- INDENG240 - FINAL PROJECT: LINEAR OPTIMIZATION

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CONTENTS

1 INTRODUCTION

For our project, our team is looking at the case study of Lake Saddleback Development Corporation, also referred to as LSDC. LSDC is a company that plans to develop 300 acres of land in Lake Saddleback Texas. With the information provided by LSDC, our team looked at the variables and parameters set by the company in the problem statement, created an objective function, set constraints, and ran our problem through AMPL, and found a way to maximize profit from the development while offering a diverse amount of options for home plans to build the community. LSDC's initial plan is to build 4 different types of homes. These include Grand Estates, The Glen Wood Collection, Lakeview Patio Homes, and Country Condominiums. Each of these building plans has three or four different floor plans, described below.

2
PROBLEM STATEMENT (FOR MANAGEMENT)

2.1 Base problem

There are many different Variables/Parameters to be taken into consideration in this problem. Below is a brief overview of each and the implementation into our problem formulation.

- **Plan** - describes the floorplans within each of the main four types of homes; there are 15 total floorplans described in the list.
- **Price** - The Selling Prices vary greatly for the costs of each home. From the low end of $140,000 for Condominiums to upwards of $700,000 for the Grand estates.
- **Size** - Measured in square feet, each floorplan ranges from 1000-4000 sq.ft.
- **Bedrooms** - The smallest homes have 2 bedrooms, and the largest homes have 5 bedrooms and a den.
- **Bathrooms** - the bathrooms of each floorplan range from 1.5 in the smallest homes to 4 in the largest.
- Stories - Each floorplan is either 1 or 2 stories. The larger homes tend to be 2 stories while the smaller ones are usually 1 story.
- **Garage Size** - In the Country Condominiums, there is no garage. The remaining floorplans feature 2 or 3 car garages, again varying based on home type and size of the home.

The construction plan is also subject to various constraints:

- **Lot Sizes**
  - All models in the Grand Estate are built on $\frac{1}{2}$ acre lots, and 50 of the $\frac{1}{5}$ acre lots are only to be used by the Grand Estate series homes built on the lake. Of the 300 acres of land, at least 50 "$\frac{1}{2}$ acre" plots are reserved for Grand Estate lake houses, making up $\frac{1}{6}$ of the total available land.
  - The selling price of the Grand Estate series homes on the lake will sell for $50,000 + 30% more than Grand Estate homes that are not on the lake.
  - There must be at least 8 units of Grand Estate Series located on the lake
  - Grand Cypress models may be built on "premium" $\frac{1}{4}$ acre lots. Premium lot grand cypress homes sell for $40,000 more than those built on standard lots.

  - Some Bayview models may be built on "premium" $\frac{1}{6}$ acre lots. These sell for \$30,000 more than Bayview models built on standard lots.
  - No more than 25% of all Grand Cypress models and 25% of all Bayview models may be built on premium lots.
  - Country Condominium lot sizes = 1500 sq.ft. This is a fixed value.
  - Minimum standard lot for homes in Glen Wood and Lakeview series homes is $\frac{1}{10}$ acre. This does not include those homes built on "premium" lots. Some models can be higher if the following calculation is greater than $\frac{1}{10}$ acre: $Lot\ Size = (ground\ area\ of\ house) + (yard\ size) + (garage\ size)$

- **Ground Area**
  - Ground area of single-story houses = 100 % of advertised square footage
  - Ground area of two-story houses = 75 % of advertised square footage

- **Yard size**
  - Glen Wood series homes that are single-story feature yards that are 1200 sqft., and they have the same ground area of the house as two-story Glen Wood homes
  - Lakeview Patio single-story homes feature yard sizes of 900 sqft, and two-story homes feature yard sizes that are 600 sqft + 50

- **Garage size**
  - Two-car garages = 500 sqft of home's advertised ground area space
  - Three-car garages = 750 sqft of the home's advertised ground area space
  - Recall: no garages for Country Condominiums

- **Parking**
  - 1 parking space/bedroom for each unit built. Outside parking space for two cars is required for a four bedroom house with a two-car garage.
    $nb\ outdoor\ parking\ spots = nb\ bedrooms\breve{}nb\ cars\ in\ garage$
  - 1 outdoor parking space = 200 sqft
  - Maximum 15 acres (= 653,400 sqft) can be used for outside parking. (This means there is a maximum of 3,267 outdoor parking spaces).
  - All parking spaces for country condominiums are outdoors.

- **Roads, greenbelts**
  - 1,000 sqft per each house is set aside for roads, greenbelts, and small parks (With maximum of 3,000 acres of land, about 6.887 acres must be allotted to roads, greenbelts, and small parks)

- **Variety**
  - There must be at least 15% and at most 25% two-bedroom homes
  - At least 25% and at most 40% three-bedroom homes
  - At least 25% and at most 40% four-bedroom homes
  - At least 5% and at most 15% five-bedroom homes
  - None of the four main home products will make up up $\geq$ 35% or $\leq$ 15% of the units in the development
  - At most 70% of single-family homes (all except the Country Condominiums) may be two-story homes.
- **Affordability**
  - At least 15% of all homes will cost at most $200,000
- **Profit**
  - 22% of Grand Estates sales prices = net profit
  - 18% of Glen Woods sales prices = net profit
  - 20% of Lakewood sales prices = net profit
  - 25% of Country Condominiums sales prices = net profit

2.2 Modified problem

The modified problem has the same core constraints with the following variations:

- There are no quotas anymore on the number of bedrooms
- At least 3 of each of the Grand Estate series plan homes must have at least 8 units on the lake, instead of all of them
- There is a new luxury tax as follow:
  - Tax of 8% of selling price per unit is imposed on each unit sold.
  - LSDC pays the following luxury tax for each unit of the Country Condos sold. This tax is $2i$%, with i the position of the number of Country Condo units sold compared to other 3 types. (E.g., If country condos are sold the most of the four types, then i = 1.). This may motivate LSDC to sell more Country Condominiums and fake the removed affordability quotas.
- The possibility to build a sport recreational complex, which would reduce the usable land of 10 acres, represent an initial fixed fee of $8 million, and enable the following price increase:

- Grand Estates - 5%
- Glen Wood – 3%
- Lakeview – 2%
- Country Condominiums – 3%

Note that all of the increases are considered as additional profit to net profit figures from original problem.

3 MATHEMATICAL FORMULATION

The mathematical formulation of the problem contains decision variables which enable setting the optimal construction plan, based on different parameters coming from LSDC data.

3.1 BASE PROBLEM

3.1.1 • SETS :

In order to define the problem, we have used different sets to regroup the different models of houses:

- $G\_Estates$: the base Grand Estates models
- $G\_Estates\_PREMIUM$: the premium version of the Grand Estates models
- $Glen\_Wood$: the Glen Wood models (including the Grand Cypress Premium)
- $Lakeview$: the Lakeview models (including the Bayview Premium)
- $Condominiums$: the Condominiums models

- all_models: the union of all the previous sets

- $two\_bedrooms$: the models having 2 bedrooms
- $three\_bedrooms$: the models having 3 bedrooms
- $four\_bedrooms$: the models having 4 bedrooms
- $five\_bedrooms$: the models having 5 bedrooms

- $two\_stories\_without\_condominiums$: the two-stories models excluding Condominiums ones in order to respect two-stories quotas
- $with\_outside\_parking$: the models requiring at least one outside parking spot
- $affordable\_houses$: the models under $200,000

3.1.2 • Parameters :

We defined the following parameters:

- $total\_surface$: the total land surface available for the project (in sq. ft.)
- $outside\_parking\_size$: the surface of a parking spot (in sq. ft.)
- $max\_parking\_space$: the maximum surface available for parking for the project (in sq. ft.)
- $(p_m)_{m \in all\_models}$: the profit table for each type of house (in \$)
- $(s_m)_{m \in all\_models}$: the size table for each type of house (in sq. ft.)
- $outside\_parking\_spots$: the number of outside parking spots required for each house
- $bedrooms$: the table giving the required minimum and maximum percentage of houses for each number of bedrooms
- $series$: the table giving the required minimum and maximum proportion of each series to bring diversity in the project
- $model$: the table giving the required minimum and maximum proportion of each model within its series to bring diversity in the project
- $max\_G\_cypress\_premium$: the required maximum proportion of Grand Cypress Premium compared with basic Grand Cypress
- $max\_bayview\_premium$: the required maximum proportion of Bay View Premium compared with basic Bay View
- $max\_two\_stories$: the required maximum proportion of two-stories houses compared with one-story houses
- $min\_affordability$: the required minimum proportion of affordable houses under $200,000

3.1.3 • Variables :

We defined the following variables:

- $(N_m)_{m \in all\_models}$: the table giving the number of houses to build for each model
- N_TOTAL: an auxiliary variable we will often use for quotas, being the sum of all the models we build

3.1.4 • Objective function:

In this problem, we want to maximize the total profit for Lake Saddleback Development Corporation. It is defined like this:

$$max \sum_{m \in all\_models} p_m N_m$$

3.1.5 • Constraints :

This problem is subject to several constraints regarding the available space and the different quotas. Even if most constraints are self-explanatory, some of them result from a personal interpretation of the problem that will be explained.

- Total Houses :
$$N\_TOTAL = \sum_{m \in all\_models} N_m$$
- Grand Estates premium spots :
$$\sum_{m \in all\_G\_Estates\_PREMIUM} N_m \geq 50$$
We assume that the constraints "50 half-acre lots on the lake are to be used exclusively by the Grand Estate Series homes" means that at least 50 half-acres lots are reserved for Grand Estate Premium homes. We also assume that the roads and outside parking related to this houses are not taking in account in these 50 reserved half-acre.
- Limited surface :
$$\sum_{m \in all\_models} s_m \times N_m \leq total\_surface$$
- Diversity Grand Estate Premium :
$$\forall m \in G\_Estates\_PREMIUM : N_m \geq 8$$
- Quotas bedrooms :
  - Quotas minimum number of k-bedrooms homes :
  $$\forall k \in \{2,3,4,5\} : \sum_{m \in k\_bedrooms} N_m \geq bedrooms_{k,min} \times N\_TOTAL$$
  - Quotas maximum number of k-bedrooms homes :
  $$\forall k \in \{2,3,4,5\} : \sum_{m \in k\_bedrooms} N_m \leq bedrooms_{k,max} \times N\_TOTAL$$

- Quotas series :
  - Quotas minimum proportion of Grand Estates homes :

$$\sum_{m \in \{G\_Estates \cup G\_Estates\_PREMIUM\}} N_m \geq series_{min} \times N\_TOTAL$$

  - Quotas maximum proportion of Grand Estates homes :

$$\sum_{m \in \{G\_Estates \cup G\_Estates\_PREMIUM\}} N_m \leq series_{max} \times N\_TOTAL$$

  - Quotas minimum proportion of homes from other series :

$$\forall S \in \{Glen\_Wood, Lakeview, Condominiums\}$$

$$\sum_{m \in S} N_m \geq series_{min} \times N\_TOTAL$$

  - Quotas maximum proportion of homes from other series :

$$\forall S \in \{Glen\_Wood, Lakeview, Condominiums\}$$

$$\sum_{m \in S} N_m \leq series_{max} \times N\_TOTAL$$

- Quotas models :
  - Quotas minimum model Grand Estates : $\forall i \in \{1, 2, 3, 4\}$:

$$N_{(G\_Estates,i)} + N_{(G\_Estates\_PREMIUM,i)} \geq model_{min} \times \sum_{m \in G\_Estates \cup G\_Estates\_PREMIUM} N_m$$

  - Quotas maximum model Grand Estates : $\forall i \in \{1, 2, 3, 4\}$:

$$N_{(G\_Estates,i)} + N_{(G\_Estates\_PREMIUM,i)} \leq model_{max} \times \sum_{m \in G\_Estates \cup G\_Estates\_PREMIUM} N_m$$

  - Quotas minimum model Glen Wood :

$$N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM} \geq model_{min} \times \sum_{m \in Glen\_Wood} N_m$$

$$\forall m \in Glen\_Wood, m \neq G\_Cypress \text{ and } m \neq G\_Cypress\_PREMIUM :$$

$$N_m \geq model_{min} \times \sum_{m' \in Glen\_Wood} N_{m'}$$

- Quotas maximum model Glen Wood :

$$N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM} \leq model_{max} \times \sum_{m \in Glen\_Wood} N_m$$

$$\forall m \in Glen\_Wood, m \neq G\_Cypress \text{ and } m \neq G\_Cypress\_PREMIUM :$$

$$N_m \leq model_{max} \times \sum_{m' \in Glen\_Wood} N_{m'}$$

- Quotas minimum model Lakeview :

$$N_{Bayview} + N_{Bayview\_PREMIUM} \geq model_{min} \times \sum_{m \in Lakeview} N_m$$

$$\forall m \in Lakeview, m \neq Bayview \text{ and } m \neq Bayview\_PREMIUM :$$

$$N_m \geq model_{min} \times \sum_{m' \in Lakeview} N_{m'}$$

- Quotas maximum model Lakeview :

$$N_{Bayview} + N_{Bayview\_PREMIUM} \leq model_{max} \times \sum_{m \in Lakeview} N_m$$

$$\forall m \in Lakeview, m \neq Bayview \text{ and } m \neq Bayview\_PREMIUM :$$

$$N_m \leq model_{max} \times \sum_{m' \in Lakeview} N_{m'}$$

- Quotas minimum model Condominiums : $\forall m \in Condominiums$

$$N_m \geq model_{min} \times \sum_{m' \in Condominiums} N_{m'}$$

- Quotas maximum model Condominiums : $\forall m \in Condominiums$

$$N_m \leq model_{max} \times \sum_{m' \in Condominiums} N_{m'}$$

- Quotas Grand Cypress Premium :

$$N_{G\_Cypress\_PREMIUM} \leq max\_G\_cypress\_PREMIUM \times (N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM})$$

- Quotas Bayview Premium :

$$N_{Bayview\_PREMIUM} \leq max\_bayview\_PREMIUM \times (N_{Bayview} + N_{Bayview\_PREMIUM})$$

- Limit surface parking :

$$\sum_{m \in with\_outside\_parking} outside\_parking\_size \times N_m \leq max\_parking\_space$$

- Quotas Two Stories houses :

$$\sum_{m \in two\_stories\_without\_condominiums} N_m \leq max\_two\_stories \times \sum_{m \in all\_models, m \notin Condominiums} N_m$$

  We assume that the constraints "no more than 70% of the single-family homes (all homes except the Country Condominiums) may be two-story homes." means that less than 70 of the houses that are not Condominiums houses (all houses expect Condominiums houses) should have more than two stories.

- Quotas affordable houses :

$$\sum_{m \in affordable\_houses} N_m \geq min\_affordability \times N\_TOTAL$$

3.2 Modified Problem

3.2.1 • Sets :

In order to define the modified problem, we have used the same sets than previously:

- $G\_Estates$: the base Grand Estates models
- $G\_Estates\_PREMIUM$: the premium version of the Grand Estates models
- $Glen\_Wood$: the Glen Wood models (including the Grand Cypress Premium)
- $Lakeview$: the Lakeview models (including the Bayview Premium)
- $Condominiums$: the Condominiums models

- all_models: the union of all the previous sets

- $two\_bedrooms$: the models having 2 bedrooms
- $three\_bedrooms$: the models having 3 bedrooms
- $four\_bedrooms$: the models having 4 bedrooms
- $five\_bedrooms$: the models having 5 bedrooms

- $two\_stories\_without\_condominiums$: the two-stories models excluding Condominiums ones in order to respect two-stories quotas
- $with\_outside\_parking$: the models requiring at least one outside parking spot
- $affordable\_houses$: the models under $200,000

3.2.2 • Parameters :

We keep the following parameters from the previous part:

- $total\_surface$: the total land surface available for the project (in sq. ft.)
- $outside\_parking\_size$: the surface of a parking spot (in sq. ft.)
- $max\_parking\_space$: the maximum surface available for parking for the project (in sq. ft.)
- $(p_m)_{m \in all\_models}$: the profit table for each type of house (in \$)
- $(s_m)_{m \in all\_models}$: the size table for each type of house (in sq. ft.)
- $outside\_parking\_spots$: table that gives the number of outside parking spots required for each house
- $series$: the table giving the required minimum and maximum proportion of each series to bring diversity in the project
- $model$: the table giving the required minimum and maximum proportion of each model within its series to bring diversity in the project
- $max\_G\_cypress\_premium$: the required maximum proportion of Grand Cypress Premium compared with basic Grand Cypress
- $max\_bayview\_premium$: the required maximum proportion of Bay View Premium compared with basic Bay View
- $max\_two\_stories$: the required maximum proportion of two-stories houses compared with one-story houses
- $min\_affordability$: the required minimum proportion of affordable houses under $200,000

We add the following parameters :

- tax_rate : the county government imposes a "Luxury Tax" on each unit sold of 8% of the selling price per unit.
- $sport\_extra\_cost$: the LSDC can build a 10-acre sports/recreational complex on the property, this would cost $8 million to build.
- $sport\_space\_loss$: the LSDC can build a 10-acre sports/recreational complex on the property, this would reduce the usable land by 10 acres.
- $extra\_profit$: the table gives the extra profit of each model the LSDC would earn if the sport field is built.
- $selling\_price$: table that gives the new selling price of each model.
- N_max : upper bound of the number of houses that can be built.

3.2.3 • Variables :

We keep the following variables from the previous part:

- $(N_m)_{m \in all\_models}$: the table giving the number of houses to build for each model
- N_TOTAL: an auxiliary variable we will often use for quotas, being the sum of all the models we build

We add the following variables :

- $sport\_build$: binary variable that is equal to 1 if the sport complex is built and that is equal to 0 otherwise.
- N_extra_profit : table that gives the extra profit of each model in case where the sport complex is built.
- $(z_m)_{m \in G\_Estates\_PREMIUM}$: binary variables that is equal to 1 if there is at least 8 units of model m built from Grand Estates Premium series, and that is equal to zero otherwise.
- i : the position of the number of Country Condominiums units sold compared to the other three types (tax rate of Condominiums houses).
- $more\_cond$: an auxiliary boolean table such that $more\_cond_S = 0$ if the series S has less houses than the Condominium one, 1 else.
- $(N\_tax_{S,m})$: an auxiliary variable to apply the variable taxation rate in the objective function. $\forall m \in Condominiums,\ N\_tax_{S,m} = 0$ if the series S has less houses than the Condominiums series. Else,

$$\forall m \in Condominiums,\ N\_tax_{S,m} = N_m$$

so much so that we can define the flexible taxation term for Condominiums houses in the objective function as below.

3.2.4 • Objective function:

In this modified problem, we want again to maximize the total profit for Lake Saddleback Development Corporation. Now, it is defined like this:

$$max \sum_{m \in all\_models} p_m N_m$$

$$+ \sum_{m \in all\_models} extra\_profit_m \times N\_extra\_profit_m - sport\_build \times sport\_extra\_cost$$

$$- \sum_{m \in all\_models, m \notin Condominiums} tax\_rate \times selling\_price_m \times N_m$$

$$- \sum_{m \in Condominiums, s \in G\_Estates, GlenWood, Lakeview} 0.02 \times selling\_price_m \times N\_tax_{s,m}$$

3.2.5 • Base Constraints :

This problem is subject to several constraints regarding the available space and the different quotas. Even if most constraints are self-explanatory, some of them result from a personal interpretation of the problem that will be explained.

We keep the following constraints from the previous part:

- Total Houses :
$$N\_TOTAL = \sum_{m \in all\_models} N_m$$
- Grand Estates premium spots :
$$\sum_{m \in all\_G\_Estates\_PREMIUM} N_m \geq 50$$
As previously, we assume that the constraints "50 half-acre lots on the lake are to be used exclusively by the Grand Estate Series homes" means that at least 50 half-acres lots are reserved for Grand Estate Premium homes. We also assume that the roads and outside parking related to this houses are not taking in account in these 50 reserved half-acre.
- Limited surface :
$$\sum_{m \in all\_models} s_m \times N_m \leq total\_surface$$
- Quotas series :
  - Quotas minimum proportion of Grand Estates homes :
$$\sum_{m \in \{G\_Estates \cup G\_Estates\_PREMIUM\}} N_m \geq series_{min} \times N\_TOTAL$$
  - Quotas maximum proportion of Grand Estates homes :
$$\sum_{m \in \{G\_Estates \cup G\_Estates\_PREMIUM\}} N_m \leq series_{max} \times N\_TOTAL$$
  - Quotas minimum proportion of homes from other series :
$$\forall S \in \{Glen\_Wood, Lakeview, Condominiums\}$$
$$\sum_{m \in S} N_m \geq series_{min} \times N\_TOTAL$$
  - Quotas maximum proportion of homes from other series :
$$\forall S \in \{Glen\_Wood, Lakeview, Condominiums\}$$
$$\sum_{m \in S} N_m \leq series_{max} \times N\_TOTAL$$

- Quotas models :
  - Quotas minimum model Grand Estates : $\forall i \in \{1,2,3,4\}$:

    $$N_{(G\_Estates,i)} + N_{(G\_Estates\_PREMIUM,i)} \geq model_{min} \times \sum_{m \in G\_Estates \cup G\_Estates\_PREMIUM} N_m$$

  - Quotas maximum model Grand Estates : $\forall i \in \{1,2,3,4\}$:

    $$N_{(G\_Estates,i)} + N_{(G\_Estates\_PREMIUM,i)} \leq model_{max} \times \sum_{m \in G\_Estates \cup G\_Estates\_PREMIUM} N_m$$

  - Quotas minimum model Glen Wood :

    $$N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM} \geq model_{min} \times \sum_{m \in Glen\_Wood} N_m$$

    $$\forall m \in Glen\_Wood, m \neq G\_Cypress \text{ and } m \neq G\_Cypress\_PREMIUM :$$
    $$N_m \geq model_{min} \times \sum_{m' \in Glen\_Wood} N_{m'}$$

  - Quotas maximum model Glen Wood :

    $$N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM} \leq model_{max} \times \sum_{m \in Glen\_Wood} N_m$$

    $$\forall m \in Glen\_Wood, m \neq G\_Cypress \text{ and } m \neq G\_Cypress\_PREMIUM :$$
    $$N_m \leq model_{max} \times \sum_{m' \in Glen\_Wood} N_{m'}$$

  - Quotas minimum model Lakeview :

    $$N_{Bayview} + N_{Bayview\_PREMIUM} \geq model_{min} \times \sum_{m \in Lakeview} N_m$$

    $$\forall m \in Lakeview, m \neq Bayview \text{ and } m \neq Bayview\_PREMIUM :$$
    $$N_m \geq model_{min} \times \sum_{m' \in Lakeview} N_{m'}$$

  - Quotas maximum model Lakeview :

    $$N_{Bayview} + N_{Bayview\_PREMIUM} \leq model_{max} \times \sum_{m \in Lakeview} N_m$$

    $$\forall m \in Lakeview, m \neq Bayview \text{ and } m \neq Bayview\_PREMIUM :$$
    $$N_m \leq model_{max} \times \sum_{m' \in Lakeview} N_{m'}$$

  - Quotas minimum model Condominiums : $\forall m \in Condominiums$

    $$N_m \geq model_{min} \times \sum_{m' \in Condominiums} N_{m'}$$

– Quotas maximum model Condominiums : $\forall m \in Condominiums$

$$N_m \leq model_{max} \times \sum_{m' \in Condominiums} N_{m'}$$

- Quotas Grand Cypress Premium :

$$N_{G\_Cypress\_PREMIUM} \leq max\_G\_cypress\_PREMIUM \times (N_{G\_Cypress} + N_{G\_Cypress\_PREMIUM})$$

- Quotas Bayview Premium :

$$N_{Bayview\_PREMIUM} \leq max\_bayview\_PREMIUM \times (N_{Bayview} + N_{Bayview\_PREMIUM})$$

- Limit surface parking :

$$\sum_{m \in with\_outside\_parking} outside\_parking\_size \times N_m \leq max\_parking\_space$$

- Quotas Two Stories houses :

$$\sum_{m \in two\_stories\_without\_condominiums} N_m \leq max\_two\_stories \times \sum_{m \in all\_models, m \notin Condominiums} N_m$$

We assume that the constraints "no more than 70% of the single-family homes (all homes except the Country Condominiums) may be two-story homes." means that less than 70 of the houses that are not Condominiums houses (all houses expect Condominiums houses) should have more than two stories.

3.2.6 • New constraints

We add the following constraints :

- To decide if we build the sport complex :

$$(1)\ \ N\_extra\_profit_m \leq sport\_build \times N\_MAX$$

$$(sport\_build = 0 \Rightarrow N\_extra\_profit = 0)$$

$$(2)\ \ N\_extra\_profit_m - N_m \leq \frac{sport\_build}{N\_max} + (1 - sport\_build) N\_max$$

$$(sport\_build = 1 \Rightarrow N\_extra\_profit_m = N_m)$$

- Depending whether we build the sport complex or not, we have to update the limited surface constraint:

$$\sum_{m \in all\_models} s_m \times N_m \leq total\_surface - sport\_build \times sport\_space\_loss$$

- To define the new diversity constraint among Grand Estate Premium : at least three of the Grand Estate series plan must have at least eight units on the lake.

$$(1)\ \ N_m \geq 8z_m \ \ (z_m = 1 \Rightarrow N_m \geq 8)$$

$$(2)\ \ \sum_{m \in G\_Estates\_PREMIUM} z_m \geq 3$$

$$\text{(for three m at least : } z_m = 1 \Rightarrow N_m \geq 8)$$

- To compute the rank for new tax rate for Condominiums :
  - Comparison with Grand Estates :

$$more\_cond_{G\_Estates} \geq \frac{\sum_{m \in G\_Estates \cup G\_Estates\_PREMIUM} N_m - \sum_{m \in Condominiums} N_m}{N\_max}$$

    (more G_Estates than Condominiums $\Rightarrow more\_cond_{G\_Estates} = 1$)
  - Comparison with Glen Wood :

$$more\_cond_{Glen\_Wood} \geq \frac{\sum_{m \in GlenWood} N_m - \sum_{m \in Condominiums} N_m}{N\_max}$$

    (more Glen Wood than Condominiums $\Rightarrow more\_cond_{Glen\_wood} = 1$)
  - Comparison with Glen Wood :

$$more\_cond_{Lakeview} \geq \frac{\sum_{m \in Lakeview} N_m - \sum_{m \in Condominiums} N_m}{N\_max}$$

    (more Lakeview than Condominiums$\Rightarrow more_cond_{Lakeview} = 1$)
  - To compute the tax rate, we can calculate i as it follows.

$$i = 1 + \sum_{s \in G\_Estates, Glen\_Wood, Lakeview} more\_cond_s$$

    It is an easy way to control i, but we can't use i directly in objective due to non-linearity. We add the to following constraints instead.

$$(1)\ \ N\_tax_{s,m} \leq more\_cond_s \times N\_MAX$$

$$(more\_cond_s = 0 \Rightarrow N\_tax_{s,m} = 0)$$

$$(2)\ \ N\_tax_{s,m} - N_m \leq \frac{more\_cond_s}{N\_MAX} + (1 - more\_cond_s)N\_MAX$$

$$(more\_cond_s = 1 \Rightarrow N\_tax_{s,m} = N_m)$$

3.3 Implementation

You will find the AMPL implementation in the figures in the Appendix.

4 OPTIMAL SOLUTION

As mentioned above, our goal is to determine the optimal number of units to build under the aforementioned constraints, that will provide the company with the highest profit Note that, because this is a construction problem, we must build an integer number of houses. Therefore, we round down our mathematical output to the nearest unit to obtain our optimal solution for this linear programming problem.

4.1 Base Problem

The optimal solution is presented in the following table:

Grand Estates Lake	
The Trump	0
The Vanderbilt	0
The Hughes	0
The Jackson	0
PREMIUM Grand Estates Lake	
The Trump Lake	91
The Vanderbilt Lake	65
The Hughes Lake	52
The Jackson Lake	52
Glen Wood Collection	
Grand Cypress	152
Grand Cypress Premium	0
Lazy Oak	213
Wind Row	122
Orangewood	122

Lakeview Patio Homes	
Bayview	102
Baywiew Premium	0
Storeline	67
Docks Edge	64
Golden Pier	58
Country Condominiums	
Country Stream	203
Weeping Willow	203
Picket Fence	174

Note that the absence of basic Grand Estates is due to our interpretation of the problem: we assumed that 50 spots were reserved for Premium models, but that we could build more. By counting the Premium with the basic models for the different quotas, the better profitability of the premium models yielded to this optimal solution. After following through the above building plan, the company can make a profit of $132,173,000.

4.2 Modified Problem

The optimal construction plan for the modified problem is presented as followed in integers:

Grand Estates Lake	
The Trump	0
The Vanderbilt	0
The Hughes	0
The Jackson	0
PREMIUM Grand Estates Lake	
The Trump Lake	92
The Vanderbilt Lake	66
The Hughes Lake	53
The Jackson Lake	53
Glen Wood Collection	
Grand Cypress	153
Grand Cypress Premium	0
Lazy Oak	215
Wind Row	123
Orangewood	123

Lakeview Patio Homes	
Bayview	92
Baywiew Premium	0
Storeline	57
Docks Edge	61
Golden Pier	53
Country Condominiums	
Country Stream	215
Weeping Willow	215
Picket Fence	184

The company will achieve its maximum profit of $88,362,600 if it follows the construction plan detailed above. It is interesting to note that building the sport complex is not profitable: the optimal solution suggests not to build it. As a sanity check, we solved the problem with forcing the sport complex to be built. The optimal solution is slightly adapted, but most importantly its maximum objective function is now around $82,173,100: we therefore verified that it's lower.

Finally, we notice than Condominiums houses are the main series, enabling a lower taxation rate at 2%. This is what we intuitively guessed, and it is confirmed. The luxury tax is therefore an effective way to implement affordability quotas which are "self-implemented" by the company, for its own interest.

5
SENSITIVITY ANALYSES

All the sensitivity analysis tables are in the Appendix.

5.1 Base Problem

5.1.1 • Changed right-hand side

If we decrease the maximum quota of the five-bedroom homes from 15% to 10%, the optimal construction plan is still the same. Thus, the optimal objective value/profit is still $132,173,000. The reason that the optimal plan is not affected is that the constraint regarding the maximum five-bedroom homes quota is non-binding. As long as the change is within the acceptable range, the optimal plan will stay the same. The acceptable range for this variable is -169.80% to infinity. Since in the real world, there is no negative percentage quota, the range is from 0 to infinity.

5.1.2 • Drop a variable

If the option Grand Cypress Premium is dropped as an option to build, the current solution presented above is still optimal for the new problem. Therefore, the optimal objective value (profit) is still the same, which is $132,173,000. The reason that the optimal solution stays the same is that in the optimal solution, the unit of Grant Cypress Premium to be built is 0. The optimal solution being 0 means that the resources are best utilized in building the other models, and because the Grand Cypress Premium model won't be built, it won't affect the optimal solution and optimal objective value if it were to be dropped. Dropping this variable can be an asset for the company, because it would reduce the complexity of its management (less option, less management to do).

5.1.3 • Drop a constraint

If the constraint on the minimum quota on the three-bedroom models is dropped, the optimal solution presented above is still the same for the new problem. The optimal objective value, the maximum profit that the firm can achieve, is unchanged. The optimal solution and objective value stay the same because in the optimal solution situation, the corresponding value for the dual solution is 0. In other words, the constraint is non-binding. As the non-binding constraint is dropped, the optimal solution is unaffected. Thus, the maximum profit that the firm can achieve after the constraint on the minimum quota on the three-bedroom models is dropped is still $132,173,000.

5.2 Modified Problem

5.2.1 • Changed right-hand side

Because the variety section of the constraint is dropped from the modified problem, the analysis about decreasing the maximum quota of the five-bedroom homes from 15% to 10% is no longer applicable. Therefore, we will examine the constraint on minimum quota for Grandestate models for the sensitivity analysis here. If the quota for minimum units of Grandestate models stays within the range of 0 to 41 units, then the optimal plan and optimal objective value stay the same. The optimal plan is still the one presented above and the corresponding profit is $88,362,600.

5.2.2 • Drop a variable

Similar to the sensitivity analysis above for the original problem, when the variable Grand Cypress Premium is dropped, the optimal construction plan and the optimal objective value are not affected. The reasoning behind this is that the optimal solution for the Grand Cypress Premium model is still 0 for the modified problem. The optimal solution being 0 means that the resources are best utilized in building the other models, and because the Grand Cypress Premium model won't be built, it won't affect the optimal solution and optimal objective value if it were to be dropped. The maximum profit the company can earn is still $88,362,600.

5.2.3 • Drop a constraint

Similar to the analysis on the changing right-hand side above, because the variety section of the constraint is dropped from the modified problem, analysis of dropping the minimum quota on the three-bedroom models is not applicable in the modified problem. For the sensitivity analysis in the modified problem, we will look at the minimum quota for the model "Lazy Oak." Because the shadow price for the constraint is 0, the constraint is non-binding. Thus, when the constraint with 0 shadow price is dropped, the optimal solution and optimal objective value are unchanged. The optimal objective value is $88,362,600.

6 CONCLUSION

In the original problem our team's optimal solution resulted in a profit of $132,173,000. In this solution, the LSDC built zero standard Grand Estates Lakes homes, 260 premium Grand Estates Lakes homes, 607 homes from the Glen Wood Collection, 288 Lakeview Patio homes, and 580 Country Condominiums. Each of these estates allows for the LSDC to make a generous profit while following all constraints put in place to maintain lot size requirements, create an appropriate amount of garage space, provide an ample number of parking spots, leave space for pre-designated roads, reserve some affordable housing, and maintain variety. Not only does this solution hit all those goals, but it maximizes profit.

When adding the constraints from the modified problem, the total profit decreased significantly from the original problem given the added constraints. While there are still no standard Great Estates Lake homes built, this time LSDC built 260 Premium Great Estates Lake homes, 611 Glen Woods homes, 260 Lakeview Patio homes, and 611 Country Condominiums. These numbers did not change drastically from the original problem. The only variations were the number of Glen Woods homes decreased by 4, the Lakeview Patio homes increased by 28, and the number of Country Condominiums decreased by 31. The minor change in the number of houses drastically affected the outcome of the modified problem, resulting in a profit of $88,362,600, which is $40M-$50M less than the original problem's profit. In addition, the decision of whether or not to build a sports/recreational complex was also a major point in the modified problem. Ultimately our analysis from AMPL showed that it was financially unwise to build it so the sports complex was not built.

Some limits of this model is that we assume there are unlimited premium spots possible for Grand Estate. This is actually false, as the best locations around the lake may be unequal. Also, in price sensitivity analyses, we have made it so the ranges of each price don't change the optimal solution. But this scenario doesn't take into account people's preferences. This could be either a situation where a better valued house opens up for a lower price, which creates a situation where nobody wants the lesser of the two. This model also doesn't take into account extra expenses for construction work, delays and their consequences for the full project timeline, the learning curve when we build the same model, models which generate more problems during construction, etc.

Therefore, the LSDC has a lot to look forward to with this property. There are an abundance of options with many alternatives for different scenarios. The real estate market is very complex, with many different factors to take into account for valuation purposes. What we have learned this semester has provided us the tool set to be able to make reasonable assumptions for problems like this and generate a very theoretical optimal solution. This gives management insight about a draft direction to follow, but being flexible and adapting the strategy to the reality on the field is much more important than following this plan by any means.

7 APPENDIX

7.1 AMPL Models

7.1.1 • Base Problem

Figure 1: AMPL Base Model 1/4

```
# DEFINE SETS
# Auxiliary set
set BOUNDS := {"MIN", "MAX"};

# Models per series
set Grand_Estates = {'The Trump', 'The Vanderbilt', 'The Hughes', 'The Jackson'} ordered;
set Grand_Estates_PREMIUM = {'The Trump Lake', 'The Vanderbilt Lake', 'The Hughes Lake', 'The Jackson Lake'} ordered;
set Glen_Wood = {'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row', 'Orangewood'};
set Lakeview = {'Bayview', 'Bayview Premium', 'Storeline', 'Docks Edge', 'Golden Pier'};
set Condominiums = {'Country Stream', 'Weeping Willow', 'Picket Fence'};

set all_models = Grand_Estates union Grand_Estates_PREMIUM union Glen_Wood union Lakeview union Condominiums;

# Models per number of rooms
set two_bedrooms = {'Golden Pier',
                    'Weeping Willow', 'Picket Fence'};
set three_bedrooms = {'The Jackson',
                      'The Jackson Lake',
                      'Wind Row', 'Orangewood',
                      'Storeline', 'Docks Edge',
                      'Country Stream'};
set four_bedrooms = {'The Vanderbilt', 'The Hughes',
                     'The Vanderbilt Lake', 'The Hughes Lake',
                     'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak',
                     'Bayview', 'Bayview Premium'};
set five_bedrooms = {'The Trump',
                     'The Trump Lake'};

# Models with other characteristics
set two_stories_without_condominiums = {'The Trump', 'The Vanderbilt',
                   'The Trump Lake', 'The Vanderbilt Lake',
                   'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row',
                   'Bayview', 'Bayview Premium', 'Storeline'};
set with_outside_parking = {'The Trump', 'The Vanderbilt', 'The Hughes',
                            'The Trump Lake', 'The Vanderbilt Lake', 'The Hughes Lake',
                            'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row', 'Orangewood',
                            'Bayview', 'Bayview Premium', 'Storeline', 'Docks Edge',
                            'Country Stream', 'Weeping Willow', 'Picket Fence'};

set affordable_houses = {'Golden Pier', 'Weeping Willow', 'Picket Fence'};
```

Figure 2: AMPL Base Model 2/4

```
# Variables
var N{all_models} >= 0;
var N_TOTAL;

# Parameters
param profit{all_models};
param size{all_models};

param total_surface;

param outside_parking_spots{with_outside_parking};
param outside_parking_size;
param max_parking_space;

param quotas_bedrooms{nb_bedrooms in 2..5, BOUNDS};
param quotas_series{BOUNDS};
param quotas_models{BOUNDS};


param max_quotas_grandcypress_premium;
param max_quotas_bayview_premium;
param max_quotas_two_stories;
param min_quotas_affordability;


# Objective function
maximize total_profit: sum{m in all_models} profit[m]* N[m];
```

Figure 3: AMPL Base Model 3/4

```
# Constraints
subject to total_houses: N_TOTAL = sum{m in all_models} N[m];
subject to Grandestates_premium_spots: sum{m in Grand_Estates_PREMIUM} N[m] >= 50;
subject to limited_surface: sum{m in all_models} size[m]*N[m] <= total_surface;

subject to diversity_grandestate_premium{m in Grand_Estates_PREMIUM}: N[m] >= 8;

    # Quotas number of bedrooms
subject to quotas_min_two_bedrooms: sum{m in two_bedrooms} N[m] >= quotas_bedrooms[2, 'MIN']*N_TOTAL;
subject to quotas_max_two_bedrooms: sum{m in two_bedrooms} N[m] <= quotas_bedrooms[2, 'MAX']*N_TOTAL;

subject to quotas_min_three_bedrooms: sum{m in three_bedrooms} N[m] >= quotas_bedrooms[3, 'MIN']*N_TOTAL;
subject to quotas_max_three_bedrooms: sum{m in three_bedrooms} N[m] <= quotas_bedrooms[3, 'MAX']*N_TOTAL;

subject to quotas_min_four_bedrooms: sum{m in four_bedrooms} N[m] >= quotas_bedrooms[4, 'MIN']*N_TOTAL;
subject to quotas_max_four_bedrooms: sum{m in four_bedrooms} N[m] <= quotas_bedrooms[4, 'MAX']*N_TOTAL;

subject to quotas_min_five_bedrooms: sum{m in five_bedrooms} N[m] >= quotas_bedrooms[5, 'MIN']*N_TOTAL;
subject to quotas_max_five_bedrooms: sum{m in five_bedrooms} N[m] <= quotas_bedrooms[5, 'MAX']*N_TOTAL;


    # Quotas series repartition
subject to quotas_min_grandestate: sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_grandestate: sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_glenwood: sum{m in Glen_Wood} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_glenwood: sum{m in Glen_Wood} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_lakeview: sum{m in Lakeview} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_lakeview: sum{m in Lakeview} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_condominiums: sum{m in Condominiums} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_condominiums: sum{m in Condominiums} N[m] <= quotas_series['MAX']*N_TOTAL;
```

Figure 4: AMPL Base Model 4/4

```
    # Quotas models within each series
subject to quotas_min_models_grandestate{i in 1..4}:  N[member(i, Grand_Estates)] + N[member(i, Grand_Estates_PREMIUM)]
                                                       >= quotas_models['MIN']*sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m];
subject to quotas_max_models_grandestate{i in 1..4}:  N[member(i, Grand_Estates)] + N[member(i, Grand_Estates_PREMIUM)]
                                                       <= quotas_models['MAX']*sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m];


        # Treat Grand Cypress and Grand Cypress Premium together
subject to quotas_min_models_glenwood{m in Glen_Wood diff {'Grand Cypress', 'Grand Cypress Premium'}}:  N[m] >= quotas_models['MIN']*sum{M in Glen_Wood} N[M]
subject to quotas_max_models_glenwood{m in Glen_Wood diff {'Grand Cypress', 'Grand Cypress Premium'}}:  N[m] <= quotas_models['MAX']*sum{M in Glen_Wood} N[M]
subject to quotas_min_models_glenwood_cypress:  N['Grand Cypress'] + N['Grand Cypress Premium'] >= quotas_models['MIN']*sum{M in Glen_Wood} N[M];
subject to quotas_max_models_glenwood_cypress:  N['Grand Cypress'] + N['Grand Cypress Premium'] <= quotas_models['MAX']*sum{M in Glen_Wood} N[M];

        # Treat Bayview and Bayview Premium together
subject to quotas_min_models_lakeview{m in Lakeview diff {'Bayview', 'Bayview Premium'}}:  N[m] >= quotas_models['MIN']*sum{M in Lakeview} N[M];
subject to quotas_max_models_lakeview{m in Lakeview diff {'Bayview', 'Bayview Premium'}}:  N[m] <= quotas_models['MAX']*sum{M in Lakeview} N[M];
subject to quotas_min_models_lakeview_bayview:  N['Bayview'] + N['Bayview Premium'] >= quotas_models['MIN']*sum{M in Lakeview} N[M];
subject to quotas_max_models_lakeview_bayview:  N['Bayview'] + N['Bayview Premium'] <= quotas_models['MAX']*sum{M in Lakeview} N[M];

subject to quotas_min_models_condominiums{m in Condominiums}:  N[m] >= quotas_models['MIN']*sum{M in Condominiums} N[M];
subject to quotas_max_models_condominiums{m in Condominiums}:  N[m] <= quotas_models['MAX']*sum{M in Condominiums} N[M];


    # Other quotas
subject to quotas_grandcypress_premium : N['Grand Cypress Premium'] <= max_quotas_grandcypress_premium*(N['Grand Cypress'] + N['Grand Cypress Premium']);
subject to quotas_bayview_premium : N['Bayview Premium'] <= max_quotas_bayview_premium*(N['Bayview'] + N['Bayview Premium']);
subject to surface_parkings: sum{m in with_outside_parking} outside_parking_size*N[m] <= max_parking_space;
subject to quotas_two_stories: sum{m in two_stories_without_condominiums} N[m] <= max_quotas_two_stories*sum{m in all_models diff Condominiums} N[m];
subject to quotas_affordability: sum{m in affordable_houses} N[m] >= min_quotas_affordability*N_TOTAL;


option solver cplex;
data final_project.dat;
option cplex_options 'sensitivity';
option presolve 0;

solve;

display N, N.up, N.down;
display _conname, _con.current, _con.down, _con.up;

reset;
```

7.1.2 • Modified Problem

Figure 5: AMPL Modified Model 1/4

```
# DEFINE SETS
    # Auxiliary set
set BOUNDS := {"MIN", "MAX"};

    # Models per series
set Grand_Estates = {'The Trump', 'The Vanderbilt', 'The Hughes', 'The Jackson'} ordered;
set Grand_Estates_PREMIUM = {'The Trump Lake', 'The Vanderbilt Lake', 'The Hughes Lake', 'The Jackson Lake'} ordered;
set Glen_Wood = {'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row', 'Orangewood'};
set Lakeview = {'Bayview', 'Bayview Premium', 'Storeline', 'Docks Edge', 'Golden Pier'};
set Condominiums = {'Country Stream', 'Weeping Willow', 'Picket Fence'};

set all_models = Grand_Estates union Grand_Estates_PREMIUM union Glen_Wood union Lakeview union Condominiums;


    # Models with other characteristics
set two_stories_without_condominiums = {'The Trump', 'The Vanderbilt',
                   'The Trump Lake', 'The Vanderbilt Lake',
                   'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row',
                   'Bayview', 'Bayview Premium', 'Storeline'};
set with_outside_parking = {'The Trump', 'The Vanderbilt', 'The Hughes',
                            'The Trump Lake', 'The Vanderbilt Lake', 'The Hughes Lake',
                            'Grand Cypress', 'Grand Cypress Premium', 'Lazy Oak', 'Wind Row', 'Orangewood',
                            'Bayview', 'Bayview Premium', 'Storeline', 'Docks Edge',
                            'Country Stream', 'Weeping Willow', 'Picket Fence'};

# Variables
    # General variables
var N{all_models} >= 0;
var N_TOTAL;

    # NEW VARIABLES: For Condominiums tax rate
var more_than_condominiums{{'Grand Estates', 'Glen Wood', 'Lakeview'}} binary;
var i integer; # tax rate
var N_tax{{'Grand Estates', 'Glen Wood', 'Lakeview'}, Condominiums}>=0;

    # NEW VARIABLES: For new Grand Estates Premium quotas
var z{Grand_Estates_PREMIUM} binary;

    # NEW VARIABLES: For sport field
var sport_build binary;
var N_extra_profit{all_models} >= 0;
```

Figure 6: AMPL Modified Model 2/4

```
# Parameters
    # Parameters base problem
param profit{all_models};
param selling_price{all_models};
param size{all_models};

param total_surface;
param N_MAX;

param outside_parking_spots{with_outside_parking};
param outside_parking_size;
param max_parking_space;

param quotas_series{BOUNDS};
param quotas_models{BOUNDS};

param max_quotas_grandcypress_premium;
param max_quotas_bayview_premium;
param max_quotas_two_stories;

    # NEW PARAMETER: For new tax rate
param tax_rate;

    # NEW PARAMETERS: For sport field
param sport_extra_cost;
param sport_space_loss;
param extra_profit{all_models};


# NEW Objective function
maximize total_profit: sum{m in all_models} profit[m]*N[m]
                     - sum{m in all_models diff Condominiums} tax_rate*selling_price[m]*N[m]
                     - sum{m in Condominiums} 0.02*selling_price[m]* N[m]
                     - sum{s in {'Grand Estates', 'Glen Wood', 'Lakeview'}, m in Condominiums} 0.02*selling_price[m]*N_tax[s,m]
                     + sum{m in all_models} extra_profit[m]*N_extra_profit[m] - sport_build*sport_extra_cost;
```

Figure 7: AMPL Modified Model 3/4

```
# Constraints
    # Base problem constraints
        ##
subject to total_houses: N_TOTAL = sum{m in all_models} N[m];
subject to Grandestates_premium_spots: sum{m in Grand_Estates_PREMIUM} N[m] >= 50;
subject to limited_surface: sum{m in all_models} size[m]*N[m] <= total_surface - sport_build*sport_space_loss; # Corrected with potential sport field surface reduction

        ##
subject to quotas_min_grandestate: sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_grandestate: sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_glenwood: sum{m in Glen_Wood} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_glenwood: sum{m in Glen_Wood} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_lakeview: sum{m in Lakeview} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_lakeview: sum{m in Lakeview} N[m] <= quotas_series['MAX']*N_TOTAL;

subject to quotas_min_condominiums: sum{m in Condominiums} N[m] >= quotas_series['MIN']*N_TOTAL;
subject to quotas_max_condominiums: sum{m in Condominiums} N[m] <= quotas_series['MAX']*N_TOTAL;

        ##
subject to quotas_min_models_grandestate{k in 1..4}:  N[member(k, Grand_Estates)] + N[member(k, Grand_Estates_PREMIUM)]
                                                      >= quotas_models['MIN']*sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m];
subject to quotas_max_models_grandestate{k in 1..4}:  N[member(k, Grand_Estates)] + N[member(k, Grand_Estates_PREMIUM)]
                                                      <= quotas_models['MAX']*sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m];

        ##
subject to quotas_min_models_glenwood{m in Glen_Wood diff {'Grand Cypress', 'Grand Cypress Premium'}}:  N[m] >= quotas_models['MIN']*sum{M in Glen_Wood} N[M];
subject to quotas_max_models_glenwood{m in Glen_Wood diff {'Grand Cypress', 'Grand Cypress Premium'}}:  N[m] <= quotas_models['MAX']*sum{M in Glen_Wood} N[M];
subject to quotas_min_models_glenwood_cypress:  N['Grand Cypress'] + N['Grand Cypress Premium'] >= quotas_models['MIN']*sum{M in Glen_Wood} N[M];
subject to quotas_max_models_glenwood_cypress:  N['Grand Cypress'] + N['Grand Cypress Premium'] <= quotas_models['MAX']*sum{M in Glen_Wood} N[M];

        ##
subject to quotas_min_models_lakeview{m in Lakeview diff {'Bayview', 'Bayview Premium'}}:  N[m] >= quotas_models['MIN']*sum{M in Lakeview} N[M];
subject to quotas_max_models_lakeview{m in Lakeview diff {'Bayview', 'Bayview Premium'}}:  N[m] <= quotas_models['MAX']*sum{M in Lakeview} N[M];
subject to quotas_min_models_lakeview_bayview:  N['Bayview'] + N['Bayview Premium'] >= quotas_models['MIN']*sum{M in Lakeview} N[M];
subject to quotas_max_models_lakeview_bayview:  N['Bayview'] + N['Bayview Premium'] <= quotas_models['MAX']*sum{M in Lakeview} N[M];

subject to quotas_min_models_condominiums{m in Condominiums}:  N[m] >= quotas_models['MIN']*sum{M in Condominiums} N[M];
subject to quotas_max_models_condominiums{m in Condominiums}:  N[m] <= quotas_models['MAX']*sum{M in Condominiums} N[M];
```

Figure 8: AMPL Modified Model 4/4

```
        ##
subject to quotas_grandcypress_premium : N['Grand Cypress Premium'] <= max_quotas_grandcypress_premium*(N['Grand Cypress'] + N['Grand Cypress Premium']);
subject to quotas_bayview_premium : N['Bayview Premium'] <= max_quotas_bayview_premium*(N['Bayview'] + N['Bayview Premium']);
subject to surface_parkings: sum{m in with_outside_parking} outside_parking_size*N[m] <= max_parking_space;
subject to quotas_two_stories: sum{m in two_stories_without_condominiums} N[m] <= max_quotas_two_stories*sum{m in all_models diff Condominiums} N[m];


    # NEW CONSTRAINTS: Compute rank for new tax rate for Condominiums
subject to condominiums_vs_grandestates_1: more_than_condominiums['Grand Estates']
                                           >= (sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] - sum{m in Condominiums} N[m])/N_MAX; # more GE => binary = 1
subject to condominiums_vs_grandestates_2: more_than_condominiums['Grand Estates']
                                           <= 1 + (sum{m in Grand_Estates union Grand_Estates_PREMIUM} N[m] - sum{m in Condominiums} N[m])/N_MAX; # less GE => binary = 0

subject to condominiums_vs_glenwood_1: more_than_condominiums['Glen Wood'] >= (sum{m in Glen_Wood} N[m] - sum{m in Condominiums} N[m])/N_MAX; # more GW => binary = 1
subject to condominiums_vs_glenwood_2: more_than_condominiums['Glen Wood'] <= 1 + (sum{m in Glen_Wood} N[m] - sum{m in Condominiums} N[m])/N_MAX; # less GW => binary = 0

subject to condominiums_vs_lakeview_1: more_than_condominiums['Lakeview'] >= (sum{m in Lakeview} N[m] - sum{m in Condominiums} N[m])/N_MAX; # more LV => binary = 1
subject to condominiums_vs_lakeview_2: more_than_condominiums['Lakeview'] <= 1 + (sum{m in Lakeview} N[m] - sum{m in Condominiums} N[m])/N_MAX; # less LV => binary = 0

# easy way to control i, but can't use i directly in objective due to non-linearity. Two next constraints instead
subject to set_tax_rate: i = 1 + sum{s in {'Grand Estates', 'Glen Wood', 'Lakeview'}} more_than_condominiums[s];
subject to build_tax_1{s in {'Grand Estates', 'Glen Wood', 'Lakeview'}, m in Condominiums}:
                    N_tax[s, m] <= more_than_condominiums[s]*N_MAX; # more_than_condominiums[s] = 0 => N_tax[s, m] = 0
subject to build_tax_2{s in {'Grand Estates', 'Glen Wood', 'Lakeview'}, m in Condominiums}:
                    N_tax[s, m] - N[m] <= more_than_condominiums[s]/N_MAX + (1 - more_than_condominiums[s])*N_MAX; # more_than_condominiums[s] = 1 => N_tax[s, m] = N[m]


    # NEW CONSTRAINTS: New way to define diversity among Grand Estate Premium
subject to at_least_three_diversity: sum{m in Grand_Estates_PREMIUM} z[m] >= 3;
subject to diversity_grandestate_premium{m in Grand_Estates_PREMIUM}: N[m] >= 8*z[m];


    # NEW CONSTAINTS: Decide if we build sports field
subject to build_sport_1{m in all_models}: N_extra_profit[m] <= sport_build*N_MAX; # sport_build = 0 => N_extra_profit = 0
subject to build_sport_2{m in all_models}: N_extra_profit[m] - N[m] <= sport_build/N_MAX + (1 - sport_build)*N_MAX; # sport_build = 1 => N_extra_profit = N


option solver cplex;
data final_project_modified.dat;
option cplex_options 'sensitivity';
option presolve 0;

solve;
display N, N.up, N.down, i, sport_build;
display _conname, _con.current, _con.down, _con.up;
```

7.2 AMPL Data

7.2.1 • Base Problem

Figure 9: AMPL Base Model data

```
param total_surface := 13068000;
param outside_parking_size := 200;
param max_parking_space := 653400;

param quotas_bedrooms: MIN MAX :=
2 0.15 0.25
3 0.25 0.4
4 0.25 0.4
5 0.05 0.15;

param quotas_series :=
MIN 0.15
MAX 0.35;

param quotas_models :=
MIN 0.2
MAX 0.35;

param max_quotas_grandcypress_premium := 0.25;
param max_quotas_bayview_premium := 0.25;
param max_quotas_two_stories := 0.7;
param min_quotas_affordability := 0.15;

param profit :=
'The Trump' 154000
'The Vanderbilt' 149600
'The Hughes' 143000
'The Jackson' 129800
'The Trump Lake' 211200
'The Vanderbilt Lake' 205480
'The Hughes Lake' 196900
'The Jackson Lake' 179740
'Grand Cypress' 75600
'Grand Cypress Premium' 82800
'Lazy Oak' 68400
'Wind Row' 57600
'Orangewood' 50400
'Bayview' 60000
'Bayview Premium' 66000
'Storeline' 54000
'Docks Edge' 48000
'Golden Pier' 40000
'Country Stream' 55000
'Weeping Willow' 40000
'Picket Fence' 35000;
```

```
param size :=
'The Trump' 23180
'The Vanderbilt' 22980
'The Hughes' 22980
'The Jackson' 22780
'The Trump Lake' 23180
'The Vanderbilt Lake' 22980
'The Hughes Lake' 22980
'The Jackson Lake' 22780
'Grand Cypress' 6850
'Grand Cypress Premium' 12090
'Lazy Oak' 6100
'Wind Row' 5556
'Orangewood' 5556
'Bayview' 5756
'Bayview Premium' 8660
'Storeline' 5556
'Docks Edge' 5556
'Golden Pier' 5556
'Country Stream' 3100
'Weeping Willow' 2900
'Picket Fence' 2900;

param outside_parking_spots :=
'The Trump' 2
'The Vanderbilt' 1
'The Hughes' 1
'The Trump Lake' 2
'The Vanderbilt Lake' 1
'The Hughes Lake' 1
'Grand Cypress' 1
'Grand Cypress Premium' 1
'Lazy Oak' 2
'Wind Row' 1
'Orangewood' 1
'Bayview' 2
'Bayview Premium' 2
'Storeline' 1
'Docks Edge' 1
'Country Stream' 3
'Weeping Willow' 2
'Picket Fence' 2;
```

7.2.2 • Modified Problem

Figure 10: AMPL Modified Model data

```
# Parameters of the base problem
param total_surface := 13068000; # 11979000 if 275 acres
param outside_parking_size := 200;
param max_parking_space := 653400;
param N_MAX = 10000;

param quotas_series :=
MIN 0.15
MAX 0.35;

param quotas_models :=
MIN 0.2
MAX 0.35;

param max_quotas_grandcypress_premium := 0.25;
param max_quotas_bayview_premium := 0.25;
param max_quotas_two_stories := 0.7;

# New parameters
param tax_rate := 0.08;
param sport_extra_cost := 8000000;
param sport_space_loss := 435600;

param profit := # Base problem parameter
'The Trump' 154000
'The Vanderbilt' 149600
'The Hughes' 143000
'The Jackson' 129800
'The Trump Lake' 211200
'The Vanderbilt Lake' 205480
'The Hughes Lake' 196900
'The Jackson Lake' 179740
'Grand Cypress' 75600
'Grand Cypress Premium' 82800
'Lazy Oak' 68400
'Wind Row' 57600
'Orangewood' 50400
'Bayview' 60000
'Bayview Premium' 66000
'Storeline' 54000
'Docks Edge' 48000
'Golden Pier' 40000
'Country Stream' 55000
'Weeping Willow' 40000
'Picket Fence' 35000;
```

```
param extra_profit := # New parameter
'The Trump' 7700
'The Vanderbilt' 7480
'The Hughes' 7150
'The Jackson' 6490
'The Trump Lake' 10560
'The Vanderbilt Lake' 10274
'The Hughes Lake' 9845
'The Jackson Lake' 8987
'Grand Cypress' 2268
'Grand Cypress Premium' 2484
'Lazy Oak' 2052
'Wind Row' 1728
'Orangewood' 1512
'Bayview' 1200
'Bayview Premium' 1320
'Storeline' 1080
'Docks Edge' 960
'Golden Pier' 1200
'Country Stream' 1650
'Weeping Willow' 1200
'Picket Fence' 1050;

param selling_price := # New parameter
'The Trump' 700000
'The Vanderbilt' 680000
'The Hughes' 650000
'The Jackson' 590000
'The Trump Lake' 960000
'The Vanderbilt Lake' 934000
'The Hughes Lake' 895000
'The Jackson Lake' 817000
'Grand Cypress' 420000
'Grand Cypress Premium' 460000
'Lazy Oak' 380000
'Wind Row' 320000
'Orangewood' 280000
'Bayview' 300000
'Bayview Premium' 330000
'Storeline' 270000
'Docks Edge' 240000
'Golden Pier' 200000
'Country Stream' 220000
'Weeping Willow' 160000
'Picket Fence' 140000;
```

```
param size := # Base problem parameter
'The Trump' 23180
'The Vanderbilt' 22980
'The Hughes' 22980
'The Jackson' 22780
'The Trump Lake' 23180
'The Vanderbilt Lake' 22980
'The Hughes Lake' 22980
'The Jackson Lake' 22780
'Grand Cypress' 6850
'Grand Cypress Premium' 12090
'Lazy Oak' 6100
'Wind Row' 5556
'Orangewood' 5556
'Bayview' 5756
'Bayview Premium' 8660
'Storeline' 5556
'Docks Edge' 5556
'Golden Pier' 5556
'Country Stream' 3100
'Weeping Willow' 2900
'Picket Fence' 2900;

param outside_parking_spots := # Base problem parameter
'The Trump' 2
'The Vanderbilt' 1
'The Hughes' 1
'The Trump Lake' 2
'The Vanderbilt Lake' 1
'The Hughes Lake' 1
'Grand Cypress' 1
'Grand Cypress Premium' 1
'Lazy Oak' 2
'Wind Row' 1
'Orangewood' 1
'Bayview' 2
'Bayview Premium' 2
'Storeline' 1
'Docks Edge' 1
'Country Stream' 3
'Weeping Willow' 2
'Picket Fence' 2;
```

7.3 AMPL Primal Variables

7.3.1 • Base Problem

Figure 11: Base Problem - Primal Variables

```
ampl: model final_project.mod;
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 132172601.2
19 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
:                              N         N.up       N.down
Bayview                     101.591    89233.5    56016.7
'Bayview Premium'             0        89371.7    -1e+20
'Country Stream'            203.182   299156      24040.1
'Docks Edge'                 63.8571   54000      45420
'Golden Pier'                58.0519   73709.4   -13609.6
'Grand Cypress'             152.386    75989.1    70612.5
'Grand Cypress Premium'       0       128599      -1e+20
'Lazy Oak'                  213.341   300929      68019
Orangewood                  121.909    56439.3    -4721.47
'Picket Fence'              174.156    40000      -3585.03
Storeline                    66.7597   56580      48000
'The Hughes'                  0       196900      -1e+20
'The Hughes Lake'            52.2467  199480     143000
'The Jackson'                 0       179740      -1e+20
'The Jackson Lake'           52.2467  197443     129800
'The Trump'                   0       211200      -1e+20
'The Trump Lake'             91.4317  292957     207498
'The Vanderbilt'              0       205480      -1e+20
'The Vanderbilt Lake'        65.3084  209173     202900
'Weeping Willow'            203.182   284156      35000
'Wind Row'                  121.909    62453.6     2478.53
;
```

7.3.2 • Modified Problem

Figure 12: Modified Problem - Primal Variables

```
ampl: model final_project_modified.mod;
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal integer solution; objective 88362616.82
94 MIP simplex iterations
0 branch-and-bound nodes

suffix up OUT;
suffix down OUT;
suffix current OUT;
:                          N         N.up       N.down       :=
Bayview                  91.9743    38317.8     33749.2
'Bayview Premium'         0         55636.1    -1e+20
'Country Stream'        214.607    109436       33496.2
'Docks Edge'             61.3162    32400       27139.6
'Golden Pier'            52.5568    28056.1   -298310
'Grand Cypress'         153.291     43080.8     40730.4
'Grand Cypress Premium'   0         77431.6    -1e+20
'Lazy Oak'              214.607     96835.8     36941.7
Orangewood              122.632     29630.5     23820.7
'Picket Fence'          183.949     36800      -42794.6
Storeline                56.9365    34041       28800
'The Hughes'              0        125300      -1e+20
'The Hughes Lake'        52.5568   127160       91000
'The Jackson'             0        114380      -1e+20
'The Jackson Lake'       52.5568   125798       82600
'The Trump'               0        134400      -1e+20
'The Trump Lake'         91.9743   229553      132109
'The Vanderbilt'          0        130760      -1e+20
'The Vanderbilt Lake'    65.6959   133045      128900
'Weeping Willow'        214.607     95635.8     32200
'Wind Row'              122.632     33235.3     27820.7
;

i = 1
sport_build = 0
```

7.4 AMPL Dual Variables

7.4.1 • Base Problem

Figure 13: Base Problem - Dual Variables 1/2

_conname	_con.current	_con.down	_con.up
total_houses	0	-39.3657	20.3074
Grandestates_premium_spots	50	-1e+20	261.234
limited_surface	13068000	2501210	26171900
"diversity_grandestate_premium['The Trump Lake']"	8	-1e+20	91.4317
"diversity_grandestate_premium['The Vanderbilt Lake']"	8	-1e+20	65.3084
"diversity_grandestate_premium['The Hughes Lake']"	8	-1e+20	52.2467
"diversity_grandestate_premium['The Jackson Lake']"	8	-1e+20	52.2467
quotas_min_two_bedrooms	0	-1e+20	174.156
quotas_max_two_bedrooms	0	-25.8187	13.1392
quotas_min_three_bedrooms	0	-1e+20	194.474
quotas_max_three_bedrooms	0	-66.7597	1e+20
quotas_min_four_bedrooms	0	-1e+20	149.484
quotas_max_four_bedrooms	0	-111.75	1e+20
quotas_min_five_bedrooms	0	-1e+20	4.35389
quotas_max_five_bedrooms	0	-169.802	1e+20
quotas_min_grandestate	0	-12.6346	19.6064
quotas_max_grandestate	0	-348.311	1e+20
quotas_min_glenwood	0	-1e+20	348.311
quotas_max_glenwood	0	-23.7168	20.1177
quotas_min_lakeview	0	-1e+20	29.0259
quotas_max_lakeview	0	-319.285	1e+20
quotas_min_condominiums	0	-1e+20	319.285
quotas_max_condominiums	0	-29.0259	1e+20
'quotas_min_models_grandestate[1]'	0	-1e+20	39.185
'quotas_min_models_grandestate[2]'	0	-1e+20	13.0617
'quotas_min_models_grandestate[3]'	0	-8.70778	5.80519
'quotas_min_models_grandestate[4]'	0	-8.70662	5.8057
'quotas_max_models_grandestate[1]'	0	-4.35418	13.0591
'quotas_max_models_grandestate[2]'	0	-26.1234	1e+20
'quotas_max_models_grandestate[3]'	0	-39.185	1e+20
'quotas_max_models_grandestate[4]'	0	-39.185	1e+20
"quotas_min_models_glenwood['Lazy Oak']"	0	-1e+20	91.4317
"quotas_min_models_glenwood['Wind Row']"	0	-60.5888	30.5695
"quotas_min_models_glenwood['Orangewood']"	0	-8.70028	5.80853
"quotas_max_models_glenwood['Lazy Oak']"	0	-60.742	30.5306
"quotas_max_models_glenwood['Wind Row']"	0	-91.4317	1e+20
"quotas_max_models_glenwood['Orangewood']"	0	-91.4317	1e+20
quotas_min_models_glenwood_cypress	0	-1e+20	30.4772
quotas_max_models_glenwood_cypress	0	-60.9545	1e+20
"quotas_min_models_lakeview['Storeline']"	0	-1e+20	8.70778
"quotas_min_models_lakeview['Docks Edge']"	0	-1e+20	5.80519
"quotas_min_models_lakeview['Golden Pier']"	0	-13.1392	4.73953

Figure 14: Base Problem - Dual Variables 2/2

"quotas_max_models_lakeview['Storeline']"	0	-34.8311	1e+20
"quotas_max_models_lakeview['Docks Edge']"	0	-37.7337	1e+20
"quotas_max_models_lakeview['Golden Pier']"	0	-43.5389	1e+20
quotas_min_models_lakeview_bayview	0	-1e+20	43.5389
quotas_max_models_lakeview_bayview	0	-34.8497	8.70662
"quotas_min_models_condominiums['Country Stream']"	0	-1e+20	87.0778
"quotas_min_models_condominiums['Weeping Willow']"	0	-1e+20	87.0778
"quotas_min_models_condominiums['Picket Fence']"	0	-1e+20	58.0519
"quotas_max_models_condominiums['Country Stream']"	0	-25.8289	13.1365
"quotas_max_models_condominiums['Weeping Willow']"	0	-29.0259	58.0519
"quotas_max_models_condominiums['Picket Fence']"	0	-29.0259	1e+20
quotas_grandcypress_premium	0	-38.0966	1e+20
quotas_bayview_premium	0	-25.3977	1e+20
surface_parkings	653400	326252	1e+20
quotas_two_stories	0	-8.70778	5.80519
quotas_affordability	0	-1e+20	174.156

7.4.2 • Modified Problem

Figure 15: Modified Problem - Dual Variables 1/3

_conname	_con.current	_con.down	_con.up
total_houses	0	0	91.3097
Grandestates_premium_spots	50	-1e+20	262.784
limited_surface	13068000	2486460	25925300
quotas_min_grandestate	0	0	41.4443
quotas_max_grandestate	0	-350.378	1e+20
quotas_min_glenwood	0	-1e+20	350.378
quotas_max_glenwood	0	0	1e+20
quotas_min_lakeview	0	0	43.8625
quotas_max_lakeview	0	-350.378	1e+20
quotas_min_condominiums	0	-1e+20	350.378
quotas_max_condominiums	0	0	89.4533
'quotas_min_models_grandestate[1]'	0	-1e+20	39.4176
'quotas_min_models_grandestate[2]'	0	-1e+20	13.1392
'quotas_min_models_grandestate[3]'	0	-4.37973	8.75946
'quotas_min_models_grandestate[4]'	0	-4.37944	8.76063
'quotas_max_models_grandestate[1]'	0	-26.2889	13.1365
'quotas_max_models_grandestate[2]'	0	-26.2784	1e+20
'quotas_max_models_grandestate[3]'	0	-39.4176	1e+20
'quotas_max_models_grandestate[4]'	0	-39.4176	1e+20
"quotas_min_models_glenwood['Lazy Oak']"	0	-1e+20	91.9743
"quotas_min_models_glenwood['Wind Row']"	0	-60.9462	30.7515
"quotas_min_models_glenwood['Orangewood']"	0	-4.37783	8.76706
"quotas_max_models_glenwood['Lazy Oak']"	0	-61.1012	30.7121
"quotas_max_models_glenwood['Wind Row']"	0	-91.9743	1e+20
"quotas_max_models_glenwood['Orangewood']"	0	-91.9743	1e+20
quotas_min_models_glenwood_cypress	0	-1e+20	30.6581
quotas_max_models_glenwood_cypress	0	-61.3162	1e+20
"quotas_min_models_lakeview['Storeline']"	0	-1e+20	4.37973
"quotas_min_models_lakeview['Docks Edge']"	0	-1e+20	8.75946
"quotas_min_models_lakeview['Golden Pier']"	0	-30.6581	8.75946
"quotas_max_models_lakeview['Storeline']"	0	-35.0378	1e+20
"quotas_max_models_lakeview['Docks Edge']"	0	-30.6581	1e+20
"quotas_max_models_lakeview['Golden Pier']"	0	-39.4176	1e+20
quotas_min_models_lakeview_bayview	0	-1e+20	39.4176
quotas_max_models_lakeview_bayview	0	-35.0566	4.37944
"quotas_min_models_condominiums['Country Stream']"	0	-1e+20	91.9743
"quotas_min_models_condominiums['Weeping Willow']"	0	-1e+20	91.9743
"quotas_min_models_condominiums['Picket Fence']"	0	-1e+20	61.3162
"quotas_max_models_condominiums['Country Stream']"	0	-30.6725	61.2587
"quotas_max_models_condominiums['Weeping Willow']"	0	-30.6581	61.3162
"quotas_max_models_condominiums['Picket Fence']"	0	-30.6581	1e+20
quotas_grandcypress_premium	0	-38.3226	1e+20
quotas_bayview_premium	0	-22.9936	1e+20

Figure 16: Modified Problem - Dual Variables 2/3

surface_parkings	653400	329356	1e+20
quotas_two_stories	0	-4.37973	8.75946
condominiums_vs_grandestates_1	0	-1e+20	0.0350378
condominiums_vs_grandestates_2	1	0.0350378	1e+20
condominiums_vs_glenwood_1	0	-1e+20	0
condominiums_vs_glenwood_2	1	0	1e+20
condominiums_vs_lakeview_1	0	-1e+20	0.0350378
condominiums_vs_lakeview_2	1	0.0350378	1e+20
set_tax_rate	1	1	1
"build_tax_1['Grand Estates','Country Stream']"	0	0	1e+20
"build_tax_1['Grand Estates','Weeping Willow']"	0	0	1e+20
"build_tax_1['Grand Estates','Picket Fence']"	0	0	1e+20
"build_tax_1['Glen Wood','Country Stream']"	0	0	1e+20
"build_tax_1['Glen Wood','Weeping Willow']"	0	0	1e+20
"build_tax_1['Glen Wood','Picket Fence']"	0	0	1e+20
"build_tax_1['Lakeview','Country Stream']"	0	0	1e+20
"build_tax_1['Lakeview','Weeping Willow']"	0	0	1e+20
"build_tax_1['Lakeview','Picket Fence']"	0	0	1e+20
"build_tax_2['Grand Estates','Country Stream']"	10000	-214.607	1e+20
"build_tax_2['Grand Estates','Weeping Willow']"	10000	-214.607	1e+20
"build_tax_2['Grand Estates','Picket Fence']"	10000	-183.949	1e+20
"build_tax_2['Glen Wood','Country Stream']"	10000	-214.607	1e+20
"build_tax_2['Glen Wood','Weeping Willow']"	10000	-214.607	1e+20
"build_tax_2['Glen Wood','Picket Fence']"	10000	-183.949	1e+20
"build_tax_2['Lakeview','Country Stream']"	10000	-214.607	1e+20
"build_tax_2['Lakeview','Weeping Willow']"	10000	-214.607	1e+20
"build_tax_2['Lakeview','Picket Fence']"	10000	-183.949	1e+20
at_least_three_diversity	3	-1e+20	3
"diversity_grandestate_premium['The Trump Lake']"	0	-1e+20	83.9743
"diversity_grandestate_premium['The Vanderbilt Lake']"	0	-1e+20	57.6959
"diversity_grandestate_premium['The Hughes Lake']"	0	-1e+20	44.5568
"diversity_grandestate_premium['The Jackson Lake']"	0	-1e+20	52.5568
"build_sport_1['The Trump']"	0	0	10000
"build_sport_1['The Vanderbilt']"	0	0	10000
"build_sport_1['The Hughes']"	0	0	10000
"build_sport_1['The Jackson']"	0	0	10000
"build_sport_1['The Trump Lake']"	0	0	10092
"build_sport_1['The Vanderbilt Lake']"	0	0	10065.7
"build_sport_1['The Hughes Lake']"	0	0	10052.6
"build_sport_1['The Jackson Lake']"	0	0	10052.6
"build_sport_1['Grand Cypress']"	0	0	10153.3
"build_sport_1['Grand Cypress Premium']"	0	0	10000
"build_sport_1['Lazy Oak']"	0	0	10214.6

Figure 17: Modified Problem - Dual Variables 3/3

"build_sport_1['Wind Row']"	0	0	10122.6
"build_sport_1['Orangewood']"	0	0	10122.6
"build_sport_1['Bayview']"	0	0	10092
"build_sport_1['Bayview Premium']"	0	0	10000
"build_sport_1['Storeline']"	0	0	10056.9
"build_sport_1['Docks Edge']"	0	0	10061.3
"build_sport_1['Golden Pier']"	0	0	10052.6
"build_sport_1['Country Stream']"	0	0	10214.6
"build_sport_1['Weeping Willow']"	0	0	10214.6
"build_sport_1['Picket Fence']"	0	0	10183.9
"build_sport_2['The Trump']"	10000	0	1e+20
"build_sport_2['The Vanderbilt']"	10000	0	1e+20
"build_sport_2['The Hughes']"	10000	0	1e+20
"build_sport_2['The Jackson']"	10000	0	1e+20
"build_sport_2['The Trump Lake']"	10000	-91.9743	1e+20
"build_sport_2['The Vanderbilt Lake']"	10000	-65.6959	1e+20
"build_sport_2['The Hughes Lake']"	10000	-52.5568	1e+20
"build_sport_2['The Jackson Lake']"	10000	-52.5568	1e+20
"build_sport_2['Grand Cypress']"	10000	-153.291	1e+20
"build_sport_2['Grand Cypress Premium']"	10000	0	1e+20
"build_sport_2['Lazy Oak']"	10000	-214.607	1e+20
"build_sport_2['Wind Row']"	10000	-122.632	1e+20
"build_sport_2['Orangewood']"	10000	-122.632	1e+20
"build_sport_2['Bayview']"	10000	-91.9743	1e+20
"build_sport_2['Bayview Premium']"	10000	0	1e+20
"build_sport_2['Storeline']"	10000	-56.9365	1e+20
"build_sport_2['Docks Edge']"	10000	-61.3162	1e+20
"build_sport_2['Golden Pier']"	10000	-52.5568	1e+20
"build_sport_2['Country Stream']"	10000	-214.607	1e+20
"build_sport_2['Weeping Willow']"	10000	-214.607	1e+20
"build_sport_2['Picket Fence']"	10000	-183.949	1e+20