

# Curved Boundary Conditions in lattice Boltzmann method

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# Outline

- 1 Introduction
  - General aspects
  - Motivation
  - Possible approaches
- 2 Problem statement
  - Off-grid populations
  - Interpolated Boundary Conditions
- 3 Summary

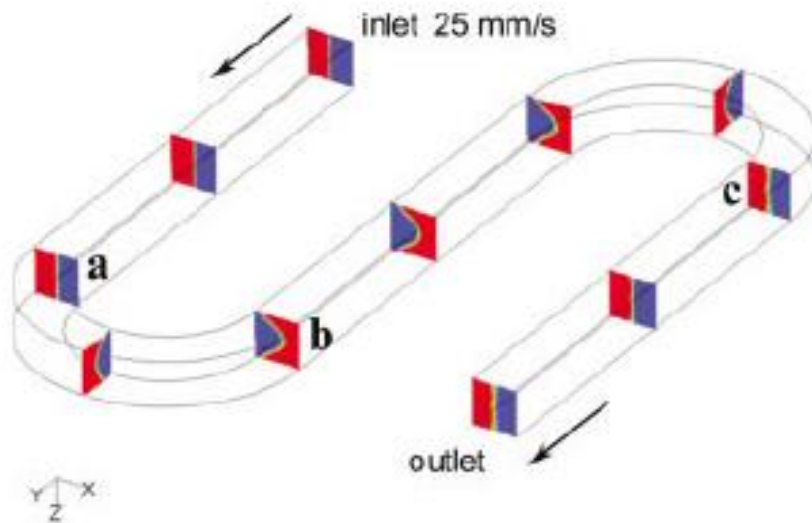
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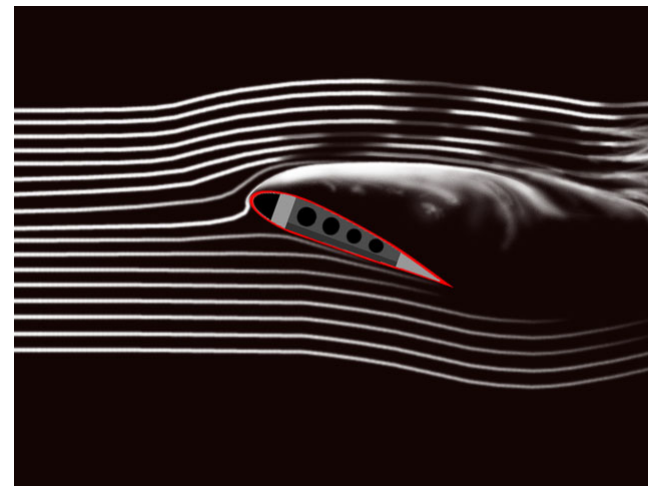
# Irregular geometries in fluid flow problems

Most practical fluid flow problems of interest in engineering or science involve:

1) **irregular** domains



2) **irregular** shaped objects



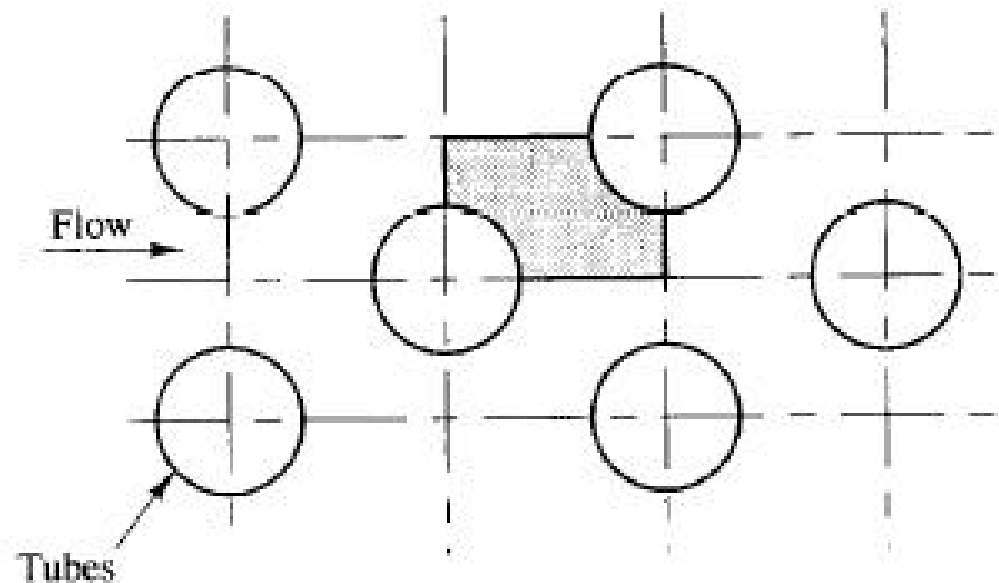
<http://engineeringskills.wikidot.com/concepts#toc27>

Yamaguchi *et al.* AIChE, 50(7), 1530 (2004)

# Computational Fluid Dynamics: non-orthogonal boundaries

Representing irregular domains in Computational Fluid Dynamics

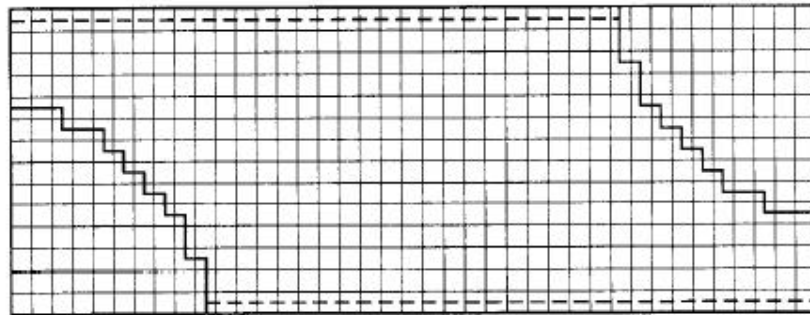
**Example:** Flow in a tube bank



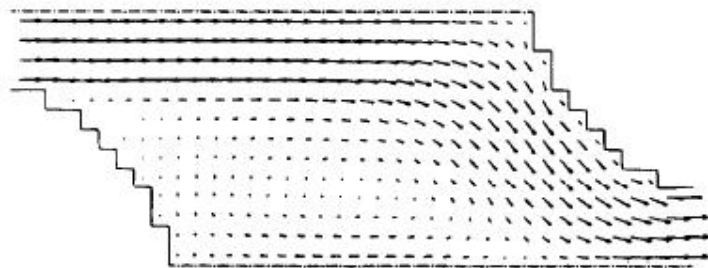
Versteeg & Malalasekera, An Introduction to CFD. Prentice Hall, 1995

# Computational Fluid Dynamics: non-orthogonal boundaries

Cartesian grid

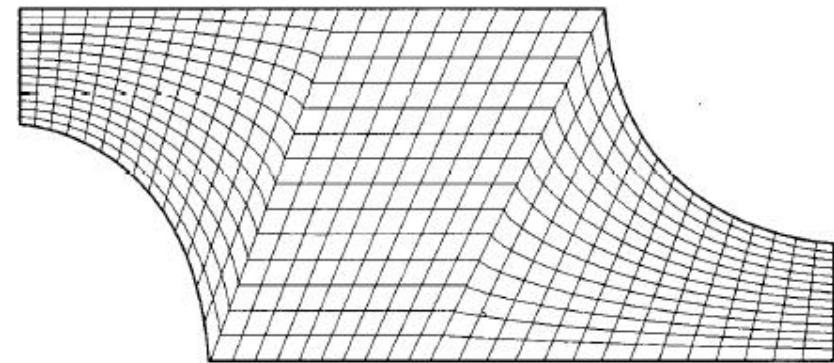


(40 × 15 grid)

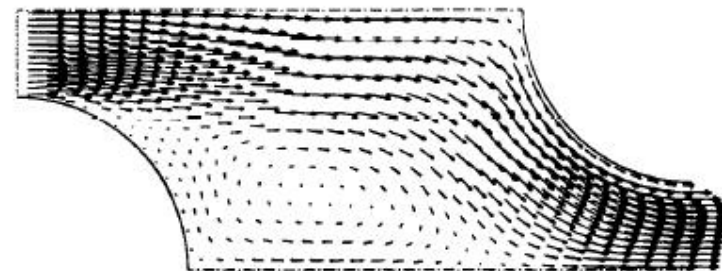


Versteeg & Malalasekera, An Introduction to CFD.  
Prentice Hall, 1995

Non-orthogonal body-fitted grid



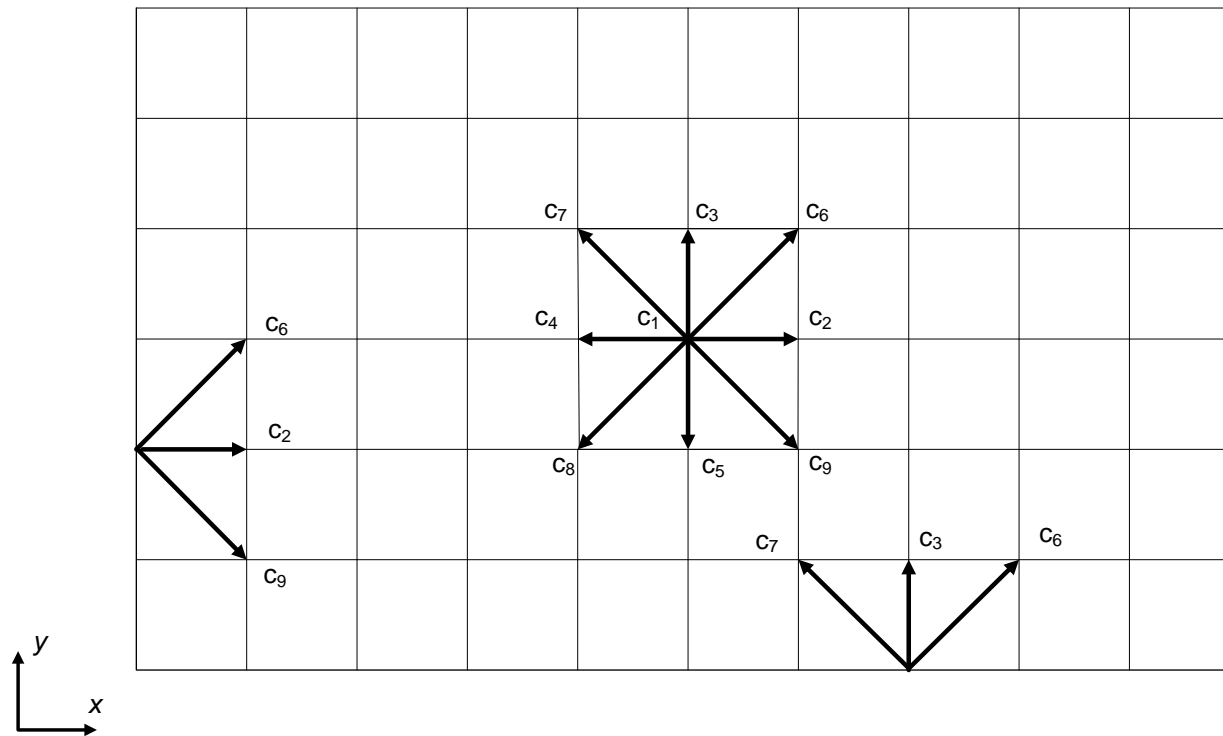
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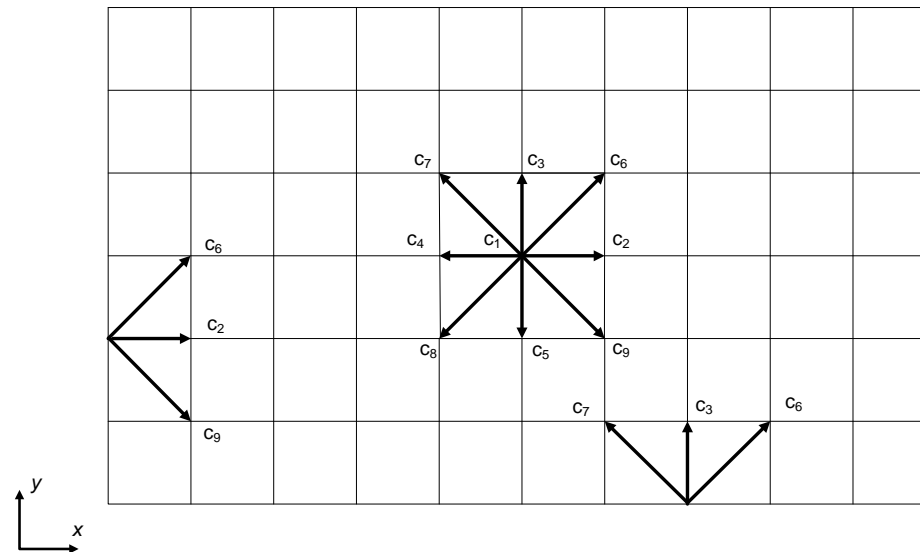
# Lattice Boltzmann method: non-orthogonal boundaries

Lattice Boltzmann algorithm consists of...  
...stream along links and equilibrate at nodes



# Lattice Boltzmann method: non-orthogonal boundaries

- ⇒ In "stream along links and equilibrate at nodes" philosophy the **streaming operation is exact**
- **Advantage:** No interpolations are needed → virtually **no numerical dissipation** is introduced in streaming
  - **Disadvantage:** Lattice structure, *i.e.* velocity space discretization, constrains the configuration space discretization, *i.e.* the location of spatial nodes is prescribed by lattice



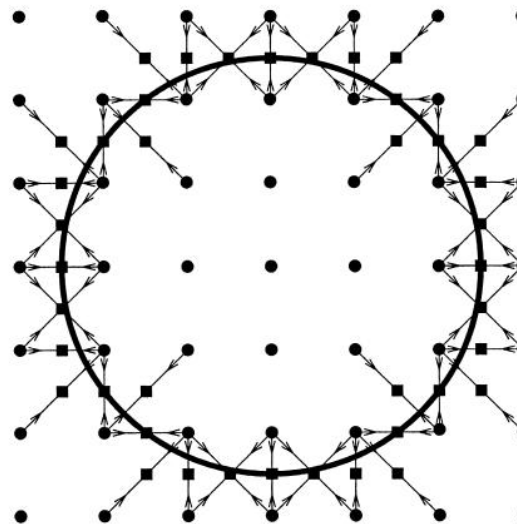


# Lattice Boltzmann method: non-orthogonal boundaries

## Conclusion:

Because the lattice Boltzmann method is an "on-grid" scheme it is restricted to **uniform cartesian grids**

Are we limited to the stepwise representation of irregular shapes?



Ladd & Verbeg 2001, J. Stat. Phys. 104, 1191-1251

# Lattice Boltzmann method: non-orthogonal boundaries

## The fact is...

...if we intend to represent non-orthogonal shapes we have to use numerical approximations (e.g. interpolations) somewhere

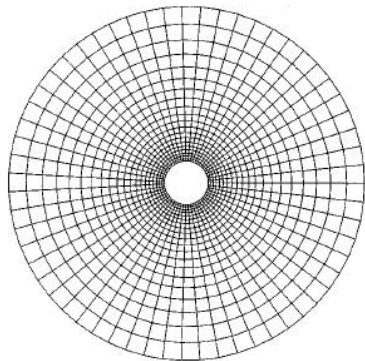
## Two different approaches:

- Interpolate: Everywhere
- Preserve: Boundary scheme
- **Non-orthogonal body-fitted grids**
- Interpolate: Boundary scheme
- Preserve: Everywhere
- **Interpolated boundary conditions**

# Lattice Boltzmann method: non-orthogonal boundaries

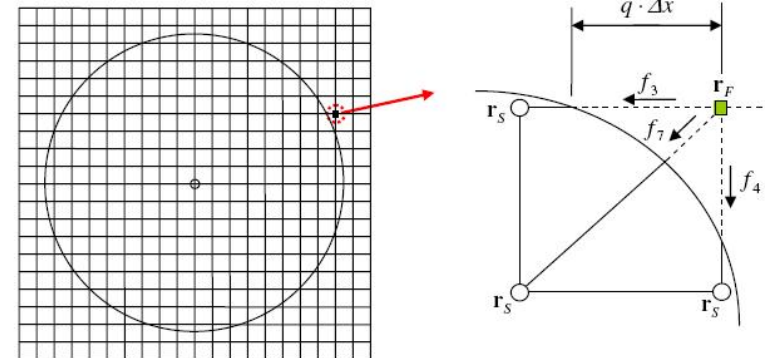
**Example:** Flow domain containing a solid circle represented through each approach:

- Non-orthogonal body-fitted grids



He & Doolen 1997, Phys. Rev. E 56, 434

- Interpolated boundary conditions



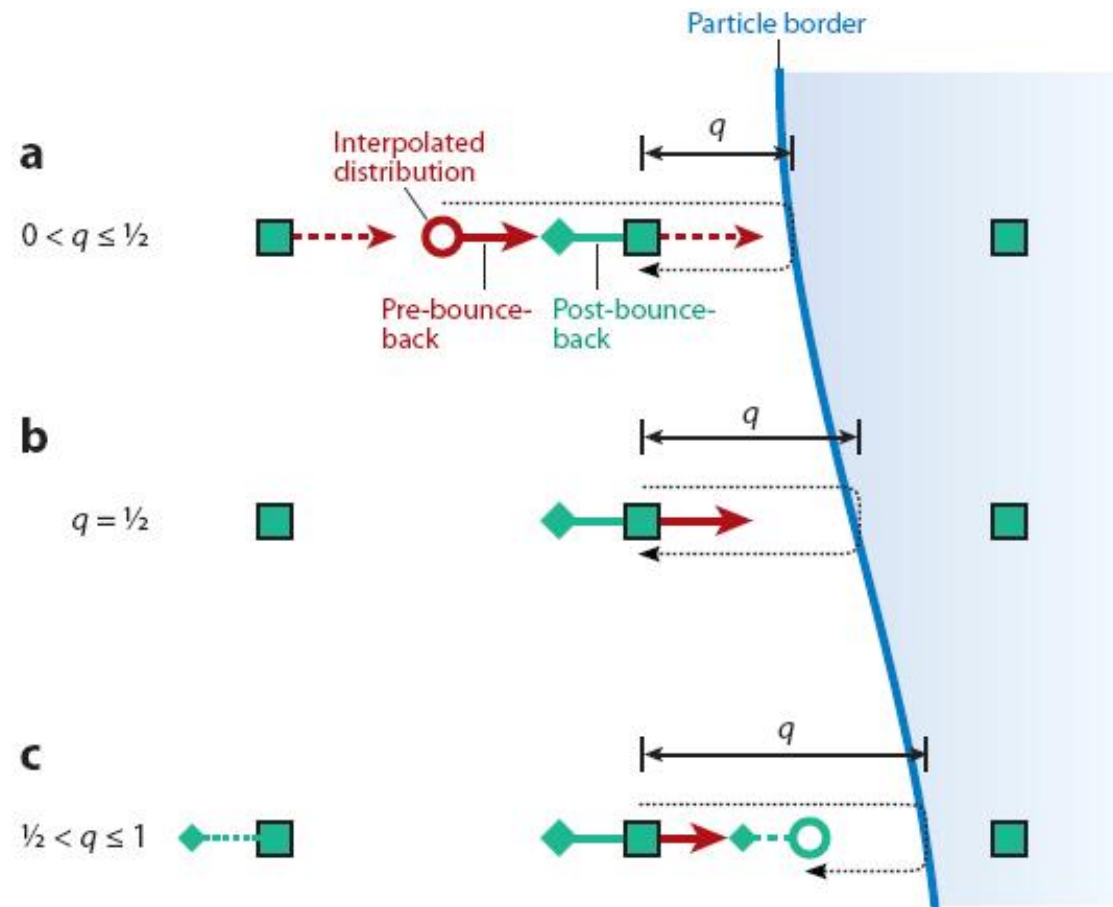
Kao & Yang 2008, J. Comp. Phys. 227, 5671-5690

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# Half-way bounceback

Example:

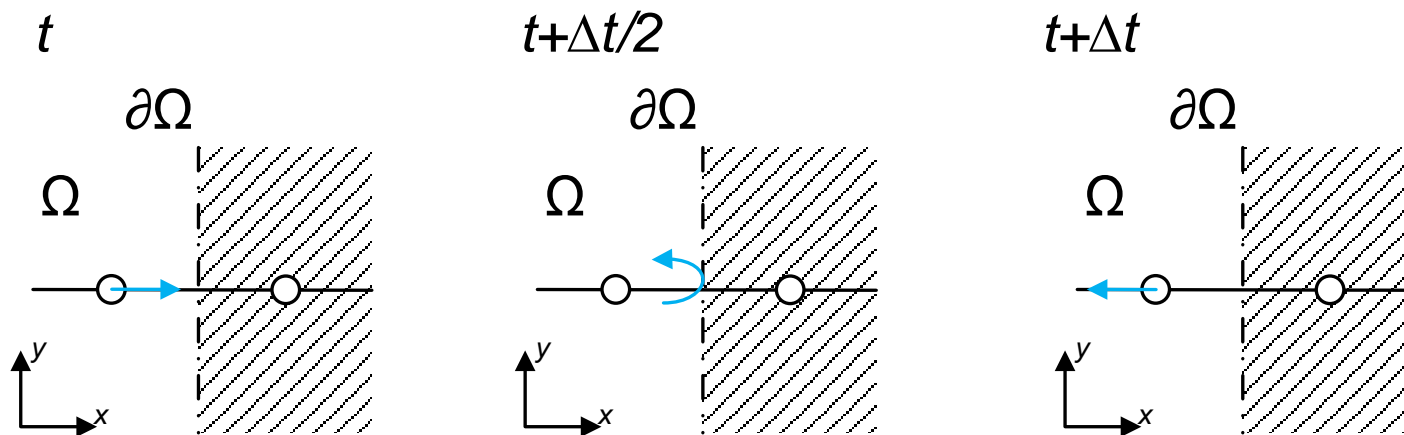


Aidun & Clausen 2010, Annu. Rev. Fluid Mech. 42, 439-472

# Half-way bounceback

**Remember:** When the wall is half-way

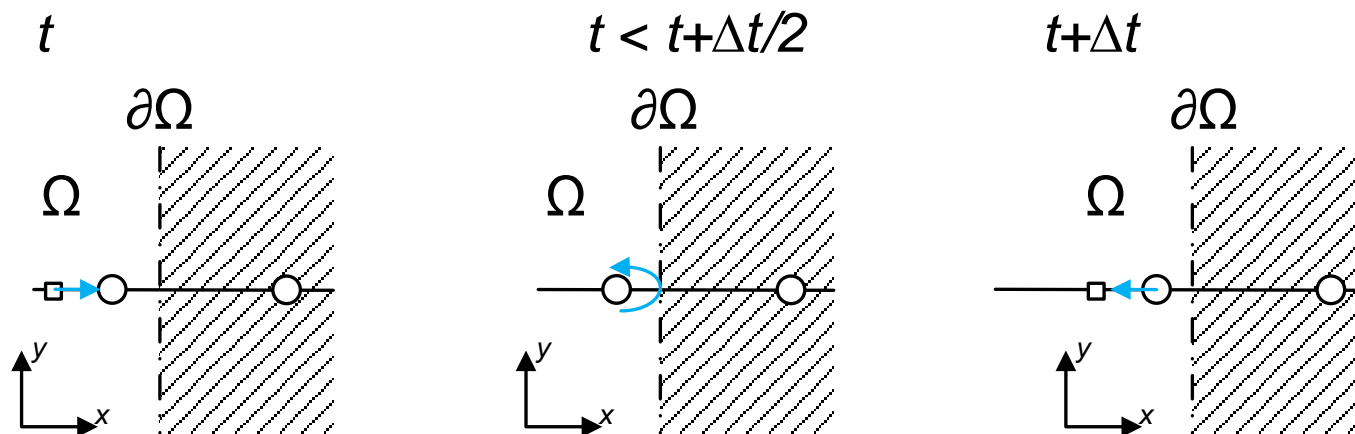
- **On-grid** population at time  $t$  is propagated
- Bounces-back at the wall
- Returns to the same **on-grid** location at time  $t + \Delta t$



# Half-way bounceback

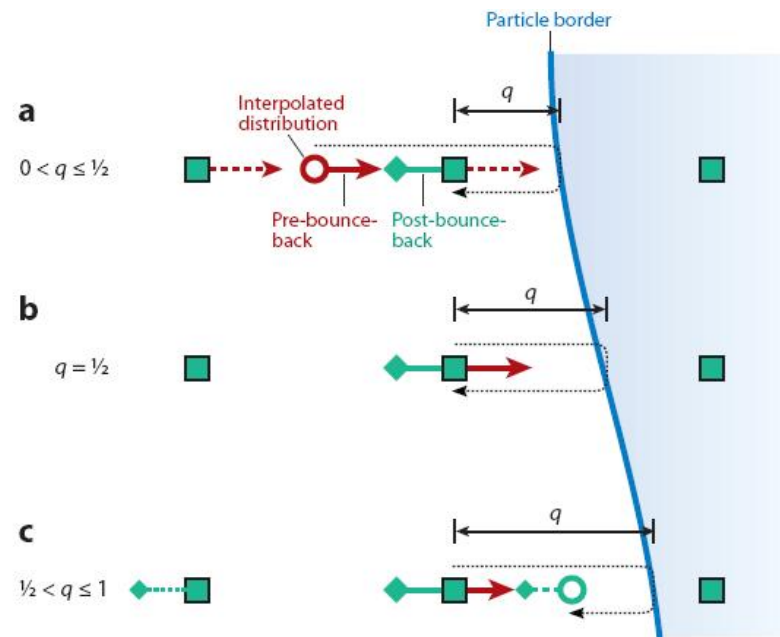
**However:** When the wall is **NOT** half-way

- Construct **off-grid** population at time  $t$  from where is propagated
- Bounces-back at the wall
- Returns to an **on-grid** location at time  $t + \Delta t$



# Half-way bounceback

Coming back to the example:



Aidun & Clausen 2010, Annu. Rev. Fluid Mech. 42, 439-472

Idea of interpolating the boundary condition:

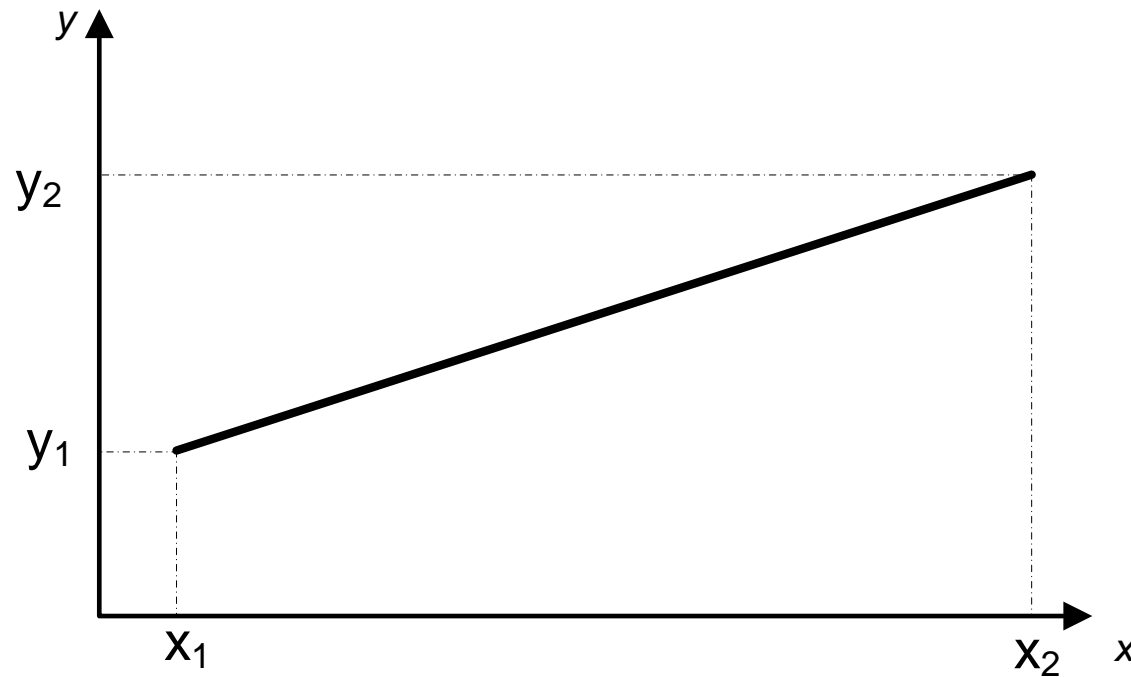
Reconstruct the **off-grid** population by interpolating the known **on-grid** data



# Linear interpolation

**Recall** the linear equation formula

$$y = \left( \frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1) + y_1$$



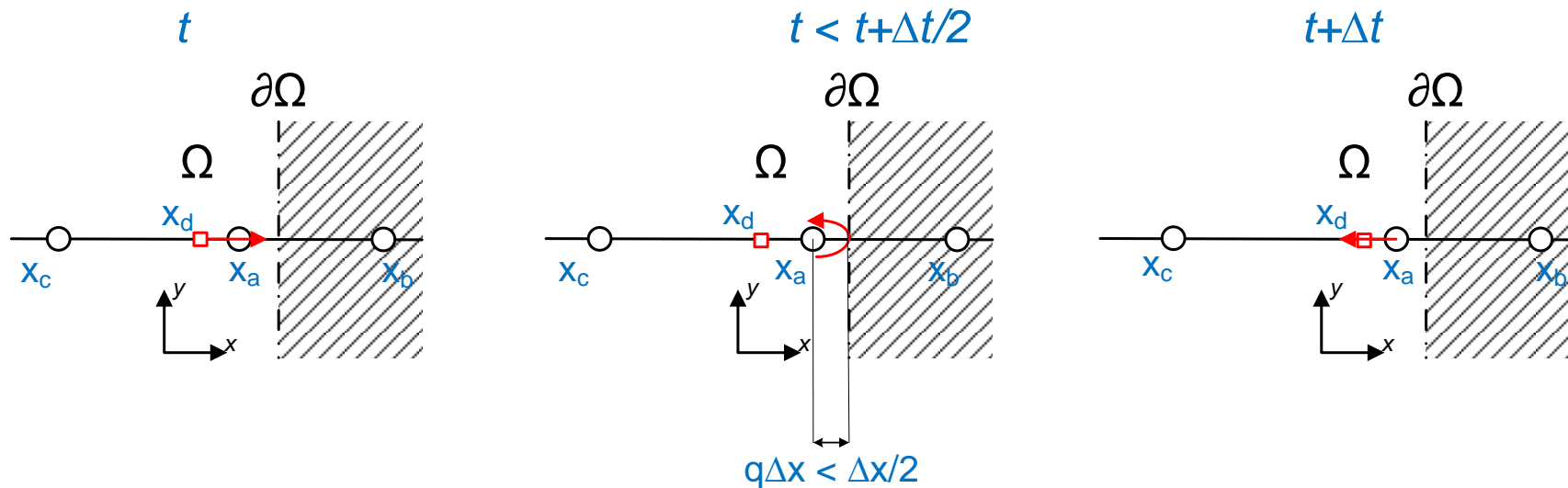
# Linear interpolation

- Use linear interpolation based on known populations:

→ known:  $\tilde{f}_R(\mathbf{x}_a, t)$

→ known:  $\tilde{f}_R(\mathbf{x}_c, t)$

→ **unknown**:  $\tilde{f}_R(\mathbf{x}_d, t)$

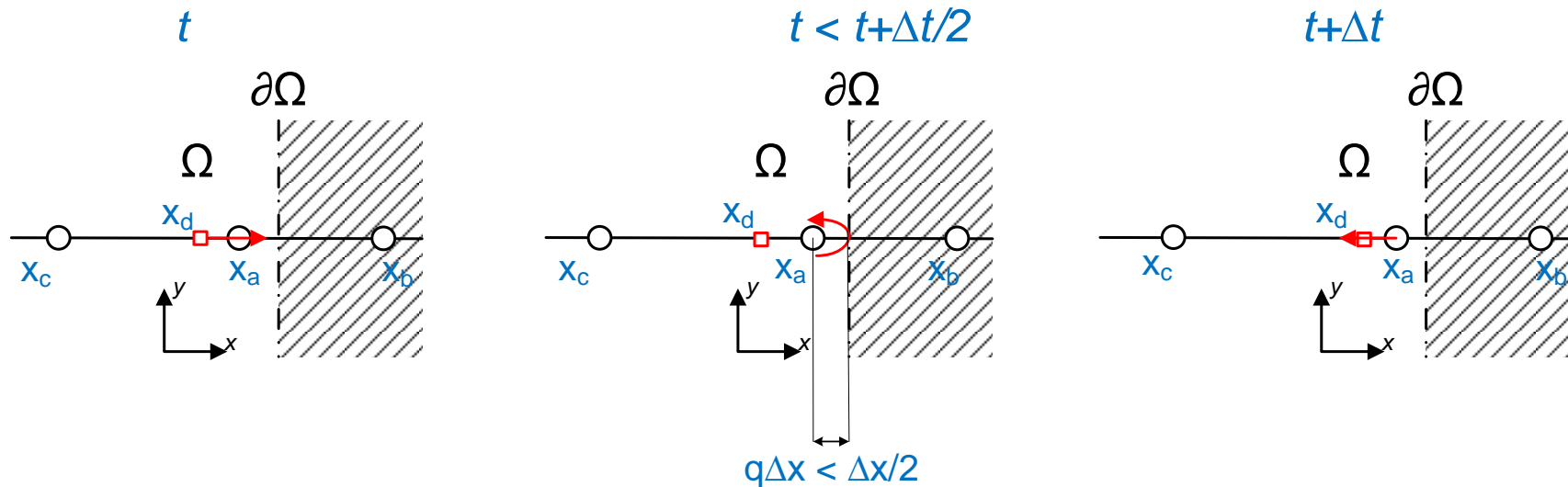


Note:  $\|\mathbf{x}_c - \mathbf{x}_a\| = \|\mathbf{x}_a - \mathbf{x}_b\| = \Delta\mathbf{x}$

# Linear interpolation: Exercise

## Question:

Compute  $\tilde{f}_R(\mathbf{x}_d, t)$  through linear interpolation of  $\tilde{f}_R(\mathbf{x}_a, t)$  and  $\tilde{f}_R(\mathbf{x}_c, t)$



Note:  $\|\mathbf{x}_c - \mathbf{x}_a\| = \|\mathbf{x}_a - \mathbf{x}_b\| = \Delta \mathbf{x}$

# Linear interpolation: Exercise

## Question:

Compute  $\tilde{f}_R(\mathbf{x}_d, t)$  through linear interpolation of  $\tilde{f}_R(\mathbf{x}_a, t)$  and  $\tilde{f}_R(\mathbf{x}_c, t)$

## Solution:

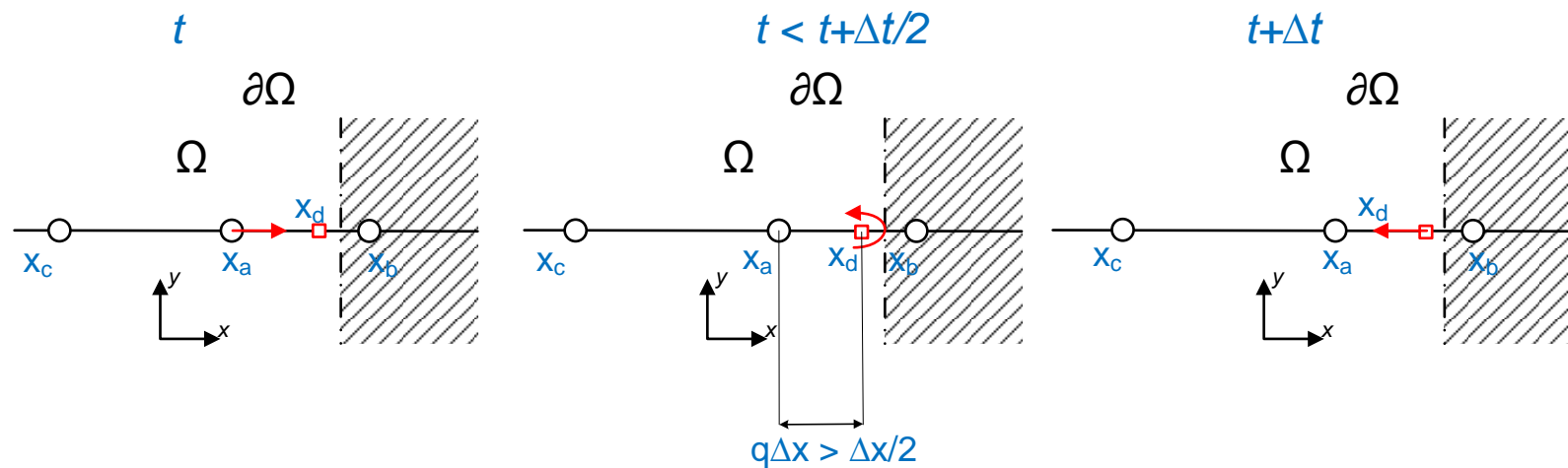
$$\tilde{f}_R(\mathbf{x}_d, t) = (1 - 2q)\tilde{f}_R(\mathbf{x}_c, t) + 2q\tilde{f}_R(\mathbf{x}_a, t)$$

that yields:

$$f_L(\mathbf{x}_a, t + \Delta t) = \tilde{f}_R(\mathbf{x}_d, t)$$

# Half-way bounceback

When the **wall is beyond the half-way location**, *i.e.*  $q > \frac{1}{2}$ , the previous procedure leads to an extrapolation instead of an interpolation scheme



## Solution:

Interpolate the post-streaming populations

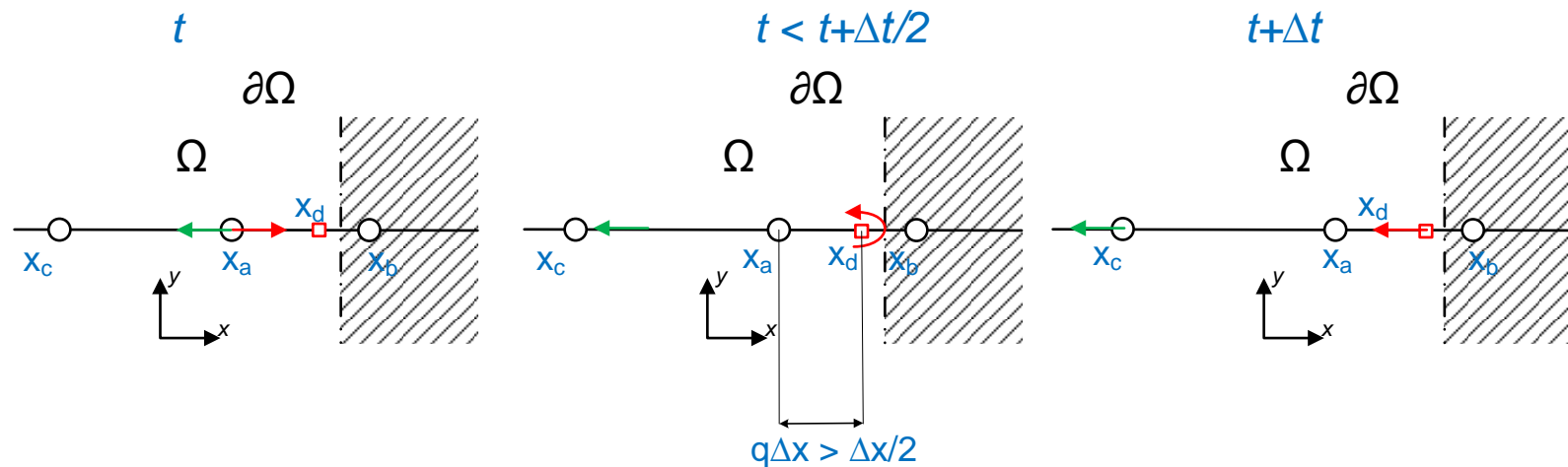
# Half-way bounceback

- From the time evolution sketch of the populations we observe:

→ known:  $f_L(\mathbf{x}_d, t + \Delta t) = \tilde{f}_R(\mathbf{x}_a, t)$

→ known:  $f_L(\mathbf{x}_c, t + \Delta t) = \tilde{f}_L(\mathbf{x}_a, t)$

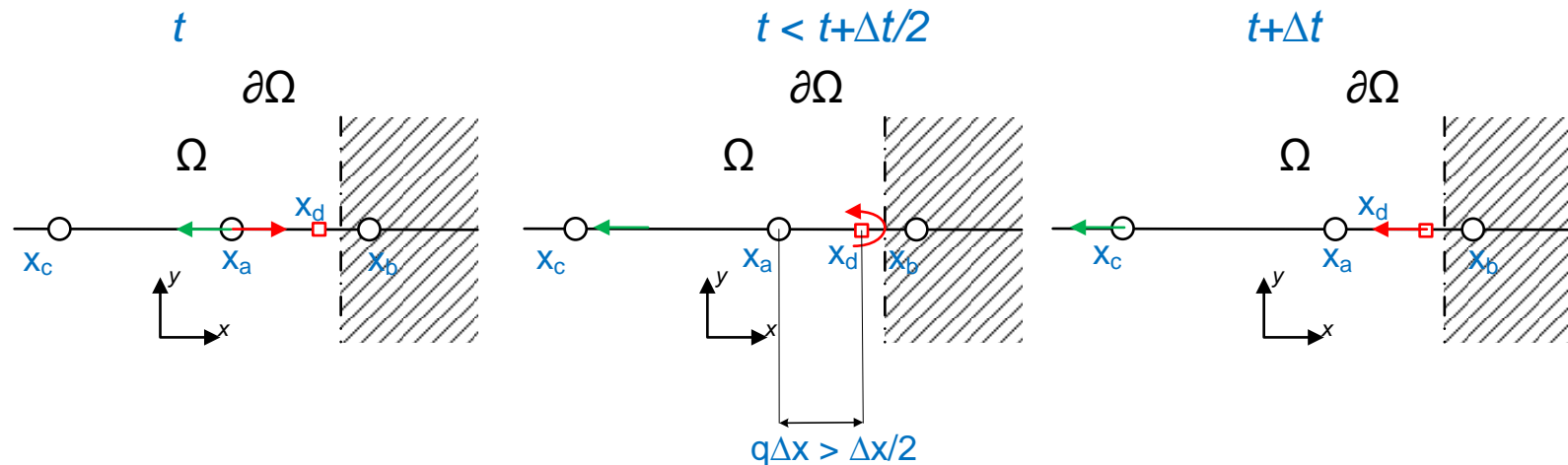
→ unknown:  $f_L(\mathbf{x}_a, t + \Delta t)$



# Linear interpolation: Exercise

## Question:

Compute  $f_L(\mathbf{x}_a, t + \Delta t)$  through linear interpolation of  $f_L(\mathbf{x}_d, t + \Delta t)$  and  $f_L(\mathbf{x}_c, t + \Delta t)$



Note:  $\|\mathbf{x}_c - \mathbf{x}_a\| = \|\mathbf{x}_a - \mathbf{x}_b\| = \Delta x$

# Linear interpolation: Exercise

## Question:

Compute  $f_L(\mathbf{x}_a, t + \Delta t)$  through linear interpolation of  $f_L(\mathbf{x}_d, t + \Delta t)$  and  $f_L(\mathbf{x}_c, t + \Delta t)$

## Solution:

$$f_L(\mathbf{x}_a, t + \Delta t) = \left( \frac{2q-1}{2q} \right) f_L(\mathbf{x}_d, t + \Delta t) + \frac{1}{2q} f_L(\mathbf{x}_d, t + \Delta t)$$

or

$$f_L(\mathbf{x}_a, t + \Delta t) = \left( \frac{2q-1}{2q} \right) \tilde{f}_R(\mathbf{x}_a, t) + \frac{1}{2q} \tilde{f}_L(\mathbf{x}_a, t)$$



# Linear interpolation: 2D lattices

## Linear interpolation bounceback:

- $q < \frac{1}{2}$

$$f_{\bar{\alpha}}(\mathbf{x}, t + \Delta t) = 2q\tilde{f}_{\alpha}(\mathbf{x}, t) + (1 - 2q)\tilde{f}_{\alpha}(\mathbf{x} - \mathbf{c}\Delta t, t)$$

- $q \geq \frac{1}{2}$

$$f_{\bar{\alpha}}(\mathbf{x}, t + \Delta t) = \frac{1}{2q}\tilde{f}_{\alpha}(\mathbf{x}, t) + \left(\frac{2q-1}{2q}\right)\tilde{f}_{\bar{\alpha}}(\mathbf{x}, t)$$

# Quadratic interpolation: 2D lattices

## Quadratic interpolation bounceback:

- $q < \frac{1}{2}$

$$f_{\bar{\alpha}}(\mathbf{x}, t + \Delta t) = q(2q + 1)\tilde{f}_{\alpha}(\mathbf{x}, t) \\ + (1 + 2q)(1 - 2q)\tilde{f}_{\alpha}(\mathbf{x} - \mathbf{c}\Delta t, t) - q(1 - 2q)\tilde{f}_{\alpha}(\mathbf{x} - 2\mathbf{c}\Delta t, t)$$

- $q \geq \frac{1}{2}$

$$f_{\bar{\alpha}}(\mathbf{x}, t + \Delta t) = \frac{1}{q(2q+1)}\tilde{f}_{\alpha}(\mathbf{x}, t) + \frac{2q-1}{q}\tilde{f}_{\bar{\alpha}}(\mathbf{x}, t) \\ + \frac{1-2q}{1+2q}\tilde{f}_{\bar{\alpha}}(\mathbf{x} - \mathbf{c}\Delta t, t)$$

## Exercise IV

### Exercise IV: Flow around circular cylinder

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# Curved boundary conditions in LBM

- LBM uses cartesian uniform grids
- Non-orthogonal shapes require numerical approximations, e.g. interpolations
- In order to preserve the advantages of LBM it is preferable to only interpolate the solution at boundary
- Bounceback provides a good framework to extend to curved boundaries through interpolations