. We evaluate the overall cost of a path by the expected end-to-end delay and the expected perceived receiving rate as follows.

a. **End-to-end Delay** The end-to-end delay refers to the overall delay for packets of one flow being transmitted in the CRNs from source to destination. A general function is given to characterize the expected end-to-end delay for flow i routing all of its packets on path p, as follows, which is a l_1-norm aggregation function.

- Under the protocol interference model, the end-to-end delay cost can be calculated by

$$\begin{aligned} PC_i(A) &= PC_i(A_i, A_{-i}) \\ &= \sum_{e \in A_i} j_1(i, e) \\ &= \sum_{e \in E} j_1(i, e) X_e^i \end{aligned} \tag{13}$$

  where $A_i = p_i \in \mathcal{P}_i$ and $X_e^i = 1, \forall e \in A_i$.

- Similarly, for the physical interference model, the end-to-end delay cost can be calculated by

$$\begin{aligned} PC_i(A) &= PC_i(A_i, A_{-i}) \\ &= \sum_{e \in A_i} j_2(i, e) \\ &= \sum_{e \in E} j_2(i, e) X_e^i \end{aligned} \tag{14}$$

  where $A_i = p_i \in \mathcal{P}_i$ and $X_e^i = 1, \forall e \in A_i$.

b. **Perceived Receiving Rate** We can also evaluate the path cost as the maximum link delay (i.e., the bottleneck) on the path, which is a a l_∞-norm aggregation function.

For the ith flow, packets are received by its destination node t_i at an expected rate. This expected rate is bounded by the maximal link delay on path p, which is regarded as a bottleneck of the receiving rate. Let $K(i,p)$ be the expected maximal link latency which is given by [9]

$$\hat{PC}_i(A) = E(\min_{e \in E}(j_k(i,e))X_e^i), \quad k = 1 \text{ } or \text{ } 2 \tag{15}$$

4.3 Game Formulation

Problem 1. *Consider the Cognitive Radio Networks environment, all fictitious flow generators need to start source routing to determine new routes from their sources to destinations respectively. Given the selfish nature, how can a flow choose its routing path to minimize its own cost?*

We can know from the problem definition that the route-selection of each flow depends on the decision of the other flows. So we formulate the problem as a game where each flow generator acts as a player and selects its route selfishly to minimize its path cost.

We use a tuple $\mathcal{G} = \{G, \mathcal{N}, \mathbf{P}, PC\}$ to formulate a routing game in the CRNs. Recall that G is the potential communication graph, $\mathcal{N}$ is the N-player set corresponding to the flows, and $\mathbf{P}$ is the strategy space. Each player $i \in \mathcal{N}$ has a strategy set $\mathcal{P}_i$ and a (path) cost function $PC_i : \mathbf{P} \to \mathbb{R}$. We give the definition of the pure-Nash equilibrium as follows.

Definition 1 (pure-Nash Equilibrium, pure-NE)**.** *A strategy profile A^* is called a pure-Nash Equilibrium if for all players $i \in \mathcal{N}$*

$$PC_i(A_i^*, A_{-i}^*) \leq PC_i(A_i, A_{-i}^*), \forall A_i \neq A_i^*,$$

which means that no player can reduce its cost by unilaterally changing its strategy at the equilibrium.

4.4 Existence of pure-NE

In this section, we show that the proposed routing game has pure-NE by proving that it is an ordinal potential game when considering end-to-end delay.

4.4.1 Potential Game

The potential game is a game model which generalizes a number of games, and its definition is given as follows.

[9] When $E(min(\cdot))$ needs to be calculated, the distribution of $(\cdot)$ must be given.

Definition 2 (Potential Game)**.** *A finite N-player game with strategy sets $\{\mathcal{P}_i\}_{i=1}^N$ and cost function sets $\{PC_i\}_{i=1}^N$ is a potential game if and only if for some potential function $\phi : \mathbf{P} \to \mathbb{R}$*

$$PC_i(A'_i, A_{-i}) - PC_i(A''_i, A_{-i}) = \phi(A'_i, A_{-i}) - \phi(A''_i, A_{-i})$$

for any player $i \in \mathcal{N}$, $A_{-i} \in \Pi_{j \neq i}\mathbf{P}_j$ and $A'_i, A''_i \in \mathcal{P}_i$.

Next we give the definition of ordinal potential game which is a more general class of potential games.

Definition 3 (Ordinal Potential Game)**.** *A finite player game is an ordinal potential game if and only if for some potential function $\phi : \mathbf{P} \to \mathbb{R}$*

$$\begin{aligned} & PC_i(A'_i, A_{-i}) - PC_i(A''_i, A_{-i}) < 0 \\ \Longrightarrow\ & \phi(A'_i, A_{-i}) - \phi(A''_i, A_{-i}) < 0 \end{aligned}$$

for any player $i \in \mathcal{N}$, $A_{-i} \in \times_{j \neq i}\mathbf{P}_j$ and $A'_i, A''_i \in \mathcal{P}_i$.

The properties of potential games have been well studied, and we illustrate the following particular one which we will refer to later on.

Lemma 1. *Every finite ordinal potential game possesses a pure-strategy Nash Equilibrium.*

Before finding a pure-Nash Equilibrium, one should prove the existence of an equilibrium in the game. This property gives us an aspect from which we can say there is a pure-NE. Note that there may be more than one pure-NEs in a game.

4.4.2 Existence of pure-NE in routing game

In this part, we show that our proposed routing game is a potential game, and thereby possesses a pure-NE.

Theorem 1. *Under the protocol interference model, given $j_1(i,e) = D(i,e) + F_1(i,e)$, $D(i,e) = \mathcal{I}(e,e)Q_i(e) + \mathcal{I}(e,\bar{e})Q_i(\bar{e})$, $F_1(i,e) = \mathcal{M}_i(e) - D(i,e)$, $\forall e \in E$, the routing game in CRNs is a finite ordinal potential game.*

Proof of Theorem 1. First of all, our proposed routing game has a finite number of players (flow generators) and each player has a finite number of strategies (paths). So we have Claim 1.

Claim 1. *The routing game in CRNs is a finite game.*

Next we prove that the game is an ordinal potential game. We denote one specific strategy profile (which corresponds to a point in space $\mathbf{P}$) as $A(a) = \{A_1(a), A_2(a), \cdots, A_N(a)\}$, where $A_i(a) = \{e | e \in A_i(a), X_e^i(a) = 1\}$ is the ith flow's strategy. Similarly, let $A_{-i}(a)$ denote the same strategy profile excluding the player i, and we have $A(a) = (A_i(a), A_{-i}(a))$. Then, the path cost is denoted as $PC_i(A_i(a), A_{-i}(a))$.

Lemma 2. *Under the protocol interference model, there is an ordinal potential function in the routing game in CRNs. The ordinal potential function can be*

$$\phi(a) = \sum_{i\in\mathcal{N}} \alpha_i PC_i(A(a)) \tag{16}$$

Proof of Lemma 2. Suppose for two strategy profile $A(a)$ and $A(b)$, we have

$$PC_k(A_k(a), A_{-k}(a)) < PC_k(A_k(b), A_{-k}(b)) \tag{17}$$

where $A_k(a) \neq A_k(b)$ and $A_{-k}(a) = A_{-k}(b)$.

Under the protocol interference model, we assume $\mathcal{I}(e,e') = \mathcal{I}(e',e)$. According to the formulae in Theorem 1, we can rewrite the path cost of the kth flow

$$PC_k(A) = \sum_{e\in A_i} j_1(k,e) = \sum_{e\in p}\sum_{e'\in E}\sum_{i\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_i X_{e'}^i \tag{18}$$

$$\begin{aligned}\phi(a) - \phi(b) &= \sum_{i\in\mathcal{N}} \alpha_i PC_i(A(a)) - \sum_{i\in\mathcal{N}} \alpha_i PC_i(A(b)) \\ &= L_1 + L_2\end{aligned} \tag{19}$$

where

$$\begin{aligned}L_1 &= \alpha_k PC_k(A(a)) - \alpha_k PC_k(A(b)) \\ &= \alpha_k \sum_{e\in A_k(a)}\sum_{e'\in E}\sum_{j\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_j X_{e'}^j(a) - \alpha_k \sum_{e\in A_k(b)}\sum_{e'\in E}\sum_{j\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_j X_{e'}^j(b) \\ &= \alpha_k \sum_{e\in E} X_e^k(a) \sum_{e'\in E}\sum_{j\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_j X_{e'}^j(a) - \alpha_k \sum_{e\in E} X_e^k(b) \sum_{e'\in E}\sum_{j\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_j X_{e'}^j(b)\end{aligned} \tag{20}$$

and

$$\begin{aligned}L_2 &= \sum_{i\in\mathcal{N}_k} \alpha_i PC_i(A(a)) - \sum_{i\in\mathcal{N}_k} \alpha_i PC_i(A(b)) \\ &= \sum_{i\in\mathcal{N}_k} \alpha_i \sum_{e\in A_i(a)}\sum_{e'\in E}\sum_{j\in\mathcal{N}_i} \mathcal{I}(e,e')\alpha_j X_{e'}^j(a) - \sum_{i\in\mathcal{N}_k} \alpha_i \sum_{e\in A_i(b)}\sum_{e'\in E}\sum_{j\in\mathcal{N}_i} \mathcal{I}(e,e')\alpha_j X_{e'}^j(b)\end{aligned} \tag{21}$$

Since $A_i(a) = A_i(b), \forall i \in \mathcal{N}_k$, we have

$$\begin{aligned}L_2 &= \sum_{i\in\mathcal{N}_k} \alpha_i \sum_{e\in A_i(a)}\sum_{e'\in E} [\sum_{j\in\mathcal{N}_i} \mathcal{I}(e,e')\alpha_j X_{e'}^j(a) - \sum_{j\in\mathcal{N}_i} \mathcal{I}(e,e')\alpha_j X_{e'}^j(b)] \\ &= \sum_{i\in\mathcal{N}_k} \alpha_i \sum_{e\in A_i(a)}\sum_{e'\in E} \mathcal{I}(e,e')\alpha_k [X_{e'}^k(a) - X_{e'}^k(b)] \\ &= \alpha_k \sum_{e'\in E} X_{e'}^k(a) \sum_{e\in E}\sum_{i\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_i X_e^i(a) - \alpha_k \sum_{e'\in E} X_{e'}^k(b) \sum_{e\in E}\sum_{i\in\mathcal{N}_k} \mathcal{I}(e,e')\alpha_i X_e^i(b) \\ &= L_1\end{aligned} \tag{22}$$

Then we have

$$\begin{aligned}\phi(a) - \phi(b) &= L_1 + L_2 \\ &= 2\alpha_k[PC_k(A(a)) - PC_k(A(b))] < 0\end{aligned} \tag{23}$$

According to Inequalities 17 and 23, and Definition 3, Lemma 2 is proved. □

Based on the above claim and lemmas, Theorem 1 is thereby proved. □

To study the game in the physical interference model, we present the following theorem.

Theorem 2. *Under the physical interference model, given*

$$\begin{cases} j_2(i,e) = D(i,e) + F_2(i,e) \\ D(i,e) = \mathcal{I}(e,e)Q_i(e) + \mathcal{I}(e,\bar{e})Q_i(\bar{e}) \\ F_2(i,e) = \begin{cases} \mathcal{M}_i(e) - D(i,e), & \forall e \in E_Y \quad \& \quad IF \leq \dfrac{1}{\beta} \\ 0, & otherwise \end{cases} \end{cases} \tag{24}$$

, the routing game in CRNs is a finite ordinal potential game.

Proof. Since the type of path cost function under the physical model is similar to that under the protocol model, the proof proceeds in the same fashion as that of Theorem 1. We omit it here because of limited space. □

Therefore, we conclude that there is at least one pure-NE in our proposed routing game in CRNs.

The existence of the pure-NE when considering perceived receiving rate in the proposed game is left for the future work.

5 Learning Design

In this section, we present **Algorithm 1** which simulates the *Fictitious Play* (FP) process in game theory under the protocol interference model. The general idea would be that each player locally runs this algorithm to determine its own best strategy by simulating other players' possible actions. Under complete information, every player's information is completely exposed to others, so the Fictitious Play will converge to the real play process. The game with incomplete information is beyond the scope of discussion in this paper. In **Algorithm 1**, we convert the reduction of the potential function into finding the shortest path in an directed graph, where such a path is found by applying *Dijkstra's Algorithm*. After finding the shortest path for flow i in one iteration, we update $\mathcal{X}$, the interference matrix $\mathcal{I}$ and the matrix for expected number of packets $\mathcal{Q}$. Then we compute the path cost PC_i for flow i accordingly. The weight of edge e for flow i is give by:

$$j(i,e) = D(i,e) + F(i,e) \tag{25}$$

where $D(i,e) = \mathcal{I}(e,e)\mathcal{Q}_i(e) + \mathcal{I}(e,\bar{e})\mathcal{Q}_i(\bar{e})$, $F(i,e) = \mathcal{M}_i(e) - D(i,e)$, $\forall e \in E$ PC_i (cost of the path that player i selects) in terms of end-to-end delay is given according to (12), where m is set to 1.

Algorithm 1 Find the best strategy A_i for flow i at the Nash Equilibrium (executed by the generator of flow i, $\forall i \in N$)

1: Initialize $\forall i \in \mathcal{N}$, $e^c_{k(u,v)}, e^c_{j(u',v')} \in E$, $\mathcal{X}^i_e = 0$, $\mathcal{I}(k,j) = 0$, $\mathcal{Q}_i(e) = 0$, $\forall c \in \mathcal{C}$, $\phi_0 = \infty$, $n = 0$, $i = 0$, $m = 0$
2: **while** $m < N$ **do**
3: Update iteration counter: $n = n + 1$;
4: Call *Dijsktra's Algorithm* to find the shortest path p_i from s_i to t_i;
5: Update $\mathcal{X}$, $\mathcal{I}$, $\mathcal{Q}$ according to the selected path p_i;
6: Compute PC_i according to (12), where the cost $j(i,e)$ for each edge along p_i is updated according to (25);
7: Compute ϕ_n according to the shortest path;
8: **if** $\phi_n < \phi_{n-1}$ **then**
9: Update the action of flow i accordingly;
10: Set: $m = 0$;
11: **else**
12: Set: $m = m + 1$, $\phi_n = \phi_{n-1}$;
13: **end if**
14: Update flow index: $i = (i + 1) mod N$
15: **end while**
16: $A_i = \{e|\mathcal{X}^i_e = 1, e \in E\}$;

Variable m in **Algorithm 1** acts like a counter that records the consecutive times for which players cannot reduce the potential function, and the stop condition implies that all N players cannot reduce the potential function, which indicates the Nash Equilibrium is reached. From step 8 to 12 we can see that a player's strategy is updated only when it can reduce the potential function, otherwise it keeps its previous strategy.

As for the time complexity, during each iteration, *Dijkstra's Algorithm* is of $O(|V|^2)$ time, while updating $\mathcal{X}$, $\mathcal{I}$, $\mathcal{Q}$ for flow i takes $(O|E|)$. In addition, when computing PC_i, we need to go through all other $(N-1)$ players' strategies, which amounts to $O(|E|N)$ of time. Therefore, the overall time complexity for each iteration is $O(|V|^2 + |E|N)$.

6 Price of Stability

In this section we will measure the inefficiency of equilibria for the game in terms of *Price of Stability* (PoS) by comparing the Nash Equilibrium with the socially optimal result. In our routing game, social cost is defined as following:

Definition 4 (*Social Cost*)**.** *Social Cost is the sum of all player's overall costs,* *i.e.*,

$$SoC(A(a)) = \sum_{i \in \mathcal{N}} PC_i(A(a)) \tag{26}$$

Then we introduce the definition of *Price of Stability* in the game.

Definition 5 (*Price of Stability*)**.** *Price of Stability is the ratio of social costs between the best Nash Equilibrium and the the optimality in centralized schemes, i.e.,*

$$PoS = \frac{SoC(A*)}{min_A SoC(A)} \tag{27}$$

7 Simulations

7.1 *Simulation Settings*

In this paper we use Java as the simulation tool. For the network topology, we generate a random network in which each node is assigned with a point (x,y), where x and y are integers randomly selected from the range 0 to 500. The transmission range for each node is 200, while the interference range is 250. The source node and destination node for each flow are picked randomly from all nodes. In addition, the number of channels and nodes are fixed to be 5 and 20 respectively.

7.2 *Simulation Results*

a. Finite Improvement Property

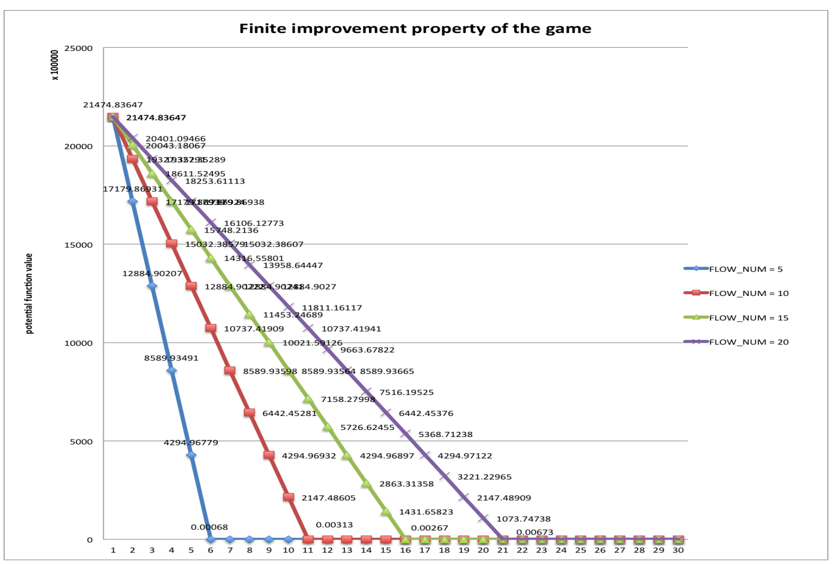

Figure 1. Finite Improvement Property of the proposed game, average from 50 experiments

We first simulate the Finite Improvement Property of the Route Switching Game, shown in Figure 1. We set the flow number to 5, 10, 15, 20 respectively. Initially, each flow generator is assigned with a route in the graph with specific source and destination nodes, and each of their route cost is set to infinity, which results in a infinitely large potential value.

After each improvement step, with each flow generator updating its routes according to the learning strategy, the potential value is gradually reduced, and eventually converges to a small value close to zero. Although the game with more flows converge more slowly, the Nash Equilibrium can still be reached quickly (around 21 iterations for 20 flows).

b. Comparison with socially optimal results (PoS Analysis)

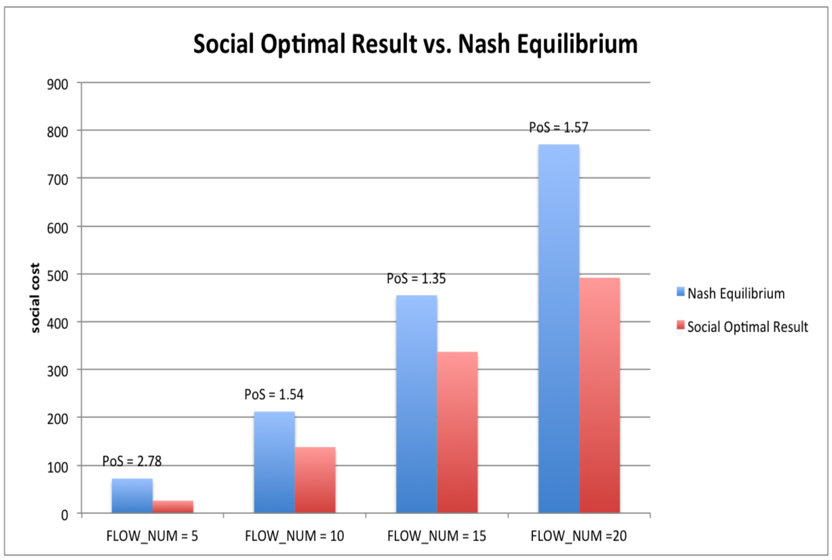

Figure 2. Social Optimal Result vs. Nash Equilibria in terms of social costs, average from 50 experiments

In this section, we compare the socially optimal results with Nash Equilibria of the game. The socially optimal results are obtained by exhaustive search, while the Nash Equilibria are obtained by Algorithm 1. We use the average Nash Equilibrium obtained from 50 simulations for each flow number respectively. From Figure 2 we can observe that though Nash Equilibria are greater then social optimal results, the average PoS is below 1.9, which indicates that the performance gap is small.

8 Conclusion

In this paper, we investigate the *spectrum mobility* problem in multi-hop CRNs, where a routing scheme that combines channel switching and route re-selection is proposed. Under both protocol and physical interference models, we form the routing problem into a routing game, which is then proved to be a potential game with a pure Nash Equilibrium (NE). Next, we present an efficient algorithm as the learning design for finding the NE, and analyze the *Price of Stability* of the proposed game. At last, simulation results show the validity of our theoretical analysis and feasibility of the proposed game model.

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