Phys 641 Assignment 4

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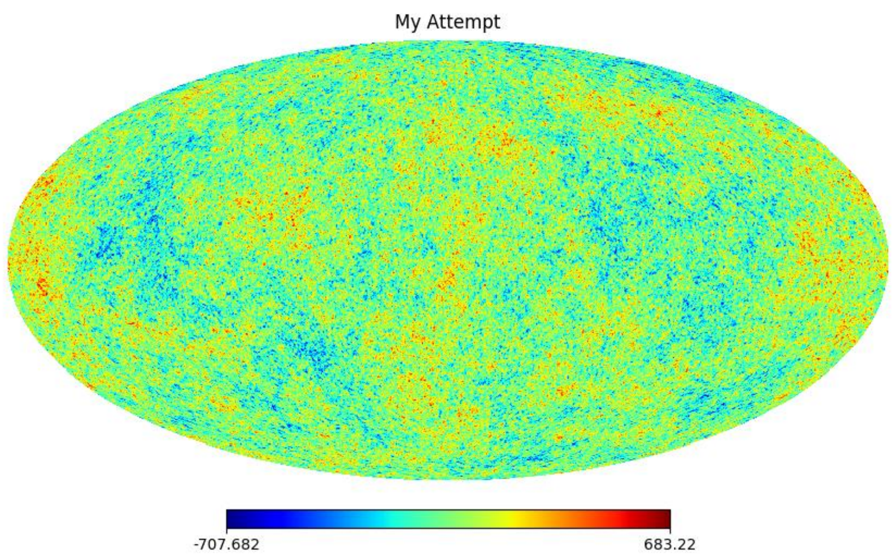
1. Note for the purposes of making my code run faster I used l=2500 so I cut the number of l given for the power spectrum in half.
   (a) For calculating the variance I first convert the $l(l+1)C_l/2pi$. I could think of two ways of doing the variance, the first naïve way is just to sum the C_ls , but when I do part (b) and (d) it seems that I have to multiply this by a factor of 4 to get the true estimate of the variance around 12568. I do not really like that method the better way I think is to do what was kind of in the lecture 10 notes where if it was a constant C_l the variance of the fluctuations was $sum(C_l*(2l+1))/2pi$. When I do parts (b) and (d), by the first method of (b) this is the same, but for the adjusted one I have to divide by 2. This gives a variance of about 11027. When I take the square root and look at the standard deviation of all of my answers I do get that they all agree within about $\Delta\sigma=6$.

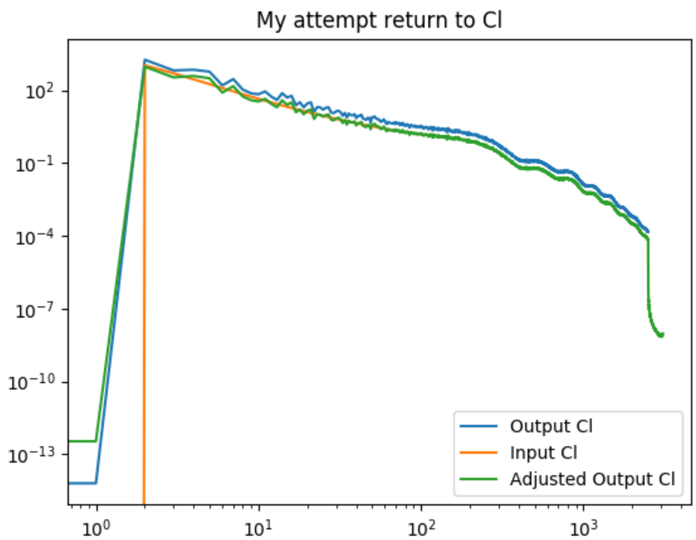
   (b) Since it said in the notes that <alm^2>=Cl and that it is assumed to be gaussian that means that my best possible guess from a single power spectrum would just be to say that the a_{lm} are gaussian distributed with a mean of 0 and a variance of Cl. From the $e^{im\phi}$ term in the Ylm's I know that there are no imaginary terms for the m=0 modes, but besides that I generate it through knowing that there. Ordered such that every m=0 a_{lm} is listed first with l's ranging from m to the maximum l and then going through the other m's.

   Then mapping with using alm2map gives the map



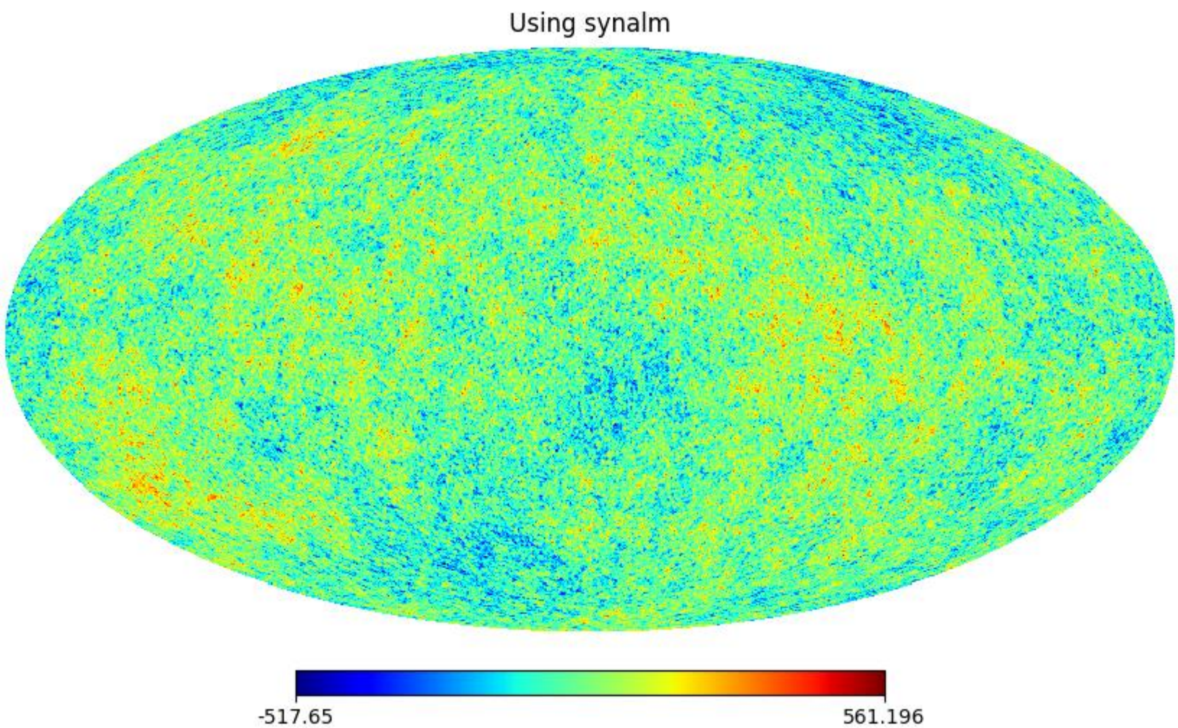
   So the variance I can do 2 ways one is just the straight variance without doing to $\sqrt{2}$ factor in the a_{lm}'s is 23924.
   When I make that adjustment I get 12094 which is acceptably similar to what I found in part (a)

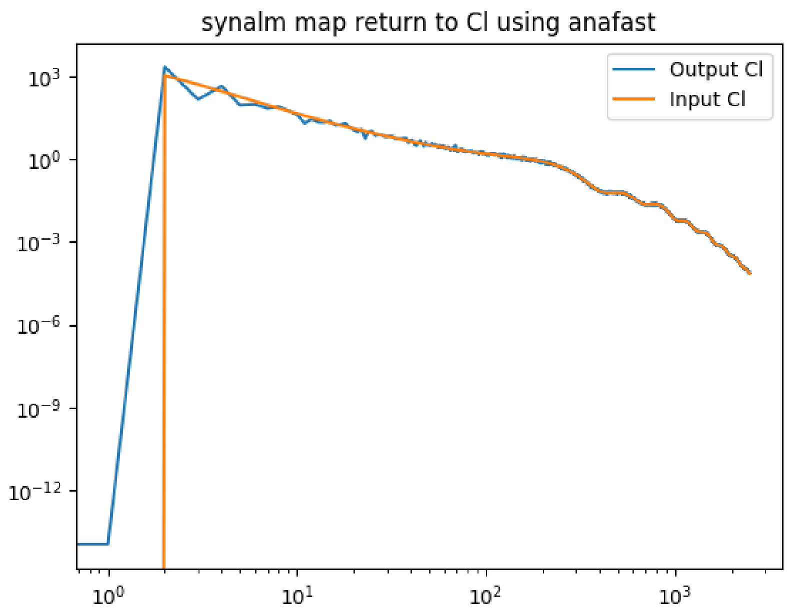
(c)



Putting the $\sqrt{2}$ factor in the a_{lm}'s gives the Adjusted Output Cl which fit the input furve better. Otherwise if I don't put in that factor it is essentially the same shape, but it just appears higher up which you see as the blue curve. So the two curves agree very well.

(d) Using the same Cls but using synalm, alm2map and then converting back with anafast gave the following plots. The variance of the map was calculated and gave a variance of 11920. In these plots it is easy to see that the Cl match up really well to the input.


Using synalm
-517.65
561.196


synalm map return to Cl using anafast
Output Cl
Input Cl
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^0
10^1
10^2
10^3

2.

(a) Using the equation l=360°*k/x where x is in degrees

So l=(360°/20°)k giving a rough conversion of l=18k

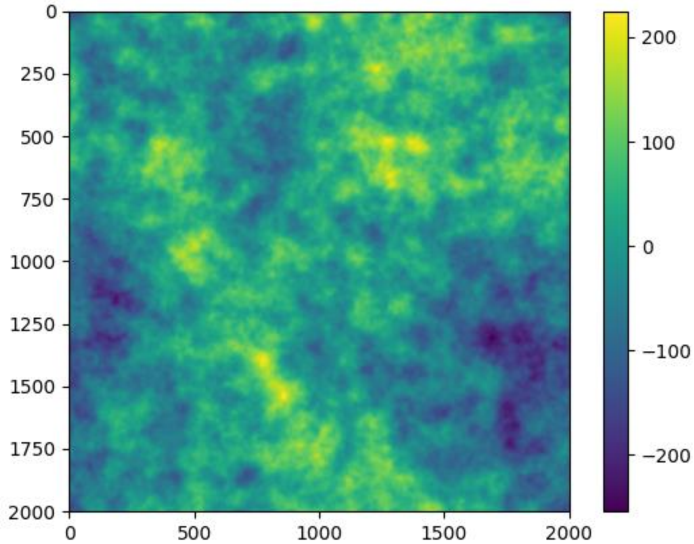
(b) So if I have some l_{max} which corresponds to $k_{max}=l_{max}/18$, from the class notes the total $\#a_{lm}=(l_{max}+1)(l_{max}+2)/2$ and just by definition there are $2k_{max}+1$ modes, if k_{max} is large this is essentially $2k_{max}$ modes.

Convert to a power spectrum by first creating an array of kx and ky covering the entire flat sky area (so #k=#pixels=1000 in my case) creating a 1000 x 1000 grid. Then calculate the effective k and round to the nearest integer. For every k grab the corresponding l from the C_l power spectrum and assign it as the power spectrum value for that element of the matrix creating a power spectrum in k(I just called it P in my code). If the k is higher then k_{max} of the power spectrum then just set it to zero. Create a same size matrix of random Gaussian distributed values and then multiply sqrt(P) by that value. It also involves some kind of amplitude adjustment which I am still not clear on. As far as I understand I should multiple F(k)=sqrt(P) by sqrt(# of alm's/# of k modes) to get a scaling. What I am still not clear on is whether this is a function of the k at that specific point or whether it is just a constant I multiply everything by. Either way I ran out of time to get this to work properly so I am more explaining what I would have done knowing that this makes my variance of my plot and the plot itself inaccurate. So I tried making the factor sqrt(# of alm's/# of k modes). Based on our discussions I know I have npix=k so $k_{max}=1000$ this would correspond to $(18k_{max}+1)(18k_{max}+2)/2$ alm for the full sky and $2k_{max}+1$ modes so I tried using these two values and multiplying it by my F(k) . Something in that logic is probably flawed since I probably npix**2 k modes but this method gave me the closest variance I could find. I maybe should have done it with the original number of alm's from question 1. I think maybe it should be #pix**2, but that would also have given my a value that was too small.

(c) So I cut off my power spectrum at l=2500 just to save computation time. Using the same equation as in (a), the maximum pixel size to get all of the information is 0.144°.

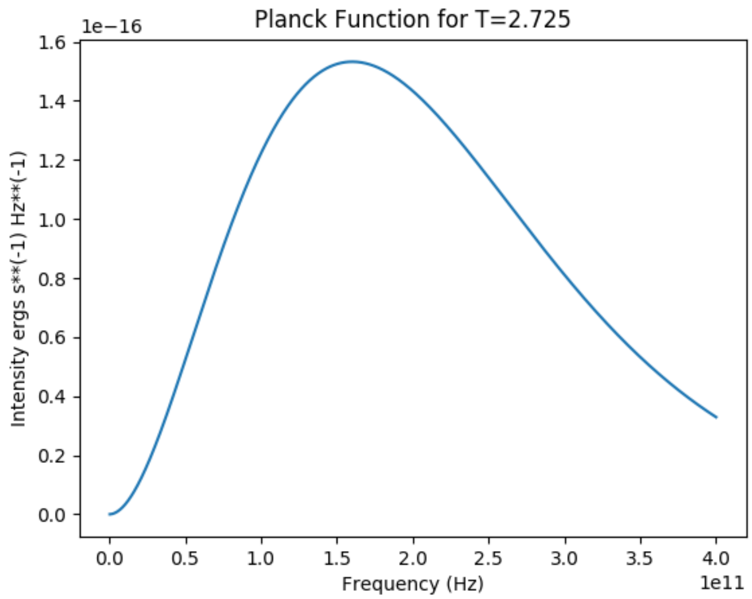
So just picking a pixel size less than this. Say 0.02°. That way the image is exactly 1000 pixels wide

Using the method I described in (b) I got the following map. I don't think the scale is right



(d) Just looking at the map and calculating its variance according to my plot which I am pretty sure is not correct I get 5772 which is almost exactly a power of 2 off of what I got before so I may just need to multiply by a constant but I do not know whether my method is even right so whether I should count this.

3. (a) Taking λ^2 as the wavelength equivalent to the central frequency of the band of 150 GHz, the $\mathrm{B}(\mathrm{v},\mathrm{T}) = \frac{2\mathrm{hv}^3}{\mathrm{c}^2}\left(e^{\frac{hv}{kT}} - 1\right)$ which has units of ergs s^{-1} cm^{-2} Hz^{-1} sr^{-1} using the constants h=6.626068x10^{-27} erg s, k=1.38066 erg/K , c=2.997925 x 10^{10} cm/s.
   So if I say that the solid angle is 1 sr and the area is $\lambda^2=c^2/v^2$
   Then $\frac{2hv^3}{c^2}\left(e^{\frac{hv}{kT}} - 1\right)\lambda^2$=$\frac{2hv^3}{c^2}\left(e^{\frac{hv}{kT}} - 1\right)\left(\frac{c^2}{150GHz^2}\right)$ is the total number of ergs s^{-1} Hz^{-1}
   To answer how many erg/s/Hz from the CMB is a function of the



(b) To convert energy to photons I must now know, at each frequency that I am using convert the units using E/photon=hv so if I just want the total number of photons per second in the bandwidth I need to integrate the equation $\int \frac{2hv^3}{c^2}\left(e^{\frac{hv}{kT}} - 1\right)\left(\frac{c^2}{150GHz^2}\right)\left(\frac{1}{hv}\right)$ over the v range centered on 150GHz for a 20 GHz bandwidth. So say that I am integrating from 135GHz to 165 GHz.
The number of photons per second that I get from this is 4.6 billion photons/s. So we sample at 150GHz, but photons only appear on average every 30 samples or so. So it is likely in the shot noise, but shot-noise seems to require that you do not really get 2 photons at once coming in which is still a possibility so there is a continuous possibility in that sense. I would say it is more so shot-noise.

(c) Since I want the temperature representation for noise, instead of using δn/n using the temperature representation $\frac{\delta T}{T} = \frac{1}{\sqrt{nt}}$ so with t=1s and T=2.725, δT=40.2 μK. Based on the value given in the problem set, this δT is better than even the best Planck detectors. However, what we calculated for is the perfect detector where it is seeing exactly the CMB temperature and there is nothing either raising that temperature and no other sources of noise. There is always ways to improve detectors, but just because it would be extremely difficult if not impossible to get a detector working so ideally, it could be possible in the future to get deeper maps, but it would probably not reach the level of this ideal detector.