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NUCLEAR PHYSICS

NAME: CLASS:

Contents

1 *Binding Energy and the Mass Defect*

Binding energy gives a measure of how **strongly bound** protons and neutrons are inside nuclei, and therefore how **stable** they are.

1 atomic mass unit (1 u) = 1.66×10^{-27} kg

- m_p: 1.0073 u $= 1.672 \times 10^{-27}$ kg
- m_n: 1.0087 u $= 1.675 \times 10^{-27}$ kg

Consider a Helium atom ${}^4_2$He, which has 2 protons and 2 neutrons. If we add up the mass of the individual protons and neutrons we arrive at the following mass for the Helium nucleus:

- Mass of protons: $2 \times m_p = 2 \times 1.0072 \text{ u} = 2.0144 \text{ u}$
- Mass of neutrons: $2 \times m_n = 2 \times 1.0087 \text{ u} = 2.0174 \text{ u}$
- Total Helium nucleus mass = mass of protons + mass of neutrons $m_{\text{He}} = 2.0144 \text{ u} + 2.0174 \text{ u} =$ **4.0318 u**

However, if we look up the mass of a Helium nucleus measured from experiment we find:

- $m_{\text{He}} =$ **4.0015 u**

Somehow the mass of a Helium atom is less when the protons and neutrons are bound together than when they are separated i.e. the mass of a Helium atom is less than the mass of the protons and neutrons that make it up.

It turns out this is a general feature of the nuclei of atoms: the **measured mass of the nucleus is less than total mass of the protons and neutrons it's made from**. This observation is called **the mass defect**.

The mass defect is the difference in mass between the sum of the protons and neutrons in a nucleus and the actual (measured) mass of that nucleus.

In the case of Helium-4 above, the mass defect is:

- $4.0318 \text{ u} - 4.0015 \text{ u} = 0.0303 \text{ u}$

Using mass-energy equivalence $E = mc^2$, this tiny mass defect corresponds to an energy of:

$$
\begin{aligned}
0.0303 \text{ u} &= 0.0303 \times 1.66 \times 10^{-27} \text{ kg} \\
&= 5.03 \times 10^{-28} \text{ kg} \\
\implies E = mc^2 &= 5.03 \times 10^{-28} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 = 4.5 \times 10^{-12} \text{ J} \\
&= \frac{4.5 \times 10^{-12}}{1.6 \times 10^{-19}} \\
&= 28.3 \text{ MeV}
\end{aligned}
$$

This energy is called **the binding energy**, and it is usual to express binding energies on a per nucleon basis to get the **binding energy per nucleon**. In the case of Helium-4, this would be $28.3 \text{ MeV}/4 = 7.1$ MeV per nucleon since there are 4 *nucleons* (protons + neutrons) in Helium-4.

In the next sections we will discuss the origin of the mass **mass defect** and investigate its consequences.

1.1 *The Strong Force*

Protons and neutrons experience **the strong force**. The strong force is a strong **attractive force** when protons and neutrons are close together, but it is also short-ranged: as soon as you separate a proton and a neutron by a distance around their own size the strong force decays nearly to zero. In other words, protons and neutrons are strongly attracted to each other as long as they're close together. If they get too close, they repel each other.

Figure 1 below shows how the force varies with distance between protons and neutrons.

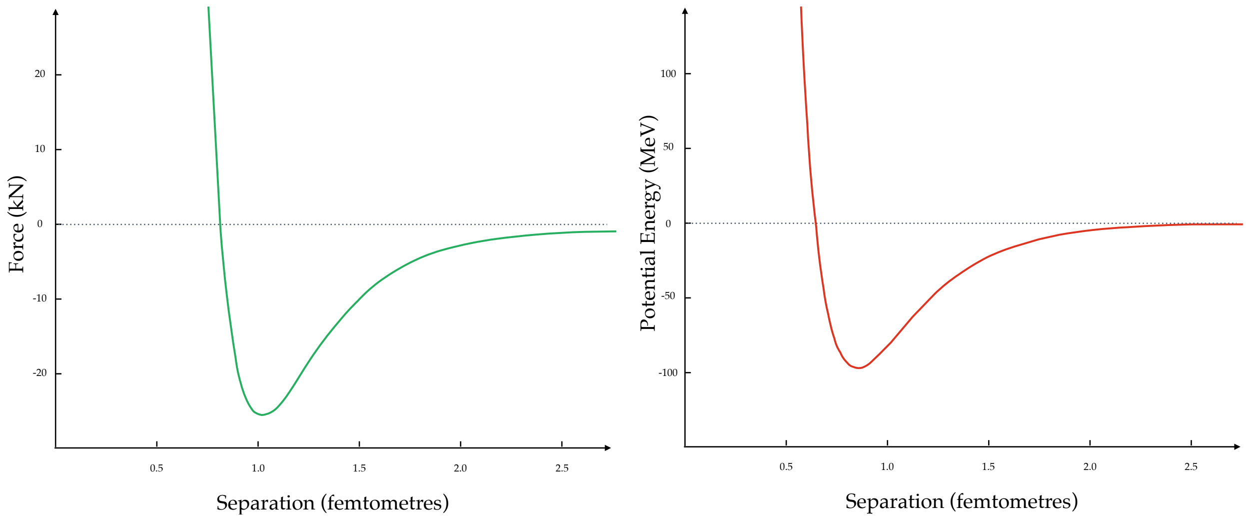


Figure 1: **Left**: the force is *attractive* (negative) unless the nuclei get too close. The force rapidly decays to 0 N as the particle separation increases. **Right**: the potential energy of particles is *negative* when they are close together.

Most importantly, the potential energy graph shows that particles that are **close to each other** have **lower energy** than if they are separated.

1.2 *What is binding energy, physically?*

Consider a single proton and single neutron, separated by a large distance. If we now move that proton and neutron together, they have lower energy than when they were apart (due to the negative potential energy of the strong force shown in Figure 1). Using mass-energy equivalence ($E = mc^2$) this also means that the proton and neutron stuck together must have less mass than when they are separated.

Now the situation in reverse: let's take our proton and neutron that are stuck together and try to separate them. We have to **provide** energy in order to pull them apart, and this energy corresponds to a mass, which explains why the separated proton and neutron have a higher mass than the stuck-together proton and neutron.

> **The binding energy of a nucleus is the average energy required to fully separate its constituent protons and neutrons.**

In the Helium-4 example above, we would require 28.3 MeV of energy to break the Helium-4 nucleus into 2 protons and 2 neutrons so that they're far enough apart to escape the strong force.

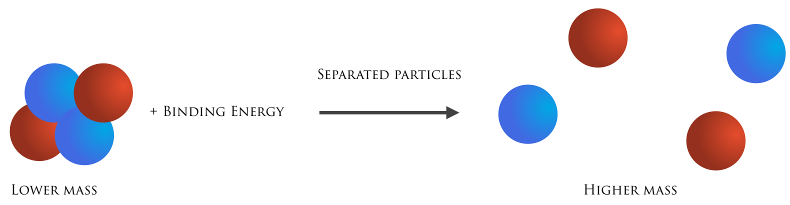


Figure 2: Separating the protons and neutrons in a nucleus requires energy. According to $E = mc^2$, this also means we are "adding mass" to the system, and therefore the separated protons and neutrons have a higher mass than the bound protons and neutrons.

More energy is needed to separate the constituent protons and neutrons in nuclei with **high** binding energies, and therefore these nuclei are more stable.

1.3 Binding Energy per Nucleon

When physicists calculated the **binding energy per nucleon** for a range of nuclei and plotted a graph of binding energy per nucleon vs mass number, they obtained Figure 3 below:

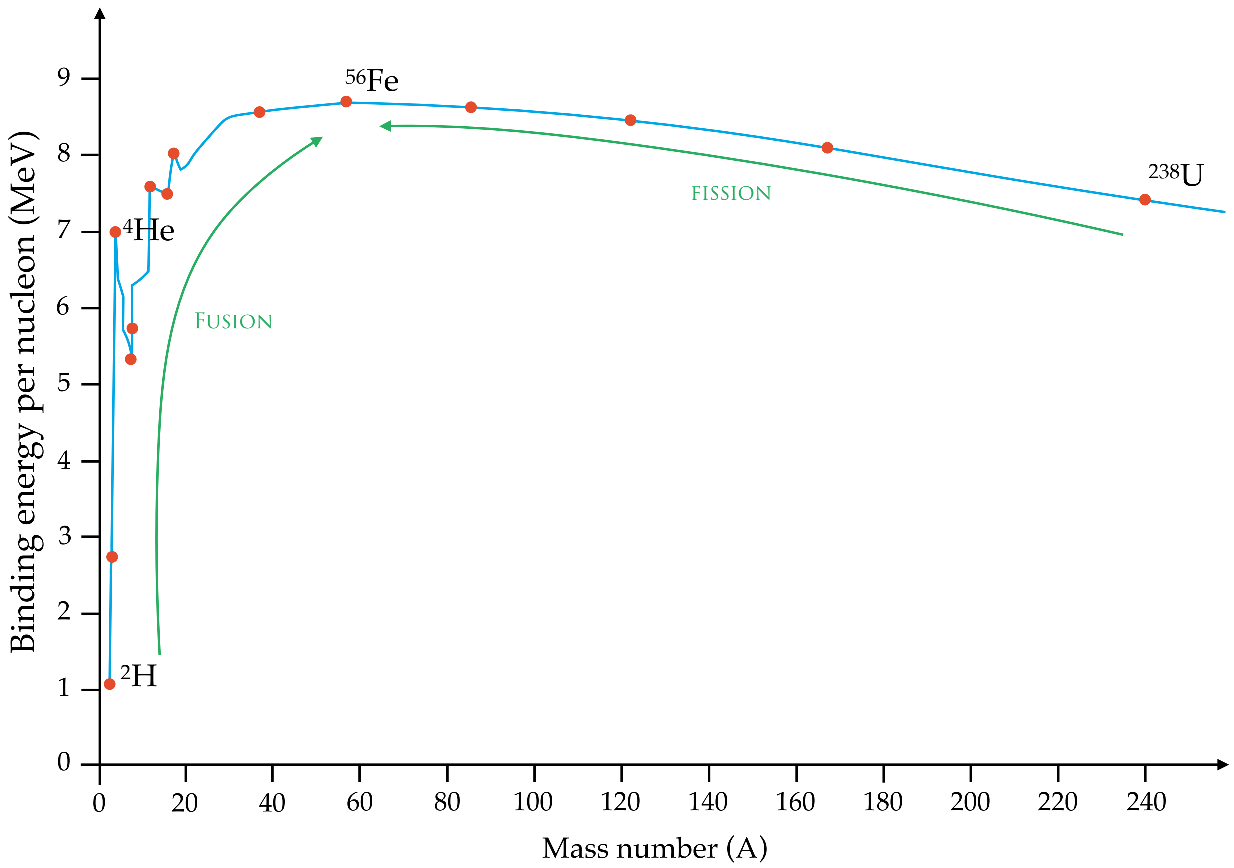


Figure 3: The binding energy per nucleon reaches a maximum for Iron-56, making it the most stable element requiring the most energy to separate its protons and neutrons. Elements to the left of Iron-56 can fuse to achieve a higher binding energy, and elements to the right of Iron-56 can be split (fission) to achieve higher binding energies. Both processes release energy.

The implications of Figure 3 above are profound: it explains why it is possible to release energy by both sticking atoms together (fusion) and by splitting them apart (fission).

Figure 3 shows that the binding energy per nucleon increases as the mass number of the nuclei increase, until it reaches a maximum at iron (Fe-56). Iron is the most stable nucleus, and due to its high binding energy, requires the most energy to separate its protons and neutrons. After Fe-56, the binding energy per nucleon decreases, and these atoms are slightly less stable than Fe-56.

Helium-4 seems to have an anomalously high binding energy per nucleon, which naturally leads us to question why the binding energy per nucleon is different for different nuclei.

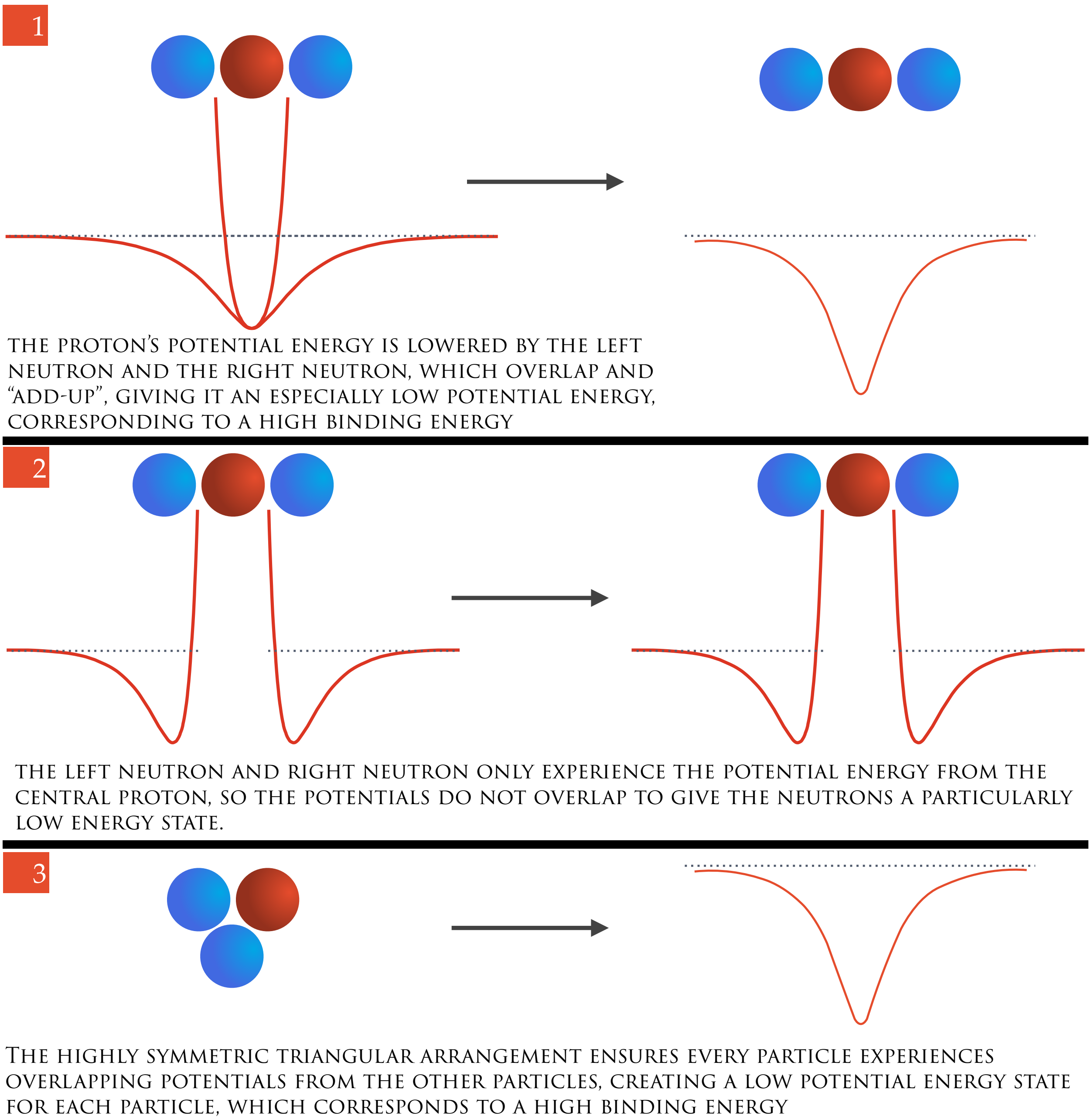


Figure 4: How the *same number* of particles can lead to different binding energies. **1.** Particles that are close to each other have lower *potential energy*. **2.** Three particles in a line are not in the lowest possible energy state. **3.** Three particles in a triangle are in the lowest energy state, leading to a high binding energy.

We know that the binding energy is related to the strong force: if atoms can get close together, they can lower their potential energy and this in turns lowers their mass; separating them again requires more energy, which is the binding energy.

Figure 4 shows two possible arrangements of 3 particles - 2 neutrons and a single proton. If we put them in a line with the proton in the middle, the middle proton has potential energy with the both the neutron on the left and the neutron on the right, and these potential energies combine to give the central proton a very low potential energy (frame 1). The neutrons on the left and right, however, only have potential energy with the proton in the middle, and so do not manage to achieve a very low energy state (frame 2).

If we simply move the neutrons so that they are in a triangular structure (frame 3), this allows all the particles to overlap their potential energies with neighbouring particles and achieve this very low energy state. Therefore, the triangular arrangement will have a higher binding energy (it will require more energy to separate the neutrons and protons) than the chain state.

This simple experiment gives us a clue as to why the binding energy per nucleon varies from atom to atom: some atoms have a particularly low energy arrangement of their protons and neutrons and this depends on the number of neutrons and protons. For example, it is easier to arrangement 4 nucleons into a symmetric low energy state (Helium-4) than 6 particles (Lithium-6), which results in Helium-4 having an especially high binding energy per nucleon.

As we increase the number of nucleons, it becomes harder and harder to arrange the nucleons into very low energy and symmetric states, and Iron-56 and a few nearby isotopes hit the sweet-spot in this regard, making them the most stable isotopes.

Worked Example 1-1 - Calculate the binding energy

Q: Calculate the average binding energy per nucleon for an $^{56}_{26}\mathrm{Fe}$ nucleus:

A: The mass of 26 individual protons is:

- 26×1.0078 u $= 26.203$ u

and the mass of the $56 - 26 = 30$ individual neutrons is:

- 30×1.0087 u = 30.260 u

leading to combined mass of:

- 26.203 u + 30.260 u = 56.463 u

corresponding to mass defect of:

- 56.463 u - 55.935 u = 0.528 u

which, using mass-energy equivalence $E = mc^2$ corresponds to an energy of:

- $E = mc^2 = 0.19 \text{ u} \times c^2 = 0.528 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 8.77 \times 10^{-28} \times 9 \times 10^{16} = 7.9 \times 10^{-11} \text{ J} = \dfrac{7.9 \times 10^{-11}}{1.6 \times 10^{-19}} = 493 \text{ MeV} = 493 \text{ MeV}/56 = 8.8 \text{ MeV}$ per nucleon.

Practice Questions 1-1 - Calculate the binding energy of the following atoms

1. $^{23}_{11}$Na.

2. $^{63}_{29}$Cu

3. $^{197}_{79}$Au

Practice Questions 1-2 - Calculate the binding energy per nucleon of the following atoms

1. $^{212}_{84}$Po.

2. $^{16}_{8}$O

3. $^{7}_{3}$Li

Nuclide	Mass (u)
n	1.008665
p	1.007825
$^{2}_{1}$H	2.014102
$^{3}_{1}$H	3.016050
$^{3}_{2}$He	3.016030
$^{4}_{2}$He (α)	4.002603
$^{6}_{3}$Li	6.015125
$^{7}_{3}$Li	7.016004
$^{10}_{5}$B	10.012939
$^{12}_{6}$C	12.000000
$^{13}_{6}$C	13.003354
$^{14}_{6}$C	14.003242
$^{13}_{7}$N	13.005738
$^{14}_{7}$N	14.003074
$^{15}_{7}$N	15.000109
$^{16}_{8}$O	15.994915
$^{23}_{11}$Na	22.989771
$^{39}_{19}$K	38.964710
$^{62}_{24}$Cr	51.940508
$^{56}_{26}$Fe	55.939395
$^{63}_{29}$Cu	62.929592
$^{92}_{36}$Kr	91.926156
$^{96}_{37}$Rb	95.934272
$^{114}_{42}$Mo	113.94492
$^{116}_{45}$Rh	115.92406
$^{107}_{47}$Ag	106.90509
$^{115}_{47}$Ag	114.90876
$^{101}_{49}$In	100.92634
$^{101}_{50}$Sn	100.93606
$^{123}_{50}$Sn	122.90572
$^{137}_{55}$Cs	136.90708
$^{138}_{55}$Cs	137.91101
$^{137}_{56}$Ba	137.90524
$^{141}_{56}$Ba	140.91441
$^{174}_{69}$Tm	173.94216
$^{174}_{70}$Yb	173.93886
$^{196}_{78}$Pt	195.96495
$^{197}_{79}$Au	196.96654
$^{198}_{80}$Hg	197.96677
$^{202}_{82}$Pb	201.97216
$^{208}_{82}$Pb	207.97665
$^{212}_{84}$Po	211.98963
$^{222}_{86}$Rn	222.01753
$^{226}_{88}$Ra	226.02536
$^{235}_{92}$U	235.04393
$^{238}_{92}$U	238.04861
$^{237}_{93}$Np	237.04817
$^{234}_{94}$Pu	234.04332
$^{242}_{94}$Pu	242.05874
$^{241}_{95}$Am	241.05683
$^{238}_{96}$Cm	238.05303
$^{254}_{97}$Bk	254.09060
$^{258}_{98}$Es	258.09952

2 *Fission and Fusion*

This remarkable feature of the binding energy per nucleon vs nucleon number graph (Figure 3) is that it tells us that both **fission** and **fusion** can result in the release of energy: if we start with elements with a lower atomic number than Iron-56 we can push them together and create a lower energy nucleus, causing the release of energy; if we start with elements with a higher atomic number than Iron-56 we can split them apart into two lower energy nuclei, releasing energy in the process.

2.1 *Fusion*

Nuclei are positively charged, meaning that they repel each other, particularly when they get close to each other. In order for fusion to occur, we need to overcome this electrostatic repulsion until the nuclei are close enough for the **attractive** strong force to overcome the electrostatic repulsion. This conditions needed for this to occur are:

- **High temperatures**: high temperatures mean large nuclei kinetic energies. If the nuclei are travelling fast enough when they collide, they can fuse together.
- **High densities**: increases the probability of collision between two particles, making fusion more likely.

Stars happen to have the ideal conditions for fusion to occur: gravitational attraction squeezes the hydrogen atoms of a star together resulting in high densities, and this squeezing also increases the temperature. Eventually the hydrogen is squeezed so much that fusion

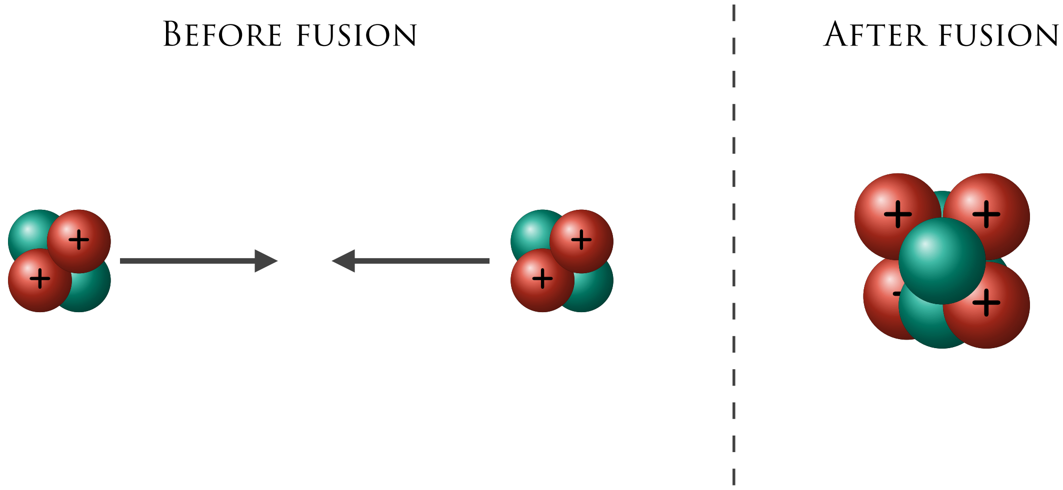


Figure 5: If two particles have high enough velocity, they can overcome the electrostatic repulsion and get close enough for the strong force to fuse the nucleons together. The binding energy of the fused nucleus is higher than of the two particles that fuse together, releasing energy.

begins in the star and the star "ignites". The energy released from fusion increases the temperature further and fusion becomes self-sustaining until the star runs out of hydrogen.

Worked Example 2-1 - Calculate the energy released during this fusion reaction

Q: A particular fusion reaction that occurs in the Sun is:

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}.$$

How much energy is released in this reaction?

A: The mass of the nuclides before fusion is:

- 2.014 u + 3.016 u = 5.030 u

and the mass of the nuclides after fusion is:

- 4.003 u + 1.009 u = 5.012 u

leading to a mess defect of:

- 5.030 u - 5.012 u = 0.018 u

which, using mass-energy equivalence $E = mc^2$ corresponds to an energy of:

- $E = mc^2 = 0.018\ u \times c^2 = 0.018 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 3.13 \times 10^{-29} \times 9 \times 10^{16} = 2.8 \times 10^{-12}\ \text{J} = \dfrac{2.8 \times 10^{-12}}{1.6 \times 10^{-19}} = 18\ \text{MeV}$

Practice Questions 2-1 - Calculate the energy released in these fusion reactions

Isotope	Mass/10^{-27} kg
$^{3}_{2}$He	5.008238
$^{4}_{2}$He	6.646483
$^{1}_{1}$H	1.673534

1. ${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \longrightarrow {}^{4}_{2}\text{He} + 2\,{}^{1}_{1}\text{H}$
2. ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$
3. ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \longrightarrow 2\,{}^{4}_{2}\text{He}$

2.2 *Fission*

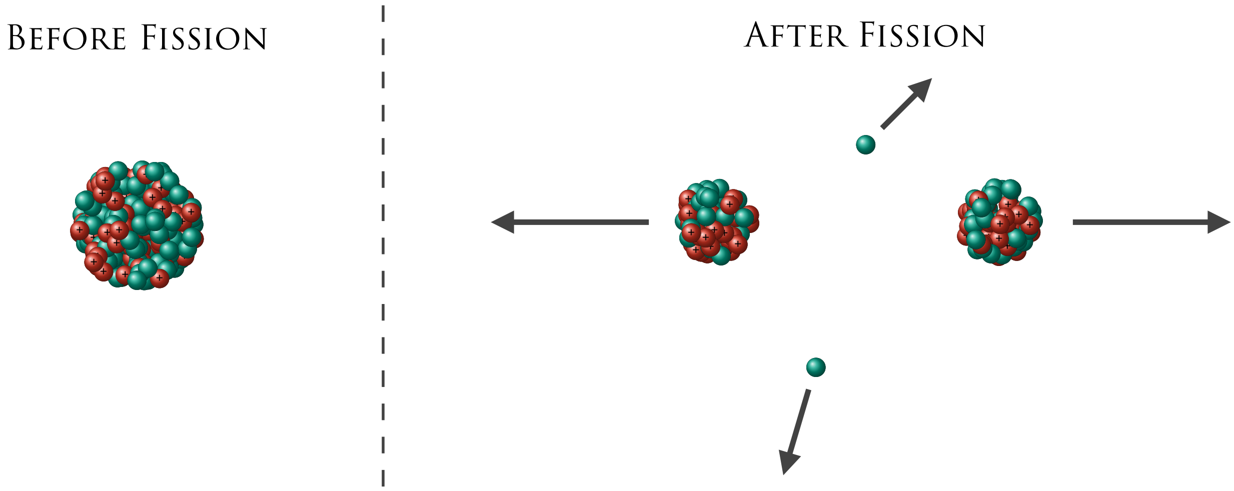


Figure 6: During fission, a particle splits into two smaller nuclei, usually accompanied by the emission of 1 or more neutrons. The overall binding energy of the two smaller nuclei is higher than the original nucleus, which means the process releases energy.

We deal with **induced** fission in more detail in the radioactivity chapter.

Fission is the process of a nucleus **splitting** into two or more smaller nuclei. Fission can occur **spontaneously** or it can be **induced**.

The two or more smaller "daughter" nuclei have a higher binding energy than the "parent" nucleus and so this process releases energy.

As a nucleus gets heavy and contains more and more nucleons, it becomes harder for the strong force to keep the nucleus bound together and it becomes more and more likely that the nucleus will **spontaneously** fission. This is one of the reasons that we do not find atoms with very large nucleon numbers in nature.

Worked Example 2-2 - Calculating the energy released in a fission reaction

Q: A possible fission reaction with Uranium-235 is:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n}.$$

How much energy is released in this reaction?

A: The mass of the nuclides before fission is:

- 235.04 u + 1.009 u = 236.05 u

and the mass of the nuclides after fission is:

- 140.91 u + 91.93 u + 3 × 1.009 u = 235.87 u

leading to a mess defect of:

- 236.05 u - 235.87 u = 0.19 u

which, using mass-energy equivalence $E = mc^2$ corresponds to an energy of:

- $E = mc^2 = 0.19\ u \times c^2 = 0.019 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 3.1 \times 10^{-28} \times 9 \times 10^{16} = 2.8 \times 10^{-11}\ \mathrm{J} = \dfrac{2.8 \times 10^{-11}}{1.6 \times 10^{-19}} = 173\ \mathrm{MeV}$

This energy is shared between the daughter nuclei and the neutrons as kinetic energy.

Practice Questions 2-2 - Calculate the energy released in the following fission reactions

1. $^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{138}_{55}\mathrm{Cs} + ^{96}_{37}\mathrm{Rb} + 3^{1}_{0}\mathrm{n}$
2. $^{233}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{115}_{47}\mathrm{Ag} + ^{116}_{45}\mathrm{Rh} + 3^{1}_{0}\mathrm{n}$
3. $^{237}_{93}\mathrm{Np} + ^{1}_{0}\mathrm{n} \longrightarrow ^{174}_{69}\mathrm{Tm} + ^{62}_{24}\mathrm{Cr} + 2^{1}_{0}\mathrm{n}$
4. $^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{123}_{50}\mathrm{Sn} + ^{114}_{42}\mathrm{Mo} + 2^{1}_{0}\mathrm{n}$

Worked Example 2-3 - Calculating the energy released in the following beta decay

Q: Calculate the energy released in the following beta decay:

$$^{137}_{55}\mathrm{Cs} \longrightarrow {}^{137}_{56}\mathrm{Ba} + \mathrm{e}^- + \overline{\nu_e}$$

A: The mass before the decay is:

- 136.907 u

and the after is:

- 136.906 u

leading to mass defect of:

- 136.907 u - 136.906 u = 0.001 u

which, using mass-energy equivalence $E = mc^2$ corresponds to an energy of:

- $E = mc^2 = 0.001\ u \times c^2 = 0.001 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 2.075 \times 10^{-30} \times 9 \times 10^{16} = 1.87 \times 10^{-13}\,\mathrm{J} = \dfrac{1.87 \times 10^{-13}}{1.6 \times 10^{-19}} = 1.17\,\mathrm{MeV}$

This energy is shared between the daughter nuclei and the electron as kinetic energy.

Practice Questions 2-3 - Calculate the energy released in these beta decays

1. $^{101}_{50}\mathrm{Sn} \longrightarrow {}^{101}_{49}\mathrm{In} + \mathrm{e}^+ + \nu_e$
2. $^{96}_{37}\mathrm{Rb} \longrightarrow {}^{96}_{38}\mathrm{Sr} + \mathrm{e}^- + \overline{\nu_e}$
3. $^{174}_{69}\mathrm{Th} \longrightarrow {}^{174}_{70}\mathrm{Yb} + \mathrm{e}^- + \overline{\nu_e}$

Worked Example 2-4 - Calculate the energy released in this alpha decay

Q: Calculate the energy released in the following alpha decay:

$$^{241}_{95}\mathrm{Am} \longrightarrow ^{237}_{93}\mathrm{Np} + ^{4}_{2}\alpha$$

A: The mass before the decay is:

- 241.056822 u

and the after is:

- 237.048166 u + 4.002603 u $=$ 241.050769 u

corresponding to a mass defect of:

- 241.056822 u - 241.050769 u = 0.006053 u

which, using mass-energy equivalence $E = mc^2$ corresponds to an energy of:

- $E = mc^2 = 0.006053\, u \times c^2 = 0.006053 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 1.004798 \times 10^{-29} \times 9 \times 10^{16} = 9.043182 \times 10^{-13}\,\mathrm{J} = \dfrac{9.043182 \times 10^{-13}}{1.6 \times 10^{-19}} =$ 5.65 MeV

This energy is shared between the daughter nuclei and the alpha particle as kinetic energy.

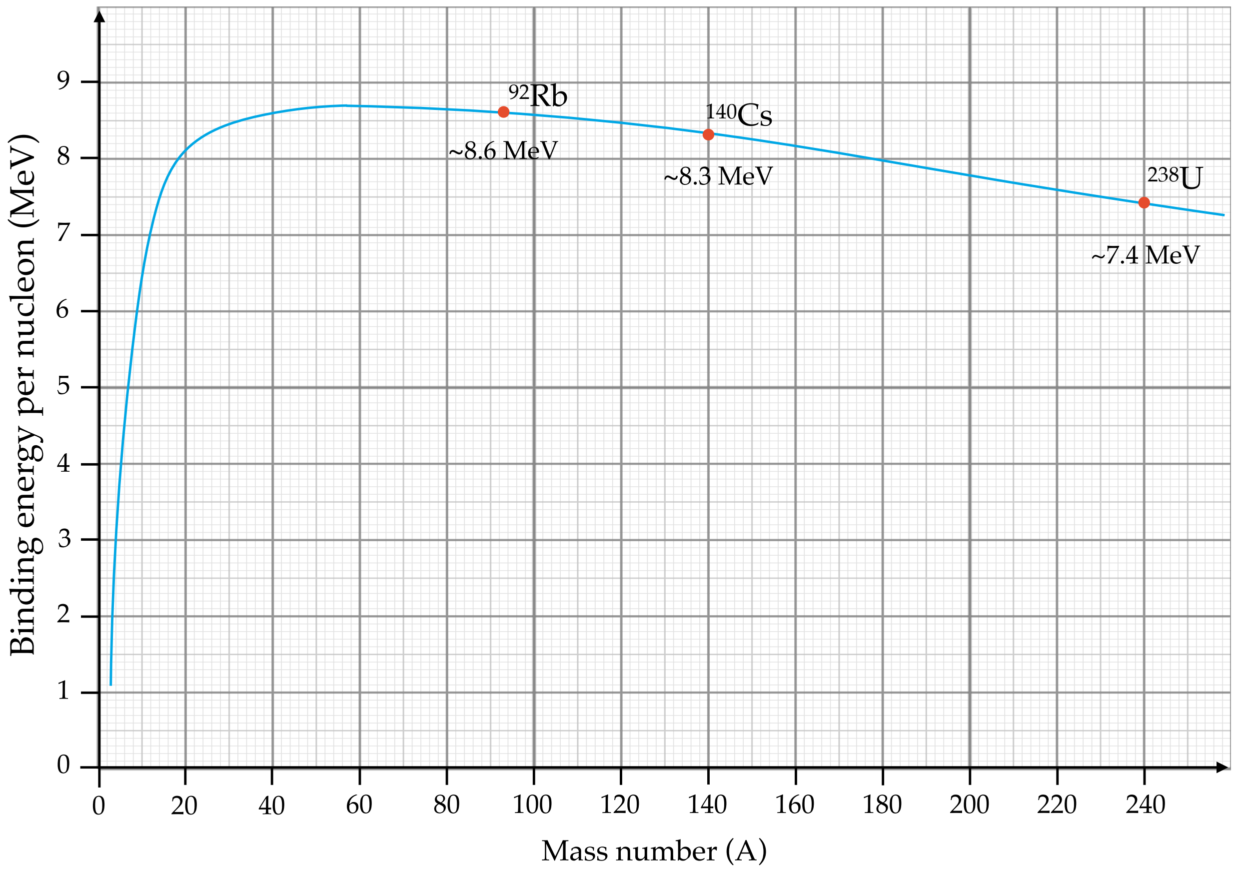
Practice Questions 2-4 - Calculate the energy released in these alpha decays

1. $^{258}_{99}\mathrm{Es} \longrightarrow ^{4}_{2}\alpha + ^{254}_{97}\mathrm{Bk}$
2. $^{238}_{96}\mathrm{Cm} \longrightarrow ^{4}_{2}\alpha + ^{234}_{94}\mathrm{Pu}$
3. $^{202}_{82}\mathrm{Pb} \longrightarrow ^{4}_{2}\alpha + ^{198}_{80}\mathrm{Hg}$

Worked Example 2-5 - Calculate the energy released in this fission reaction

Q: Use the graph to calculate the energy released in the following fission reaction:

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \longrightarrow ^{92}_{37}\text{Rb} + ^{140}_{55}\text{Cs} + 3\,^{1}_{0}\text{n}$$



A: Reading from the graph, we find that the binding energy per nucleon of 92-Rb is roughly 8.6 MeV which gives it a total binding energy of:

$8.6 \text{ MeV} \times 92 = 791 \text{ MeV}$

Likewise, the binding energy per nucleon of 140-Cs is roughly 8.3 MeV which gives it a total binding energy of:

$8.3 \text{ MeV} \times 140 = 1162 \text{ MeV}$

Therefore, the total binding energy after the reaction is $1162 + 791 = 1953$ MeV.

The binding energy per nucleon of 235-U is roughly 7.4 MeV which gives it a total binding energy of:

$7.4 \text{ MeV} \times 235 = 1739 \text{ MeV}$

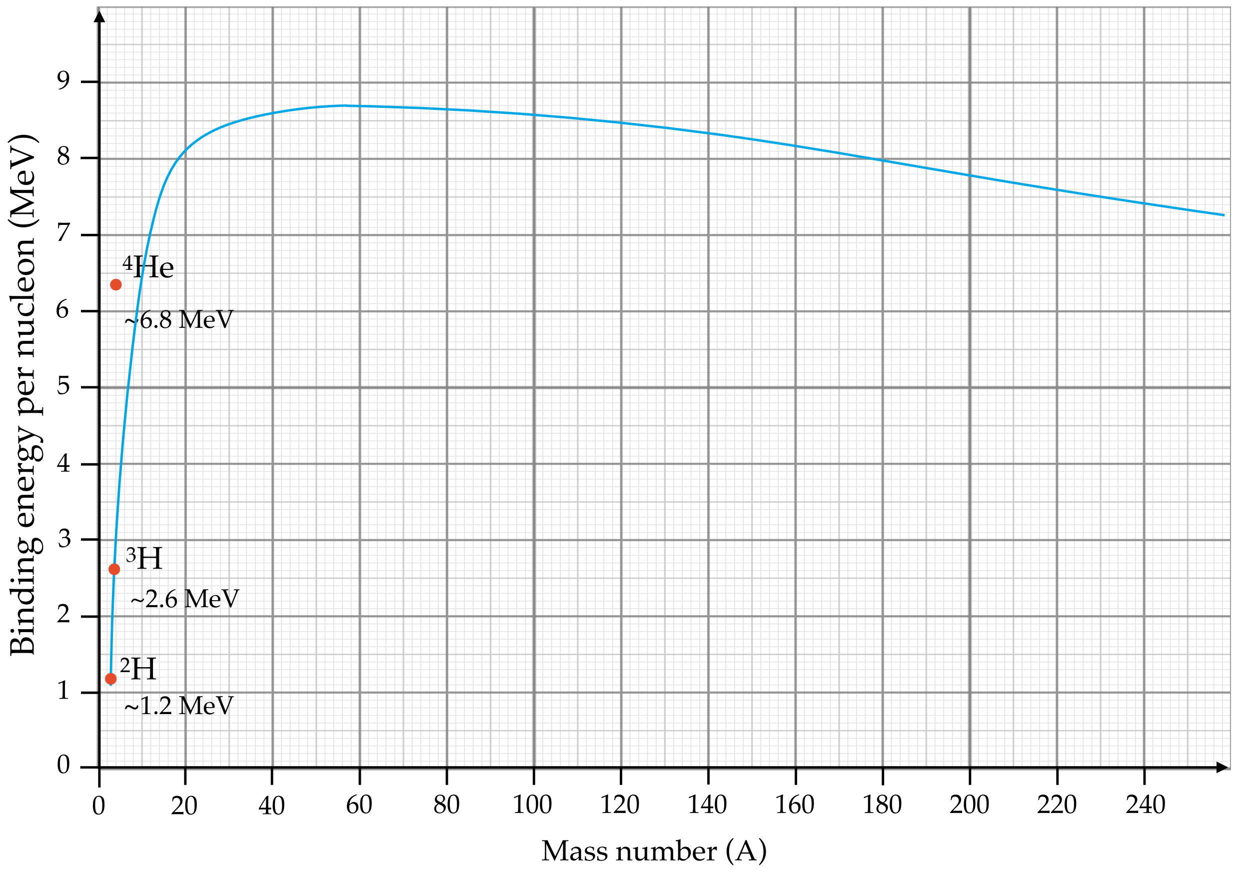
Therefore, the binding energy after the reaction is less than the binding energy before the reaction, resulting in an energy release of:

$(1953 - 1739) \text{ MeV} = 214 \text{ MeV} \approx 200 \text{ MeV}$

Worked Example 2-6 - Calculate the energy released in this fusion reaction

Q: Use the graph to calculate the energy released in the following fusion reaction:

$$^{3}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \longrightarrow ^{4}_{2}\mathrm{He}$$



A: Reading from the graph, we find that the binding energy per nucleon of 3-H is roughly 2.6 MeV which gives it a total binding energy of:

$2.6\ \mathrm{MeV} \times 3 = 7.8\ \mathrm{MeV}$

Likewise, the binding energy per nucleon of 2-H is roughly 1.2 MeV which gives it a total binding energy of:

$1.2\ \mathrm{MeV} \times 2 = 2.4\ \mathrm{MeV}$

Therefore, the total binding energy before the reaction is $(7.8 + 2.4)\ \mathrm{MeV} = 10.2\ \mathrm{MeV}$.

The binding energy per nucleon of 4-He is roughly 6.8 MeV which gives it a total binding energy of:

$6.8\ \mathrm{MeV} \times 4 = 27.2\ \mathrm{MeV}$

Therefore, the binding energy after the fusion is more than the binding energy before the reaction, resulting in an energy release of:
$(27.2 - 10.2)\ \mathrm{MeV} = 17\ \mathrm{MeV}$

Practice Questions 2-5 - Estimate the energy released in the following reactions

Q: Use the graph to estimate the energy released in the following fission/fusion reactions:

1. $^{237}_{93}\mathrm{Np} + ^{1}_{0}\mathrm{n} \longrightarrow ^{174}_{69}\mathrm{Tm} + ^{62}_{24}\mathrm{Cr} + 2\,^{1}_{0}\mathrm{n}$

2. $^{7}_{3}\mathrm{Li} + ^{1}_{1}\mathrm{H} \longrightarrow 2\,^{4}_{2}\mathrm{He}$

Compare your estimates with a calculation based on the mass deficit.