Problem Set
CS 234 Homework 11
April 17, 2017

1. Consider a Turing Machine that allows moves $R, L, R*, L*$ in which $R*$ and $L*$ move to the left and right respectively *until* a blank space is found. That is they scan left/right stopping at the first blank. This transition will replace the very first symbol with the desired replacement and then keep each subsequent symbol unchanged as it moves left or right. Give the formal definition of this variation and then prove it is not more powerful than a regular Turing Machine.

2. Read the wikipedia page on Alan Turing and provide brief (about one sentence or so) responses to the following questions.

   (a) Describe Alan Turing's contribution to World War 2. Describe the impact his work had on winning and shortening the war.

   (b) Describe Alan Turing's personal lifestyle and the impact that had on his work, the way his country treated him, and his death.

   (c) What was the title of Alan Turing's famous 1936 paper? What is the definition of the long word in that title?

   (d) What two other people's work does Alan Turing's 1936 paper build on?

   (e) What is the name of the 1950 paper he published where he described the "Turing Test". What is the nature of the Turing Test?

3. Section 13.1: 4
   Give a bijection between the set of even integers and the set of odd integers to show that the two sets have equal cardinality.

4. Section 13.1: 7
   Give a bijection between $\mathbb{Z}$ and the set $S = \{..., \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, ...\}$.

5. Section 13.1: 9
   Give a bijection between $\{0, 1\} \times \mathbb{N}$ and $\mathbb{N}$.

6. Section 13.1: 10
   Give a bijection between $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$.

7. Section 13.2: 1
   Prove that the set $A = \{\ln(n) : n \in \mathbb{N}\} \subseteq \mathcal{R}$ is countably infinite.

8. Section 13.2: 2
   Prove that the set $\{(m, n) \in \mathbb{N} \times \mathbb{N} : m \leq n\}$ is countably infinite.

9. Present Cantor's "Diagonal Slash" proof that shows there are no surjections from $\mathbb{N} \to \mathcal{R}$.