Potential Outcomes and Randomized Experiments

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23 July 2019

Preliminaries

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- PS1 for today
- PS2 for tomorrow
- `.Rmd`

Unit Selection	Treatment Assignment: Randomized	Treatment Assignment: Not Randomized
Randomized	Randomized Experiment (gold standard)	Survey Sampling (allows population inference)
Not Randomized	Controlled Experiment (allows causal inference)	Observational Study (large potential for bias)

Figure 1: Ramsey & Schafer, *The Statistical Sleuth*

Review Questions

1. When can we observe both $Y_i(1)$ and $Y_i(0)$?
2. What is this fact called?
3. When does the empirical difference-in-means estimator exactly equal the true, underlying ATE?
4. What are the two parts of SUTVA?

Exercise in Potential Outcomes

Statistical Refreshment

Expectation

$$E(Y) = \sum_{i=1}^{n} [y_i \cdot p(y_i)]$$

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Calculate $E(\text{Ideology})$.

Respondent	Ideology
1	3
2	-2
3	3
4	1
5	1

Conditional Expectation

$$E(Y|X = x) = \sum_{i=1}^{n}[y_i \cdot p(y_i|X = x)]$$

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Calculate $E(\text{Ideology}|\text{Dem} = 0)$.

Respondent	Ideology	Dem
1	3	0
2	-2	1
3	3	0
4	1	0
5	1	1

Conditional Expectation

$$E(Y|X = x) = \sum_{i=1}^{n}[y_i \cdot p(y_i|X = x)]$$

Calculate $E(\text{Ideology}|\text{Dem} = 0)$

		Dem?	
		0	1
	3	2	0
Ideology	1	1	1
	-2	0	1

(“wide” data, cells: counts of units with row/column scores)

Conditional Expectation

$$E(Y|X = x) = \sum_{i=1}^{n} [y_i \cdot p(y_i|X = x)]$$

Calculate $E(\text{Ideology}|\text{Dem} = 0)$

		Dem?	
		0	1
	3	0.4	0.0
Ideology	1	0.2	0.2
	-2	0.0	0.2

("wide" data, cells: probability unit has row/column scores)

Unbiasedness

$$E(\hat{\theta}) = \theta$$

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- $E\left[\widehat{\overline{Y_1 - Y_0}}\right] = \overline{Y_1 - Y_0}$

Unbiasedness

$$E(\hat{\theta}) = \theta$$

- $E\left[\widehat{\overline{Y_1 - Y_0}}\right] = \overline{Y_1 - Y_0}$
- $E\left[\widehat{\beta_1}\right] = \beta_1$

Unbiasedness

What estimator used to estimate $\overline{Y_1 - Y_0}$?

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The difference-in-means estimator.

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The difference-in-means estimator.

$$\widehat{\overline{Y_1 - Y_0}} = (Y_1 | T_i = 1) - (Y_0 | T_i = 0)$$

Probability

The Three Axioms

1. $P(A) \geq 0$
2. $P(\Omega) = 1$
3. If events *mutually exclusive* (or, sets *disjoint*), then

$$P(A \text{ or } B) = P(A) + P(B)$$

Conditional Probability

For events A and B,

$$
\begin{aligned}
P(A \text{ and } B) &= P(A)P(B|A) \\
&= P(B)P(A|B)
\end{aligned}
$$

Divide both sides by marginal probability $P(B)$ yields

$$
P(A|B) = \frac{P(A \text{ and } B)}{P(B)}
$$

Statistical Independence

A and B are independent iff both

$$P(A|B) = P(A)$$

and

$$P(B|A) = P(B)$$

Conditional Independence

A and B can be independent *conditional on* C.

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Many times, *only* independent given C.

E.g., Y_1, Y_0 indep of T, given X.

$$P(A \text{ and } B|C) = P(A|C)P(B|C)$$

Potential Outcomes Model: Estimands

- Individual TE

$$\tau_i = Y_i(1) - Y_i(0)$$

- Average treatment effect

$$ATE = E(Y_1 - Y_0) = E(Y_1) - E(Y_0)$$

- Average treatment effect for the treated

$$ATT = E(Y_1|T = 1) - E(Y_0|T = 1)$$

Potential Outcomes Model: Estimands

The average treatment effect (ATE):

$$
\begin{aligned}
E(Y_1 - Y_0) &= \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i1} - Y_{i0}) \\
&= \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i1}) - \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i0}) \\
&= E(Y_1) - E(Y_0)
\end{aligned}
$$

Potential Outcomes Model: Estimands

- If we know (Y_1, Y_0) indep of T
- Then,

$$\begin{aligned} E\left(Y_1\right) &= E(Y_1|T=1) \\ E\left(Y_0\right) &= E(Y_0|T=0) \end{aligned}$$

- Then, can substitute

$$\begin{aligned} ATE &= E(Y_1) - E(Y_0) \\ &= E(Y_1|T=1) - E(Y_0|T=0) \end{aligned}$$

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Observed diff in Tr and Co group means gives ATE!

Potential Outcomes Model: Estimands

Holland (1986): "*prima facie effect*":

$$E(Y_t|S = t) - E(Y_c|S = c)$$

Potential Outcomes Model: Estimands

Holland (1986): "*prima facie effect*":

$$E(Y_t|S = t) - E(Y_c|S = c)$$

"It is important to recognize that $E(Y_t)$ and $E(Y_t|S = t)$ are *not* the same thing ..."

Potential Outcomes Model: Estimands, Interpretation

Gerber & Green:

When $Y(1)$ and $Y(0)$ indep of T,

$$ATE = E(Y_i(1)|T_i = 1) - E(Y_i(0)|T_i = 0)$$

Potential Outcomes Model: Estimands, Interpretation

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When $Y(1)$ and $Y(0)$ indep of T,

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When $Y(1)$ and $Y(0)$ indep of T,

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ATE &= E(Y_i(1)|T_i = 1) - E(Y_i(0)|T_i = 0) \\
&= E(Y_i(1)|T_i = 1) - E(Y_i(0)|T_i = 1) + \\
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&= [E(Y_i(1)|T_i = 1) - E(Y_i(0)|T_i = 1)] + \\
&\quad\; [E(Y_i(0)|T_i = 1) - E(Y_i(0)|T_i = 0)] \\
&= E(Y_i(1) - Y_i(0)|T_i = 1) + \\
&\quad\; E(Y_i(0)|T_i = 1) - E(Y_i(0)|T_i = 0)
\end{aligned}$$

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When $Y(1)$ and $Y(0)$ indep of T,

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&= [E(Y_i(1)|T_i = 1) - E(Y_i(0)|T_i = 1)] + \\
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&= E(Y_i(1) - Y_i(0)|T_i = 1) + \\
&\quad E(Y_i(0)|T_i = 1) - E(Y_i(0)|T_i = 0)
\end{aligned}$$

$$\underbrace{E(Y_i(1) - Y_i(0)|T_i = 1)}_{\text{ATT}} + \underbrace{E(Y_i(0)|T_i = 1) - E(Y_i(0)|T_i = 0)}_{\text{Selection Bias}}$$

Assignment of Treatment

Ideas

- Causal effects: relative to some other condition

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- Causal effects: relative to some other condition
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  - timing defined
  - must be excludable
  - cannot be “attribute”
- Covariates: causally prior to treatment
- Dose-response “biological gradient” evidence

Attributes

"Causal effect of race"?

```
resume <- read_csv("http://j.mp/2sDjsHI")
dim(resume)
```

```
## [1] 4870    4
```

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```r
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dim(resume)
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```
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```r
kable(table(resume$race, resume$call))
```

	0	1
black	2278	157
white	2200	235

Attributes

"Causal effect of race"?

```
resume <- read_csv("http://j.mp/2sDjsHI")
dim(resume)
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```
## [1] 4870    4
```

```
kable(table(resume$race, resume$call))
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	0	1
black	2278	157
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Sen and Wasow (2016) : "Race as a Bundle of Sticks: Designs that Estimate Effects of Seemingly Immutable Characteristics" (*elements* of attributes varyingly manipulable)

The Potential Outcomes Model: Assignment

Observed outcome: $Y_i = Y_i(1) \cdot T_i + Y_i(0) \cdot (1 - T_i)$

The assignment mechanism *selects* which potential outcome we observe.

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The assignment mechanism *selects* which potential outcome we observe.

(Gerber & Green use $Y_i = Y_i(1) \cdot d_i + Y_i(0) \cdot (1 - d_i)$ to highlight that we observe pot outcome from treatment actually taken, not hypothetical or assigned treatment.)

The Potential Outcomes Model: Assignment

"Assignment mechansims" are really missing-data-generating procedures.

Ignorability

Assignment mechanism is *ignorable* if Y_{obs} conditnly indep of T

$$P(T|X, Y_{obs}, Y_{mis}) = P(T|X, Y_{obs})$$

Ignorability

Assignment mechanism is *ignorable* if Y_{obs} conditnly indep of T

$$P(T|X, Y_{obs}, Y_{mis}) = P(T|X, Y_{obs})$$

Nothing in unobserved Y_{mis} informs relationship between Y_{obs}, T.

Unconfoundedness

Some ignorable mechanisms are *unconfounded*, too.

$$P(T|X, Y_{obs}, Y_{mis}) = P(T|X)$$

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Nothing in Y informs T.

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Nothing in Y informs T.

These are special cases of conditional independence.

An Assignment Mechanism

Little & Rubin (2000):

Patient	Y	T
1	6	1
2	12	1
3	9	0
4	11	0

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Clearly, treatment is harmful. $\overline{Y(1)} - \overline{Y(0)} = 9 - 10 = -1$.

An Assignment Mechanism

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Patient	Y(0)	Y(1)	τ	T
1		6		1
2		12		1
3	9			0
4	11			0
Mean	10	9		

Clearly, treatment is harmful.

$$\overline{Y(1)|T=1} - \overline{Y(0)|T=0} = 9 - 10 = -1$$

An Assignment Mechanism: Perfect Doctor

Little & Rubin (2000):

Patient	Y(0)	Y(1)	τ	T
1	(1)	6		1
2	(3)	12		1
3	9	(8)		0
4	11	(10)		0
Mean	10	9		

An Assignment Mechanism: Perfect Doctor

Little & Rubin (2000):

Patient	Y(0)	Y(1)	τ	T
1	(1)	6	(5)	1
2	(3)	12	(9)	1
3	9	(8)	(-1)	0
4	11	(10)	(-1)	0
Mean	10	9	(3)	

An Assignment Mechanism: Perfect Doctor

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Patient	Y(0)	Y(1)	τ	T
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Mean	10	9	(3)	

Clearly, treatment is beneficial:

$$\overline{Y(1)} - \overline{Y(0)} = 9 - 6 = 3$$

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Little & Rubin (2000):

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1	(1)	6	(5)	1
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4	11	(10)	(-1)	0
Mean	10	9	(3)	

Clearly, treatment is beneficial:

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This assg mechanism is non-ignorable, confounded.

Random Assignment Mechanisms

- Simple/complete randomization
  (Bernoulli trial, prob π)

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Random Assignment Mechanisms

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- Complete randomization / random allocation
  (fixed proportion to tr)
- Blocked randomizations
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- Cluster randomizations
  (assignment at higher level)

Randomization (Design-based) Inference

A volunteer?

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The task: select the 2 folders with messages

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  - "No x-ray vision. No ESP. Effect of messages on choice = 0."

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  - "Some way to detect messages. Message location $\rightarrow$ choice."

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- What is our baseline expectation/model for this process?
    - "No x-ray vision. No ESP. Effect of messages on choice = 0."
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Select!

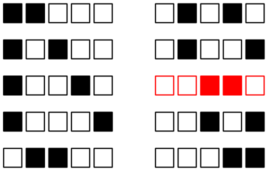
Randomization Inference

The possible choices:

■■□□□	□■□■□
■□■□□	□■□□■
■□□■□	□□■■□
■□□□■	□□■□■
□■■□□	□□□■■

Randomization Inference

The possible choices:



- You chose _____ and _____. Let X = number found.

Randomization Inference

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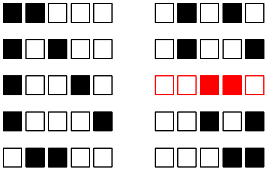
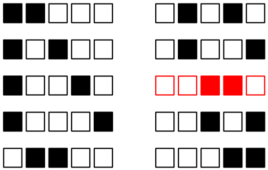


- You chose _____ and _____. Let X = number found.
- What was $P(X \geq 2 | \text{no ESP})$?

Randomization Inference

The possible choices:



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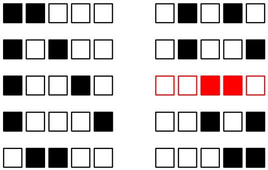

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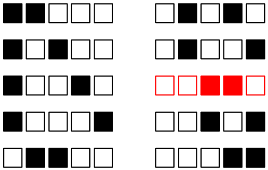

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- What was $P(X \geq 2|\text{no ESP})$? $\frac{1}{10} = 0.1$
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Randomization Inference

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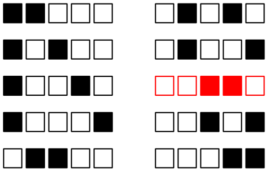

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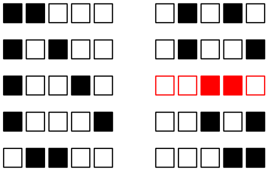
- What is "prob result at least this extreme, given model of no effect"?

Randomization Inference

The possible choices:



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- Definition of p-value!

Randomization Inference

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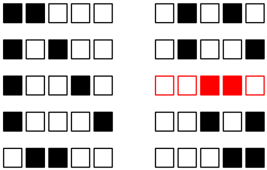

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- Valid, exact, with no distributional assumption, no large n.

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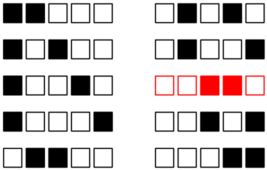

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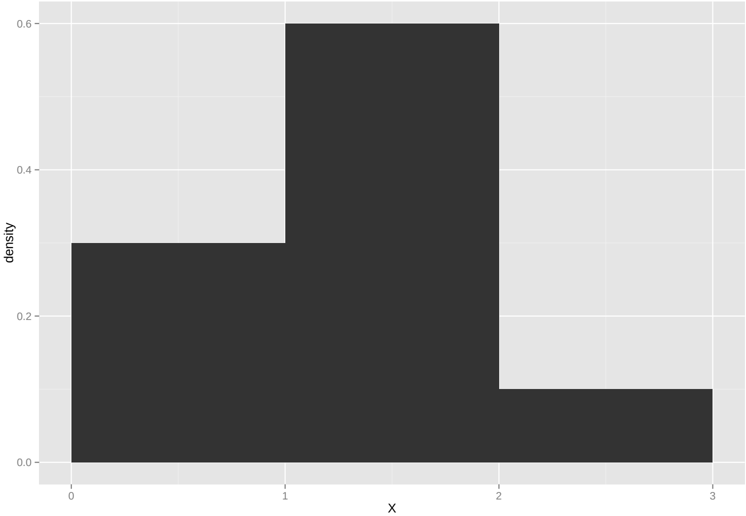
- What is "prob result at least this extreme, given model of no effect"?
- Definition of p-value!
- Valid, exact, with no distributional assumption, no large n.
- *Randomization* creates dist'n of possible numbers correct

The Randomization Distribution of X



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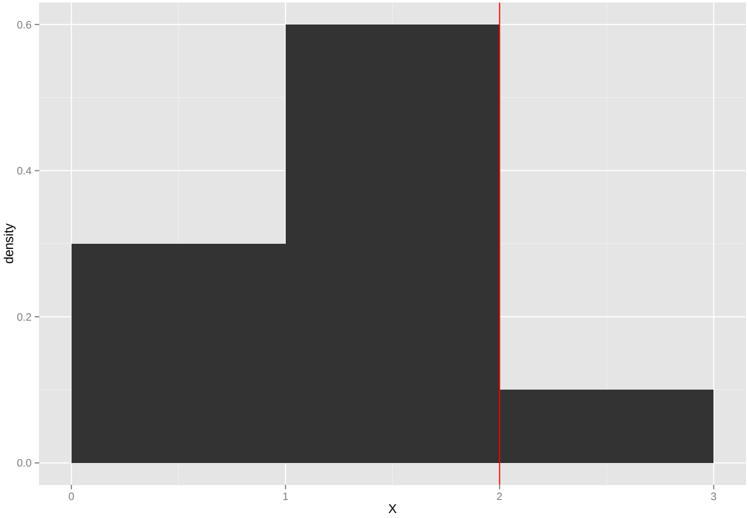

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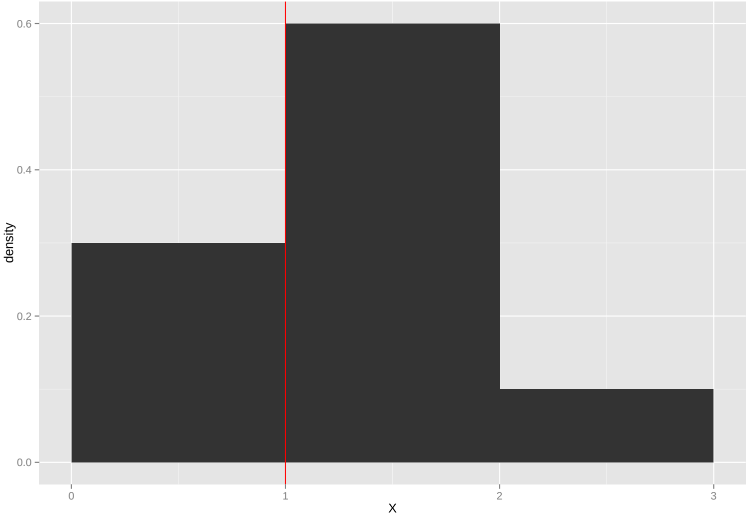

Parametric Null Hypothesis Significance Testing

- Specify and assume H_0
- Define H_A
- Examine reference dist'n (t, χ^2, $\ldots$) under H_0
- Calculate p-value
- Compare to some α; reject H_0 if $p < \alpha$

Randomization Inference

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  (sharp null of no treatment effect)

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  (or at least a big sample of them)

Randomization Inference

- Specify and assume H_0
  (sharp null of no treatment effect)
- Define H_A
- Create reference dist'n from all possible values of X under H_0
  (or at least a big sample of them)
- What prop. of possible "at least as extreme as" observed?
  $\rightsquigarrow$ p-value!

Randomization Inference

- Specify and assume H_0
  (sharp null of no treatment effect)
- Define H_A
- Create reference dist'n from all possible values of X under H_0
  (or at least a big sample of them)
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- CA ballot ordering effects (JASA 2006)

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How many randomizations are there?

Combinations: Counting selected sets

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$$ \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252 $$

Common Assumptions, Null Hypotheses

- Constant effect:

$$\tau_i = Y_{i1} - Y_{i0} = \tau \quad \forall i$$

- Null hypothesis of no average effect:

$$ATE = \overline{\tau} = 0$$

- Sharp null hypothesis of no effect:

$$\tau_i = 0$$

An Assignment Mechanism: Perfect Doctor

Calculate RI p-value for Perfect Doctor, under sharp null.

Patient	Y(0)	Y(1)	τ	T
1	(1)	6	(5)	1
2	(3)	12	(9)	1
3	9	(8)	(-1)	0
4	11	(10)	(-1)	0
Mean	10	9	(3)	

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(See `02-ri-perfect-dr.R`)

RI versus the t-test

Perfect Doctor:

- RI: $p = 1$
- `t.test()`: $p \approx 0.8$
- "If no tr effect, then this result typical"

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(Odd logic of NHST: "assume false thing, how strange is data?"")

Randomization Inference

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	0	1
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```
resume %>% group_by(race) %>% summarise(call_rate = me
```

```
## # A tibble: 2 x 2
##   race  call_rate
##   <fct>     <dbl>
## 1 black    0.0645
## 2 white    0.0965
```

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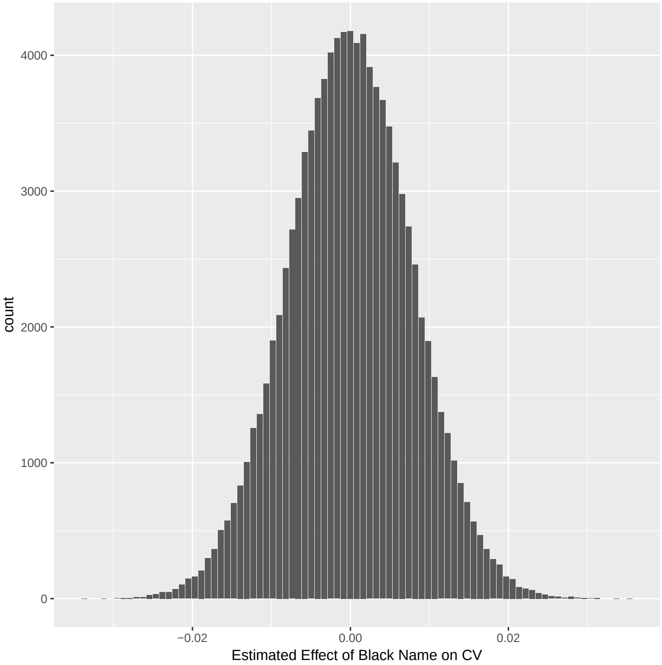
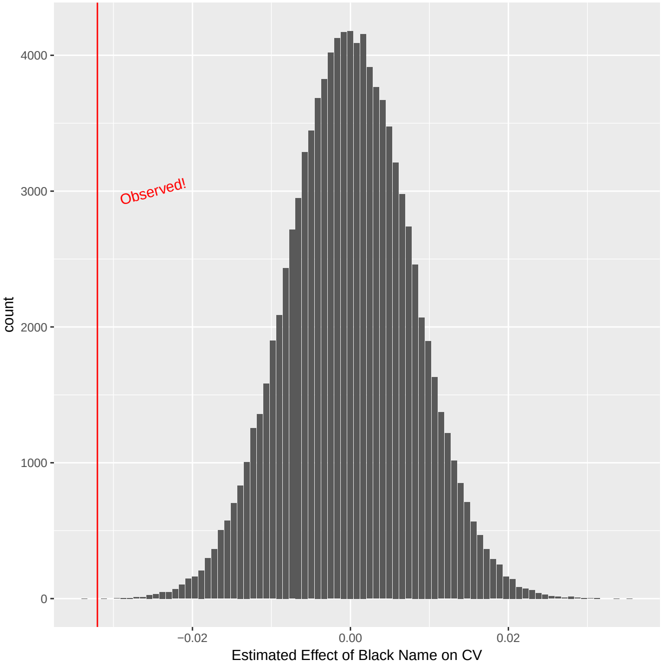


4000
3000
2000
1000
0
count
−0.02
0.00
0.02
Estimated Effect of Black Name on CV


Observed!
4000
3000
2000
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0
count
−0.02
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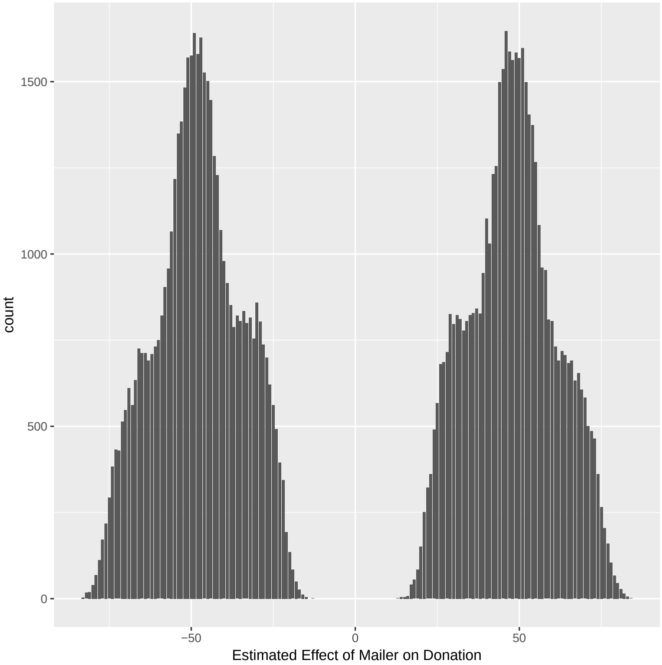
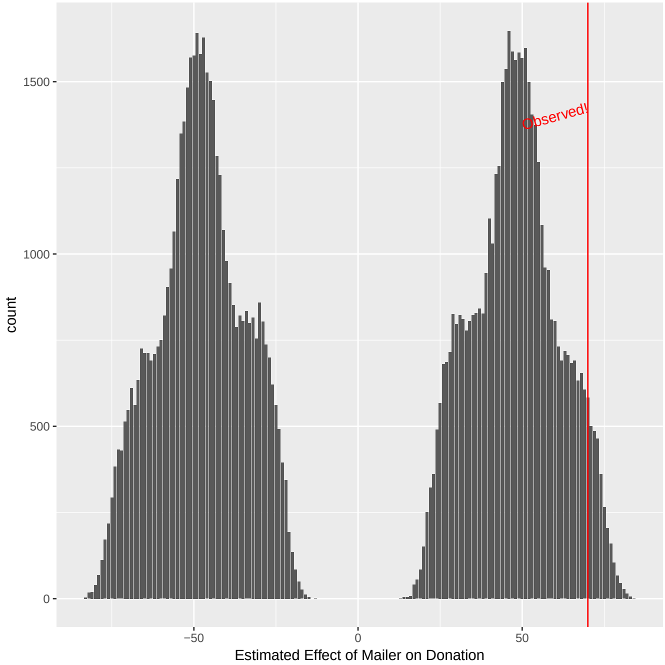
Randomization Inference

- Gerber & Green donations example, p. 65
- Possible values $\tau_i \in (-\infty, \infty)$
- Y_1, Y_0, τ likely very skewed
- See `02-ri-resume-donate.R`


1500
1000
500
0
count
−50
0
50
Estimated Effect of Mailer on Donation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Next:
Covariates in Experiments
PS2 due

Bertrand, Marianne, and Sendhil Mullainathan. 2004. "Are Emily and Greg More Employable Than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination." *American Economic Review* 94 (4): 991–1013.

Sen, Maya, and Omar Wasow. 2016. "Race as a Bundle of Sticks: Designs That Estimate Effects of Seemingly Immutable Characteristics." *Annual Reviews of Political Science* 19: 499–522.