DR. G. B. DAVIES

RADIOACTIVITY

NAME: CLASS:

Contents

1 **Radioactive Decay** **4**
1.1 Alpha Decay 4
1.2 Beta Decay 5
1.3 Positron Decay 6
1.4 Gamma Decay 8

2 **Ionisation** **10**

3 **Penetration** **11**

4 **Half-Life** **13**
4.1 Activity and Becquerels 16
4.2 Background Radiation and Corrected Count Rate . . . 17
4.3 Parents and Daughters - What are the radioactive atoms decaying into? 17

5 **Half Life Graphs** **19**

6 **Half-Life II** **22**
6.1 A more intuitive mathematical understanding of half-life 24

7 **Nuclear Fission** **34**
7.1 Nuclear Fission Equations 34

The Periodic Table

1 **H** Hydrogen **1**																	4 **He** Helium **2**
7 **Li** Lithium **3**	9 **Be** Beryllium **4**											11 **B** Boron **5**	12 **C** Carbon **6**	14 **N** Nitrogen **7**	16 **O** Oxygen **8**	19 **F** Flourine **9**	20 **Ne** Neon **10**
23 **Na** Sodium **11**	24 **Mg** Magnesium **12**											27 **Al** Aluminium **13**	28 **Si** Silicon **14**	31 **P** Phosphorus **15**	32 **S** Sulphur **16**	35.5 **Cl** Chlorine **17**	40 **Ar** Argon **18**
39 **K** Potassium **19**	40 **Ca** Calcium **20**	45 **Sc** Scandium **21**	48 **Ti** Titanium **22**	51 **V** Vanadium **23**	52 **Cr** Chromium **24**	55 **Mn** Manganese **25**	56 **Fe** Iron **26**	59 **Co** Cobalt **27**	59 **Ni** Nickel **28**	64 **Cu** Copper **29**	65 **Zn** Zinc **30**	70 **Ga** Gallium **31**	73 **Ge** Germanium **32**	75 **As** Arsenic **33**	79 **Se** Selenium **34**	80 **Br** Bromine **35**	84 **Kr** Krypton **36**
86 **Rb** Rubidium **37**	88 **Sr** Strontium **38**	89 **Y** Yttrium **39**	91 **Zr** Zirconium **40**	93 **Nb** Niobium **41**	96 **Mo** Molybdenum **42**	96 **Tc** Technetium **43**	101 **Ru** Ruthenium **44**	103 **Rh** Rhodium **45**	106 **Pd** Palladium **46**	108 **Ag** Silver **47**	112 **Cd** Cadmium **48**	114 **In** Indium **49**	119 **Sn** Tin **50**	122 **Sb** Antimony **51**	128 **Te** Tellurium **52**	127 **I** Iodine **53**	131 **Xe** Xenon **54**
133 **Cs** Caesium **55**	137 **Ba** Barium **56**	**La-Lu** Lanthanide **57-71**	179 **Hf** Hafnium **72**	181 **Ta** Tantalum **73**	184 **W** Tungsten **74**	186 **Re** Rhenium **75**	190 **Os** Osmium **76**	192 **Ir** Iridium **77**	195 **Pt** Platinum **78**	197 **Au** Gold **79**	201 **Hg** Mercury **80**	204 **Tl** Thallium **81**	207 **Pb** Lead **82**	209 **Bi** Bismuth **83**	209 **Po** Polonium **84**	210 **At** Astatine **85**	222 **Rn** Radon **86**
223 **Fr** Francium **87**	226 **Ra** Radium **88**	**Ac-Lr** Actinide **89-103**	261 Rf Rutherfordium **104**	262 Db Dubnium **105**	266 Sg Seaborgium **106**	264 Bh Bohrium **107**	277 Hs Hassium **108**	268 Mt Meitnerium **109**	281 Ds Darmstadtium **110**	280 Rg Roentgenium **111**	285 Uub Ununbium **112**	284 Uut Ununtrium **113**	289 Uuq Ununquadium **114**	288 Uup Ununpentium **115**	293 Uuh Unumhexium **116**	292 Uus Ununseptium **117**	294 Uuo Ununoctium **118**
			139 **La** Lanthanum **57**	140 **Ce** Cerium **58**	141 **Pr** Praseodymium **59**	144 **Nd** Neodymium **60**	145 **Pm** Promethium **61**	150 **Sm** Samarium **62**	152 **Eu** Europium **63**	157 **Gd** Gadolinium **64**	159 **Tb** Terbium **65**	163 **Dy** Dysprosium **66**	165 **Ho** Holmium **67**	167 **Er** Erbium **68**	169 **Tm** Thulium **69**	173 **Yb** Ytterbium **70**	175 **Lu** Lutetium **71**
A **Symbol** Name **Z**			227 **Ac** Actinium **89**	232 **Th** Thorium **90**	231 **Pa** Protactinium **91**	238 **U** Uranium **92**	237 Np Neptunium **93**	244 Pu Plutonium **94**	243 Am Americium **95**	247 Cm Curium **96**	247 Bk Berkelium **97**	251 Cf Californium **98**	252 Es Einsteinium **99**	257 Fm Fermium **100**	258 Md Mendelevium **101**	259 No Nobelium **102**	262 Lr Lawrencium **103**

1 Radioactive Decay

Radioactive decays occur when an atom of an isotope emits a radioactive particle. There are many types of radioactive decay, though the three main types of decay you will already be familiar with are: **alpha (α) decay**, **beta (β) decay**, and **gamma (γ) decay**. For A-level, there is also positron decay (also known as **beta+ (β^+) decay**).

1.1 Alpha Decay

Alpha decays occur when an atom emits an alpha particle.

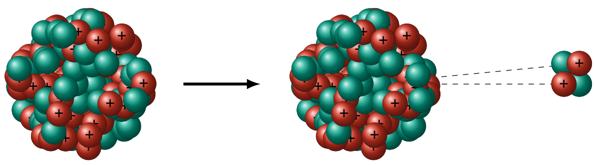


Figure 1: A representation of alpha decay. The atom emits 2 protons and 2 neutrons in the form of an alpha particle. The atom's mass number decreases by 4 and its atomic number decreases by 2.

Since an alpha particle contains 2 protons and 2 neutrons, the atom that emits the alpha particle **loses** 2 protons and 2 neutrons. This emission means that the atom actually changes into a different element! This process of turning one element into another through radioactive decay is called **transmutation**. Because we know exactly how many protons and neutrons the isotope loses, we can work out what the new element must be.

Let's use the example of Uranium-238. The atomic symbol for Uranium-238 is $^{238}_{92}\mathrm{U}$ and we can use this to work out that it has **92 protons** and **238-92 = 146 neutrons**.

Uranium-238 undergoes alpha decay, emitting 2 protons and 2 neutrons. After the decay, the atom therefore has **90** protons and **150** neutrons, which means its atomic number is 90 and its mass number is 234.

The element in the periodic table with 90 protons is **Thorium**. Our new isotope is therefore $^{234}_{90}\mathrm{Th}$. In other words, Uranium-238 is turning into Thorium-234 by emitting an alpha particle!

We can summarise this radioactive decay in a decay equation:

$$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$$

$$^{238}_{92}\mathrm{U} \longrightarrow \overbrace{{}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha}^{234\,+\,4\,=\,238}$$

$$\underbrace{\phantom{{}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha}}_{90\,+\,2\,=\,92}$$

Figure 2: The mass numbers and the atomic numbers on the right hand side must equal the mass number on the left hand side.

$$^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\alpha$$

Figure 3: You may come across this more formal, abstract notation for alpha decay.

Practice Questions 1-1 - Complete the full alpha decay equations.

1. $^{176}_{80}\text{Hg} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

2. $^{202}_{85}\text{At} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

3. $^{210}_{86}\text{Rn} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

4. $^{165}_{74}\text{W} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

5. $^{254}_{99}\text{Es} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

6. $^{218}_{87}\text{Fr} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

Practice Questions 1-2 - Which of these are valid alpha decay equations?

1. $^{177}_{80}\text{Hg} \longrightarrow {}^{4}_{2}\alpha + {}^{171}_{78}\text{Pt}$
2. $^{191}_{84}\text{Po} \longrightarrow {}^{4}_{2}\alpha + {}^{187}_{82}\text{Pb}$
3. $^{211}_{86}\text{Rn} \longrightarrow {}^{4}_{2}\alpha + {}^{208}_{83}\text{Bi}$
4. $^{209}_{87}\text{Fr} \longrightarrow {}^{4}_{2}\alpha + {}^{203}_{84}\text{Po}$
5. $^{209}_{85}\text{At} \longrightarrow {}^{4}_{2}\alpha + {}^{206}_{83}\text{Bi}$
6. $^{175}_{79}\text{Au} \longrightarrow {}^{4}_{2}\alpha + {}^{170}_{78}\text{Pt}$
7. $^{258}_{101}\text{Md} \longrightarrow {}^{4}_{2}\alpha + {}^{255}_{97}\text{Bk}$
8. $^{238}_{96}\text{Cm} \longrightarrow {}^{4}_{2}\alpha + {}^{234}_{94}\text{Pu}$
9. $^{202}_{82}\text{Pb} \longrightarrow {}^{4}_{2}\alpha + {}^{196}_{78}\text{Pt}$

1.2 *Beta Decay*

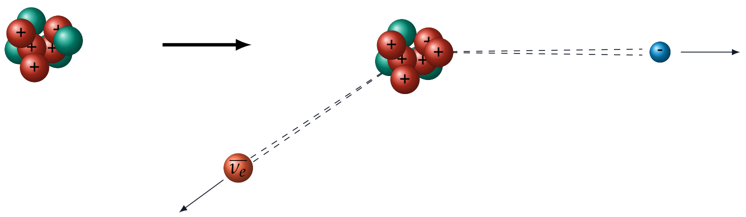


Figure 4: A representation of beta decay. In the nucleus, a neutron (green) turns into a proton (red) and an electron, and the atom emits the electron from the atom. When the neutron turns into a proton, an anti-electron neutrino is created, conserving Lepton number.

Beta particles are electrons. During beta decay, a **neutron** decays into a **proton**, an **electron**, and an **anti-electron neutrino**. The proton **stays in the nucleus** but the electron and anti-electron neutrino are ejected **ejected from the nucleus**.

Let's use the example of Carbon-14 with atomic symbol $^{14}_{6}$C. This isotope of carbon has a mass number of 14 and an atomic number of 6, and therefore has **6** protons and **14-6=8** neutrons.

During beta decay, one of the neutrons turns into a proton. This means that after beta decay the atom has **6+1 = 7** protons and **8-1=7 neutrons**. Crucially, since both protons and neutrons have a relative atomic mass of 1, the mass number of the new atom **stays the same** i.e. the mass number is still 14 in this case.

$$^{14}_{6}\text{C} \longrightarrow \underbrace{\overbrace{^{14}_{7}\text{N} + {}^{0}_{-1}\beta}^{14 + 0 = 14}}_{7 + -1 = 6} + \overline{\nu_e}$$

Figure 5: The mass numbers and the atomic numbers on the right hand side must equal the mass number on the left hand side.

Since the number of protons in the nucleus has increased to 7 the atom is no longer a Carbon atom and has transmuted into a new element. The element in the periodic table with atomic number 7 is **Nitrogen**. Therefore Carbon-14 turns into Nitrogen-14 when it undergoes beta decay.

An anti-electron neutrino ($\overline{\nu_e}$) is produced in the decay in order to conserve **lepton number**, and the anti-neutrino must be of the electron flavour because the flavour of the lepton number must be conserved. Anti-electron neutrinos have **tiny mass** and **no charge** so they do not change any of the mass or atomic numbers in the decay equation.

$$^{A}_{Z}\text{X} \longrightarrow {}^{A}_{Z+1}\text{Y} + {}^{0}_{-1}\beta + \overline{\nu_e}$$

Figure 6: You may come across this more formal, abstract notation for beta decay.

We can summarise this with the following decay equation:

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta + \overline{\nu_e}$$

Practice Questions 1-3 - Complete the full beta decay equations

1. $^{31}_{14}\text{Si} \longrightarrow {}^{\boxed{}}_{\boxed{}}\boxed{} + {}^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$
2. $^{140}_{57}\text{La} \longrightarrow {}^{\boxed{}}_{\boxed{}}\boxed{} + {}^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$
3. $^{162}_{65}\text{Tb} \longrightarrow {}^{\boxed{}}_{\boxed{}}\boxed{} + {}^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$

Practice Questions 1-4 - Are these valid beta decay equations?

1. $^{115}_{49}\text{In} \longrightarrow {}^{0}_{-1}\beta + {}^{113}_{48}\text{Cd} + \nu_\mu$
2. $^{214}_{82}\text{Pb} \longrightarrow {}^{0}_{-1}\beta + {}^{215}_{81}\text{Tl} + \overline{\nu_e}$
3. $^{91}_{36}\text{Kr} \longrightarrow {}^{0}_{-1}\beta + {}^{91}_{35}\text{Br} + \overline{\nu_e}$
4. $^{33}_{15}\text{P} \longrightarrow {}^{0}_{-1}\beta + {}^{31}_{15}\text{P} + \overline{\nu_e}$
5. $^{154}_{63}\text{Eu} \longrightarrow {}^{0}_{-1}\beta + {}^{154}_{65}\text{Tb} + \nu_e$
6. $^{116}_{46}\text{Pd} \longrightarrow {}^{0}_{-1}\beta + {}^{117}_{46}\text{Pd} + \overline{\nu_e}$
7. $^{63}_{28}\text{Ni} \longrightarrow {}^{0}_{-1}\beta + {}^{63}_{27}\text{Co} + \overline{\nu_e}$
8. $^{98}_{37}\text{Rb} \longrightarrow {}^{0}_{-1}\beta + {}^{98}_{37}\text{Rb} + \overline{\nu_e}$

1.3 *Positron Decay*

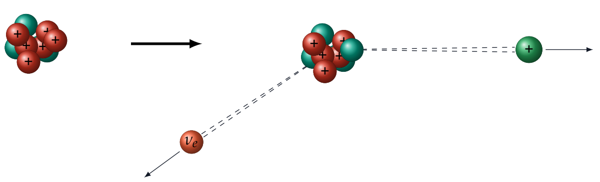


Figure 7: A representation of positron decay. In the nucleus, a proton (red) turns into a neutron (green) and a positron, and the atom emits the positron.

Beta+ particles are positrons (anti-electrons). During beta+ decay, a **proton** decays into a **neutron**, a **positron**, and an **electron neutrino**. The neutron **stays in the nucleus** but the positron and electron neutrino are **ejected from the nucleus**.

Let's use the example of Fluorine-18 with atomic symbol $^{18}_{9}\mathrm{F}$. This isotope of Fluorine has a mass number of 18 and an atomic number of 9, and therefore has **9** protons and **18-9 = 9** neutrons.

During beta+ decay, one of the protons turns into a neutron. This means that after beta+ decay the atom has **9-1 = 8** protons and **9+1 = 10 neutrons**. Crucially, since both protons and neutrons have a relative atomic mass of 1, the mass number of the new atom **stays the same** i.e. the mass number is still 18 in this case.

$$^{18}_{9}\mathrm{F} \longrightarrow \overbrace{^{18}_{8}\mathrm{O} + \,^{0}_{+1}\beta}^{18+0=18} + \nu_e \qquad \underbrace{\phantom{^{18}_{8}\mathrm{O} + \,^{0}_{+1}\beta}}_{8+1=9}$$

Figure 8: The mass numbers and the atomic numbers on the right hand side must equal the mass number on the left hand side.

Since the number of protons in the nucleus has decreased to 8, the atom is no longer a Fluorine atom and has transmuted into a new element. The element in the periodic table with atomic number 8 is **Oxygen**. Therefore Carbon-18 turns into Oxygen-18 when it undergoes beta+ decay.

An electron neutrino (ν_e) is produced in the decay in order to conserve **lepton number**, and the neutrino must be of the electron flavour because the flavour of the lepton number must be conserved.

Like electron anti-neutrinos, electron neutrinos have **tiny mass** and **no charge** so they do not change any of the mass or atomic numbers in the decay equation.

$$^{A}_{Z}\mathrm{X} \longrightarrow \,^{A}_{Z-1}\mathrm{Y} + \,^{0}_{+1}\beta + \nu_e$$

Figure 9: You may come across this more formal, abstract notation for beta+ decay.

We can summarise this with the following decay equation:

$$^{18}_{9}\mathrm{F} \longrightarrow \,^{18}_{8}\mathrm{O} + \,^{0}_{+1}\beta + \nu_e$$

Practice Questions 1-5 - Complete the full positron decay equations

1. $^{31}_{14}\mathrm{Si} \longrightarrow \,^{\boxed{}}_{\boxed{}}\boxed{} + \,^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$

2. $^{140}_{57}\mathrm{La} \longrightarrow \,^{\boxed{}}_{\boxed{}}\boxed{} + \,^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$

3. $^{162}_{65}\mathrm{Tb} \longrightarrow \,^{\boxed{}}_{\boxed{}}\boxed{} + \,^{\boxed{}}_{\boxed{}}\boxed{} + \boxed{}$

Practice Questions 1-6 - Are these valid positron decay equations?

1. $^{100}_{40}\text{Zr} \longrightarrow {}^{0}_{+1}\beta + {}^{98}_{40}\text{Zr} + \nu_{\mu}$	5. $^{69}_{29}\text{Cu} \longrightarrow {}^{0}_{+1}\beta + {}^{70}_{30}\text{Zn} + \nu_{e}$
2. $^{240}_{92}\text{U} \longrightarrow {}^{0}_{+1}\beta + {}^{238}_{91}\text{Pa} + \overline{\nu_{e}}$	6. $^{126}_{53}\text{I} \longrightarrow {}^{0}_{+1}\beta + {}^{124}_{54}\text{Xe} + \nu_{e}$
3. $^{81}_{32}\text{Ge} \longrightarrow {}^{0}_{+1}\beta + {}^{79}_{33}\text{As} + \nu_{e}$	7. $^{80}_{32}\text{Ge} \longrightarrow {}^{0}_{+1}\beta + {}^{81}_{32}\text{Ge} + \nu_{\tau}$
4. $^{136}_{52}\text{Te} \longrightarrow {}^{0}_{+1}\beta + {}^{134}_{52}\text{Te} + \nu_{e}$	8. $^{106}_{45}\text{Rh} \longrightarrow {}^{0}_{+1}\beta + {}^{106}_{45}\text{Rh} + \nu_{e}$

1.4 *Gamma Decay*

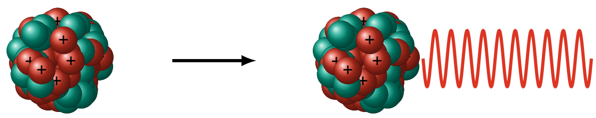


Figure 10: A representation of gamma decay. An atom emits an electromagnetic wave, lowering the overall energy of the atom.

Gamma decay is the simplest of the three decays. During gamma decay, an atom emits an electromagnetic wave in the gamma frequency part of the electromagnetic spectrum. Since gamma waves have no charge and no mass, the proton number and atomic number of the atom stays the same; the atom simply has less energy overall than it did before it emitted the gamma wave.

$$^{152}_{66}\text{Dy} \longrightarrow \underbrace{\overbrace{^{52}_{66}\text{Dy} + {}^{0}_{0}\gamma}^{152\,+\,0\,=\,152}}_{66+\,0\,=\,66}$$

Figure 11: The mass numbers and the atomic numbers on the right hand side must equal the mass number on the left hand side.

Let's look at Dysprosium-152 with atomic symbol $^{152}_{66}\text{Dy}$. Dysprosium has **66** protons and **152-66 = 86** neutrons. After emitting a gamma wave, it still has **66** protons and **86** neutrons. Therefore, its decay equation can be written:

$$^{152}_{66}\text{Dy} \longrightarrow {}^{152}_{66}\text{Dy} + {}^{0}_{0}\gamma$$

$$^{A}_{Z}\text{X} \longrightarrow {}^{A}_{Z}\text{Y} + {}^{0}_{0}\gamma$$

Figure 12: You may come across this more formal, abstract notation for gamma decay.

> Only α and β decays result in **transmutation** i.e. the turning of one element into another.

Practice Questions 1-7 - Complete the full gamma decay equations

1. $^{133}_{54}\text{Xe} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$
2. $^{202}_{86}\text{Rn} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$
3. $^{180}_{72}\text{Hf} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$
4. $^{77}_{33}\text{As} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$
5. $^{159}_{64}\text{Gd} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$
6. $^{252}_{98}\text{Cf} \longrightarrow {}^{\square}_{\square}\square + {}^{\square}_{\square}\square$

Practice Questions 1-8 - Are these valid gamma decay equations?

1. $^{151}_{62}Sm \longrightarrow {}^{0}_{0}\gamma + {}^{151}_{62}Sm$
2. $^{133}_{53}I \longrightarrow {}^{0}_{0}\gamma + {}^{132}_{53}I$
3. $^{84}_{34}Se \longrightarrow {}^{0}_{0}\gamma + {}^{84}_{33}As$
4. $^{102}_{43}Tc \longrightarrow {}^{0}_{0}\gamma + {}^{102}_{42}Mo$
5. $^{135}_{52}Te \longrightarrow {}^{0}_{0}\gamma + {}^{134}_{51}Sb$
6. $^{231}_{89}Ac \longrightarrow {}^{0}_{0}\gamma + {}^{230}_{89}Ac$
7. $^{39}_{17}Cl \longrightarrow {}^{0}_{0}\gamma + {}^{38}_{17}Cl$
8. $^{208}_{81}Tl \longrightarrow {}^{0}_{0}\gamma + {}^{207}_{80}Hg$
9. $^{103}_{43}Tc \longrightarrow {}^{0}_{0}\gamma + {}^{103}_{43}Tc$

Practice Questions 1-9 - Review: Mixed Decay Equations

1. $^{172}_{69}Tm \longrightarrow {}^{0}_{-1}\beta + {}^{\square}_{\square}\square + \square$
2. $^{204}_{85}At \longrightarrow {}^{4}_{2}\alpha + {}^{\square}_{\square}\square + \square$
3. $^{246}_{98}Cf \longrightarrow {}^{4}_{2}\alpha + {}^{\square}_{\square}\square + \square$
4. $^{71}_{30}Zn \longrightarrow {}^{0}_{+1}\beta + {}^{\square}_{\square}\square + \square$
5. $^{160}_{74}W \longrightarrow {}^{0}_{0}\gamma + {}^{\square}_{\square}\square + \square$
6. $^{28}_{12}Mg \longrightarrow {}^{0}_{-1}\beta + {}^{\square}_{\square}\square + \square$
7. $^{175}_{80}Hg \longrightarrow {}^{0}_{+1}\beta + {}^{\square}_{\square}\square + \square$
8. $^{36}_{17}Cl \longrightarrow {}^{0}_{0}\gamma + {}^{\square}_{\square}\square + \square$
9. $^{213}_{87}Fr \longrightarrow {}^{4}_{2}\alpha + {}^{\square}_{\square}\square + \square$
10. $^{40}_{17}Cl \longrightarrow {}^{0}_{-1}\beta + {}^{\square}_{\square}\square + \square$

Practice Questions 1-10 - Review: What type of radioactive decay has occurred?

1. $^{127}_{50}\text{Sn} \longrightarrow {}^{127}_{49}\text{In} + {}^{\square}_{\square}\square + \square$

2. $^{228}_{89}\text{Ac} \longrightarrow {}^{228}_{90}\text{Th} + {}^{\square}_{\square}\square + \square$

3. $^{47}_{18}\text{Ar} \longrightarrow {}^{47}_{18}\text{Ar} + {}^{\square}_{\square}\square + \square$

4. $^{211}_{83}\text{Bi} \longrightarrow {}^{207}_{81}\text{Tl} + {}^{\square}_{\square}\square + \square$

5. $^{101}_{40}\text{Zr} \longrightarrow {}^{101}_{41}\text{Nb} + {}^{\square}_{\square}\square + \square$

6. $^{170}_{69}\text{Tm} \longrightarrow {}^{170}_{68}\text{Er} + {}^{\square}_{\square}\square + \square$

7. $^{244}_{94}\text{Pu} \longrightarrow {}^{240}_{92}\text{U} + {}^{\square}_{\square}\square + \square$

8. $^{219}_{85}\text{At} \longrightarrow {}^{215}_{83}\text{Bi} + {}^{\square}_{\square}\square + \square$

9. $^{187}_{81}\text{Tl} \longrightarrow {}^{183}_{79}\text{Au} + {}^{\square}_{\square}\square + \square$

10. $^{156}_{62}\text{Sm} \longrightarrow {}^{156}_{62}\text{Sm} + {}^{\square}_{\square}\square + \square$

2 *Ionisation*

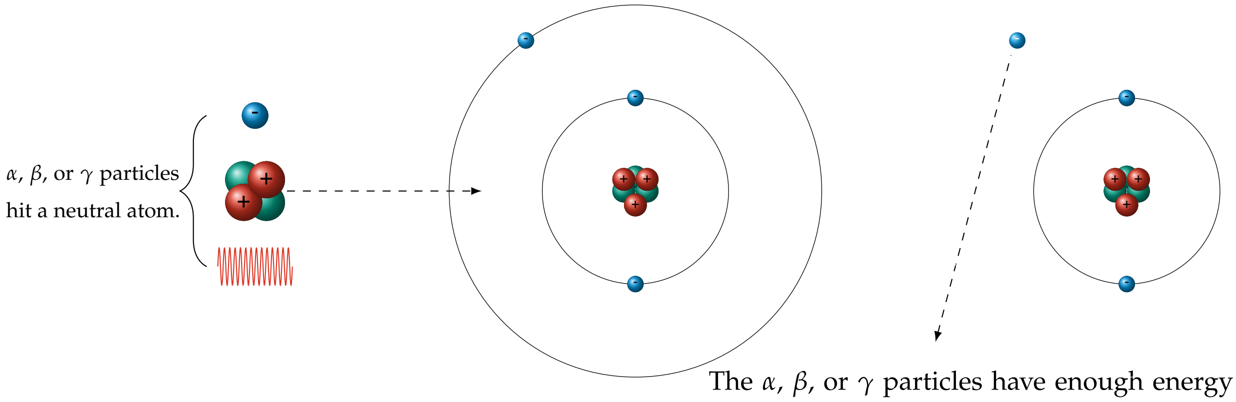
When a radioactive particle hits a neutral atom, it can provide enough energy to **remove an electron** from the atom. When the atom loses an electron, it has a net positive charge, and is therefore called an **ion**. **Ionisation** is the name given to this process.

The danger of radioactive particles is closely related to their ionising ability (i.e. their ability to knock electrons off of neutral atoms). The order of ionising ability from most to least ionising amongst the radioactive particles is as follows:

$\beta+$ particles (positrons) are a more nuanced case that we will discuss in a moment

1. Alpha (α) Particles.
2. Beta (β) Particles.
3. Gamma (γ) Waves.

α **particles** are most ionising because they have a +2 overall charge: they **pull** electrons off other atoms very easily, ionising them.



The α, β, or γ particles have enough energy to remove electrons from atoms, **ionising** them.

β**- particles** are the next most ionising because they have a high velocity: they carry large amounts of kinetic energy that can be transferred to other electrons, giving these electrons enough energy to escape from their host atoms

γ **rays** are weakly ionising and will pass through most materials without ionising them. If conditions are just right, their energy can be transferred to electrons in the neutral atom causing the electrons to escape from their host atom.

β**+ particles** are more complicated. Since β+ particles are identical to electrons except they have opposite charge, in theory they should have the exact same ionising ability. However, as soon as a β+ particle meets an electron it annihilates to form 2 γ rays. There is a huge asymmetry of matter to anti-matter in the universe - i.e. there are far more electrons than positrons - and so positrons usually do not manage to travel very far before meeting an electron and annihilating.

3 *Penetration*

The penetrative ability of radioactive particles is strongly related to their ionisation ability: the most highly ionising particles do not penetrate materials very far because they ionise atoms very easily! The order of penetrative ability from most to least penetrating amongst the radioactive particles is as follows:

1. Gamma (γ) waves (photons).
2. Beta- (β^-) particles (electrons).
3. Alpha (α) particles (Helium nuclei).
4. Beta+ (β^+) particles (positrons).

Notice that this is the reverse of the ionisation ability list, as expected.

Alpha particles are stopped very easily by almost any material; remember, they only travel a few centimetres in air before ionising and so their penetrative ability in anything more dense than air will be even less. Outside of the body, alpha particles pose little risk to humans: the layer of dead skin on the surface of our bodies is thick

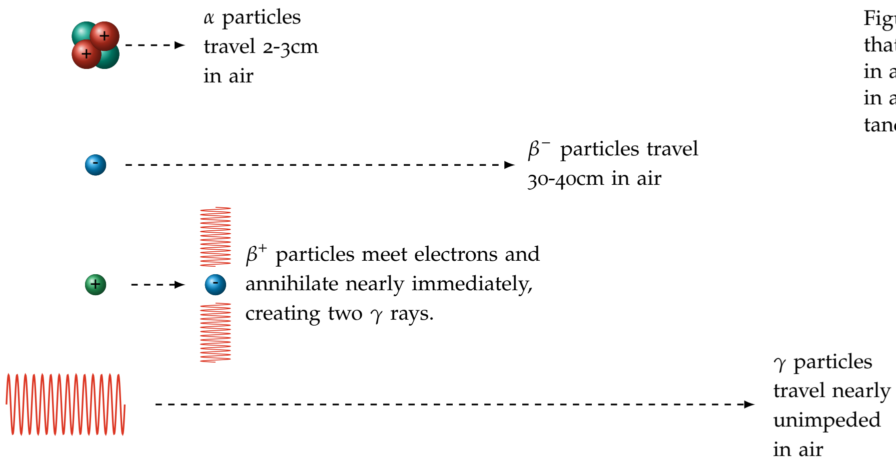
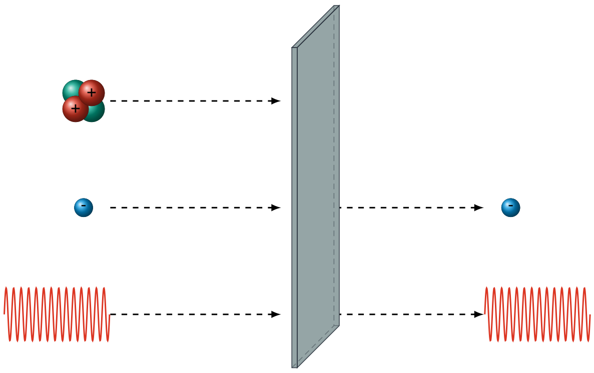


Figure 13: α particles are so ionising that they only travel a few centimetres in air. β particles can travel up to 40cm in air, and γ waves can travel large distances in air before ionising an atom.

enough to ionising the alpha particles and stop them travelling further.

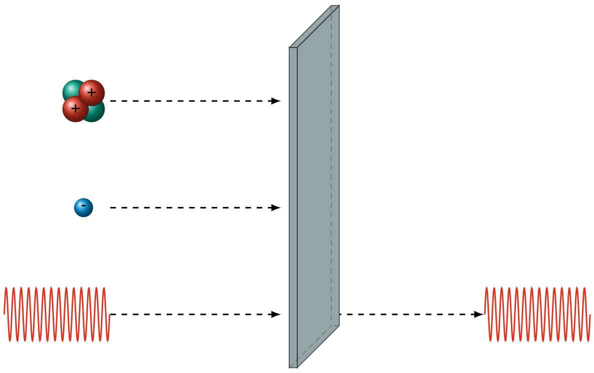
Beta particles penetrate 10x further than alpha particles in air, and therefore can penetrate small thicknesses of some materials e.g. a few mm of aluminium foil. Beta particles can therefore be harmful outside of the body provided you are close enough to the source.

Only thick lead (2 inches or more) or thick concrete (several metres) stops gamma waves by any significant amount. Because they're electromagnetic waves with no mass, no charge, and a tiny wavelength, they pass through most things without ionising them. Strictly speaking it's impossible to fully stop gamma waves. Thick lead and concrete just reduce the chances of a gamma wave getting through, but some gamma waves will still get through. We can only reduce the intensity of the gamma rays i.e. the number arriving per second in a given area. Gamma rays can therefore be harmful if the source is outside the body, but their probability of hitting an atom and ionising it is quite low.



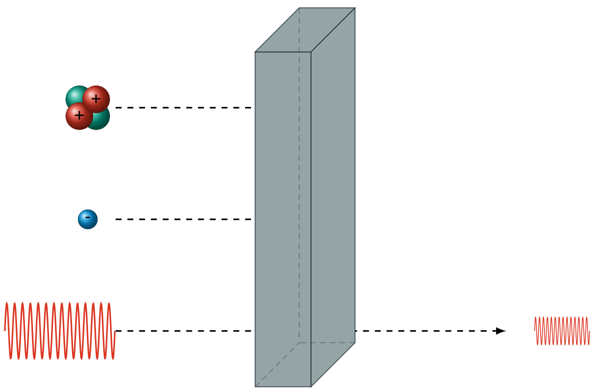
Thin paper

α particles are so ionising that even a thin sheet of paper is enough to stop them.



Thin Aluminium

Thin Aluminium foil stops α particles easily and can also stop β^- particles if the thickness is more than 5mm.



Thick Concrete/Lead

Thick lead or concrete stops α and β^- particles easily. It can also stop gamma rays and reduces their intensity significantly.

4 *Half-Life*

A radioactive isotope is an isotope that is **unstable** and one that will emit a radioactive particle. Unfortunately, radioactive particles decay **randomly** and we have no way of knowing when an individual nucleus of a radioactive isotope will decay and emit a radioactive particle. Radioactive decays are also **spontaneous**, meaning they cannot be triggered by e.g. temperature, pressure, or other physical conditions.

- **Unstable**: unstable nuclei are likely to **randomly**, and **spontaneously** decay because they have **too much energy** and/or **too many nucleons**.

- **Random**: it is not possible to predict when an individual nucleus will decay, we can only predict the probability or fraction of nuclei that will decay in a given time.
- **Spontaneous**: radioactive decay cannot be controlled by external factors such as temperature or pressure.

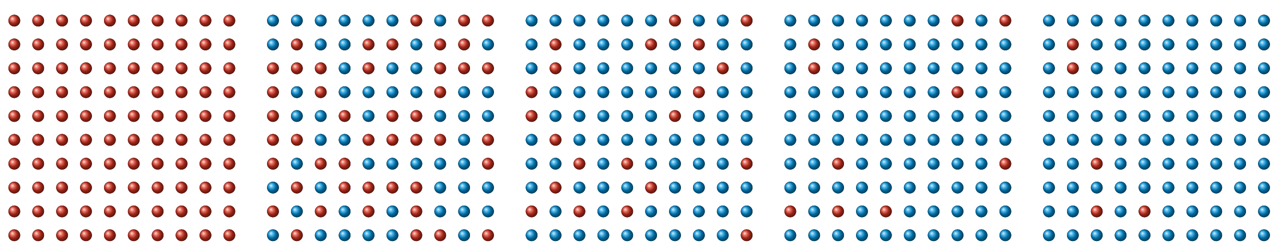
However, physicists discovered that if they had enough nuclei together in one place they could predict how long it would take a certain proportion of them to decay. In particular, they discovered that the time it takes for 50% of the radioactive nuclei to decay was always the same, and they called this the half-life.

The Half-Life of an isotope is the **average time taken** for **50%** of the radioactive atoms in a sample to decay.

Let's say we have **100** atoms of the isotope Rubidium-84. After 1 half life (HL), half of the atoms will decay, leaving **50** atoms. After another half-life, half of those 50 atoms will have decayed, leaving **25** atoms. After another half-life, half of those 25 atoms will have decayed, leaving **12.5** atoms. Since we can't have half an atom, in reality there will be either **12** or **13** atoms remaining, on average. After another half-life, we expect 50% of the 12 or 13 atoms to have decayed, leaving **6** or **7** atoms. We can summarise this using the following decay chain:

$$100\,\text{Atoms} \xrightarrow[+\,1\,\text{HL}]{1\,\text{HL}} 50\,\text{Atoms} \xrightarrow[+\,1\,\text{HL}]{2\,\text{HLs}} 25\,\text{Atoms} \xrightarrow[+\,1\,\text{HL}]{3\,\text{HLs}} 13\,\text{Atoms} \xrightarrow[+\,1\,\text{HL}]{4\,\text{HLs}} 7\,\text{Atoms}$$

It is very important to understand that since radioactive decay is completely random, the actual number of atoms remaining fluctuates a bit around the expected values; if we flip 50 coins, we do not expect **exactly** 25 to turn up heads, but rather 25 on average. The same is true of radioactive decay. Figure 14 below shows a sample of atoms from a radioactive isotope decaying at random.



Initial: 100 Atoms. 1HL: 48 Atoms 2 HLs: 22 Atoms 3 HLs: 9 Atoms 4 HLs: 5 Atoms

Figure 14: A visual representation of Half-Life. After each half-life, 50% of the remaining atoms decay.

Practice Questions 4-1 - How many atoms remain?

All half-lives are with respect to the **initial** number of atoms, not the previous box.

1. 462 Atoms $\xrightarrow{1\,HL}$ ☐ $\xrightarrow{2\,HL}$ ☐ $\xrightarrow{3\,HL}$ ☐ $\xrightarrow{4\,HL}$ ☐

2. 812 Atoms $\xrightarrow{1\,HL}$ ☐ $\xrightarrow{2\,HL}$ ☐ $\xrightarrow{3\,HL}$ ☐ $\xrightarrow{4\,HL}$ ☐

3. 198 Atoms $\xrightarrow{1\,HL}$ ☐ $\xrightarrow{2\,HL}$ ☐ $\xrightarrow{3\,HL}$ ☐ $\xrightarrow{4\,HL}$ ☐

Practice Questions 4-2 - How many atoms remain after the given half-lives?

Half-Lives	No. of Atoms
0	26
1	
2	
5	
8	
10	

Half-Lives	No. of Atoms
0	429
2	
4	
7	
9	
10	

Half-Lives	No. of Atoms
0	258
5	
7	
8	
9	
10	

So far we have talked about how atoms decay after a given number of half lives. But we haven't discussed the time element much. Remember our definition of half-life: the **time-taken** for **50%** of the atoms in a sample to decay.

Each unstable isotope has its own characteristic half-life that tells us how long it takes for 50% of its atoms to decay. Let's use the example of Rubidium-84, which has a half-life of 33 days.

You can think of a radioactive element's half-life as its own fingerprint: no two radioactive isotopes share the same half-life.

This means it takes 33 days for half of the atoms in a sample of Rubidium-84 to decay. If we start with a sample of **80** Rubidium-84 atoms, there are **40** atoms of Rubidium-84 remaining after 33 days (1 Half-Life). After a further 33 days (66 days in total, $66/33 = 2$ Half-Lives), there are **20** atoms of Rubidium-84 left. After another 33 days (99 days in total, $99/33 = 3$ Half-Lives), there are **10** atoms of Rubidium-84 remaining, and so on.

Isotope	Half-Life
Na-22	2.6 years
H-3	12 years
Se-75	120 days
Ru-105	4.44 hours
Ag-110	25 s
U-238	4.5 billion years

Figure 15: Some sample half-lives of radioactive isotopes

In order to understand how much of an isotope remains after a given time, we need to calculate many half-lives have occurred in that time. For example, if we wanted to know how much Rubidium-84 remained after 90 days, we would need to first realise that 3 half-lives (90/30) had occurred in that time.

Practice Questions 4-3 - Calculate how many half-lives have occurred

Isotope Half-Life = 147 years	
Time (years)	Half-Lives
0	0
147	1
441	3
735	
1029	
1470	

Isotope Half-Life = 644 seconds	
Time (seconds)	Half-Lives
0	
644	
1288	
3220	
5152	
5796	

Isotope Half-Life = 731 hours	
Time (hours)	Half-Lives
0	
1462	
2193	
5848	
6579	
7310	

Practice Questions 4-4 - Calculate the number of half-lives occurred to find out how many atoms remain

Isotope Half-Life = 952 days		
Time (days)	Half-Lives	No. of Atoms
0	0	7866
1904		
2856		
3808		
4760		
8568		

Isotope Half-Life = 271 seconds		
Time (seconds)	Half-Lives	No. of Atoms
0	0	2314
542		
1084		
1355		
1626		
2168		

4.1 *Activity and Becquerels*

So far we have talked about how many **atoms** decay and how many remain after a given number of half-lives. But there are other units that we can use to measure radioactive decay.

We can measure the **Activity** of an isotope, which gives us a measure of how many radioactive decays per second occur: the higher the activity, the more radioactive decays per second.

The unit used to measure activity is the Becquerel (Bq).

> The Becquerel (Bq) measures how many **disintegrations/decays per second** occur.
>
> 1 Bq = 1 radioactive disintegration/decay per second.

Number of Atoms, **Activity**, and **Mass** all follow the same behaviour and rules when it comes to radioactive decay.

The activity is quite closely related to the number of atoms: if we have more atoms decaying, there will be more radioactive decays per second, and therefore a higher activity.

Similarly, if we have a higher mass of radioactive isotope, this will contain more atoms, and therefore the sample will have a higher activity.

4.2 *Background Radiation and Corrected Count Rate*

If you turn on a Geiger counter even far away from any radioactive sources it will still click and measure an activity. This is because of **background radiation**. There are many sources of background radiation:

- **Air**: there is a small amount of radioactive radon gas in the air.
- **Buildings and materials**: the materials that make up the buildings and some of the materials around us have small amounts of radioisotopes in them.
- **Food and other organic matter**: Living things that contain Carbon contain a small amount of radioactive Carbon-14. Some of the foods we eat also contain radioisotopes e.g. Potassium in bananas.
- **Cosmic rays**: cosmic rays produced in outer space hurtle down to Earth and react with atoms in the upper atmosphere and produce radioisotopes in the process.
- **Industrial and medical**: hospitals and some industry make use of radioisotopes.

Cosmic rays are produced in highly energetic astrophysical phenomena such as when two black holes collide.

When measuring the activity of a source, **we must subtract the background radiation count rate** from the measured activity. This calculated activity is called the **corrected count rate**.

Worked Example 4-4 - Corrected count rate

Q: The activity of a sample of radioisotope is 460 Bq. 30 minutes later, the activity of the sample has fallen to 240 Bq. The background radiation is measured to have an activity of 20 Bq. Calculate the half-life of the radioisotope.

A: Before doing anything we must correct the activity for the background count rate. This gives a corrected initial activity of $460 - 20 = 440$ Bq and a corrected final activity of $240 - 20 = 220$ Bq.

We can see that the corrected activity has halved in 30 minutes, and therefore the half-life is 30 minutes.

Notice that if we did not correct for the count rate, we would have overestimated the half life. This problem gets worse the lower the count rate measured.

4.3 *Parents and Daughters - What are the radioactive atoms decaying into?*

In Figure 14 we showed the red, undecayed particles turning into the blue, decayed particles. What do the red and blue particles represent? It depends on the kind of decay the atoms are undergoing. Let's use the example of Rubidium-84, which undergoes beta- decay.

$$^{84}_{37}\mathrm{Rb} \longrightarrow {}^{84}_{38}\mathrm{Sr} + {}^{0}_{-1}\beta + \overline{\nu_e}$$

This means that the new, blue atoms are actually Strontium atoms. So while the number of Rubidium atoms is decreasing with each decay, the number of Strontium atoms is increasing. We call the initial isotope the **parent** nucleus, and the new isotope that is formed the

daughter nucleus. In this case the Rubidium is the parent isotope and Strontium the daughter isotope.

Practice Questions 4-5 - Calculate the number of atoms remaining

All arrows are with respect to the **initial** activity. Write the number of half-lives that have occurred since the **initial** value above the arrows.

Isotope Half-Life = 720 years

1. 2000 Bq $\xrightarrow[\text{720 years}]{\text{1 HL}}$ ☐ $\xrightarrow[\text{1440 years}]{\text{2 HL}}$ ☐ $\xrightarrow[\text{2160 years}]{}$ ☐ $\xrightarrow[\text{2880 years}]{}$ ☐

Isotope Half-Life = 49 seconds

2. 120 g $\xrightarrow[\text{49 seconds}]{\text{1 HL}}$ ☐ $\xrightarrow[\text{98 seconds}]{}$ ☐ $\xrightarrow[\text{147 seconds}]{}$ ☐ $\xrightarrow[\text{196 seconds}]{}$ ☐

Isotope Half-Life = 707 hours

3. 800 Atoms $\xrightarrow[\text{707 hours}]{\text{1 HL}}$ ☐ $\xrightarrow[\text{1414 hours}]{}$ ☐ $\xrightarrow[\text{2121 hours}]{}$ ☐ $\xrightarrow[\text{2828 hours}]{}$ ☐

Practice Questions 4-6 - How many atoms remain after the given time?

Write the number of half-lives that have occurred and use this to calculate the activity in each of the boxes. **The half-lives are with respect to the initial time and activity, not from the previous box.**

Isotope Half-Life = 855 hours

1. 294 Bq $\xrightarrow{\text{2565 hours}}$ ☐ $\xrightarrow{\text{3420 hours}}$ ☐ $\xrightarrow{\text{6840 hours}}$ ☐ $\xrightarrow{\text{7695 hours}}$ ☐

Isotope Half-Life = 63 years

2. 254 Bq $\xrightarrow{\text{189 years}}$ ☐ $\xrightarrow{\text{378 years}}$ ☐ $\xrightarrow{\text{441 years}}$ ☐ $\xrightarrow{\text{567 years}}$ ☐

Isotope Half-Life = 547 minutes

3. 27 Bq $\xrightarrow{\text{1094 minutes}}$ ☐ $\xrightarrow{\text{1641 minutes}}$ ☐ $\xrightarrow{\text{2188 minutes}}$ ☐ $\xrightarrow{\text{3829 minutes}}$ ☐

Practice Questions 4-7 - What's the half-life of the isotope?

Write the number of half-lives that have occurred and use this to calculate the activity in each of the boxes. **The half-lives are with respect to the initial time and activity, not from the previous box.**

1. 381 Bq $\xrightarrow[\text{Half Lives} =]{112\text{ days}}$ 23.8 Bq
Half Life =

2. 296 Atoms $\xrightarrow[\text{Half Lives} =]{2664\text{ minutes}}$ 37 Atoms
Half Life =

3. 98 Bq $\xrightarrow[\text{Half Lives} =]{5754\text{ years}}$ 1.5 Bq
Half Life =

4. 982 kg $\xrightarrow[\text{Half Lives} =]{1253\text{ minutes}}$ 7.7 kg
Half Life =

5. 516 Bq $\xrightarrow[\text{Half Lives} =]{1820\text{ seconds}}$ 32.2 Bq
Half Life =

6. 45 g $\xrightarrow[\text{Half Lives} =]{2820\text{ years}}$ 2.8 g
Half Life =

5 *Half Life Graphs*

Let's conduct our radioactive decay experiment again, this time starting with **80** atoms of an isotope with a half life of **50** years. We know that after 50 years (1 half-life) the number of atoms remaining of the original isotope should be roughly **40**. After 100 years (2 half-lives), it should be roughly **20**, after 150 years (3 half-lives) there should be around **10** atoms remaining, and after 200 years (4 half-lives), roughly **5** atoms should remain.

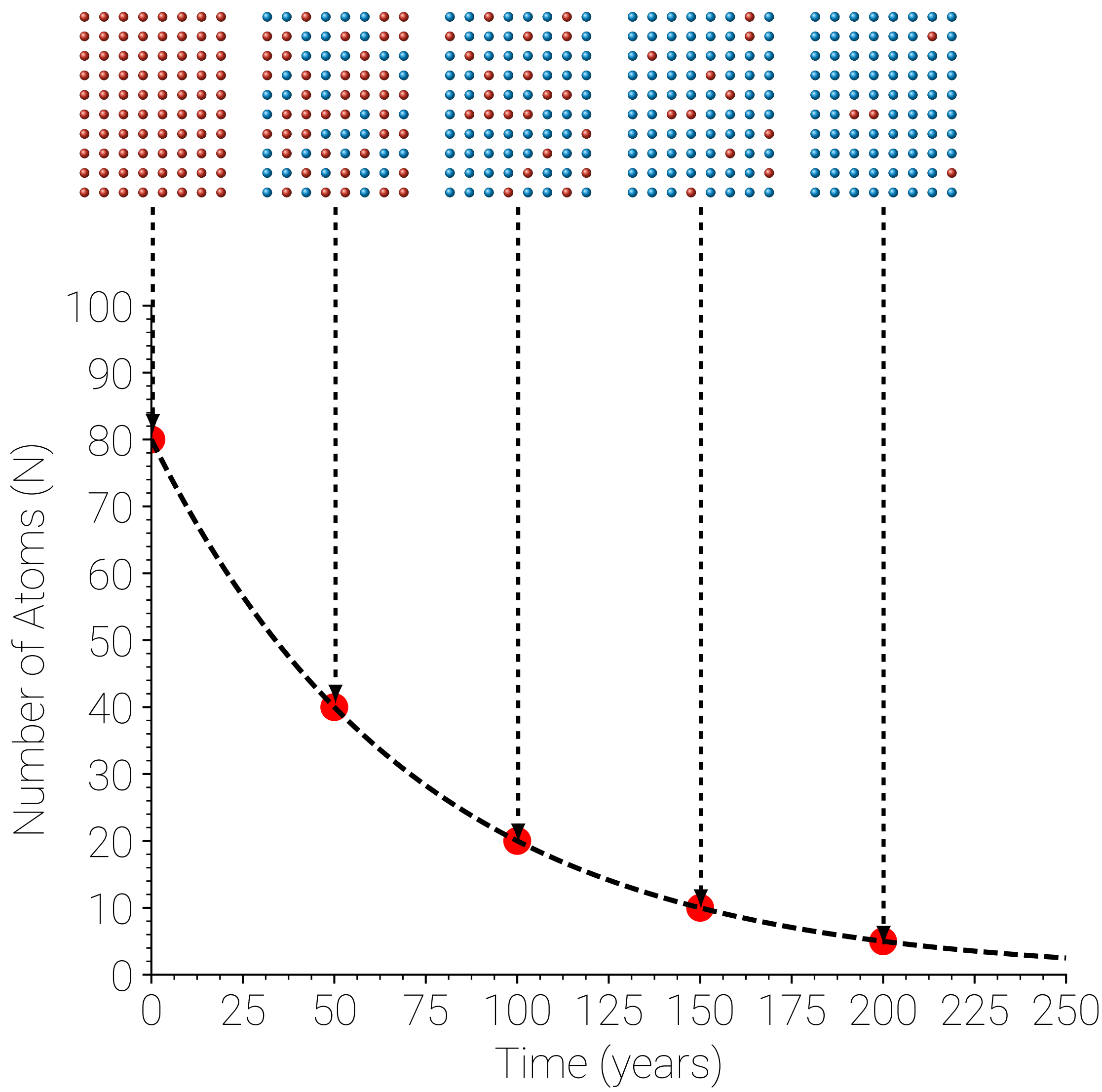


Figure 16: The classic **exponential decay** curves of radioactive decay. After each half-life, 50% of the remaining atoms decay.

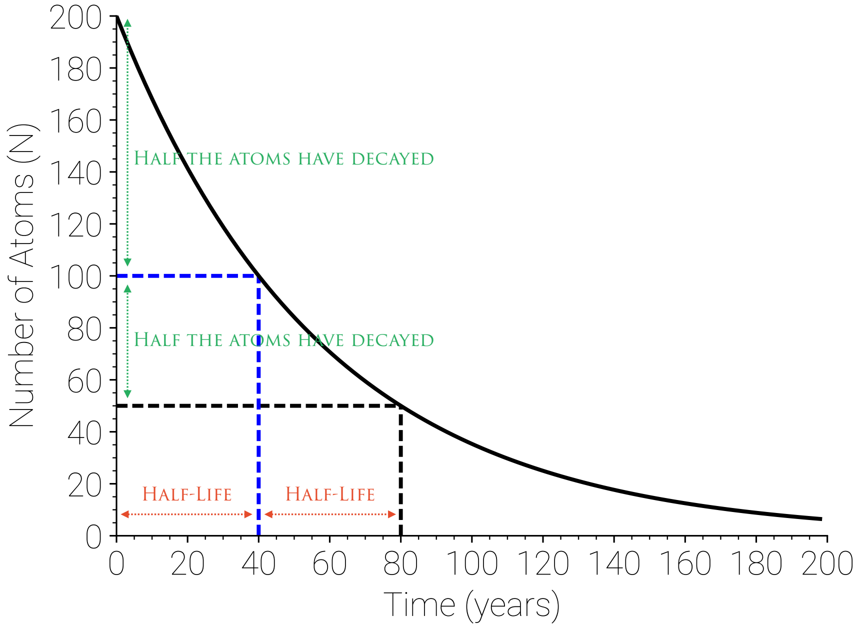


Figure 17: We find the half-live by picking an initial number of atoms/activity/mass, and finding how long it takes to get to half of that number of atoms/activity/mass.

If we plot the number of atoms vs time, we get a graph of that looks like the one in Figure 16. This type of curve is called an **exponential decay** curve. Notice that the curve gets less steep with time: this is because there are fewer radioactive particles left that are able to decay, and so the number decaying in each period of time decreases. However, the **proportion/percentage** of particles that decay in a given time period is always the same.

The amazing thing about these kinds of exponential decay curves is that we can find out the half-life of an isotope using them. Figure 18 shows the basic procedure for finding the half-life of a radioisotope using its decay graph. Remember that our y-axis units could equally well be activity/mass.

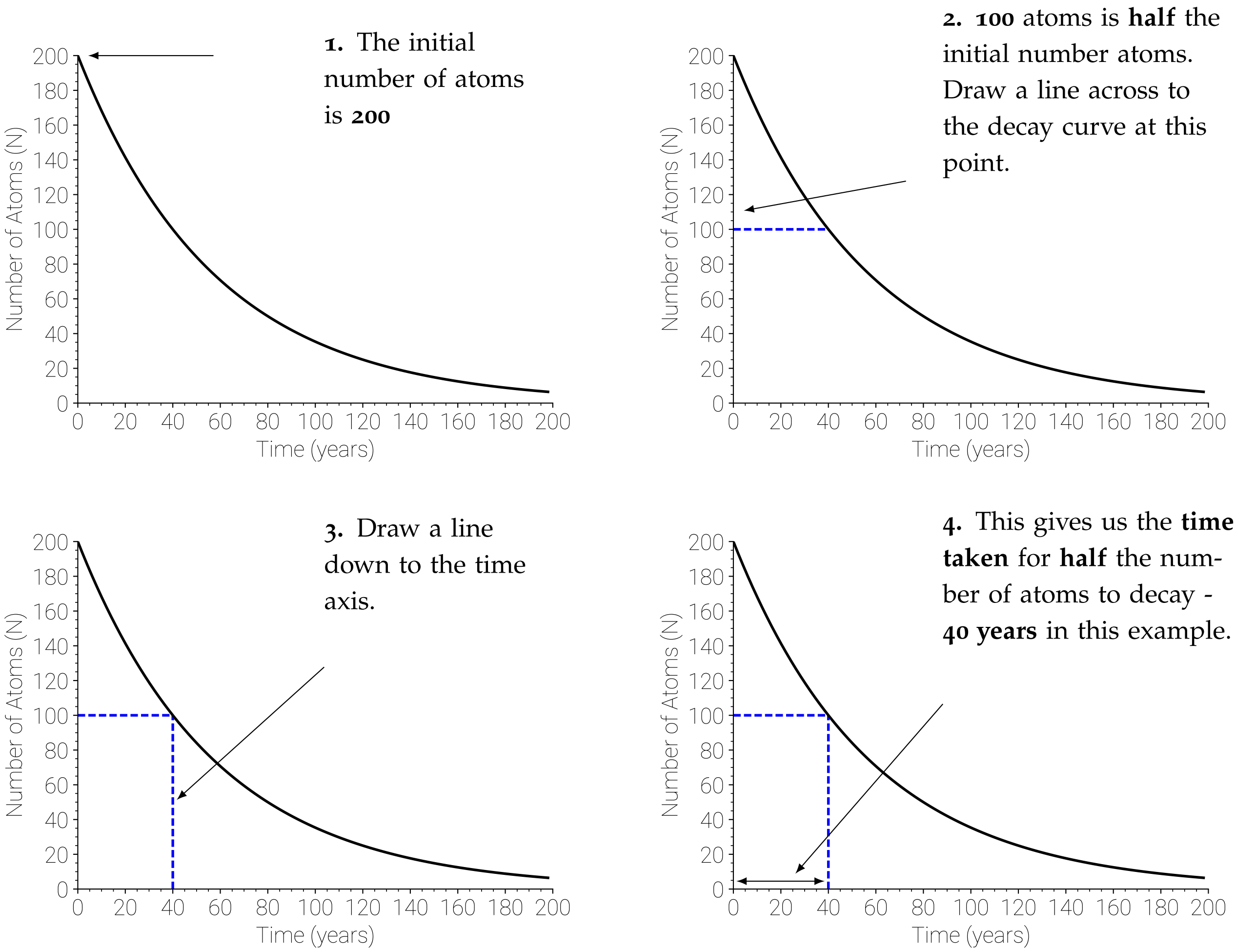


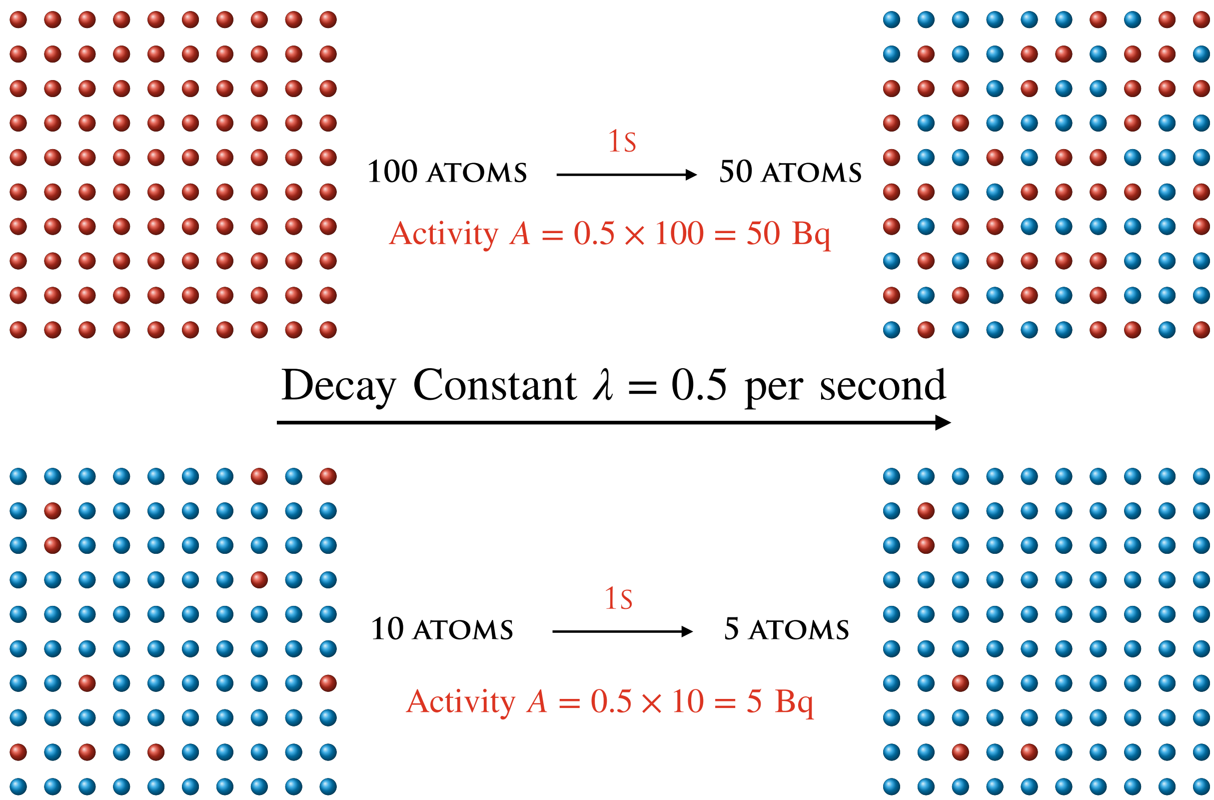
Figure 18: **How to find the half life from a radioactive decay curve**. The procedure for finding the half life from a radioactive decay curve. This also works if "Activity" or "Mass" is on the y-axis.

6 *Half-Life II*

So far we have focussed on situations where we calculate the number of half-lives remaining after a whole number of half-lives have occurred. For example, if an isotope has 100 atoms and a half-life of 25 seconds, the number of atoms remaining after 50 seconds would be 25 atoms. But what if we wanted to know the number atoms remaining after 40 seconds? Similarly, if we started with 100 atoms and find after 60 seconds that we have 20 atoms remaining, can we calculate the half-life of the isotope from that knowledge?

The answer is yes, but we need to introduce some new mathematical machinery. This mathematical machinery is the main difference between GCSE radioactivity and A-level radioactivity, and it will be the focus of this section.

The first observation we need to make is that the number of radioactive particles emitted per second by an isotope (the activity) depends on the number of atoms of the isotope.



Therefore, we can say:

$$A \propto N$$

where A is the activity and N is the number of particles of the isotope.

Using our usual rules of proportionality:

$$\boxed{\begin{aligned} A &\propto N \\ \implies A &= \lambda N \end{aligned}}$$

where λ is a constant known as the **decay constant**.

In the case of γ decay, the "daughter" nuclei is the same as the "parent" nuclei except in a lower-energy state.

We can write $A = -\dfrac{dN}{dt}$ since this simply says that the activity is the rate of change of the number of particles of the parent isotope. The negative sign is because the number of parent isotope particles decreases during the decay, while the number of daughter particles increases. Since a change from parent to daughter only occurs when a radioactive particle is emitted, it makes sense that the activity (which measures the number of radioactive decays per second) is related to the rate of change of the number of particles.

Combining these equations leads to:

$$\boxed{\begin{aligned} A = -\frac{dN}{dt} &\propto N \\ \implies A = -\frac{dN}{dt} &= \lambda N \end{aligned}}$$

This kind of equation is called a first order separable differential equation, which you learn to solve in further maths or your degree. The basic idea is that we move all N terms to one side and all t terms to one side and integrate each side of the equation with respect to those variables:

$$\begin{aligned} -\frac{dN}{dt} &= \lambda N \\ \implies -\frac{1}{N}\,dN &= \lambda\,dt \\ \implies \int \frac{1}{N}\,dN &= -\int \lambda\,dt \\ \implies \ln N &= -\lambda t + C \\ \implies N &= e^{-\lambda t + C} \\ \implies N &= N_0 e^{-\lambda t} \end{aligned}$$

If we set $t = 0$ in $N(t) = e^C e^{-\lambda t}$ we find $N(0) = N_0 = e^c$. In other words, the constant term is the initial number of particles in the sample.

It is not necessary to understand or know the derivation of this equation, and so if you can't understand the derivation simply sear the following equation into your brain:

$$\boxed{N = N_0 e^{-\lambda t}}$$

and since we know that $A = \lambda N \implies N = \dfrac{A}{\lambda}$ we can substitute that into the above equation and obtain:

$$\boxed{A = A_0 e^{-\lambda t}}$$

in terms of activity.

The **decay constant** λ tells us the number of radioactive decays per second, and the **half-life** $t_{1/2}$ tells us how long it takes for 50% of the atoms to decay. Intuitively, it feels as if there could be a link between the two quantities, and there is, which we can find by solving the above equation for the time it takes for the activity or number of particles to halve:

$$\begin{aligned}
N =& \; N_0 e^{-\lambda t} \\
\implies \frac{N_0}{2} =& \; N_0 e^{-\lambda t_{1/2}} \\
\implies \frac{1}{2} =& \; e^{-\lambda t_{1/2}} \\
\implies \ln(\frac{1}{2}) =& \; -\lambda t_{1/2} \\
\implies \ln 1 - \ln 2 =& \; -\lambda t_{1/2} \\
\implies -\ln 2 =& \; -\lambda t_{1/2} \\
\implies \frac{\ln 2}{\lambda} =& \; t_{1/2}
\end{aligned}$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.69}{\lambda}}$$

6.1 *A more intuitive mathematical understanding of half-life*

These equations look intimidating, and they are, and there is in fact a much easier way of working out how many atoms remain after a given time period.

Consider a situation in which we have 120 atoms with a half-life of 30 minutes. Using the notation from the equation above, we would say $N_0 = 120$ and $t_{1/2} = 30$ mins.

Let's calculate how many atoms remain after a given number of half-lives:

$$\begin{aligned}
N_0 \times \frac{1}{2} =& \; N_0 \times \left(\frac{1}{2}\right)^1 = 60 \text{ atoms. After } \mathbf{1 \text{ half-life.}} \\
\left(N_0 \times \frac{1}{2}\right) \times \frac{1}{2} =& \; N_0 \times \left(\frac{1}{2}\right)^2 = 30 \text{ atoms. After } \mathbf{2 \text{ half-lives.}} \\
\left(\left(N_0 \times \frac{1}{2}\right) \times \frac{1}{2}\right) \times \frac{1}{2} =& \; N_0 \times \left(\frac{1}{2}\right)^3 = 15 \text{ atoms. After } \mathbf{3 \text{ half-lives.}}
\end{aligned}$$

Clearly we can see a pattern: to work out how many atoms remain after a certain number of half-lives, you just multiply the initial number of atoms N_0 by $\left(\frac{1}{2}\right)^n$ where n is the number of half-lives:

$$N = N_0 \left(\frac{1}{2}\right)^n$$

To work out how many atoms remain when a non-integer number of half-lives occur, we use the exact same formula. For example, with a half-life of 30 minutes, if we wanted to know the number of atoms remaining after 45 minutes, that corresponds to 45 mins/30 mins = 1.5 half-lives. The number of atoms remaining would be $N = 120 \times \left(\frac{1}{2}\right)^{1.5} = 42$ atoms. Therefore, we can just use the same techniques as before and note that this equation could be written:

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Practice Questions 6-1 - Calculate the number of half-lives occurred to find out how many atoms remain

Isotope Half-Life = 884 years

Time (years)	$\frac{t}{t_{1/2}}$	No. of Atoms
0	0	1136
1594		
2182		
3666		
4475		
6658		

Isotope Half-Life = 421 days

Time (days)	$\frac{t}{t_{1/2}}$	No. of Atoms
0	0	7877
603		
924		
1332		
3370		
4365		

Practice Questions 6-2 - Calculate the number of half-lives occurred to find out how many atoms remain

1. A sample of niobium-92 has a half-life of 34.7 million years. Its initial activity is 614 Bq. What is the activity of the element after 115 million years?

2. A sample of bismuth-208 has a half-life of 368 thousand years. Its initial activity is 789 Bq. What is the activity of the element after 700 thousand years?

3. A sample of caesium-137 has a half-life of 30.17 years. Its initial activity is 896 Bq. What is the activity of the element after 80 years?

Practice Questions 6-3 - Calculate the number of half-lives occurred to find out how many atoms remain

1. The isotope iodine-129 has a half-life of 15.7 million years. Its current activity is 809 Bq. What was its activity 87.0 million years ago?

2. The isotope niobium-94 has a half-life of 20300 years. Its current activity is 665 Bq. What was its activity 43917.0 years ago?

3. The isotope cadmium-109 has a half-life of 1.267 years. Its current activity is 273 Bq. What was its activity 4.0 years ago?

Worked Example 6-3 - Carbon dating - calculating the age of a sample.

Q: The sample of the isotope technetium-97 has a half-life of 2.6 million years. Its current activity is 176 Bq, and its initial activity was 3562 Bq. What is the age of the sample?

A: Identifying the variables, we find $t_{1/2} = 2.6$ million years, $A_0 = 3562$ Bq, $A = 176$ Bq. Using $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$ and solving for t:

$$
\begin{aligned}
A &= A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies 176 &= 3562 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies 0.049 &= \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies \ln 0.049 &= \frac{t}{t_{1/2}} \ln\left(\frac{1}{2}\right) \\
\implies \frac{-3.01}{\ln 0.5} &= \frac{t}{t_{1/2}} \\
\implies 4.338 &= \frac{t}{t_{1/2}} \\
\implies t &= 4.338 \times 2.6 \text{ million years} \\
\implies t &= 11.2 \text{ million years}
\end{aligned}
$$

Practice Questions 6-4 - Calculate the number of half-lives occurred to find out how many atoms remain

1. The sample of the isotope samarium-147 has a half-life of 106 billion years. Its current activity is 616 Bq, and its initial activity was 1585 Bq. What is the age of the sample?

2. The sample of the isotope berkelium-247 has a half-life of 1380 years. Its current activity is 1555 Bq, and its initial activity was 4940 Bq. What is the age of the sample?

3. The sample of the isotope silicon-32 has a half-life of 170 years. Its current activity is 2285 Bq, and its initial activity was 3323 Bq. What is the age of the sample?

Worked Example 6-4 - Carbon dating - calculating the age of a sample.

Q: After a period of 6563 years, the activity of a sample of plutonium-240 decreases from 3884 Bq to 3005 Bq. What is the half-life of the isotope?

A: Identifying the variables, we find $t = 6563$ years, $A_0 = 3884$ Bq, $A = 3005$ Bq. Using $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$ and solving for $t_{1/2}$:

$$\begin{aligned}
A &= A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies 3005 &= 3884 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies 0.774 &= \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\
\implies \ln 0.774 &= \frac{t}{t_{1/2}} \ln \left(\frac{1}{2}\right) \\
\implies \frac{-0.257}{\ln 0.5} &= \frac{t}{t_{1/2}} \\
\implies 0.370 &= \frac{t}{t_{1/2}} \\
\implies t_{1/2} &= \frac{6563}{0.370} \\
\implies t_{1/2} &= 17,729 \text{ years}
\end{aligned}$$

Practice Questions 6-5 - Calculate the number of half-lives occurred to find out how many atoms remain

1. After a period of 717 thousand years, the activity of a sample of aluminium-26 decreases from 7004 Bq to 3561 Bq. What is the half-life of the isotope?

2. After a period of 1380 years, the activity of a sample of berkelium-247 decreases from 2728 Bq to 349 Bq. What is the half-life of the isotope?

3. After a period of 6.5 million years, the activity of a sample of palladium-107 decreases from 771 Bq to 269 Bq. What is the half-life of the isotope?

Worked Example 6-5 - Calculate the decay constant and the activity

Q: The isotope lanthanum-137 has a half-life of 60 thousand years. Calculate the decay constant λ, and hence the activity of a sample of $N = 2.3 \times 10^{15}$ atoms of the isotope

A: Using $\lambda = \frac{\ln 2}{t_{1/2}}$ we can find the decay constant straightforwardly $\lambda = \frac{0.69}{60000 \text{ years}} = 1.15 \times 10^{-5}$ year^{-1}. We can then find the activity of the sample using $A = \lambda N = 1.15 \times 10^{-5}$ year$^{-1} \times 2.3 \times 10^{15} = 2.3 \times 10^{10}$ decays per year, which can easily be converted into decays per second if required.

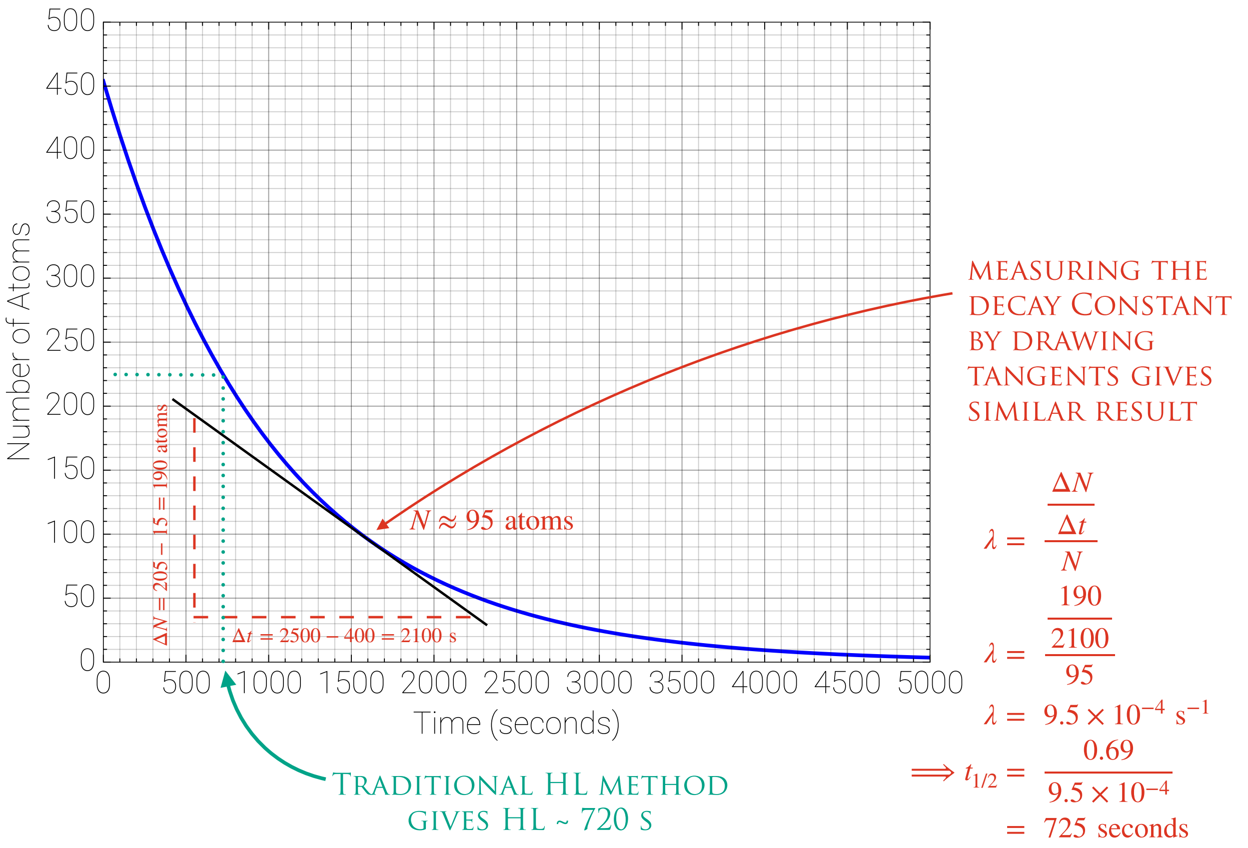
Practice Questions 6-6 - Calculate the decay constant and the activity

1. The isotope americium-241 has a half-life of 432.2 years. Calculate the decay constant λ, and hence the activity of a sample of $N = 6.4 \times 10^{10}$ atoms of the isotope

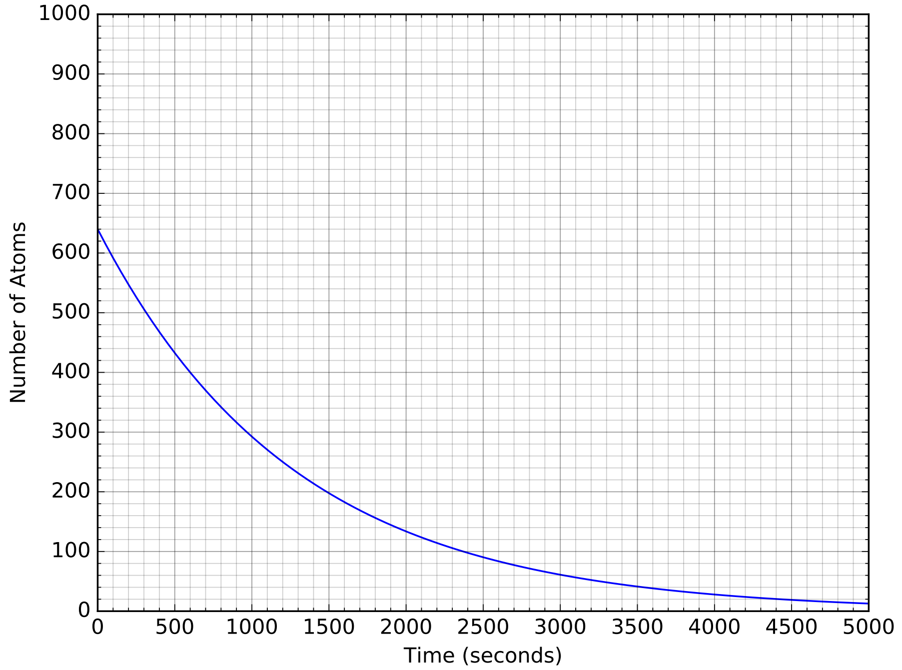
2. The isotope curium-245 has a half-life of 8500 years. Calculate the decay constant λ, and hence the activity of a sample of $N = 8.7 \times 10^{23}$ atoms of the isotope

3. The isotope curium-247 has a half-life of 15.6 million years. Calculate the decay constant λ, and hence the activity of a sample of $N = 3.1 \times 10^{18}$ atoms of the isotope

Worked Example 6-6 - Calculate the decay constant and half-life from the graph



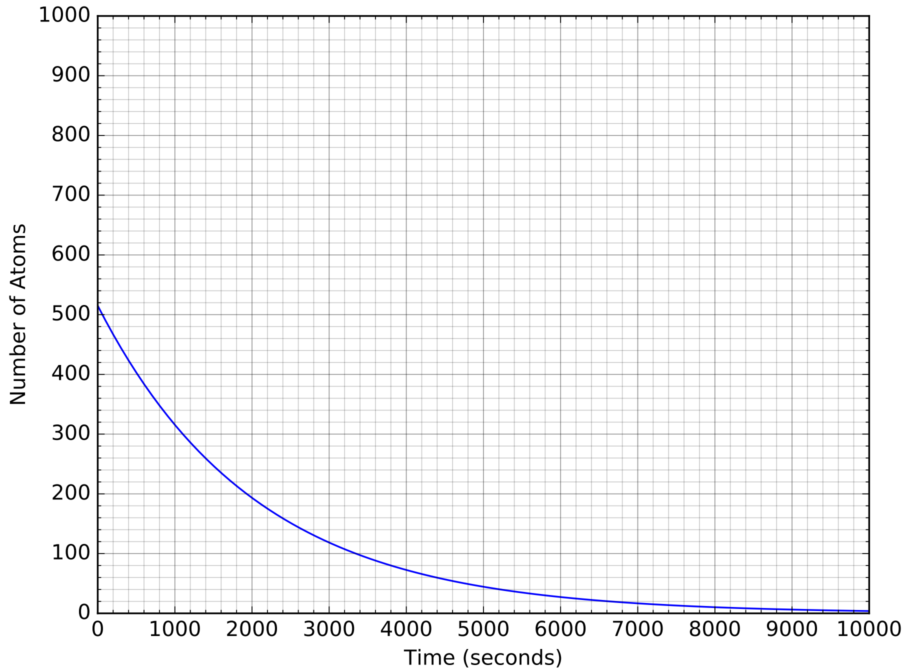
Practice Questions 6-7 - Find the decay constant and the half-life from the graph

1. 

Half-Life (from graph) =

Decay Constant (calculated) =

Decay Constant (from graph) =

2. 

Half-Life (from graph) =

Decay Constant (calculated) =

Decay Constant (from graph) =

Worked Example 6-7 - Carbon dating - calculating the age of a sample.

Q: The half-life of Carbon-14 is 5730 years. The ratio of Carbon-14 to Carbon-12 in a fossil is 60% the ratio found in living specimens. Calculate the age of the sample.

A: The ratio of C-14 to C-12 is constant in living organisms i.e. $\frac{\mathrm{C}-14_{\text{living}}}{\mathrm{C}-12_{\text{living}}} = \text{const.}$

In organisms that have died, the amount of C-14 decreases due to beta decay but the amount of C-12 remains the same.

Therefore, the statement "The ratio of Carbon-14 to Carbon-12 in a fossil is 60% the ratio found in living specimens" means we can write, mathematically:

$\frac{\mathrm{C}-14_{\text{dead}}}{\mathrm{C}-12_{\text{dead}}} = 0.6 \times \frac{\mathrm{C}-14_{\text{living}}}{\mathrm{C}-12_{\text{living}}} \implies \mathrm{C}-14_{\text{dead}} = 0.6 \times \mathrm{C}-14_{\text{living}}$ since $\mathrm{C}-12_{\text{living}} = \mathrm{C}-12_{\text{dead}}$.

We now simply plug into our half-life equations $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $N = N_0 e^{-\lambda t}$ to give:

$$\begin{aligned}
\mathrm{C}-14_{\text{dead}} &= \mathrm{C}-14_{\text{living}}\, e^{-\lambda t} \\
\implies 0.6 \times \mathrm{C}-14_{\text{living}} &= \mathrm{C}-14_{\text{living}}\, e^{-\lambda t} \\
\implies 0.6 &= e^{-\lambda t} \\
\implies \ln 0.6 &= -\lambda t \\
\implies t &= \frac{0.51}{0.69} \times 5730 \text{ years} \\
\implies t &= 4220 \text{ years}
\end{aligned}$$

Practice Questions 6-8 - Carbon dating - calculating the age of a sample.

The half-life of Carbon-14 is 5730 years.

1. The ratio of Carbon-14 to Carbon-12 in a fossil is 60% the ratio found in living specimens. Calculate the age of the sample.

2. The ratio of Carbon-14 to Carbon-12 in a fossil is 25% the ratio found in living specimens. Calculate the age of the sample.

3. The ratio of Carbon-14 to Carbon-12 in a fossil is 7% the ratio found in living specimens. Calculate the age of the sample.

Worked Example 6-8 - Carbon dating - calculating the age of a sample.

Q: The half-life of Carbon-14 is 5730 years. A bone sample from an archaeological dig has an activity of 5.2 Bq per gram. A bone sample from a modern skeleton has an activity of 6.5 Bq per gram. Calculate the age of the sample.

A: The ratio of C-14 to C-12 is constant in living organisms i.e. $\frac{\mathrm{C}-14_{\text{living}}}{\mathrm{C}-12_{\text{living}}} = \text{const}$. Using $A = \lambda N$, this means the activity is proportional in the same way.

We can therefore say that the initial activity $A_0 = 6.5$ Bq and the current activity $A = 5.2$ Bq and plug this into our $A = A_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $A = A_0 e^{-\lambda t}$ equations to give:

$$\begin{aligned} \implies 5.2 =& 6.5\, e^{-\lambda t} \\ \implies 0.8 =& e^{-\lambda t} \\ \implies \ln 0.8 =& -\lambda t \\ \implies t =& \frac{0.223}{0.69} \times 5730 \text{ years} \\ \implies t =& 1850 \text{ years} \end{aligned}$$

Practice Questions 6-9 - Carbon dating - calculating the age of a sample.

The half-life of Carbon-14 is 5730 years. The activity of a bone sample from a modern skeleton has an activity of 6.5 Bq per gram.

1. A bone sample from an archaeological dig has an activity of 3.25 Bq per gram. Calculate the age of the sample.
2. A bone sample from an archaeological dig has an activity of 2.0 Bq per gram. Calculate the age of the sample.
3. A bone sample from an archaeological dig has an activity of 0.75 Bq per gram. Calculate the age of the sample.

Worked Example 6-9 - Carbon dating - calculating the age of a sample.

Q: When examining a small sample of an old skull, scientists found that 2.3×10^{11} % of the carbon was C-14, where as in recently deceased skulls the C-14 proportion is 1.0×10^{-10} %. Calculate the age of the skull.

A: The initial proportion of C-14 is $N_0 = 1.0 \times 10^{-12}$ and the current proportion is $N = 2.3 \times 10^{13}$ (where we have converted from percentages into decimals). Plugging into $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $N = N_0 e^{-\lambda t}$ gives:

$$\begin{aligned} \implies 2.3 \times 10^{-13} =& 1.0 \times 10^{-12}\, e^{-\lambda t} \\ \implies 0.23 =& e^{-\lambda t} \\ \implies \ln 0.23 =& -\lambda t \\ \implies t =& \frac{1.47}{0.69} \times 5730 \text{ years} \\ \implies t =& 12,207 \text{ years} \end{aligned}$$

Practice Questions 6-10 - Carbon dating - calculating the age of a sample.

The proportion of C-14 in recently deceased skulls is 1.0×10^{-10} %. The half-life of C-14 is 5730 years.

1. When examining a small sample of an old skull, scientists found that 2.3×10^{11} % of the carbon was C-14. Calculate the age of the skull.

2. When examining a small sample of an old skull, scientists found that 8.5×10^{12} % of the carbon was C-14. Calculate the age of the skull.

3. When examining a small sample of an old skull, scientists found that 6.3×10^{11} % of the carbon was C-14. Calculate the age of the skull.

Worked Example 6-10 - Carbon dating - calculating the age of a sample.

Q: Carbon-14 has a half-life of 5730 years. A living organisms contains approximately 1 atom of C-14 for every 10^{18} atoms of C-12. A fossil contains 12 atoms of C-14 for every 10^{20} atoms of C-12. Estimate its age.

A: The initial proportion of C-14 is $N_0 = \dfrac{1}{1 \times 10^{18}} = 1.0 \times 10^{-18}$ and the current proportion is $N = \dfrac{12}{1 \times 10^{20}} = 1.2 \times 10^{-19}$ using $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $N = N_0 e^{-\lambda t}$ gives:

$$\begin{aligned}
\implies 1.2 \times 10^{-19} =&\ 1.0 \times 10^{-18}\, e^{-\lambda t} \\
\implies 0.12 =&\ e^{-\lambda t} \\
\implies \ln 0.12 =&\ -\lambda t \\
\implies t =&\ \frac{2.12}{0.69} \times 5730 \text{ years} \\
\implies t =&\ 17,607 \text{ years}
\end{aligned}$$

Practice Questions 6-11 - Carbon dating - calculating the age of a sample.

Carbon-14 has a half-life of 5730 years. A living organisms contains approximately 1 atom of C-14 for every 10^{18} atoms of C-12.

1. A fossil contains 32 atoms of C-14 for every 10^{20} atoms of C-12. Estimate its age.

2. A fossil contains 73 atoms of C-14 for every 10^{20} atoms of C-12. Estimate its age.

3. A fossil contains 2 atoms of C-14 for every 10^{20} atoms of C-12. Estimate its age.

Worked Example 6-11 - Carbon dating - calculating the age of a sample.

Q: In a particular rock sample, 145 of the initial 256 Carbon-14 atoms have decayed. Calculate the age of the rock.

A: If $145/256 = 57\%$ of the atoms have decayed, this means that the remaining proportion is $N = (256 - 145)/256 = 111/256 = 0.43$. Plugging into $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $N = N_0 e^{-\lambda t}$ gives:

$$
\begin{aligned}
\implies 0.43 N_0 =& \; N_0 \, e^{-\lambda t} \\
\implies 0.43 =& \; e^{-\lambda t} \\
\implies \ln 0.43 =& \; -\lambda t \\
\implies t =& \; \frac{0.844}{0.69} \times 5730 \text{ years} \\
\implies t =& \; 7{,}008 \text{ years}
\end{aligned}
$$

Practice Questions 6-12 - Carbon dating - calculating the age of a sample.

Carbon-14 has a half-life of 5730 years.

1. In a particular rock sample, 64% Carbon-14 atoms have decayed. Calculate the age of the rock.

2. In a particular rock sample, 14% Carbon-14 atoms have decayed. Calculate the age of the rock.

3. In a particular rock sample, 95% Carbon-14 atoms have decayed. Calculate the age of the rock.

Worked Example 6-12 - Carbon dating - calculating the age of a sample.

Q: Using Carbon-14 to date samples is not always possible. For dating rocks taken from the moon on the Apollo missions, scientists used the decay of potassium-40 to argon-40. The half-life of potassium-40 is 1.3×10^9 years. In one particular rock sample scientists found 0.84 μg of argon-40 and 0.10 μg of potassium-40. Calculate the age of the rock sample in years.

A: The K-40 decays into Ar-40 and therefore as the mass of K-40 decreases the mass of Ar-40 increases in exactly proportional amounts. The sample must have started with $0.84 + 0.10 = 0.94\mu$g of K-40, but there is now only 0.1μg remaining. We then simply plug this into our $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $N = N_0 e^{-\lambda t}$ equations to give:

$$
\begin{aligned}
\implies 0.1 =& \; 0.94 \, e^{-\lambda t} \\
\implies 0.106 =& \; e^{-\lambda t} \\
\implies \ln 0.106 =& \; -\lambda t \\
\implies t =& \; \frac{0.106}{0.69} \times 1.3 \times 10^9 \text{ years} \\
\implies t =& \; 4.2 \times 10^9 \text{ years}
\end{aligned}
$$

Practice Questions 6-13 - Alternatives to Carbon dating

The half-life of potassium-40 is 1.3×10^9 years.

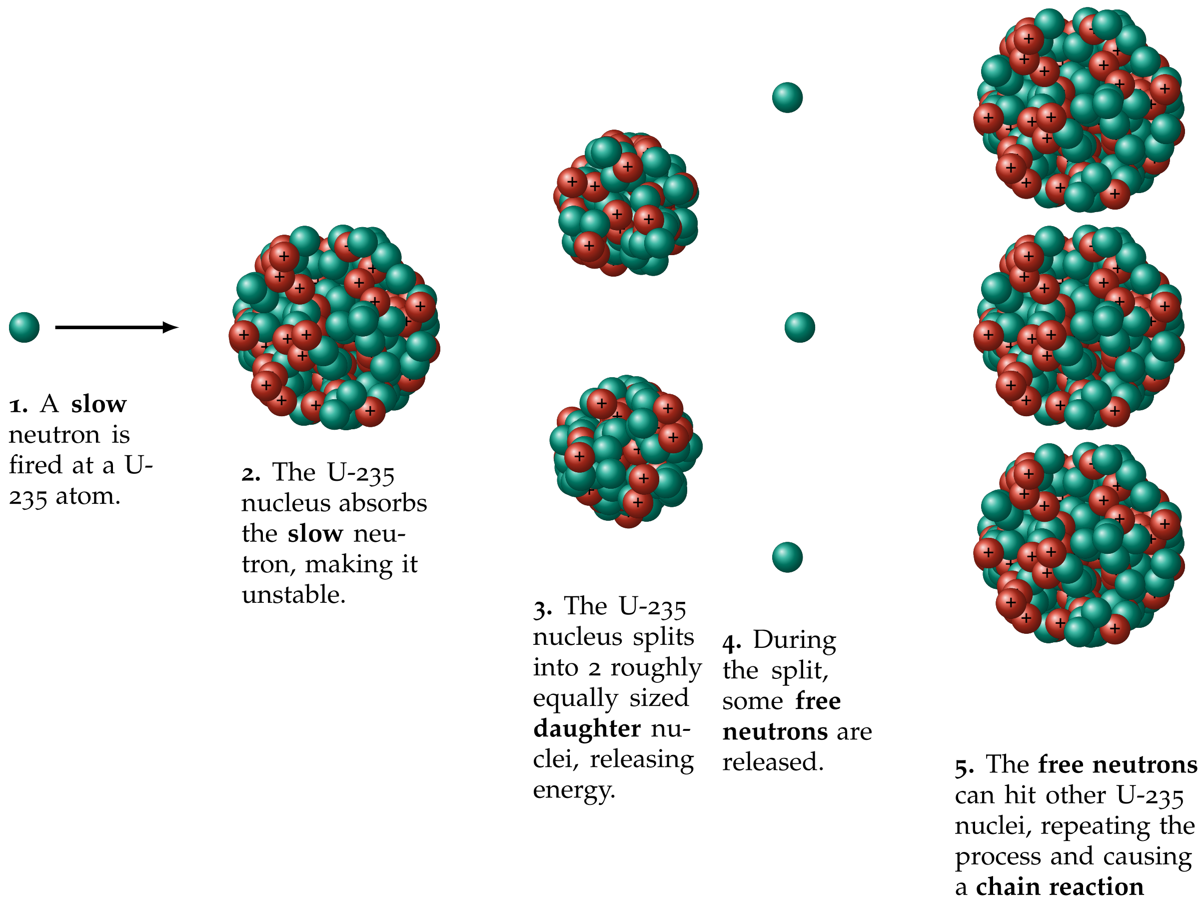
1. In one particular rock sample scientists found 0.54 μg of argon-40 and 0.20 μg of potassium-40. Calculate the age of the rock sample in years.

2. In one particular rock sample scientists found 1.4 μg of argon-40 and 0.74 μg of potassium-40. Calculate the age of the rock sample in years.

3. In one particular rock sample scientists found 0.22 μg of argon-40 and 1.38 μg of potassium-40. Calculate the age of the rock sample in years.

7 *Nuclear Fission*

Nuclear fission involves splitting atoms (usually Uranium) into smaller elements and releasing vast amounts of energy. Fission can either be controlled and used in **nuclear reactors** to produce useful energy or uncontrolled and used in **nuclear bombs** to produce explosions.



7.1 *Nuclear Fission Equations*

$$\underbrace{\overbrace{{}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n}}^{235 + 1 = 236}}_{92 + 0 = 92} \longrightarrow \underbrace{\overbrace{{}^{90}_{36}\mathrm{Kr} + {}^{143}_{56}\mathrm{Ba} + 3\,{}^{1}_{0}\mathrm{n}}^{90 + 143 + 3\mathrm{x}1 = 236}}_{36 + 56 + 3\mathrm{x}0 = 92}$$

Figure 19: The mass numbers and the atomic numbers on the right hand side must equal the mass number on the left hand side.

When the **parent** Uranium-236 nucleus splits into two **daughter** nuclei, it generates free neutrons. The number of free neutrons released depends on the **fission products** i.e. the exact two daughter nuclei the U-236 breaks into. Some reactions produce more free neutrons than others. Let's look at the example of Uranium-235 absorbing a neutron and fissioning into Krypton and Barium:

$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{90}_{36}\mathrm{Kr} + {}^{143}_{56}\mathrm{Ba} + 3\,({}^{1}_{0}\mathrm{n})$$

Practice Questions 7-1 - How many neutrons are released in the following fission reactions?

1. $^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{181}_{71}\mathrm{Lu} + ^{55}_{21}\mathrm{Sc} + \boxed{}\,^{1}_{0}\mathrm{n}$
2. $^{232}_{90}\mathrm{Th} + ^{1}_{0}\mathrm{n} \longrightarrow ^{91}_{37}\mathrm{Rb} + ^{140}_{53}\mathrm{I} + \boxed{}\,^{1}_{0}\mathrm{n}$
3. $^{239}_{94}\mathrm{Pu} + ^{1}_{0}\mathrm{n} \longrightarrow ^{100}_{38}\mathrm{Sr} + ^{139}_{56}\mathrm{Ba} + \boxed{}\,^{1}_{0}\mathrm{n}$

Practice Questions 7-2 - What is the second daughter nucleus?

1. $^{233}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{115}_{47}\mathrm{Ag} + ^{\boxed{}}_{\boxed{}}\boxed{} + 3\,^{1}_{0}\mathrm{n}$
2. $^{237}_{93}\mathrm{Np} + ^{1}_{0}\mathrm{n} \longrightarrow ^{174}_{69}\mathrm{Tm} + ^{\boxed{}}_{\boxed{}}\boxed{} + 2\,^{1}_{0}\mathrm{n}$
3. $^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{123}_{50}\mathrm{Sn} + ^{\boxed{}}_{\boxed{}}\boxed{} + 2\,^{1}_{0}\mathrm{n}$