02417: Time Series Analysis
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Assignment 2: ARMA Processes and Seasonal Processes

Be aware that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions. The assignment is to some extent specified in terms of R commands. Users of other software packages should find similar functions to answer the questions.

Question 2.1: Stability Let the process X_t be given by

$$X_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3}$$

where ε_t is a white noise process with $\sigma = 0.1$.

1. Investigate analytically the second order moment representation of the process.
2. Is the process stationary and invertible?
3. Simulate 10 realisations with 200 observations from the process and plot then (Preferably in the same plot).
4. Estimate the ACF for each realisation and plot those (Again, preferably in the same plot). Comment on the results.
5. Repeat for the PACF of the same realisations.
6. Estimate the variance of the realisations.

Compare and discuss the analytical and numerical results.

Question 2.2: Predicting temperature in district heating network In district heating networks some consumers needs high forward temperature in order to heat their houses. However, higher forward temperatures also lead to higher losses from the pipe network. In the summer time the forward temperature can be decreased relative to colder periods of the year. For strategic planning of heat production it is important to have predictions of the needed forward temperature.

Based on historical data for the forward temperature to a house the following model has been identified:

$$(1 - 0.5B + 0.3B^2)(1 - 0.9B^{12})(Y_t - \mu) = \varepsilon_t$$

where ε_t is a white-noise process with variance σ_ε^2. Based on five years of data, it is found that $\sigma_\varepsilon^2 = 0.5^2$. Furthermore, μ was estimated to 55. The table below shows the last 15 observations of Y_t:

Predict the values of Y_t corresponding to t = 2017M12 and 2018M1, together with 95% prediction intervals for the predictions.

Year	2016	2016	2016	2016	2017	2017	2017	2017	2017	2017	2017	2017	2017	2017	2017
Month	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11
Temp	46.6	49.5	60.3	59.2	59.5	61.9	59.7	60.1	57.8	49.7	49.7	50.1	48.6	54.5	62.3

Question 2.3: Simulating seasonal processes A process Y_t is said to follow a multiplicative $(p,d,q)\times(P,D,Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\varepsilon_t$$

where (ε_t) is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to definition 5.22 in the textbook.

Simulate the following models (where monthly data are assumed). Plot the simulations and the associated autocorrelation functions (ACF and PACF).

1. A $(1,0,0)\times(0,0,0)_{12}$ model with the parameter $\phi_1 = -0.85$.
2. A $(0,0,0)\times(1,0,0)_{12}$ model with the parameter $\Phi_1 = 0.85$.
3. A $(1,0,0)\times(0,0,1)_{12}$ model with the parameters $\phi_1 = -0.8$ and $\Theta_1 = 0.9$.
4. A $(1,0,0)\times(1,0,0)_{12}$ model with the parameters $\phi_1 = 0.7$ and $\Phi_1 = 0.8$.
5. A $(2,0,0)\times(1,0,0)_{12}$ model with the parameters $\phi_1 = 0.6$, $\phi_2 = -0.3$, and $\Phi_1 = 0.8$.
6. A $(0,0,1)\times(0,0,1)_{12}$ model with the parameters $\theta_1 = -0.4$ and $\Theta_1 = 0.8$.

Are all models seasonal? And stationary? Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes?

Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made.