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NAME:

THERMAL PHYSICS

CLASS:

Contents

1   *Internal Energy, States of Matter, and Phase Changes*

1.1   *States of Matter*

There are three states of matter that we need to know about at A-level: solids, liquids, and gases.

	SOLID	LIQUID	GAS
Position	PARTICLES OCCUPY FIXED POSITIONS	PARTICLES DO NOT HAVE FIXED POSITIONS	RANDOM POSITIONS
Movement	VIBRATE AROUND FIXED POSITIONS	PARTICLES "SLIDE" OVER ONE ANOTHER	RANDOM VELOCITIES
Attraction	STRONGEST ATTRACTION	STRONG ATTRACTION	NO ATTRACTION
Separation	PARTICLES ARE CLOSELY PACKED IN A LATTICE ARRANGEMENT	PARTICLES ARE CLOSE BUT WITH NO FIXED OR REGULAR ARRANGEMENT	PARTICLES WIDELY SPACED

1.2   *Kinetic Energy*

At any non-zero temperature, the particles in a substance - whether solid, liquid, or gas - have some kinetic energy: they are *moving* and kinetic energy is associated with movement.

However, the nature of this kinetic energy depends on the state of matter. For example, the motion in solids tends to be of particles vibrating around a fixed point, but in liquids and gases the motion tends to be completely random.

You may notice that the state of matter of most materials depends on the temperature: a substance is in the solid state when cold, and in the gaseous state when hot.

This is because the individual particles in the solid state have less kinetic energy than in the liquid or gas state. So in general, particles in a gas have more kinetic energy than particles in a liquid, which have more kinetic energy than particles in a solid.

1.3   *Potential Energy*

Potential energy concerns the *position* of particles. It turns out that charged particles like atoms and molecules attract each other at long range and repel each other at short range.

Ideally, the atoms would like to be in some "sweet spot" distance apart where they're not too close or too far apart from other atoms.

If they deviate from this sweet spot, then they have higher *potential energy*.

The best way to think of this is to imagine a spring. If we pull the spring apart we store elastic potential energy in the spring and if we let go the spring would transfer this elastic potential energy to kinetic energy and go back to its original length. If we push the spring together it will also store elastic potential energy and transfer it to kinetic energy when we let it go and return to its original length.

In this analogy two atoms are like a single spring: if you separate the two atoms, they store potential energy and they would come back together if nothing was stopping them from doing so.

Atoms in a solid have the lowest potential energy, followed by liquids, and gases have the highest potential energy because they have the highest average separation.

1.4   *The Zeroth Law of Thermodynamics and Heating*

When we take ice out of the freezer, it heats up and melts, yet when blow out a candle, it cools down and solidifies.

What happens in both cases is that thermal energy transfers from the object at a higher temperature to the object at lower temperature.

In the case of the ice, the *higher temperature air* transfers thermal energy to the *lower temperature ice*, causing the internal energy of the ice to increase and the internal energy of the air to decrease. This is why a room with lots of ice (i.e. a freezer) will feel cold.

In the case of the candle wax, the *higher temperature candle wax* transfers thermal energy to the *lower temperature air*, causing the internal energy of the candle wax to decrease and the internal energy of the air to increase: this is why a room with lots of candles lit will feel hot.

The transfer of thermal energy between the objects will continue until they are at the same temperature and there is no temperature difference.

This phenomena is a general rule and is never violated, and it's important to know.

> When two objects are in contact and there is a temperature difference between the objects, thermal energy flows from the higher temperature object to the lower temperature object until their temperatures are equal and no more thermal energy transfer occurs.

This concept is called the *zeroth law of thermodynamics* - it simply says that thermal energy flows from an object at higher temperature to an object at lower temperature when they are in "thermal contact" i.e.

that there is nothing to stop thermal energy flowing between them.

Another interesting concept related to temperature difference is the *rate* of thermal energy transfer: the larger the temperature difference, the *faster* the rate of thermal energy transfer.

As the objects get closer in temperature, the temperature of each object changes more slowly than when the temperature difference is large.

> Any transfer of thermal energy from one object to another **due to a temperature difference** is called **heating**

You will frequently hear the term "heat" used in books (including this one) as a noun, which means a quantity of thermal energy that has been transferred by heating; think of it as a chunk of thermal energy.

Internal Energy

We define the total **internal energy** of a system as the sum of kinetic energies and potential energies of all the particles:

$$\text{Internal Energy} = \underbrace{\sum \text{KE}}_{\text{Temperature}} + \sum \text{PE}$$

So when we supply thermal energy via heating to a system, its internal energy increases, and the internal energy includes contributions from the random motion of particles (KE) and the average separation of particles (PE).

Why is this distinction important?

In this model only the kinetic energy contributes to **temperature**: if the average kinetic energy of the particles in a system increases, its temperature increases. However, if the potential energy increases because particles move further apart from one another, then *the temperature of the system remains constant.*

TEMPERATURE (vertical axis)

THERMAL ENERGY (horizontal axis)

AVERAGE SEPARATION AND THEREFORE POTENTIAL ENERGY INCREASES BUT KINETIC ENERGY AND HENCE TEMPERATURE REMAIN CONSTANT

KINETIC ENERGY AND HENCE TEMPERATURE INCREASES

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AVERAGE SEPARATION AND THEREFORE POTENTIAL ENERGY REMAIN CONSTANT

INTERNAL ENERGY =

KINETIC ENERGY
+
POTENTIAL ENERGY

A KINETIC ENERGY VS THERMAL ENERGY GRAPH IS EQUIVALENT TO ONLY LOOKING AT HOW THE KINETIC ENERGY CHANGES AS THERMAL ENERGY IS ADDED

This situation is exactly what happens during a phase change from e.g. solid to liquid or liquid to gas.

The diagram below shows the changes to both the arrangement of particles and the changes in energy as a system's internal energy increases via heating.

1.6   *Specific Heat Capacity*

It is possible to calculate exactly how much the temperature T of a substance will rise given some thermal energy input ΔQ:

$$\Delta Q = mc\Delta T$$

where m is the mass of the substance (kg) and c is a constant called the *specific heat capacity* of the substance.

The specific heat capacity has units of $\frac{\text{J}}{\text{kg}^\circ\text{C}}$ and it basically tells you how much energy is needed to raise 1 kg of a substance by 1°C.

This formula only applies when the substance is not undergoing a phase change.

1.7   *Specific Latent Heat*

During a phase change, the concept of specific heat capacity makes no sense, since the temperature is constant.

Instead we define a quantity called the *specific latent heat L* which is related to the thermal energy input as follows:

$$\Delta Q = mL$$

The latent heat L has units of J/kg and tells us how much energy is required to completely change the phase of 1 kg of a substance.

We saw earlier that the amount of energy required to turn a solid into a liquid is different from the amount of energy required to turn a liquid into a gas: in particular, it takes more energy to turn a liquid into a gas because we have to increase the potential energy a lot to get the particles far enough apart to be considered a gas.

Therefore this is not a single latent heat but two different latent heats: the **latent heat of fusion** L_f is the energy required to turn 1 kg of a substance from solid to liquid and the **latent heat of vaporisation** is the energy required to turn 1 kg of a substance from liquid to gas. As discussed above, $L_v > L_f$.

Worked Example 1-1 - Calculations involving simple re-arrangements of the specific heat capacity equation

Q: Aluminium has a specific heat capacity of 8000 J/(kg K). Calculate the energy required to heat a 0.5 kg block of aluminium from -20°C to 50°C.

A: This requires simply recognising the variables ($c = 8000$ J/(kg K), $m = 0.5$ kg, $\Delta T = 50 - -20^\circ$C $= 70^\circ$C and substituting into $Q = mc\Delta T$:

$$Q = mc\Delta T$$
$$= 0.5 \text{ kg} \times 8000 \text{ J}/(\text{kg K}) \times 70°\text{C}$$
$$= 280 \text{ kJ}$$

Practice Questions 1-1 - Calculations involving simple re-arrangements of the specific heat capacity equation

1. Gaseous carbon dioxide has a specific heat capacity of $c = 0.8$ J/(g K). Calculate the energy required to heat 1000 kg of carbon dioxide from 30°C to 35°C.

2. 5 kg of solid copper ($c = 389$ J/kg K) at 10°C is given 50 kJ of energy. What will its final temperature be?

3. A 2 kg of aluminium with specific heat capacity $c = 800$ J/kg K is given 20 kJ of energy and its temperature rises to 5°C. What was its initial temperature?

4. 100 kJ of heat energy is supplied to a silver block ($c = 235$ J/kg K) and its temperature changes by 30°C. What is the mass of the block?

5. 112.5 kJ of heat energy is required to heat a 10 kg lump of brass from 20°C to 50°C. What is the specific heat capacity of brass?

Worked Example 1-2 - Calculations involving simple re-arrangements of the specific latent heat equation

Q: The latent heat of fusion of ammonia is 332 kJ/kg. Calculate the energy required to melt a 50 kg sample of ammonia.

A: This requires simply recognising the variables ($L_f = 332$ kJ/kg, $m = 50$ kg and substituting into $Q = mL_f$ to give $Q = 50$ kg$\times$332 kJ/kg $= 16.6$ MJ.

Practice Questions 1-2 - Calculations involving simple re-arrangements of the specific latent heat equation

1. Calculate the thermal energy required to turn 5 kg of liquid ethanol into gas if its latent heat of vaporisation is 442 kJ/kg.

2. The boiling point of liquid oxygen is $-183°$C. Calculate the thermal energy output when 5000 kg of gaseous oxygen condenses into liquid oxygen if the latent heat of vaporisation is $L = 213$ kJ/kg.

3. A 2 kg sample of hydrogen requires 985 kJ to turn from liquid to gas. Calculate the latent heat of vaporisation.

4. 6600 J of energy is released from a sample of gaseous mercury than liquifies at $-39°$C. The latent heat of fusion is 11 kJ/kg. Calculate the mass of the sample of mercury.

Worked Example 1-3 - Calculations of specific heat involving power

Q: Humans have a power output of around 80 W. 30 people get onto a bus. The total mass of air on the bus is 1000 kg. Calculate the temperature rise of the bus due to the presence of humans during a 15 minute bus journey. The specific heat capacity of air is 1010 Jkg^{-1} K^{-1}.

A:

Practice Questions 1-3 - Calculations of specific heat involving power

1. A 2300 W kettle boils 0.75 kg of water that was initially at 20°C in 2 minutes. After an additional 7

minutes, half the water has boiled away.

(a) Calculate the specific heat capacity of the water.

(b) Calculate the specific latent heat of vaporisation.

2. A 55 W heater heats a 1 kg block of aluminium for several minutes until the temperature remains constant. Once the heater is switched off, the temperature of the block falls at a maximum rate of 3.5 K per minute.

Worked Example 1-4 - Calculations involving combinations of specific heat capacity and specific latent heat

Q: The latent heat of fusion of hydrogen is 58 kJ/kg and the latent heat of vaporisation is 455 kJ/kg. The melting point of hydrogen is $-259°C$ and the boiling point is $-253°C$. The specific heat capacity of hydrogen is 14 kJ/(kg K). Calculate the energy required to heat a 400 kg sample of hydrogen from $-265°C$ to $265°C$. **A:**

Practice Questions 1-4 - Calculations involving combinations of specific heat capacity and specific latent heat

1. The graph shows the temperature of nitrogen as it absorbs thermal energy at a constant rate. The temperature rises from 53K to 63K in 160 seconds, stays at 63K for 240 seconds, and then goes from 63K to 73K in 200 seconds. Calculate the specific latent heat of fusion, the specific heat capacity of liquid nitrogen, given that the specific heat of nitrogen is 1600 J kg^{-1} K^{-1}.

2. A block of ice has a small hole in the top. 150 g of water at $100°C$ is poured into the hole, causing the some of the ice to melt. The total amount of water in the hole is 350 g. Calculate the specific latent heat of fusion of the ice. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}.

3. It's a hot sunny day and the temperature of a person's diet coke is $30°C$. They drop 3 ice cubes each of mass 10 g into the coke.

(a) What is the final temperature of the coke?

(b) Coke is best enjoyed at $5°C$. How many ice cubes would the person have to add to enjoy the coke to the max?

Worked Example 1-5 - Calculations of specific heat capacity involving mixtures

Q: A chef is making some strawberry jam in a kitchen at a temperature of $20°$ C. They place 2 kg of strawberries into a 1.25 kg iron pan and heat the pan at a rate of 500 W. Calculate the temperature of the strawberries after 10 minutes of heating.

The specific heat capacity of iron is 462 J kg^{-1} K^{-1} The specific heat capacity of strawberries is 3.5 J kg^{-1} K^{-1}

A: The thermal energy supplied to the pan is $E = 500 \times 10 \times 60 = 300,000$ J.

Given that the strawberries and pan are in good contact, we can assume that this thermal energy is shared amongst the pan and the strawberries.

In addition, we can assume they remain at the same temperature: if this wasn't the case, thermal energy would transfer from the strawberries to the pan or from the pan to the strawberries until their temperatures were equal.

Therefore, we can write:

$$\Delta Q = m_{\text{strawberries}}c_{\text{strawberries}}\Delta T + m_{\text{iron}}c_{\text{iron}}\Delta T$$

$$= \left(m_{\text{strawberries}}c_{\text{strawberries}} + m_{\text{iron}}c_{\text{iron}}\right)\Delta T$$

$$\implies \Delta T = \frac{\Delta Q}{\left(m_{\text{strawberries}}c_{\text{strawberries}} + m_{\text{iron}}c_{\text{iron}}\right)}$$

$$= \frac{300,000 \text{ J}}{2 \times 3500 + 1.25 \times 462}$$

$$= 40 \text{ K}$$

Practice Questions 1-5 - Calculations of specific heat capacity involving mixtures

1.

Worked Example 1-6 - Calculations of specific heat capacity involving mixtures

Q: A person makes a cup of tea with 200 g of hot water at 85°C. They add 0.05 kg of milk to the hot water. Calculate the final temperature of the tea and milk mixture.
The specific heat capacity of water is 4200 J kg^{-1} K^{-1} The specific heat capacity of milk is 3930 J kg^{-1} K^{-1}

A:

Practice Questions 1-6 - Calculations of specific heat capacity involving mixtures

1. A 100 g brass ball is heated by a bunsen flame for several minutes so that it is at the same temperature as the flame. It is then placed in a beaker with 500 g of water. The water was initially at 20°C but rises to 30°C. Calculate the temperature of the bunsen flame.

2. A person cools down a 1.5 kg Aluminium pan after cooking by pouring 500 g of water at 20°C onto it. The pan was initially 200°C. What is the final temperature of the pan and the water?

Worked Example 1-7 - Calculations involving flow rates and heating

Q: A hair dryer supplies 1.5 kW of thermal energy to air that passes its heating element. The hair dryer sucks in air at 20°C and it passes through the heating element at a rate of 0.05 kg/s. Calculate the temperature of the air as it leaves the hair dryer.

The specific heat capacity of air is 1010 J kg^{-1} K^{-1}

A:

### Practice Questions 1-7 - Calculations involving flow rates and heating

1. The useful output power of a nuclear reactor is 3000 MW. To cool the reactor, sea water enters the reaction chamber at a rate of 1×10^5 kg/s before being expelled back into the ocean. Calculate the temperature increase of the water and hence the total input power of the reactor and hence efficiency of the reactor.

2   *Gases, Kinetic Theory, and the Ideal Gas Law*

In A-level physics, we will often focus on gases because they are easy to model mathematically. However, in order to model a gas mathematically, we need to make some simplifying assumptions about how individual gas particles behave.

2.1   The Nature of a Gas

- Consist of a large number of particles moving with **random velocities** that obey Newton's laws of motion.
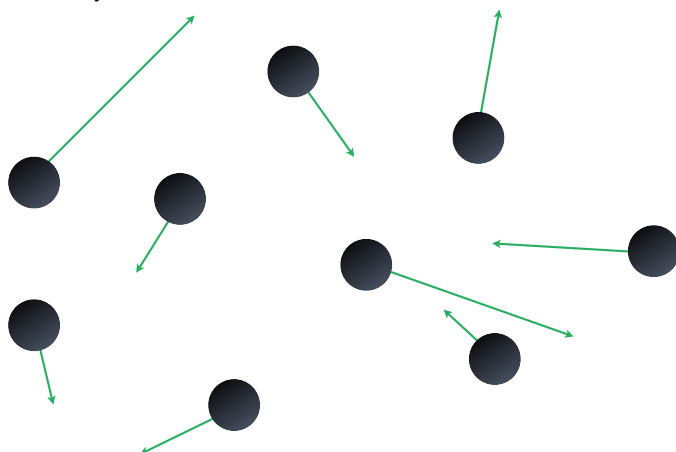
- The particles are "point" particles that occupy no physical space.

- The particles do not interact with each other (e.g. electrostatically) and therefore no forces act on the particle except for the forces during **brief** collisions with other particles and the walls of the container that the gas is kept in.

The easiest mental model to have here is that a gas is a lot of tiny particles flying around a box in random directions with random speeds and colliding with each other and the box. That mental picture is essentially enough to understand the behaviour of gases and as we will shortly, to develop a powerful mathematically theory of gases.

2.2   Kinetic Theory

Learning Objectives and Prior Knowledge
In this section you'll learn: - How to derive the equation $PV = \frac{1}{3}Nm\langle v^2 \rangle$, which tells us how the pressure P, volume V, number of particles N, mass m and average velocity $\langle v^2 \rangle$ are related.

By considering the force that a *single* gas particle exerts on a container wall, we are going to derive an equation for the total pressure of a gas. In other words, we are going to make a link between the **microscopic** world of single particles and the **macroscopic** world of billions of particles.

We start by considering a single particle that hits a wall and rebounds elastically. The momentum before the collision is $-mv$ and the momentum after is mv and so the change in momentum is $\Delta p = mv - -mu = 2mv$.

The force exerted on the wall is the rate of change of momentum. Another way of saying this is the force exerted by a particle multiplied by the number of collisions per second. If the box has length x it will take the particle $t = \frac{2x}{v_x}$ seconds between each collision and

hence there will be $1/t = \dfrac{v_x}{2x}$ collisions per second.

Combining this with our change in momentum per collision we get:

$$F = 2mv_x \times \frac{v_x}{2x} = \frac{mv_x^2}{x} \tag{1}$$

So far we have considered the force exerted by a *single* particle, but we can now extend this to many particles: we simply add up the squared velocities of lots of the particles and divide by the number of particles to get the average force:

$$\bar{F} = \frac{F}{N} = \frac{m}{x}\left(\frac{(v_{x1})^2 + (v_{x2})^2 + \ldots + (v_{xN})^2}{N}\right)$$

The quantity $\left(\dfrac{(v_{x1})^2 + (v_{x2})^2 + \ldots + (v_{xN})^2}{N}\right)$ is called the **mean squared velocity**, which is given the symbol $\langle v_x^2 \rangle = \left(\dfrac{(v_{x1})^2 + (v_{x2})^2 + \ldots + (v_{xN})^2}{N}\right)$ and so we can write our force equation as:

$$F = \frac{Nm\langle v_x^2 \rangle}{x}$$

We can get the pressure on the wall by remembering

$$P = \frac{F}{A} = \frac{Nm\langle v_x^2 \rangle}{Ax} = \frac{Nm\langle v_x^2 \rangle}{V}$$

The final step is to scale this up to 3 dimensions. The velocity of a particle is $v^2 = v_x^2 + v_y^2 + v_z^2$ and we can write $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$.

But there is nothing special about one particular direction over another: on average, particles moving in each direction should have the same mean squared velocity, and so we can say $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$ and therefore $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_x^2 \rangle + \langle v_x^2 \rangle = 3\langle v_x^2 \rangle$.

Our pressure equation now reads

$$P = \frac{F}{A} = \frac{Nm\langle v_x^2 \rangle}{Ax} = \frac{Nm\langle v^2 \rangle}{3V}$$

which we can re-arrange to:

$$PV = \frac{1}{3}Nm\langle v^2 \rangle$$

This equation is rather important because it tells us that the pressure in a gas is *solely* caused by the collisions of particles with the container walls.

Worked Example 2-1 - Calculating Root Mean Squared Speed

Q: The density of air is around $\rho = 1.3 \ \text{kg/m}^3$. Calculate the root mean squared velocity of the air particles in a room of at atmospheric pressure.

A: In our kinetic theory equation $PV = \frac{1}{3}Nm\langle v^2 \rangle$ it's important to realise that the mass m is the mass of a single particle.

The total mass of air in the room will therefore be Nm where N is the number of particles and m is the mass of each individual particles.

If we divide both sides by V we're then left with:

$$P = \frac{1}{3}\rho\langle v^2 \rangle$$

which we can re-arrange easily to give

$$\langle v^2 \rangle = 3\frac{P}{\rho}$$

which means the *root* mean squared speed in our example is:

$$
\begin{aligned}
\sqrt{\langle v^2 \rangle} &= 3\frac{P}{\rho} \\
&= 3 \times \frac{1.01 \times 10^5 \text{ N/m}^2}{1.3 \text{ kg/m}^3} \\
&= \sqrt{2.42 \times 10^5} \\
&= 492 \text{ m/s}
\end{aligned}
$$

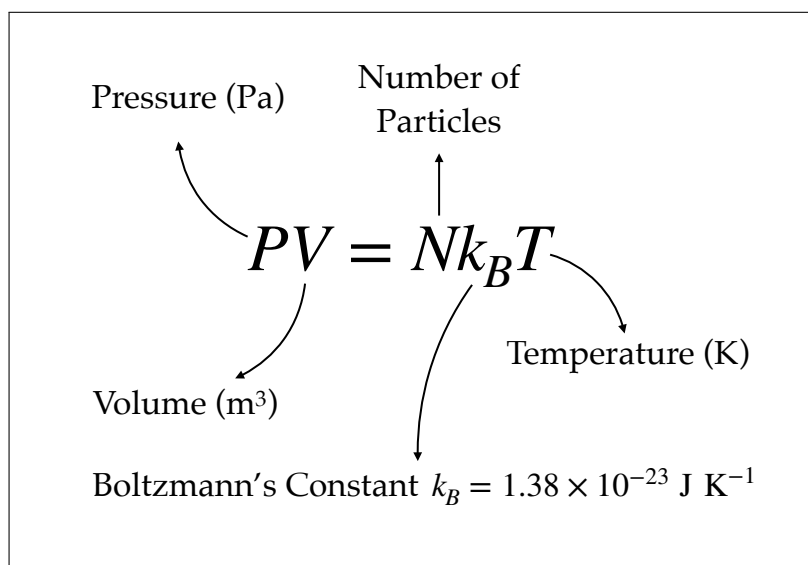### Practice Questions 2-1 - Calculating Root Mean Squared Speed

1. The rms of Helium at atmospheric pressure and 300 K is 1200 m/s. Calculate the density of Helium at this pressure and temperature.

2.3 *The Ideal Gas Law*

An alternative approach to understanding pressure in gases is to do experiments. We can use a *barometer* to measure the pressure of a gas.

Experiments early in the history of physics and chemistry showed that the pressure - as measured by a barometer - and the temperature - as measured by a thermometer - are linked to the volume V of the gas the number of particles in the gas N by the ideal gas law:

An equation linking the variables P, V, and T is called an **equation of state**. The ideal gas law is one of several possible equations of state.

$$PV = Nk_BT$$

Pressure (Pa)

Number of Particles

Volume (m³)

Temperature (K)

Boltzmann's Constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}

An alternative way of writing the ideal gas law that is more common in *chemistry* is $pV = nRT$ where R is the ideal gas constant and n is the number of moles. However, it is easier and more useful to reason about the behaviour of a gas based on the number of particles N.

We can understand the fundamental cause of temperature and pressure by considering the behaviour of individual ideal gas particles, but the earliest experiments were only able to measure these as "macroscopic" variables: temperature gives us a measure of the average kinetic energy of millions of gas particles, and pressure gives us a measure of the force and force of collisions from millions of gas particles.

Worked Example 2-2 - Calculations involving the Ideal Gas Equation

Q: A bicycle tyre has a volume of $V = 2 \times 10^{-4}$ m^3 and a pressure of $P = 2 \times 10^5$ Pa when fully inflated at a temperature of $T = 300$ K.

1. Calculate the number of moles of air in the tyre.

2. If the molar mass of the gas is 4×10^{-2} kg/mol, calculate the density of the gas.

3. Calculate the number of particles in the tyre.

A:

Practice Questions 2-2 - Calculations involving the Ideal Gas Equation

1.

2.4 *Temperature and Average Kinetic Energy*

When we compare the experimentally determined ideal gas law $PV = Nk_BT$ with the relationship we derived theoretically by considering individual gas particles colliding with a wall $PV = \frac{1}{3}Nm\langle v^2 \rangle$ we can see that the two are equivalent to each other provided:
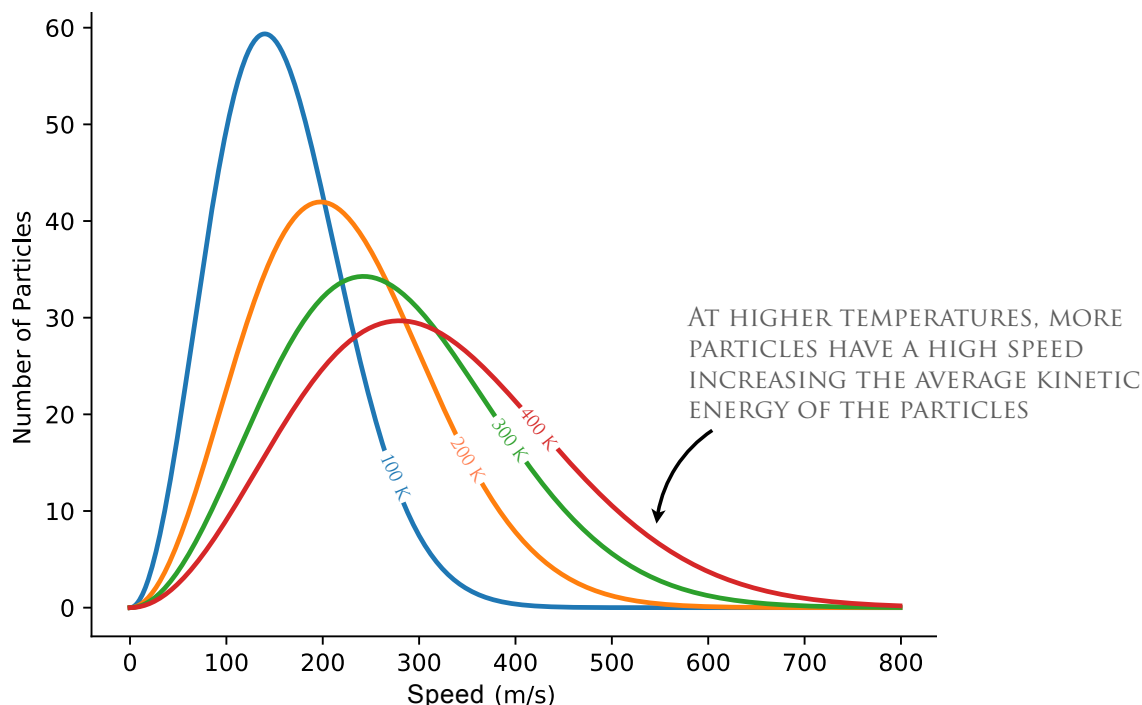
$$\frac{1}{3}m\langle v^2 \rangle = k_BT$$

and if we multiply this expression by $\frac{3}{2}$ on both sides we can see the relationship between average kinetic energy and temperature:

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_BT$$

The Maxwell-Boltzmann Distribution

If we measure the speed of individual particles and count how many particles have a particular speed, we end up with curves that look like the following:

These curves follow something called the *Maxwell-Boltzmann distribution*, but the name isn't important.

What's important to note is that as the temperature increases, the average speed of the particles increases and therefore also the average squared speed (mean squared speed) of the particles.

Since the y-axis tells you the number

Worked Example 2-3 - Average Kinetic Energy and Temperature

The average kinetic energy of particles in a sample of gas is 2×10^{-20} J. Calculate the temperature of the gas.

Practice Questions 2-3 - Average Kinetic Energy and Temperature.

Calculate the average kinetic energy of molecules in a sample of Lithium at 5000 K.

The temperature at the centre of the Sun is 3×10^6 K. Calculate the average kinetic energy of the hydrogen and helium in the centre of the Sun.

Uranium used for nuclear reactors is enriched from a gas of Uranium Hexafluoride. The minimum average kinetic energy of molecules must be at least 6.8×10^{-21} J for Uranium Hexafluoride to exist as a gas instead of a solid. What is the sublimation temperature of Uranium Hexafluoride?

Worked Example 2-4 - Average Kinetic Energy and Temperature

The average kinetic energy of particles in a sample of gas is 2×10^{-20} J. Calculate the temperature of the gas.

Practice Questions 2-4 - Average Kinetic Energy and Temperature.

The boiling point of Silver is 962°C. Calculate the root mean squared velocity of silver atoms at this temperature.

Atmospheric air is mainly nitrogen in the form of N_2. The second most abundant element in air is oxygen in the form of O_2. Calculate the root mean squared velocity of N_2 and O_2 at 300 K.

Salt turns into a gas at around 1600 K. Calculate the root mean squared speed of salt at this temperature.

An unknown substance has

Worked Example 2-5 - Calculating the mean squared speed and temperature

Calculate the mean square speed.

Practice Questions 2-5 - Calculate the mean square speed and temperature.

1. A sample of 5 Neon atoms have velocities 500 m/s, 1500 m/s, 2000 m/s, 3000 m/s, and 7000 m/s. Calculate their mean squared speed and hence the temperature of the sample.

2. The table below shows the number of particles with a particular speed in a sample of Helium atoms. Calculate the mean squared speed and the temperature of the sample.

Number of Particles	10	100	200	175	50
Speed (m/s)	50	100	150	200	300

3. The table below shows the number of particles with a particular speed in a sample of hydrogen atoms for two different temperatures: 300 K and 375 K.

Speed (m/s)	200	600	1000	1400	1800	2200	2600	3000	3400	3800	4200
Number of Particles (300 K)	4	11	16	18	16	14	9	6	3	2	1
Number of Particles (375 K)	2	9	14	16	15	13	11	8	5	4	3

(a) Plot a graph of the number of particles vs speed for the two different temperatures (on the same axes) and draw a line of best fit through each set of data.

(b) Calculate the mean squared velocity at each temperature.

(c) Show that the mean squared speed you have calculated is consistent with the temperature given in the table.

Worked Example 2-6 - Root Mean Squared Speed

Q: The root mean squared speed of a sample of Oxygen atoms at atmospheric pressure and 300 K is 400 m/s (**CHECK THIS**). Calculate the root mean squared speed at 1000 K.

Practice Questions 2-6 - Root Mean Squared Speed

Worked Example 2-7 - Root Mean Squared Speed

Q: Calculate the temperature at which the root mean squared speed of hydrogen atoms is the same their root mean squared speed at room temperature.

Practice Questions 2-7 - Root Mean Squared Speed

Worked Example 2-8 - Root Mean Squared Speed

Q: At atmospheric pressure and 300 K, the density of nitrogen is around $\rho = 1.3$ kg/m^3. Calculate the

root mean squared speed of nitrogen at 700 K.

A: Re-arranging our kinetic theory equation we obtain:

$$\langle v^2 \rangle = 3\frac{P}{\rho}$$
$$= 3 \times \frac{1.01 \times 10^5 \text{ N/m}^2}{1.3 \text{ kg/m}^3}$$
$$= 2.42 \times 10^5$$

Now we use the fact that $\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_B T$ which means that $\frac{\langle v^2 \rangle}{T} = \text{const.}$

This means that:

$$\frac{\langle v_1^2 \rangle}{T_1} = \frac{\langle v_2^2 \rangle}{T_2}$$
$$\implies \langle v_2^2 \rangle = \frac{T_2}{T_1}\langle v_1^2 \rangle$$
$$= \frac{700}{300} \times 2.42 \times 10^5$$
$$= 5.64 \times 10^5$$
$$\implies \sqrt{\langle v_2^2 \rangle} = \sqrt{5.64 \times 10^5}$$
$$= 751 \text{ m/s}$$

Practice Questions 2-8 - Root Mean Squared Speed

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3   *The Gas Laws*

There are three variables we can change in the ideal gas equation: pressure P, volume V, and temperature T. By fixing one of these variables, and varying the other two, we can investigate how two of the variables relate to one another. The possible combinations are:

- Fix temperature and vary the pressure and volume.
  $\implies PV = \text{const}$. This is called **Boyle's law**.

- Fix volume and vary the temperature and pressure.
  $\implies \frac{P}{T} = \text{const}$. This is called **The Pressure Law**.

- Fix pressure and vary the temperature and volume.
  $\implies \frac{V}{T} = \text{const}$. This is called **Charles' Law**.

3.1   *Boyle's Law*

Boyle's law $PV = \text{const}$ states that the pressure of a gas is inversely proportional to the volume of the gas $P \propto \frac{1}{V}$: if we double the volume, the pressure halves; if we increase the volume by a factor of 10, the pressure drops by a factor of 10; if we squeeze the gas into a volume 4 times smaller than the original volume then the pressure if

You may also be familiar with the ideal gas law as $PV = nRT$. The two equations are equivalent, which you can prove as an exercise.

We could also vary the number of particles, but for A-level we avoid this since it complicates the situation severely.

You don't need to know the names of the laws, but you need to understand them and apply them to situations involving ideal gases.

Although there is not a single gas that obeys these assumptions fully, amazingly they approximate the behaviour of many real gases, and nearly all real gases at high temperatures.

the gas increases by a factor of 4.

In other words, if we change the volume of the gas the pressure changes to compensate so that the new pressure and volume combine to the give the same constant $P_2 V_2 = $ const and so we can state Boyle's law equivalently as $P_1 V_1 = P_2 V_2$.

Why does the pressure drop when the volume increases? The **temperature of the gas stays the same** and so the **average velocities of the particles stays the same** too. Therefore, it cannot be due to the fact that the particles are hitting the container walls harder. Instead, it's because:

- There are **fewer collisions each second** because the particles have further to travel between collisions.

- These collisions are **spread over a larger area** because the volume has increased.

In order for Boyle's law to be true, we need to keep the temperature of the gas constant.

It's important to realise that $PV = $ const, $P \propto \frac{1}{V}$, and $P_1 V_1 = P_2 V_2$ are three ways of stating the same law, they are not three separate equations that need to be remembered

Remember the force exerted on an object during a collision is

$$F = \frac{mv - mu}{t}$$
$$= \frac{\text{change in momentum}}{\text{time}}$$
$$= \text{rate of change of momentum.}$$

To do this in practice, we keep the gas in contact with a thermal reservoir kept at a constant temperature. For example, if we expand the gas, it cools, but the thermal reservoir then transfers some

Worked Example 3-1 - Boyle's Law Calculations

Q: A gas has a pressure of 100Pa and takes up a volume of 25m^3. Its volume changes to 50m^3. What is its new pressure?

A: There are two ways to approach this: first of all we could just use the mechanical way of plugging in the values into the equation and solving for the new pressure. Let's try that first.

$$p_1 = 100\text{Pa}$$
$$V_1 = 25\text{m}^3$$
$$p_2 = ?$$
$$V_2 = 50\text{m}^3$$

Using Boyle's law:

$$p_1 V_1 = p_2 V_2$$
$$100\text{Pa} \times 25\text{m}^3 = p_2 \times 50\text{m}^3$$
$$p_2 = \frac{2500\text{m}^3\text{Pa}}{50\text{m}^3}$$
$$p_2 = 50 \text{ Pa}$$

Now, let's put our physicist hats on and think about this properly.

The first thing we need to notice is that since the volume has increased, the pressure must decrease. The new pressure we calculated is less than the original, and so our answer is standing up to scrutiny so far.

We also know that since volume is inversely proportional to pressure, if we double the volume, the pressure must halve. In this case, we have exactly doubled the volume, and so the new pressure must be exactly half of 100 Pa, which it is.

Further, we know that the product of pressure and volume must be constant: the initial product is $p_1 V_1 = 100\text{Pa} \times 25\text{m}^3 = 2500\text{m}^3$ Pa and we know that the new product therefore must equal this value: $p_2 V_2 =$

$p_2 \times 50\text{m}^3 = 2500\text{m}^3\text{'Pa}$ and we can rearrange to solve for p_2, which is completely equivalent to the method we used.

Practice Questions 3-1 - Boyle's Law calculations

1. The pressure of a gas changes from 80Pa to 60Pa. Its original volume was 225m^3. What is its new volume?

2. The pressure of a gas increases from 100Pa to 125Pa. Its original volume was 10m^3. What is its new volume?

3. The pressure of a gas is 150Pa at a volume of 20m^3. The gas is compressed to a new volume of 5m^3. What is its new pressure?

3.2   *The Pressure Law*

When we kept the volume constant in the ideal gas equation we ended up with the Pressure law: $\frac{P}{T} = $ const. This is equivalent to saying that the pressure is directly proportional to the temperature $P \propto T$ if the gas is not allowed to change its volume by expanding or contracting: if we double the temperature, the pressure doubles; if we increase the temperature by a factor of 10, the pressure increases by a factor of 10; and if we decrease the temperature by a factor of 4, the pressure decreases by a factor of 4.

Why does the pressure increase if we increase the temperature? Recalling that increasing the temperature increases the **average velocity** of the particles, we can see that the change in momentum when the particles hit the walls of the container will be larger at higher temperatures, and so the force exerted on the boundaries and hence the pressure will be larger.

Again, the pressure and temperature change in tandem, compensating for one another, and so we can say that $\frac{P_1}{T_1} = \frac{P_2}{T_2}$.

It's important that temperature is mea-

Worked Example 3-2 - Pressure Law Calculations

Q: A gases pressure decreases from 200 Pa to 50 Pa. Its initial temperature is 27°C. What is its new temperature?

A: Note firstly that since its pressure decreases, it must have cooled.

$$p_1 = 200 \text{ Pa}$$
$$T_1 = 27°\text{C} = 27 + 273 = 300 \text{ K}$$
$$p_2 = 50 \text{ Pa}$$
$$T_2 = ?$$

Using Gay-Lussac's law:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{200 \text{ Pa}}{300 \text{ K}} = \frac{50 \text{ Pa}}{T_2}$$

$$T_2 = \frac{300 \text{ K}}{200 \text{Pa}} \times 50 \text{ Pa}$$

$$T_2 = 75 \text{ K}$$

Note secondly that the pressure of the gas decreases by a factor of 4. Since pressure and temperature are directly proportional, we should expect that the temperature of the gas also fell by a factor of 4 i.e. $300/4 = 75$, as our calculations confirmed.

Practice Questions 3-2 - Pressure Law calculations

1. A gas has a pressure of 75Pa after it was heated from 50 K to 250 K. What was its initial pressure?

2. A gas's pressure increases from 10 Pa to 55 Pa after it was heated to a final temperature of 300 K. What was its initial temperature?

3.3   Charles' Law

Charles' law - where we fix the pressure of a gas and obtain $\frac{V}{T} = $ const - is the most technically interesting, because we have to disentangle two competing effects.

In an experiment, we could just as easily vary the temperature and measure the volume as vary the volume and measure the tempera-ture. Therefore, it's useful to keep in mind that $V \propto T$ can easily be flipped to give $T \propto V$.

Let's first consider the $V \propto T$ case. This form of Charles' law says that if we increased the temperature of a gas, its volume increases, provided the pressure stays constant.

If the temperature increases and the volume is fixed, the pressure of the gas increases. However, if we allow the gas to expand as we increase its temperature, the pressure of the gas decreases as it ex-pands. This allows us to keep the pressure constant. Crucially, the volume **must increased** in order for the pressure to **stay constant**. Similarly, if we decrease the temperature, the pressure decreases, and the only way to compensate for that decrease in pressure is to com-press the gas and reduce its volume. This is the basis of Charles' law.

Now let's consider the $T \propto V$. This says that if we increase the volume, the temperature increases? How can this possibly be true, given that we know that increasing the volume decreases the pres-sure? The answer is that we have to remember that we have to keep the pressure constant. If we increase the volume, the pressure of the gas drops, and so to compensate for this we need to increase the temperature. Likewise, if we decrease the volume, the pressure in-creases, and in order to keep the pressure constant we would have to cool the gas down.

Worked Example 3-3 - Calculations Involving Changing Particle Number

Q: A beach ball of volume $V = 5 \times 10^{-3}$ m^3 has a pressure of $P = 1.5 \times 10^5$ Pa when at a temperature of $T = 20°$C.

As the day gets warmer, the beach ball is exposed to direct sunlight and its temperature rises to $T = 35°$C.

1. Calculate the pressure of the air in the ball.

2. The number of molecules of air in the ball.

3. Air is then released from the ball until it returns to its original pressure of $P = 1.5 \times 10^5$ Pa. Assuming the temperature remains at $T = 35°$C and the ball remains fully inflated:

   (a) Calculate number of molecules in the ball.

   (b) Calculate the number of molecules that escaped from the ball.

A:

1. We can calculate the pressure of the ball using $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$ which gives: $p_2 = p_1 \times \dfrac{T_2}{T_1} = 1.5 \times 10^5$ Pa $\times$ $\dfrac{35 + 273}{20 + 273} = 1.58 \times 10^5$ Pa.

2. $N = \dfrac{pV}{k_B T} = \dfrac{1.58 \times 10^5 \text{ Pa} \times 5 \times 10^{-3} \text{ m}^3}{1.38 \times 10^{-23} \times 308} = 1.86 \times 10^{23}$

3. Air is then released from the ball until it returns to its original pressure of $P = 1.5 \times 10^5$ Pa. Assuming the temperature remains at $T = 35°$C and the ball remains fully inflated:

   (a) $N = \dfrac{pV}{k_B T} = \dfrac{1.5 \times 10^5 \text{ Pa} \times 5 \times 10^{-3} \text{ m}^3}{1.38 \times 10^{-23} \times 308} = 1.8 \times 10^{23}$

   (b) Calculate the number of molecules that escaped from the ball. $\Delta N = 1.86 \times 10^{23} - 1.8 \times 10^{23} = 5.86 \times 10^{21}$

Practice Questions 3-3 - Calculations involving the Ideal Gas Equation

1. A metal canister of volume $V = 0.25$ m^3 is taken from sea level where it is at a temperature of $T = 20°$C and pressure $P = 1.5 \times 10^6$ Pa to the top of a mountain where the temperature is $T = -50°$C.

2. Calculate the number of moles of gas in the cylinder.

3. Calculate the density of the gas.

4. Calculate the number of particles in the cylinder.

5. The cylinder is dropped and a small hole develops in the cylinder causing gas to escape.

   (a) Calculate the mass of gas remaining in the cylinder after it reaches equilibrium with the surroundings.