

Assignment 4: Multivariate Processes

Global warming is a great concern around the globe. Scientists at the Climatic Research Unit (CRU) at University of East Anglia have made several datasets with temperatures on the globe covering the past +150 years. The temperatures are expressed as anomalies from 1961-90 and in this assignment the focus will be on the changes in the average annual anomalies for the Southern and Northern hemispheres. The temperatures are estimated based on a number of measurement stations and do include measurement errors.

The data is provided in `A4_annual.txt` and includes three columns:

year Year for the observations

sh Temperature anomaly for the Southern hemisphere

nh Temperature anomaly for the Northern hemisphere

All observations are expected to be used in the following.

Question 4.1: Presenting the data Present the data. Do comment on what you see.

Question 4.2: Formulating state space model One assumption could be that the temperature anomalies for the two hemispheres follow independent random walks. Formulate a state space for the bi-variate dynamical system (According to Eqns 10.1 and 10.2).

Question 4.3: Kalman filtering Assume that the initial temperature is -0.4C for the Southern hemisphere and -0.3C for the Northern hemisphere. The initial variance is 0.01 and both the system noise and observation noise have a variance of 0.01 for each of the states.

Use a Kalman filter implementation to reconstruct (estimate) the temperature anomalies for the entire series including a 95% confidence interval. Present the reconstructions.

Report the log-likelihood of these parameters.

Predict the anomalies for each year up to year 2050 and present the predictions. Do include a table with the predictions for years 2020, 2030, 2040, and 2050.

Comment on the results.

Question 4.4: Optimize parameters The provided values for the initial state and the variances are not the optimal values. Find the maximum likelihood estimates of these variables and present them.

The presentation should include a table with predictions for the same four years as in the previous case.

Question 4.5: Optimize parameters - with correlation It is reasonably obvious from the data that the two hemispheres have similar anomalies. Adjust the model to allow the system noise to be correlated. Then find the maximum likelihood estimates and present as in the previous question.

Compare and comment on the results.

Question 4.6: Formulating state space model with common trend Extend the state space model so that it includes a common trend (slope). The trend should itself be a random walk and it should be added to the states for the two anomalies for each step (year). Present the matrices specifying this extended model.

Question 4.7: Optimize parameters - with correlation and common trend Find the maximum likelihood estimates in the extended model and present as in the previous questions.

Again, the presentation should include a table with predictions for the same four years as in the previous case.

Question 4.8: Comparison Comment on the performance of the different formulations of state space models.

Suggest alternative formulations of the Kalman filter that may perform better.