

$$\boxed{|\tau| = T}$$

$$\begin{aligned} |\alpha| &= \alpha \\ |\tau_1| \rightarrow |\tau_2| &= |\tau_1| \rightarrow |\tau_2| \\ |\forall \alpha. \tau| &= \forall \alpha. |\tau| \\ |\tau_1 \& \tau_2| &= (|\tau_1|, |\tau_2|) \\ |\{l:\tau\}| &= |\tau| \end{aligned}$$

Figure 1: Type translation.

$$\begin{aligned} &\boxed{\tau <: \tau \hookrightarrow C} \qquad \frac{}{\alpha <: \alpha \hookrightarrow \lambda(x:|\alpha|).x} \text{subvar} \\ &\frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \quad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \rightarrow \tau_2 <: \tau_3 \rightarrow \tau_4 \hookrightarrow \lambda(f:|\tau_1 \rightarrow \tau_2|). \lambda(x:|\tau_3|). C_2 (f (C_1 x))} \text{subfun} \\ &\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall \alpha_1. \tau_1 <: \forall \alpha_2. \tau_2 \hookrightarrow \lambda(f:|\forall \alpha. \tau_1|). \lambda \alpha. C (f \alpha)} \text{subforall} \\ &\frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \quad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \& \tau_3 \hookrightarrow \lambda(x:|\tau_1|). (C_1 x, C_2 x)} \text{suband} \\ &\frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_1 x)} \text{suband}_1 \\ &\frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_2 x)} \text{suband}_2 \\ &\frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l:\tau_1\} <: \{l:\tau_2\} \hookrightarrow \lambda(x:|\{l:\tau_1\}|). C x} \text{subrec} \end{aligned}$$

Figure 2: Coersive subtyping.

$$\begin{array}{c}
\boxed{\gamma \vdash e : \tau \hookrightarrow E} \qquad \frac{(\mathbf{x}, \tau) \in \gamma}{\gamma \vdash \mathbf{x} : \tau \hookrightarrow \mathbf{x}} \text{Evar} \\
\\
\frac{\gamma, \mathbf{x} : \tau \vdash e : \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash \lambda(\mathbf{x} : \tau). e : \tau \rightarrow \tau_1 \hookrightarrow \lambda(\mathbf{x} : |\tau|). E} \text{Elam} \\
\\
\frac{\gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \quad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1 \ e_2 : \tau_2 \hookrightarrow E_1 \ (C \ E_2)} \text{Eapp} \\
\\
\frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \lambda \alpha. e : \forall \alpha. \tau \hookrightarrow \lambda \alpha. E} \text{Eblam} \qquad \frac{\gamma \vdash e : \forall \alpha. \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash e \ \tau : [\tau/\alpha] \tau_1 \hookrightarrow E \ |\tau|} \text{Etapp} \\
\\
\frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2}{\gamma \vdash e_1, e_2 : \tau_1 \ \& \ \tau_2 \hookrightarrow (E_1, E_2)} \text{Emerge} \\
\\
\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E} \text{Erec-con} \qquad \frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C \ E} \text{Erec-sel} \\
\\
\frac{\gamma \vdash e : \tau \hookrightarrow E \quad \gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \quad \tau \blacktriangleleft \{l : \tau_1 \hookrightarrow E_1\} = \tau_2[\tau_3] \hookrightarrow C \quad \tau_1 <: \tau_3}{\gamma \vdash e \text{ with } \{l = e_1\} : \tau_2 \hookrightarrow C \ E} \text{Erec-upd} \\
\\
\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C} \qquad \overline{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(\mathbf{x} : |\{l : \tau\}|). \mathbf{x}} \text{get} \\
\\
\frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(\mathbf{x} : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_1 \mathbf{x})} \text{get}_1 \\
\\
\frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(\mathbf{x} : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_2 \mathbf{x})} \text{get}_2 \\
\\
\boxed{\tau \blacktriangleleft \{l : \tau \hookrightarrow E\} = \tau_2[\tau_3] \hookrightarrow C} \\
\\
\overline{\{l : \tau\} \blacktriangleleft \{l : \tau_1 \hookrightarrow E\} = \{l : \tau_1\}[\tau] \hookrightarrow \lambda(_ : |\{l : \tau\}|). E} \text{put} \\
\\
\frac{\tau_1 \blacktriangleleft \{l : \tau \hookrightarrow E\} = \tau_3[\tau_4] \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \blacktriangleleft \{l : \tau \hookrightarrow E\} = \tau_3 \ \& \ \tau_2[\tau_4] \hookrightarrow \lambda(\mathbf{x} : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_1 \mathbf{x})} \text{put}_1 \\
\\
\frac{\tau_2 \blacktriangleleft \{l : \tau \hookrightarrow E\} = \tau_3[\tau_4] \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \blacktriangleleft \{l : \tau \hookrightarrow E\} = \tau_1 \ \& \ \tau_3[\tau_4] \hookrightarrow \lambda(\mathbf{x} : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_2 \mathbf{x})} \text{put}_2
\end{array}$$

Figure 3: Elaboration typing from $F_{\&}$ to System F.