Cardano Economic Parameters

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Decentralized Governance of Blockchain networks allows the participation of community members to crucial technical and economic decisions for the future of the ecosystem. In Cardano, the network technology and the monetary distribution are embedded in the code of the protocol, and depend on parameters that simplify decision-making and automate implementation. To make this work well in practice, the community needs as full an understanding of the system they are governing as possible. This means building an ecosystem of decentralized knowledge, analysis, and testing for the benefit of community members and representatives.

To contribute to this goal, we present the results of a research carried out on Cardano economic parameters. In this paper we focus on pool incentives, providing a detailed analysis of the effect of parameters on the rewards of stake pools with different size and pledge, and the interplay with other ecosystem quantities, including reserves and their distribution. To the best of our knowledge, such a joint analysis was not available so far. Existing documentation focuses on different parts of the system separately, describing, for instance, the pledge parameter separately from the one that sets the desired number and size of pools, and both separately from those affecting monetary expansion and treasury dynamics. However, interactions between parameters have a significant impact, and only a joint

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analysis can fully reveal the consequences of parameter-related decisions, which is essential in the new decentralized governance setting.

Summary and Roadmap

In Section 1 we start from the description of parameters k, the desired number of pools determining the saturation level (stake above which no additional rewards are paid), and a_0, the parameter deciding the effect of the pledge (own stake provided by pool operators and not by delegators) on rewards, as given in existing documentation. Then we analyze the effects of the parameterization in more detail. We show how it reduces rewards not only for pools larger than saturation, but also for those that are smaller than that: the benefit of the pledge is extremely reduced for small and medium pools compared to very large ones. For smaller pools, having a larger pledge, at equal stake, can even reduce rewards slightly, an effect which disappears only for pools with a size of at least half of saturation. The saturation stake was around 70M ADA (Cardano native currency) in 2024.

Next, we describe the tokenomic rationale and effects. This is an incentive for pools to increase the stake until saturation level, the one corresponding to the desired number of pools. But the incentive to small pools is to increase the stake first by delegation rather than by pledge. Only large pools have a clear reward increase if they consist of own stake rather than delegated capital. This discourages large owners from trying to game the system by splitting their stake into many pools, and rewards the security value of large commitments of own money. This can increase security and efficiency, but also have a negative effect on inclusion and decentralization, by reducing the rewards of small pool operators, and of delegators of all pools. Governance must keep the balance by acting on k and a_0, or just on one of them, but being always aware of the interactions.

The effect can be appreciated synthetically through Figure 1.1, where every line shows the rewards of pools with same stake size but different percentages of pledge. The different lines represent different stake sizes. Then we confirm these results with a mathematical analysis of the rewards formula and some numerical examples, together with charts detailing further the behavior of rewards at different stakes and pledges.

In Section 2 we describe the additional adjustments made to rewards based on pool performance and active stake. Again we start from the description given in existing documentation, and then we detail and analyze the practical consequences. We point out that, for ease of understanding, the computation of the total amount of rewards can be split in two main components. One is driven by the staking incentive parameters of Section 1 (f), the other one by the ratio between the active stake and the circulating supply (p). Measuring the impact of these components in time, we see in Figure 2.1 that the second component had more impact. We show this component exists because the circulating supply is used as a measure of total stake for rewards, in contrast with what is done for consensus, where active stake is used as an aggregate measure. We describe the tokenomic rationale and the effects of this choice, that makes rewards different from those of other chains, such as Bitcoin. It gives stability and predictability to the rewards of a single pool, but less stability and predictability to the total amount of rewards. We show how this should be monitored by taking into account both the impact on the stability of the aggregate active stake, and the effect on the behavior of single validators. A detailed mathematical analysis closes the section.

In Section 3 we analyze the actual composition of the staking pools of Cardano, to investigate the interplay between the incentive parameterization and how stake and pledge are distributed across pool. We find that the global amount of pledge is rather low, and that stake and pledge are unevenly distributed among pools of various sizes, with the majority of the stake - and the portion in own stake - concentrated in the largest pools. We show that this is quite consistent with the effect of the current incentive parameters detailed in the previous two sections. At the end, we develop a rough estimate of rewards based only on very synthetic statistics extracted from the distribution of the pool (see Figure 3.3), and check that this is in line with the actual aggregate rewards, providing some further explanation about the core determinants of pool composition and aggregate rewards.

In Section 4 we analyze how the incentives seen in the previous sections influence the dynamics of the Reserves and their distribution to Rewards and Treasury, making them very different from descriptions that overlook the interactions of parameters. The release of reserves every epoch does not trivially follow the monetary expansion parameter $\rho = 0.3\%$, nor the treasury expansion parameter $\tau = 20\%$ reveals the actual partition between Treasury increases and Rewards. In reality, due to the fraction which is retained from Rewards distribution, mostly as an incentive to grow stake and pledge or to rescale rewards by the active stake, the actual release has been between 0.165% and 0.185% rather than 0.3%, as shown in Figure 4.3. Additionally, while based on τ only Rewards should receive $1 - \tau = 80\%$ of the release, they actually got just between 65% and 70%, with Treasury taking a fraction between 30% and 35%, as shown in Figure 4.1 Thus the proportion between the growth of Treasury and the Rewards, that could be expected to be $\frac{\tau}{1-\tau} = 25\%$, has often been more than 50%. These effect can be broken down further, to calculate how Reserves and the other digital allocations would behave if the above incentives were different or absent.

While this section comes at the end of the analysis, community members will find it at least as relevant as the previous ones, if not more, as it gives useful indications for realistic projections on the future of the network and its sustainability. Some of the results shown will be useful also to analyze the finite-difference or differential equations to be used in simulation, and the statistical properties and stochastic analysis that can make simulation robust, a topic to be extended in future research.

1 Pool Incentives for Stake and Pledge

We start our analysis from the core mechanism of a blockchain ecosystem, the reward system for transaction validators. Cardano's consensus algorithm is based on proof-of-stake, with stake pools aggregating own stake of pool operators, aka *pledge*, plus stake delegated by other participants that want to stake their ADA without running a node. Validation reward systems have at least two goals: making validation sustainable and economically competitive, and incentivize commitment to network security in a decentralized way. Cardano reaches this goal with a parametric system. The general description of the parameters is given in Cardano (2025). More insights are given in Kant et al. (2020). The derivation of part of the system as a Nash equilibrium can be found in Brünjes et al. (2020). Related discussions can be found in Cardano Community (2024); Cardano (2025).

Overview and Economics

Here we focus on the effect of a change of the parameters k, called 'desired number of pools', and a_0, called 'pool pledge influence', on the rewards of a pool. Both have a strong impact on Cardano tokenomics. Parameter k affects the impact on rewards of the size of the stake of a pool, that includes the own tokens pledged by pool operators, and the additional stake delegated to them. Parameter a_0 affects the impact of the pledge on rewards.

Desired number of pools. Documentation in Cardano (2025), that gives a high level overview of the workings of the system, explains that rewards increase with stake, but stop increasing once the stake reaches the point of *saturation*, given by the total possible stake (aka circulating supply S) divided by the desired number of pools k. This is aimed at penalizing pools that are larger than the desired size. We show in the following that the incentive system also penalizes pools that are smaller than the desired size. The size of this penalization depends on the interaction of a_0 and k.

Pool Pledge Influence. Documentation in Cardano (2025) explains that, if a_0 is zero, the pool reward is proportional to the pool's fraction of the total possible stake (up to the point of saturation), while for larger values of a_0, the pledge becomes more important. We show here that the effect of a_0, and therefore the incentives to increase the pledge, are different for pools of different size, and they are aimed mostly at pools that already have a stake near to the desired saturation stake. Small pools receive little or no additional rewards for increasing the pledge, as shown in Figure 1.1. Increasing the pledge displays the commitment of pool operators and makes all pools more attractive for delegators, but in the reward system smaller pools are implicitly incentivized to increase their stake by increasing first delegated stake, rather than own stake.

Description. Pools with no pledge are penalized by considering only a fraction $\frac{1}{1+a_0}$ of their share of the stake for the purpose of reward computation. Since currently $a_0 = 0.3$, we have $\frac{1}{1+a_0} \approx 77\%$. To reduce this disadvantage, it is not enough to increase the pledge. Pools need first to increase their stake until it is a significant fraction of the saturation size, to have an unambiguous increase of rewards from increasing the pledge. For example, as one can see in Figure 1.1, pools with a stake which is 10% of the saturated pool, [1] or less, receive around 7.7% of the available rewards, rather than a full 10%, even if they increase the pledge to 100% of the stake. A high pledge percentage can even marginally reduce rewards, as explained later. For larger stakes, the benefit of a higher pledge increases gradually, until the stake reaches 50% of the saturation level, when the pledge becomes beneficial in any amount. But only fully saturated pools, by increasing the pledge towards 100%, can reach their full share of rewards ($1/k$).

Economic Rationale. The current parameterization means that large, individual pools built on own stake earn a higher return on the pool's total stake, compared to small pools (no matter their pledge) or to large pools based more on delegated stake. On one hand, this can be undesirable in terms of inclusivity and decentralization. On the other hand, this can be justified by the fact that small pools, by having less skin in the game, are often less of a guarantee for security. No matter their pledge, small pools remain an easy target for an attacker, who

[1] In the last few years the average saturation level has been around 70M ADA, so a 10% pool had a stake of around 7M ADA. See also Section 3.

can easily deploy resources higher than their stake. It's more sensible for them to grow by attracting delegate capital, in which case their contribution to security also grows, thanks to the acquired capital, and to the need to attract delegators, which also works as an incentive to honest behavior. Additionally, large pools with small pledge can compensate a smaller total return with the smaller size of the own capital they commit, leading to returns on capital that can be much higher than the rewards of equivalent fully pledged pools. The level of the two parameters a_0 and k has to be determined jointly to balance these different aspects.

Some Maths

The effect of a_0 and k on the rewards of a pool depends on the incentive formula

$$f(\alpha, z, \sigma, \lambda) = \frac{1}{1+\alpha}\left(\sigma + \alpha\lambda\frac{\sigma - \lambda\frac{z-\sigma}{z}}{z}\right), \tag{1.1}$$

where we assume that pools are not saturated, but for simplicity do not write this constraint explicitly. In this formula, $\alpha = a_0$ and σ and λ are the active stake and the own stake or pledge of the pool. The quantities σ and λ are both expressed as fractions of the circulating supply S (aka total stake), and so is the saturation level $z = \frac{S}{k}$. For example, for a pool x with a stake of $stake^x$ ADA, we have $\sigma = \frac{stake^x}{S}$. The output of the formula is meant to represent the fraction of the total available rewards that the pool will receive, save for a minor adjustment that we will see later.

The formula is easier to interpret if rewrite it showing how the variables defining the composition of the pool, σ and λ, appear in this function at different orders (or powers)

$$f(\alpha, z, \sigma, \lambda) = \frac{1}{1+\alpha}\sigma + \frac{\alpha}{1+\alpha}\left[\sigma\frac{\lambda}{z} + \sigma\left(\frac{\lambda}{z}\right)^2 - z\left(\frac{\lambda}{z}\right)^2\right]. \tag{1.2}$$

We can also use more intuitive variables. Let us redefine the pledge as a fraction of the stake of the pool, $\lambda_\% = \frac{\lambda}{\sigma}$, and the stake of the pool as a fraction of the stake at saturation, $\sigma_\% = \frac{\sigma}{z}$.

Numerical Example With these new variables, if the stake at saturation is $70M$ and the stake of our pool is $7M$, with an amount of pledge of $1.4M$, then $\sigma_\% = 10\%$ and $\lambda_\% = 20\%$. The example is realistic. From maturity of the staking system around epoch 300, until epoch 470, the average stake ranged between 6,893,338 and 7,950,258 ADA. In the same period, the average pledge ranged between 1,186,745 and 1,657,484 ADA. The circulating supply moved from almost 34B ADA to almost 37B. With $k = 500$, the size of a saturated pool averaged around 70M, so the above percentages represent a realistic average Cardano pool. In the following we will use $\sigma_\% = 10\%$ and $\lambda_\% = 20\%$ as an example to fix ideas, and then we will consider what happens for smaller or larger pools.

With the new variables,

$$\begin{aligned} f(\alpha, z, \sigma_\%, \lambda_\%) &= \frac{1}{1+\alpha}\sigma_\% z + \frac{\alpha}{1+\alpha}\left[\sigma_\%^2\lambda_\% z - \sigma_\%^2\lambda_\%^2 z + \sigma_\%^3\lambda_\%^2 z\right] \\ &= \left(\frac{1}{1+\alpha}\sigma_\% + \frac{\alpha}{1+\alpha}\left[\sigma_\%^2\lambda_\% - \sigma_\%^2\lambda_\%^2 + \sigma_\%^3\lambda_\%^2\right]\right) z \end{aligned}$$

The last passage allows us to focus on the rewards of a pool as a fraction of the rewards of the desired or optimal pool, which is fully pledged and saturated, and therefore has size $z = 1/k$, and also receives $z = 1/k$ of the total available rewards. Thus we define

$$f_\%(\alpha, z, \sigma_\%, \lambda_\%) = \frac{f(\alpha, z, \sigma_\%, \lambda_\%)}{z} = \left(\frac{1}{1+\alpha}\sigma_\% + \frac{\alpha}{1+\alpha}\left[\sigma_\%^2\lambda_\% - \sigma_\%^2\lambda_\%^2 + \sigma_\%^3\lambda_\%^2\right]\right).$$

This makes it easier to understand how rewards depend on parameters. The lead term in $f_\%$ is $\frac{1}{1+\alpha}\sigma_\%$. If there is no pledge, that's all a pool receives. For a pool with $\sigma_\% = 10\%$, this means that potential rewards are reduced by a factor $\frac{1}{1+\alpha} \approx 0.77$, for a total of $0.1 * 0.77 = 0.077 = 7.7\%$ of the rewards of the optimal pool.

Having a higher pledge fraction would not help so much at this size of the pool, since all following terms are of larger order in $\sigma_{\%}$ and $\lambda_{\%}$ overall. When small fractions are multiplied or raised to powers, their values decrease further. For the above example pool, the second biggest term, of order 3 overall, is $\frac{\alpha}{1+\alpha}\sigma_{\%}^2\lambda_{\%}$. With current $\alpha = 0.3$, we get

$$\frac{\alpha}{1+\alpha}\sigma_{\%}^2\lambda_{\%} = 0.23 * 0.1^2 * 0.2 = 0.00046 = 0.046\%.$$

This term is already very small compared to the leading term. The next term is smaller and negative:

$$-\frac{\alpha}{1+\alpha}\sigma_{\%}^2\lambda_{\%}^2 = -0.23 * 10\% * 10\% * 20\% * 20\% = -0.009\%$$

The last term is of even higher order and even smaller in absolute value, albeit positive:

$$\frac{\alpha}{1+\alpha}\sigma_{\%}^3\lambda_{\%}^2 = 0.23 * 0.1^3 * 0.2^2 = 0.0009\%.$$

Thus the contribution of the 20% of own stake is to move rewards from 7.7% of the rewards of a saturated pool to 7.73%, a 0.03% addition. If either the stake or the pledge were even smaller than this average, the effect of the pledge would be even less relevant, since all non-leading terms would be nearer to zero, the only term where the order of the pledge is not smaller than the order of the stake is the negative one.[2]

Large pools. For larger pools the pledge becomes more relevant. However, it is not until the stake $\sigma_{\%}$ approaches 50% of the stake of the saturated pool that increasing the pledge has a significant effect, with the stakes near to saturation having the clearest benefit. It's easy to test that when $\sigma_{\%} = 100\%$ of the saturation

[2]As shown at the end of this section, this term can even result in an increase in the pledge having a negative impact on the rewards.

level, the formula becomes

$$f_{\%}\left(\alpha, z, \sigma, \lambda\right) = \left(\frac{1}{1+\alpha} + \frac{\alpha}{1+\alpha}\lambda_{\%}\right)$$

and increasing the pledge till 100% of the stake increases the rewards percentage linearly to 100%.

Conclusions. Based on this analysis, the effect of a change in $a_0 = \alpha$ or $k = 1/z$ is clearer. Reducing/increasing a_0 reduces/increases the penalization for pools that are smaller than the size of the saturated pool, and makes the effect of increasing the pledge less/more relevant, particularly for pools nearer to saturation stake. Reducing/increasing k increases/reduces the size of the saturation stake, therefore changing the reference size that affects the behavior of a_0. Furthermore, the saturation stake is also the maximum size above which rewards are terminated.

The charts in section 1 on page 30 confirms. The top left chart considers all possible levels of stake and pledge. For ease of analysis, the other charts zoom in more narrow stake ranges, specifically pools with stake from 0 to 2% of the saturation stake (top right), stake between 20% and 25% (bottom left), and finally stake from 50% to 100% (bottom right). We can also see that rewards decrease when pledge increases, for fixed levels of the stake lower than 50% of saturation. This effect is explained in the following.

Further Analysis

It is useful to compute the first derivative of the rewards function (1.2) with respect to pledge $\lambda_{\%}$, given by

$$\frac{\partial f_{\%}\left(\alpha, z, \sigma_{\%}, \lambda_{\%}\right)}{\partial \lambda_{\%}} = \left(\frac{\alpha}{1+\alpha}\left[\sigma_{\%}^2 - 2\sigma_{\%}^2\lambda_{\%} + 2\sigma_{\%}^3\lambda_{\%}\right]\right).$$

A function grows when its first derivative is positive, and goes down if the derivative turns negative. The first derivative turns from positive to negative when

$$\lambda_{\%} \geq \frac{1}{2\left(1-\sigma_{\%}\right)} \equiv \lambda_{\%}^{*}\left(\sigma_{\%}\right)$$

This means that rewards grow with $\lambda_{\%}$ until $\lambda_{\%} = \lambda_{\%}^{*}$, but, for pledges larger than this value, increasing the pledge reduces rewards. The value $\lambda_{\%}^{*}\left(\sigma_{\%}\right)$ at which a higher pledge proportion starts reducing rewards depends on $\sigma_{\%}$, the size of the pool relative to the saturated pool. $\lambda_{\%}^{*}\left(\sigma_{\%}\right)$ grows with $\sigma_{\%}$, and when $\sigma_{\%}$=50% we have $\lambda_{\%}^{*}$=100%. Only at this level of stake increasing the pledge always increases rewards, since in practice pledge cannot be higher than 100% of the stake. For lower stakes, there will always be a level of the pledge that it is better not to reach, since beyond that level more pledge means less rewards, as shown in section 1 on page 31. For $\sigma_{\%}$=1% the absolute effect of the pledge is so small that it's hardly relevant if the effect is positive or negative. But for $\sigma_{\%}$=10% and then $\sigma_{\%}$=20% it becomes increasingly visible that, when pledge is increased, rewards grow slightly until the point when the derivative crosses the x-axis and becomes negative. Beyond that point, rewards decrease slightly if the pledge is increased. For $\sigma_{\%} \geq$50%, the rewards become uniformly increasing with pledge. Indeed, at $\sigma_{\%}$ =50% the first derivative crosses the x-axis and becomes negative only at at 1, that is 100% pledge.

2 The effect of Stake definitions

The incentives emerging from the application of (1.1) to all pools are not the last adjustment made to available resources before paying them out as rewards. As Cardano (2025) says, "the rewards that are produced by this formula are now adjusted by pool's performance: we multiply by $\frac{\beta}{\sigma_a}$, where β is the fraction of all blocks produced by the pool during the epoch and σ_a is the stake delegated to the pool relative to the active stake ". The more technical reference Kant et al. (2020) says "The actual rewards take the apparent performance into account, and are given by $\bar{p} * f()$. Since $\bar{p} = \beta/\sigma$, this nearly amounts to replacing σ by β in $f()$, i.e. to rewarding pools based on the number of blocks that they produced".

Let us define this final adjustment precisely. It consists of multiplying the output of the incentive formula for a pool x, with a stake of $stake^x$ ADA, by

$$\frac{N^x}{N}\frac{active}{stake^x} \tag{2.1}$$

where N^x the number of blocks produced by the pool x, N is the total number of blocks given by the sum of N^x for all pools, and *active* is the total amount of stake doing validation, given by the sum of $stake^x$ for all pools. Individually, this adjustment takes into account of the technological efficiency of a pool in producing blocks. In terms of aggregate effect on the total amount of rewards distributed, the adjustment is small. According to the consensus protocol, in an idealized setting where all pools operate with perfect efficiency and the realization of randomness matches probabilistic expectations, the pool's fraction of blocks $\frac{N^x}{N}$ would track the pool's fraction of stake $\frac{stake^x}{active}$ and (2.1) would be very near to 1. In practice, this remains true on average.

Yet, introducing explicitly the pool performance and the active stake in the adjustments helps us understand them better. We saw in Section 1 that the incentive formula (1.1) depends on the relative stake defined as a fraction $\sigma = \frac{stake^x}{S}$ of the total stake or circulating supply S. This definition of the weight of a pool used for rewards is different from the one used in the protocol to select pools for validation, which is instead $\sigma_a = \frac{stake^x}{active}$, determining $\frac{N^x}{N}$. The latter are definitions of the relative weight of a pool that sum up to 1 if one considers all pools. Instead, $\sigma = \frac{stake^x}{S}$ weights do not sum up to 1 when one considers all pools, but just to

$$p = \frac{active}{S}.$$

If σ_a was used instead of σ, there would be no adjustment if all pools were fully saturated and pledged. Instead with σ, even if all pools were ideal, there would be in any case an adjustment to the total amount of resources before paying them out as rewards, given by p. Thus in practice the adjustments include not only the incentives for the pools to have the desired size, pledge and efficiency, but also a rescaling by p.

It useful to represent the aggregate effect of the adjustments, given by summing $\frac{N^x}{N}\frac{active}{stake^x} f\left(\alpha, z, \sigma^x, \lambda^x\right)$ for all pools, as $p * f$, where p is always the ratio between active stake and circulating supply, while f is derived implicitly from knowledge of the aggregate adjustment and of p. Since these quantities can change at every epoch i, we add an epoch index and write

$$Rewards_i = f_i * p_i * AvailableRewards_i.$$

At the end of this Section we will see that this is almost equivalent to compute f_i by replacing, for each epoch and each pool, $\sigma_i^x = \frac{stake_i^x}{S_i}$ with $\frac{stake_i^x}{active_i}$ in the incentive formula, then sum up these adjustments $f\left(\alpha, z, \frac{stake_i^x}{active_i}, \lambda_i^x\right)$ for all pools. Having redefined the stake, now this aggregate adjustment can reach 1 if pools have the desired size and are fully pledged, therefore it is finally rescaled by p.

Economic relevance. Why the final passage? What's the effect of rescaling by p? This final rescaling changes the behavior of Rewards when the active stake moves. Without rescaling, the total amount of rewards paid out in every epoch would be adjusted only by f and would not change when the active stake changes. If the active stake went down, the remaining stake would share the same total rewards with a higher yield for each unit of stake, and the other way around if the stake increased.

Rescaling by p neutralizes this effect, by reducing rewards proportionally when the active stake decreases, and increasing them proportionally when the active stake goes up. The advantages of this rescaling are the stability of rewards for each pool, and more importantly the elimination of any incentive for validators to reduce the active stake in order to increase their yield, for example by delaying the approval of new pools.

From a tokenomic point of view, there are other considerations to make. In many other popular blockchains such as Bitcoin, the amount of rewards is set in advance and the participation rate (e.g. hashing power in proof-of-work) does not impact the total amount distributed. In Cardano Rewards could work like in Bitcoin. The amounts released in every period are parameterized in both chains. Also in Cardano, the available reward could be all distributed after the incentive formula has been applied, without adjusting them by the level of stake, i.e. without adjusting them by p.

This could stabilize the participation of validators. If the active stake went down, the Reward yield for unit of stake would increase, attracting more validators. This could be useful in case of excessive competition for the use of stake from the DeFi market or any other successful application, not to mention stronger competition from external activities, such as validation in other chains or fiat interest rates. In this case it would be important to check that p, by reducing total rewards when stake decreases, does not lead a so-called "pro-cyclical effect", which means contributing to a further stake decrease. This needs monitoring and analysis through time, under the constraint that any possible improvement must remain free from manipulation and aware of the max cap.

Historical behavior. In Figure 2.1 we can see the value taken by both f_i and p_i during the last few years. We see that, as expected, both are lower than one. A bit more surprisingly, we can notice that the p fraction is always lower than the f fraction, therefore p reduces rewards more than the specific pool incentives f in every epoch. It is also the largest contributor to the volatility of aggregate rewards over time, with even an upward jump after epoch 325,[3] and a smoother but bigger decline after epoch 400.

[3]Jumps in staking are usually associated to large players, such as exchanges, starting or terminating their participation in staking.

We can also distinguish the effects on p of variations in active stake from the effect of variations in circulating supply, by comparing p with *active* in Figure 2.1. That is the active stake not divided by the circulating supply in every epoch, but by the circulating supply at epoch 300, the first epoch considered, to have the same starting point as p. We see that the active stake is more stable than p, which is the ratio between active stake and circulating supply. Circulating supply increases in normal times. Using it as the denominator, makes the p ratio decrease more than active stake, and contributes to its volatility. This confirms the need to analyze and monitor the definitions of stake relevant to tokenomics.

Figure 2.1 also raises the question of why f is so close to the $\frac{1}{1+\alpha}$ minimum. As we saw in Section 1, this is the level of rewards that applies to pools with zero pledge. The following mathematical description will help understand this, and a further clarification will come from the analysis of the distribution of stake and pledge across pools.

Some Maths. Here we explain the approximate equivalence between f and the aggregate effect of incentives that we obtain if we replace $\sigma^x = \frac{stake^x}{S}$ with $\sigma^x_a = \frac{stake^x}{active}$ in the incentive formula. We start from the formula as given in (1.2) and write it to show its components as

$$f(\alpha, z, \sigma^x, \lambda^x) = \sigma^x \frac{1}{1+\alpha} + \sigma^x \frac{\alpha}{1+\alpha} \left[\frac{\lambda^x}{z} + \left(\frac{\lambda^x}{z} \right)^2 \right] - \frac{\alpha}{1+\alpha} z \left(\frac{\lambda^x}{z} \right)^2 . \tag{2.2}$$

$$= \frac{stake^x}{S} \frac{1}{1+\alpha} + \frac{stake^x}{S} \gamma(\alpha, z, \lambda^x) - \varepsilon(\alpha, z, \lambda^x) .$$

Remembering the analysis in the first section, we see the stake relative to total supply multiplied first by $\frac{1}{1+\alpha}$, and then by a function $\gamma(\alpha, z, \lambda^x)$ which is likely to be smaller, and finally we have a negative term $\varepsilon(\alpha, z, \lambda^x)$, depending on the square of the relative pledge and only indirectly on the stake. This term is usually very small, or simplifies with part of γ, as we saw above. If we now add the second adjustment, we obtain

$$\frac{N^x}{N}\frac{active}{stake^x}\left[\frac{stake^x}{S}\frac{1}{1+\alpha}+\frac{stake^x}{S}\gamma\left(\alpha,z,\lambda^x\right)-\varepsilon\left(\alpha,z,\lambda^x\right)\right]$$

Noticing that $stake^x$ cancels out, we get

$$\frac{active}{S}\left[\frac{N^x}{N}\frac{1}{1+\alpha}+\frac{N^x}{N}\gamma\left(\alpha,z,\lambda^x\right)-\hat{\varepsilon}\left(\alpha,z,\lambda^x\right)\right],$$

where $\hat{\varepsilon}$, which is ε modified by the final adjustment, is still expected to be small or to nearly cancel out. Based on this, we can understand the approximation mentioned by Kant et al. (2020) "this nearly amounts to rewarding pools based on the number of blocks that they produced". Adding the convergence of $\frac{N^x}{N}$ to $\frac{stake^x}{active}$ on average, we get the following approximation of the aggregate reward adjustment

$$\begin{aligned}
&\sum_x \frac{active}{S} f\left(\alpha,z,\frac{stake^x}{active},\lambda^x\right)\\
=&\frac{active}{S}\sum_x\left[\frac{stake^x}{active}\frac{1}{1+\alpha}+\frac{stake^x}{active}\gamma\left(\alpha,z,\lambda^x\right)-\hat{\varepsilon}\left(\alpha,z,\lambda^x\right)\right]\\
=&\frac{active}{S}\left[\frac{1}{1+\alpha}+\sum_x\frac{stake^x}{active}\gamma\left(\alpha,z,\lambda^x\right)-\sum_x\hat{\varepsilon}\left(\alpha,z,\lambda^x\right)\right],
\end{aligned}$$

where in square brackets we have the approximation of f, showing its terms from the most relevant to the one expected to be the smallest.

3 Pool Distribution and Rewards

The historical evidence in Figure 2.1 provides a striking confirmation of the goodness of the approximation just computed, and of the expectation that $\frac{1}{1+\alpha}=\frac{1}{1+a_0}\approx$ 77% is the dominant component of the f adjustment. The other components of f, which should increase the rewards above the $\frac{1}{1+a_0}$ minimum, accounted for less than 2% in the last few years. This evidence that aggregate rewards were little higher than the amount that applies to pools with no pledge needs to be rooted in the distribution of the staking pools during the period. We analyze this in the following, keeping the analysis simple and based on few basic statistics.

Statistics. We see in Figure 3.1 that the average pool had a stake between 7 and 8 Million ADA. The average pledge was around 1.5 Million ADA. This amount of own tokens is not very high, but it is significantly above zero. We know from the initial analysis that the pledge has different effects on pools with different stake, so we need to know more than the average stake and pledge, to understand why there is so little effect from the pledge. Since the median is the value with 50% higher observations and 50% lowest observations, median values could add relevant information, but here we see that median stake and pledge are very small, due to the presence of very many very small pools. So median data do not help understand the real distribution of the pools in terms of stake and pledge size.

Let's build slightly more granular, and more targeted, statistics. In Figure (3.2) the upper chart shows how the stake has been distributed across different classes of stake,

$$[0 - 7M]\,, [7M - 14M]\,, [14M - 21M] \ldots [63M - 70M]\,, [70M - 77M]\,.$$

They correspond approximately to

$$[0 - 10\%]\,, [10\% - 30\%]\,, [20\% - 30] \ldots [90\% - 100]\,, [100\% - 110]$$

of the average saturation level of the period, that ranges between 67M and 73M ADA. The class of pools with highest stake, $[70M - 77M]$, was above the saturation stake in the first years used in the analysis. But the saturation stake increases with circulating supply, so the number of pools in this class grows with the increase of the saturation stake to 70M and above. Any higher stake has been added to this last class, considering that no additional rewards are given to pools larger than saturation.

The dynamics of all classes of stake during the period shows a rather complex and volatile situation, yet there is a clear evidence that cohorts associated to the largest pools are also those where most of the stake is concentrated, with an expected gradual shift from the $[63M - 70M]$ to the $[70M - 77M]$ stake cohort as the saturation level increased, passing $70M$ due to the growth of circulating supply. Yet, in the last year or so, no cohort contained more than 20% of the active stake.

In Figure 3.2, the lower chart shows how the pledge has been distributed across the above classes of stake, measured a percentage of the class stake. We notice larger percentage pledge for the class containing the smallest pools, and for those containing the largest ones. This is not surprising. Small pools typically start with own stake, and the very high percentage of pledge may be apparent, since they include pools that are not yet or no longer operative, or not yet or no longer matching their declared pledge. This may explain why in some moments the pledge appeared even higher than the associated stake. In any case, pledge in this class has little to no impact on the amount of rewards received, since for pools with stake lower than 10% of the saturation a higher pledge proportion yields little to no advantage.

Conversely, larger pools are likely to hold more pledge both because they are more likely to have been created by large holders, or because their incentive to hold more pledge is the highest among all pools. Yet recently even the largest classes have an overall percentage of pledge which is between 30% and 10%, while many others have much less.

Estimates. To keep the statistics sufficiently synthetic, we do not use information on how the pledge was distributed within a class, but just the relative pledge held in the class, over the stake held in the class. This does not tell us directly how much single pools own, but can allow us some estimates. If we want an overestimate of their fees, for example, we can assume that in every class all pools have the maximum stake. We will also assume they all have the same percentage pledge. This is likely to be an overestimate as well, since pledge has for all but the largest, saturated pools, a less than linear impact on rewards. We can even apply this logic to the time average of stake and pledge across the whole period, which is shown in the upper chart of Figure 3.3. The histogram corresponds to the following table:

	0-7M	7-14M	14-21M	21-28M	28-35M	35-42M	42-49M	49-56M	56-63M	63-70M	70-77M
stake (%of total)	6.3%	4.8%	7.1%	7.2%	9.3%	7.6%	8.2%	8.5%	9.8%	25.4%	5.8%
pledge (% of stake)	63.1%	2.5%	1.5%	1.3%	0.8%	1.6%	2.0%	2.7%	2.0%	19.6%	19.8%

This shows that on average, across the period the 0-7M class held 6.3% of the stake, while pledge in the class was on average 63.1% of the stake, while 7-14M pools held 4.8% of the stake and pledge was 2.5% of the stake, and so on. If we apply the above estimate logic to these data, we treat all the 0-7M pools as if they were 7M pool with 63.1% pledge, all 7-14M pools as if they were 14M pools with 4.8% stake and so on. Now for each class, we can compute the amount of rewards as a percentage of the maximum ($\frac{1}{k}$ of the total rewards). Since we know that the first class weighted for 6.3% of the stake, the second one for 7.1% and so on, then we can compute a weighted average. This is shown in the lower chart of Figure 3.3, where it is compared to the 76.9% earned by pools with no pledge. Because of the time average, this rough estimate is not necessarily an overestimate, yet we obtain 78.6%, which is consistent with the f_i values in Figure 2.1 showing that the aggregate incentive adjustment was within 2% of the minimum.

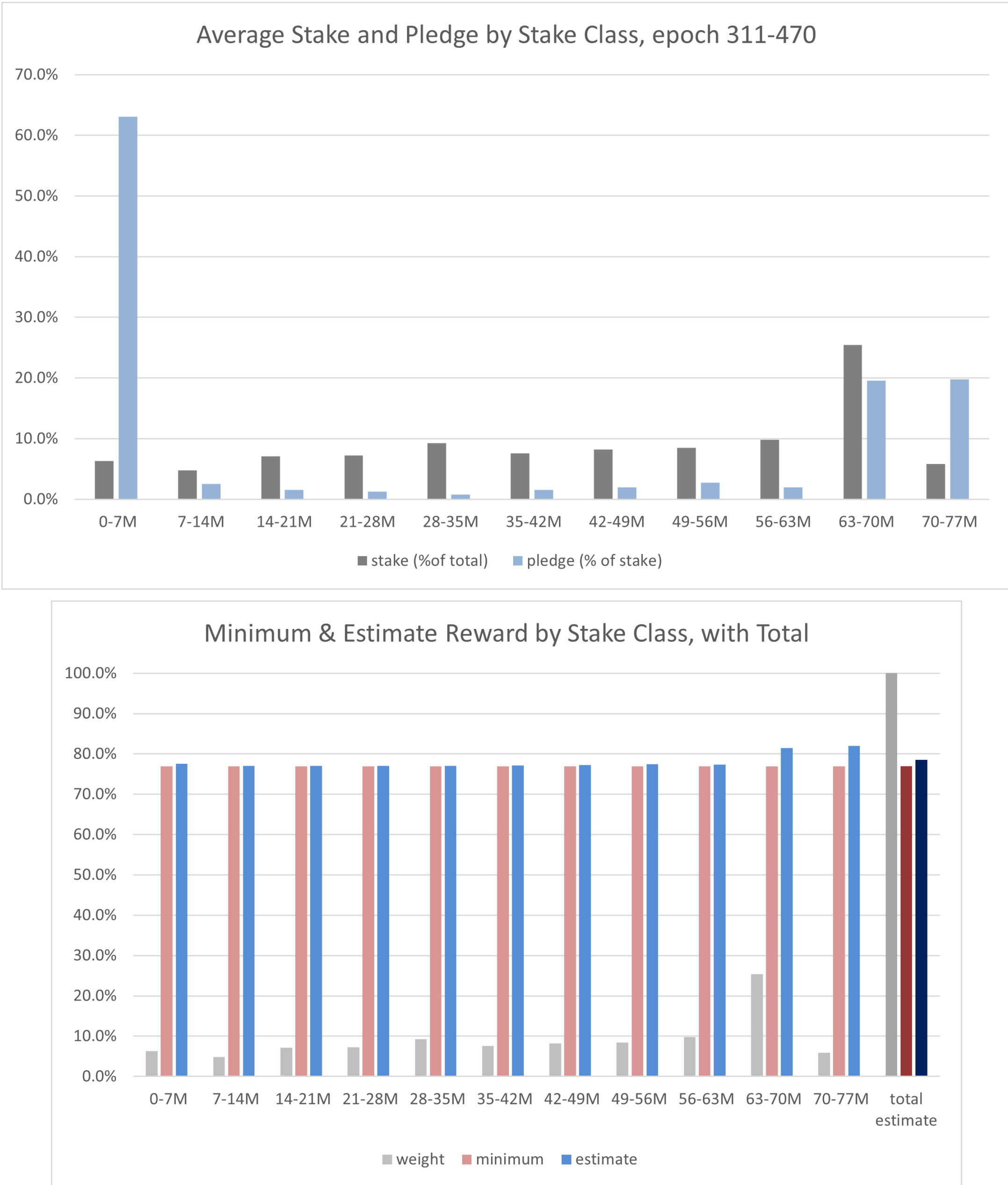

Figure 3.3: **Stake and Pledge Statistics and Reward Estimates.** In the upper chart we see the time-averages of stake and pledge. Below, we see estimates of the reward percentages.

However rough, this statistic support the assumption that the current level of own stake and its distribution across pools leads to rewards that, on average, near those of a pool with no own stake. Indeed, even for large saturated pools, financial optimization may lead to a low amount of pledge, since for them a higher pledge increases the total rewards for large pools, but the yield on own capital may be reduced when increasing the pledge. Thus even large pools may find it sub-optimal to replace delegated capital with own capital in order to get the last 23% of the full reward potential. A precise utility and cost assessment, and the interplay with parameters setting minimum costs and margins for individual pools, are left to future research.

4 Actual Reserves and Treasury Distribution

The reward incentives have effects on Cardano Tokenomics that go beyond rewarding differently pools of different stake and pledge. The documentation in Cardano (2025) introduces Cardano monetary policy by describing the Reserves as the difference between the maximal supply of ADA and the circulating supply, and explaining that "during each epoch, a fixed but parameterizable percentage of the remaining reserve is taken from the reserve and used for epoch rewards and treasury", and later on that "fees from every transaction from all blocks produced during every epoch go into a virtual 'pot'. A fixed percentage (ρ) of the remaining ADA reserves is added to that pot. A certain percentage (τ) of the pot is sent to the treasury, and the rest is used as epoch rewards".

These parameters always had values $\rho = 0.003$ and $\tau = 0.2$. The above description leads to a simple mathematical representation of the Cardano Economic Dynamics. The Pot is funded by Fees F_i and the regular release of Reserves ρReserve,

$$Pot_i = F_i + \rho \text{Reserve}_{i-1},$$

and it is then distributed to Treasury and Rewards in fixed proportions

$$\begin{aligned} \text{Rewards}_i &= (1-\tau)\, Pot_i. \\ \Delta\text{Treasury}_i &= \tau Pot_i \end{aligned} \tag{4.1}$$

where $\Delta\text{Treasury}_i$ is the increase in the value of the treasury added at every release. Quantities indexed with i are usually known through epoch i.

The above description leads to a simple fundamental equation for monetary policy, a formula for the update of Reserves given by

$$\begin{aligned} \text{Reserve}_i &= \text{Reserve}_{i-1} - \rho\text{Reserve}_{i-1} \\ &= \text{Reserve}_{i-1}\,(1-\rho)\,, \end{aligned} \tag{4.2}$$

In practice there are several additional elements that bring variability. First of all, ρ is multiplied by a quantity η_t. This is the aggregate equivalent of the adjustments for pool's performances seen above. It is given by the ratio between the actual number of blocks produced in an epoch, and the theoretical value of $21,600$ obtained as the number of slots (seconds) in an epoch multiplied by the so-called *active slots coefficient* of 5%. Also with the inclusion of η_i, the formula for the update of Reserves remains simple

$$\text{Reserve}_i = \text{Reserve}_{i-1}\,(1-\rho\eta_i)\,,$$

but we are still far from the actual distribution, since we are ignoring the adjustments to rewards seen in the previous sections. We have to take into account the effect of the incentive factor f_i, and the p_i rescaling. So the actual amount of Rewards distributed is reduced by the adjustments, and formula (4.1) becomes

$$\text{Rewards}_i = f_i p_i (1-\tau) Pot_i$$
$$\Delta\text{Treasury}_i = \tau Pot_i$$

Thus, the proportion between the Treasury increase and the Rewards is not the one between τ and $1-\tau$. The fraction of the theoretical release devoted to the Treasury has really been around $\tau = 20\%$, but reducing Rewards to a fraction $f_i p_i (1-\tau)$ rather than just $(1-\tau)$ changes the actual proportions.

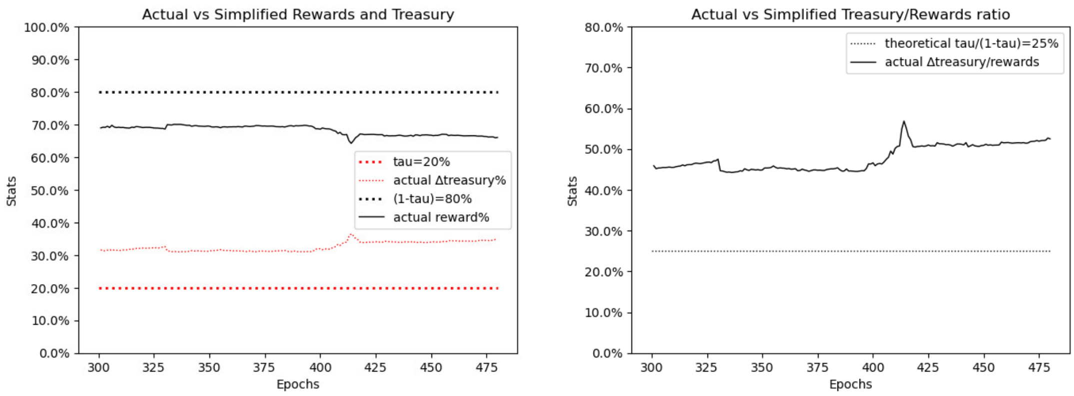

Figure 4.1: **Actual vs Theoretical Rewards and Treasury**

Look at Figure 4.1. On the left we see that, rather than 80% of the release, Rewards have been between a bit less than 65% and little more than 70%, with Treasury taking a fraction approximately between 30% and 35%. Thus, as we see on the right, the proportion between the growth of Treasury and the Rewards, that in principle should be $\frac{\tau}{1-\tau} = 25\%$, has often been more than 50%.

A more precise description of the effect of rewards adjustments is given in Figure 4.2, that breaks down the effect for f_i and p_i. We can see that without incentives, equivalent to setting $f_i = 1$, but with the p_i rescaling, rewards would be be nearer to the theoretical value of 80%, and even more if instead we had incentives f_i but no p_i rescaling. This is consistent with the results of the previous sections. If there were neither f_i nor p_i adjustments, the distribution would be very close to the theoretical values, in spite of some elements of variability, such as Fees or the treatment of unpaid rewards and unreturned deposits, that are rather small and not represented explicitly here.

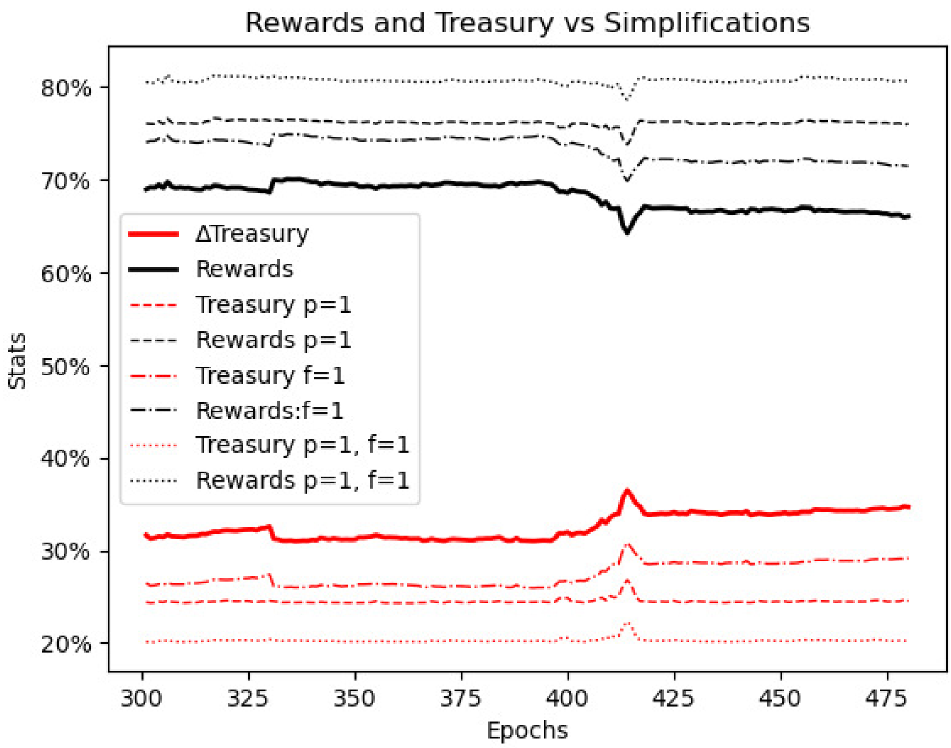

Figure 4.2: **Rewards and Treasury. Breakdown of Adjustments.**

What happens to the amounts which are not paid out as Rewards? They stay with the Reserves, making the actual release lower than the one dictated by the monetary expansion parameter ρ. This is another difference between simplified descriptions of the tokenomics and the reality. In Figure 4.3, left chart, we see the actual dynamics Reserves had in the last few years, compared to the predictions of simplified representations, where the f_i, p_i, and potentially also the η_i adjustment are ignored. The difference is material, and the most relevant contribution to this difference does not come from ignoring η_i, but from ignoring the effect of reward adjustment. This effect alone leads to a difference of almost 1.5B ADA in the amount of reserves over 2.5 years. Ignoring η_i as well leads only to an additional difference of little more than 100M ADA.

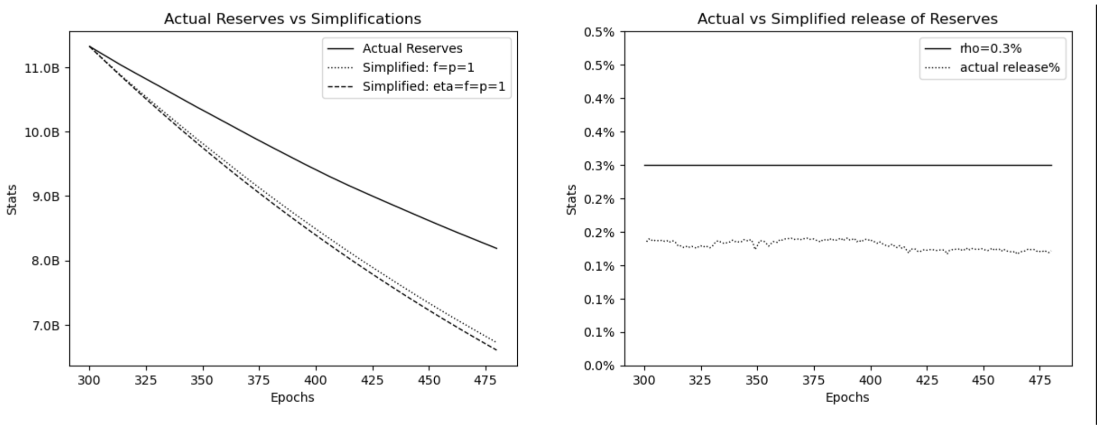

Figure 4.3: **Actual Reserves vs Simplifications**

This clarifies that the release from Reserves does not happen at a fixed rate equal to $\rho = 0.3\%$ but at a variable rate, lower than ρ and ranging, in the period analyzed, between 0.165% and 0.185%. This means that in some cases the reward adjustments can make the release be only little more than half of the theoretical value, as we can see in the right chart of Figure 4.3.

Taking into account also the impact of f_i and p_i, the formula for the update of Reserves become much more complicated, since we have to take into account the undistributed part of Pot_i returning to Reserves, including a component from fees F_i:

$$\begin{aligned}
\text{Reserve}_i &= \text{Reserve}_{i-1}\left(1-\rho\eta_i\right)+\left(1-\tau\right)\left(F_i+\rho\eta_i\text{Reserve}_{i-1}\right)\left(1-p_i f_i\right)\\
&= \text{Reserve}_{i-1}\left(1-\rho\eta_i\left(\tau-\left(1-\tau\right)p_i f_i\right)\right)+\left(1-\tau\right)F_i\left(1-p_i f_i\right)
\end{aligned}$$

It is useful to see the relevance of the different parts of this equation. So far we have seen that some simplifications, like ignoring the effect of the incentive parameterization on the monetary policy, leads to massive errors. However, there are also approximations that simplify the picture with a very limited practical impact. Here the most relevant is to exclude the F_i component from the reserve update, leading to a simpler equation

$$\begin{aligned}\text{Reserve}_i &= \text{Reserve}_{i-1} - \rho\eta_i \text{Reserve}_{i-1} + \rho\eta_i \text{Reserve}_{i-1} \left(1-\tau\right)\left(1-p_i f_i\right) \\ &= \text{Reserve}_{i-1} \left(1 - \rho\eta_i \left(\tau - \left(1-\tau\right) p_i f_i\right)\right).\end{aligned}$$

This equation has a trivial solution

$$\text{Reserve}_i = \text{Reserve}_0 \prod_{j=1}^{i} \left(1 - \rho\eta_i \left(\tau - \left(1-\tau\right) p_i f_i\right)\right),$$

expressing directly current reserves in terms of their level at an initial epoch. This is equation allows also to project easily Reserves into the future. However, we have to remember not only that we still have to consider the impact of Fees, currently very small but potentially relevant in the future, but also that the elements introduced, compared to the simplified version in (eq. (4.2) on page 22), are stochastic, which meas that they vary randomly.

We have analyzed their statistical properties and leveraged this analysis to model them as stochastic processes. We considered both a discrete-time setting, where epochs correspond to finite-difference time steps, and a continuous-time framework, obtained by taking the epoch frequency to the limit. This has a contained impact, that we assessed both analytically and in simulation, while making the system much more tractable. As anticipated above, η_i and f_i exhibit smooth dynamics that are well-represented by continuous stochastic processes. While p_i could instead be modeled as a jump-diffusion process, incorporating a jump component governed by a compound Poisson process, when considered jointly with the other processes, the resulting dynamics remains smooth.

Our analysis was conducted to gain a deeper understanding of Cardano economic parameters, particularly to clarify their interactions in the context of the current decentralization of governance. At the same time, it serves as both a foundation and a blueprint for the stochastic modeling of the system—an equally crucial requirement for the future governance of decentralized ecosystems. This topic will be explored further in the follow-up to this document.

Conclusions and Perspectives

In this paper we analyzed staking incentives, showing how the parameters for pledge influence and desired stake size have very differentiated effects on the rewards of pools with different features. We looked at the impact of stake calculations, and of the actual pool distribution, on rewards. Finally we analyzed how the dynamics of reserves, rewards and treasury is affected by incentives and stake calculations. We used simple math, economics, and data analysis, striving to keep complexity to a minimum. We have made the paper available to the community to foster discussion and improvements at https://github.com/cardano-foundation/cardano-economic-parameter-insights.

This paper is part of a larger research that extends some of the above analysis, including a more detailed investigation of the resource optimization problem faced by pools, taking into account also the parameters limiting pools choices, such as minimum costs and margins, and the long term impact of rewards. The analysis of the actual dynamics of variables such as reserves and rewards extends to measuring their statistical properties and modeling them using stochastic finite-difference or differential equations. This is facilitated by Cardano's rigorous parameterization, as highlighted also in Hoskinson (2021). This topic is relevant for projecting and simulating the effect of governance choices on the long-term evolution of the system, properly accounting for probabilistic uncertainty.

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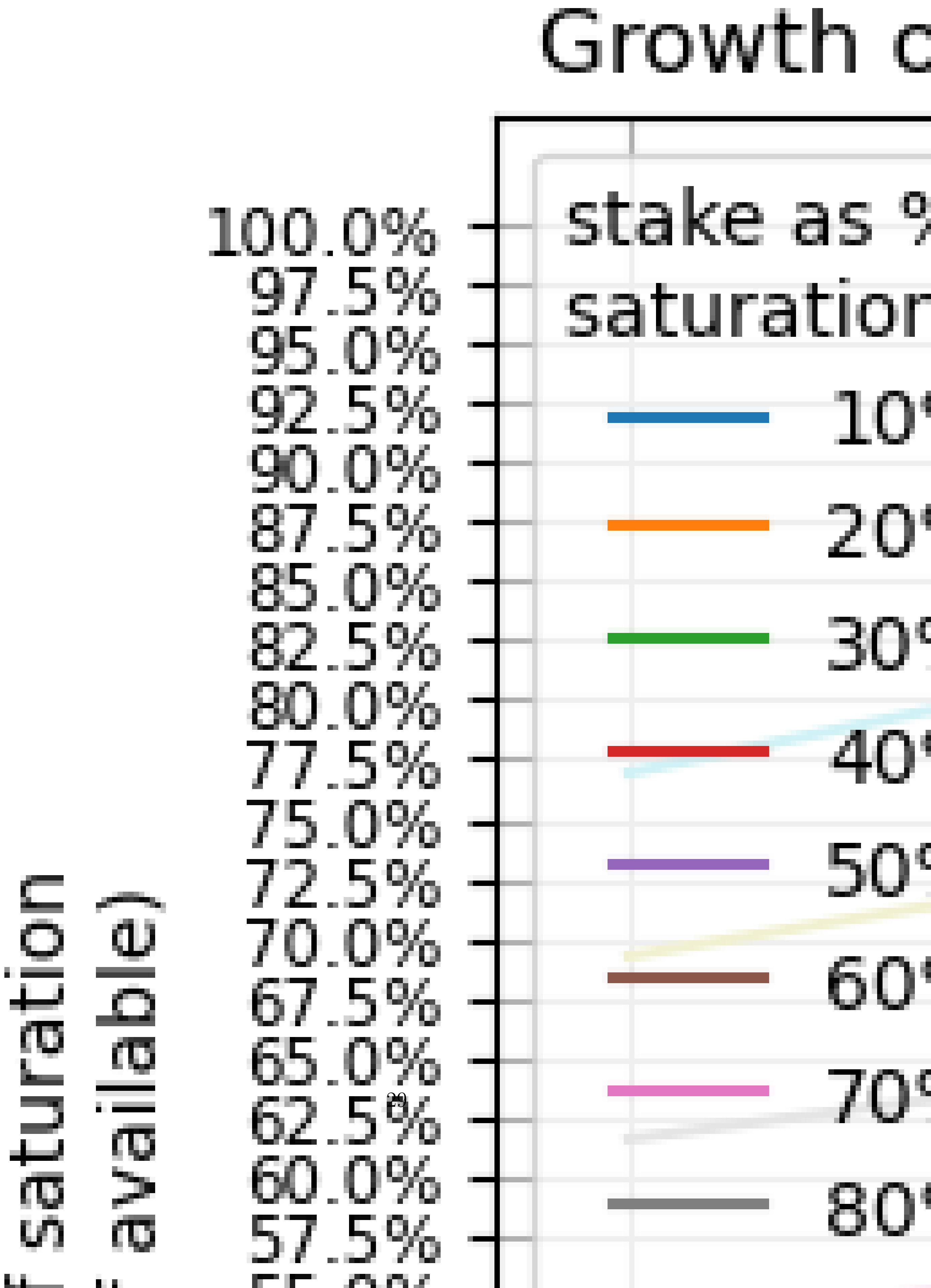
stake as
100.0%
97.5%
95.0%
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saturation
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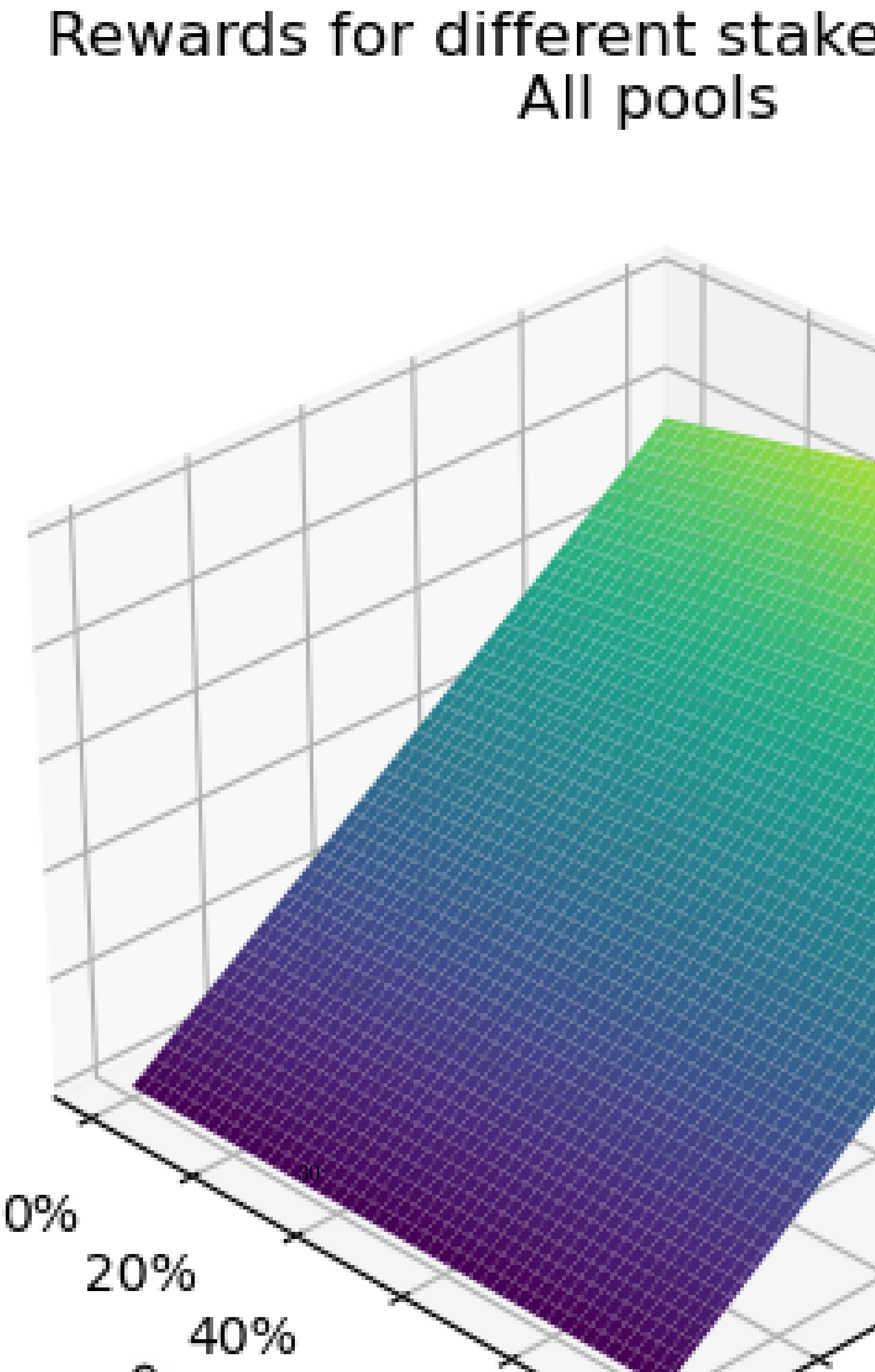

Rewards for different stake
All pools
0%
20%
40%

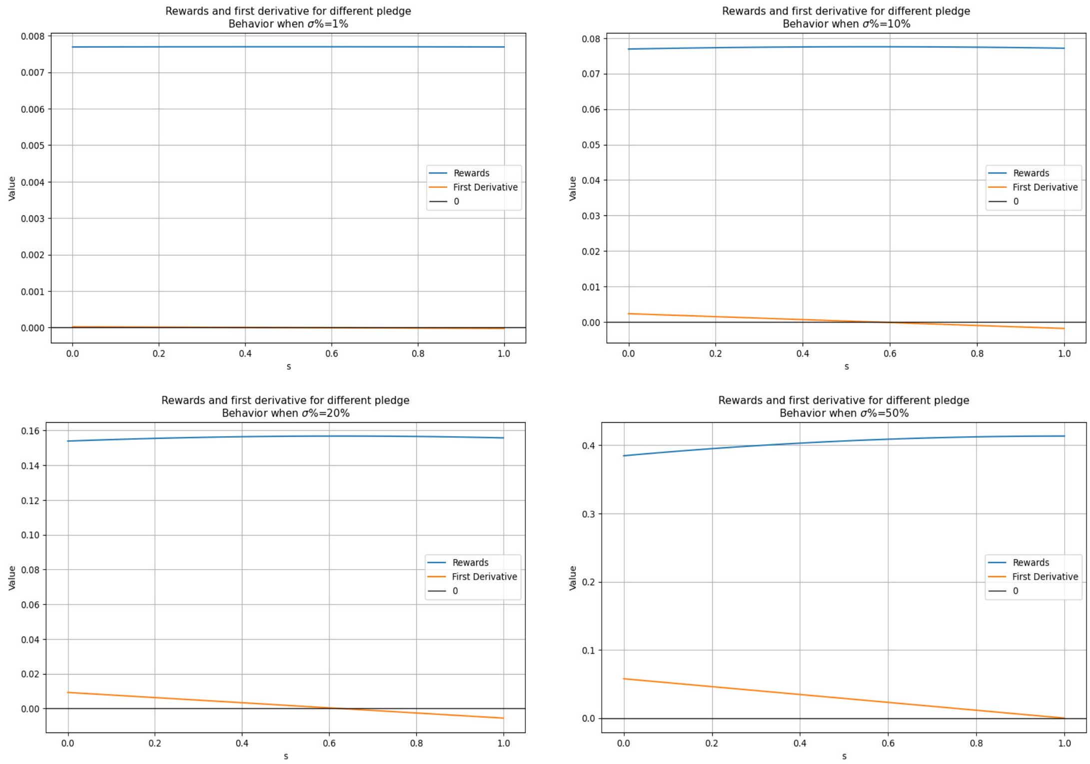

Figure 1.3: **Behavior of Rewards with pledge and its 1st derivative.** Showing different cases for different $\sigma_{\%}$, i.e. different stakes expressed as fixed percentages of saturation, showing in each chart both the function and its derivative with respect to the proportion $\lambda_{\%}$ of pledge.

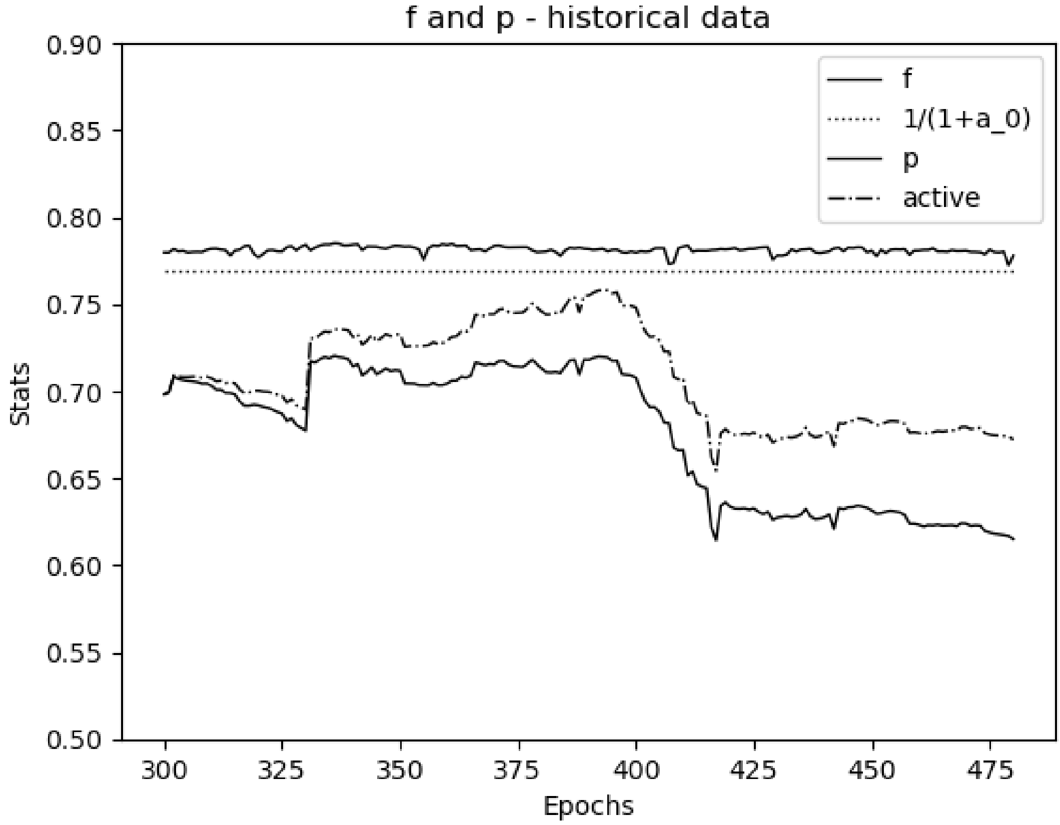

Figure 2.1: **Reward adjustment f and p in practice** The chart shows p and f as extracted from the actual level of rewards, beside the leading term in f, $\frac{1}{1+a_0} \approx 77\%$, and the active stake component of p.


9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
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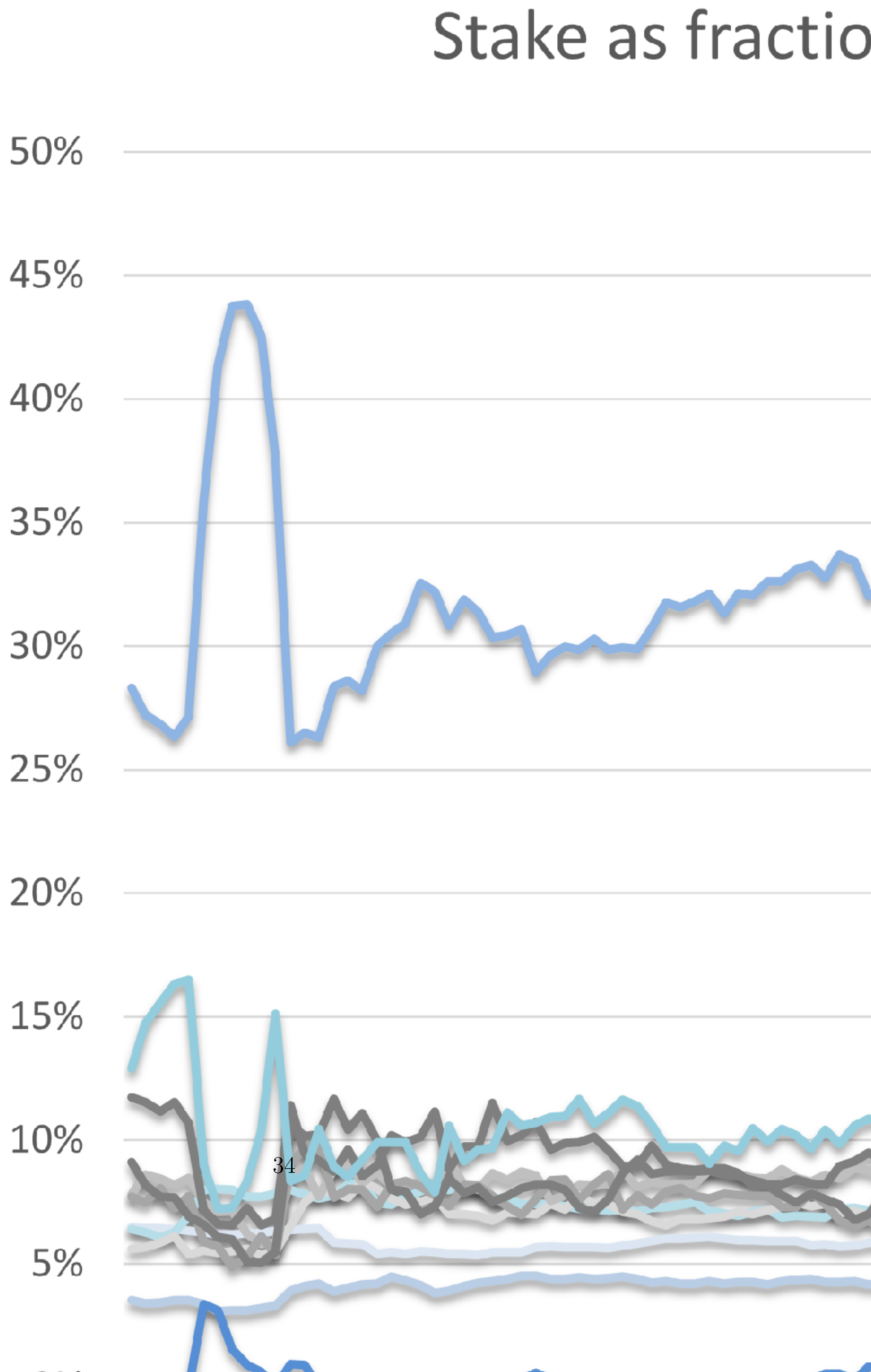
3,000,000


Stake as fractio
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30%
25%
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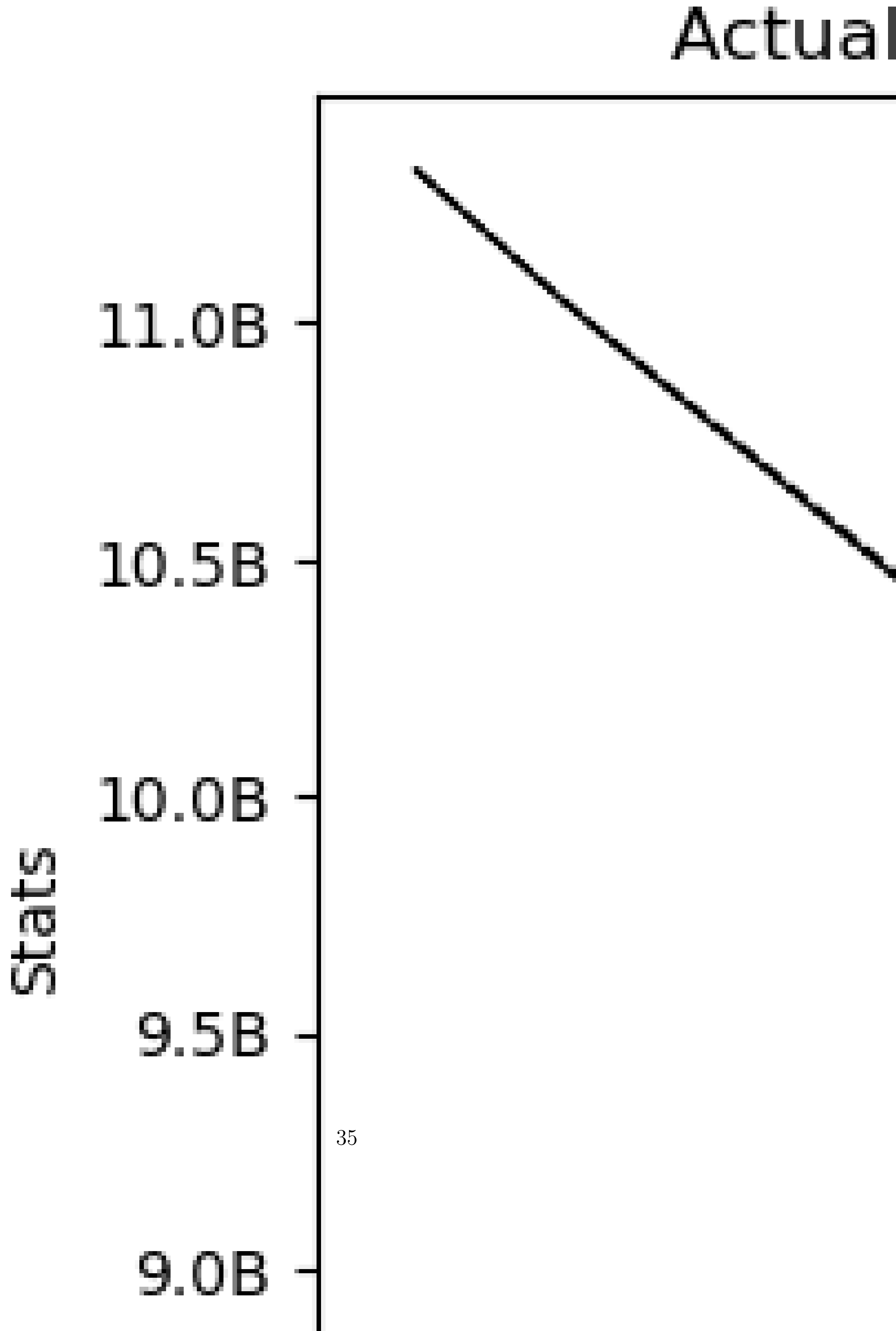
10%
5%


Actual
11.0B
10.5B
10.0B
9.5B
9.0B
Stats