Introduction and Potential Outcomes

Ryan T. Moore

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Welcome

About Me

- Associate Prof of Government, American University
- Senior Social Scientist, The Lab @ DC
- Research agenda: political methodology, causal inference, experimental design

The Course: Learning Objectives

- Identify causal effects using the potential outcomes framework

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- (Walk through syllabus)

What is Causal Inference?

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"What caused the 9/11 attacks?"

vs.

"What is the effect of foreign policy X on domestic terror attacks?"

Example: Canvassing and Program Enrollment

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- $\frac{2}{2} - \frac{0}{2} = 1$
- (For each person canvassed, expect 1 more enrollment.)

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What do we really want to know?

What would have happened under *other* conditions?

Would canvassing actually *change* anyone's enrollment?

Example: Canvassing and Enrollment

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Well . . . how do we know which?

We can never know.

Can we know for one person?

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We can never know.

But I have some ideas.

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We could not canvass, then canvass later.

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We can never know.

We can never observe both “Canvassed” and “Not Canvassed” for a unit.

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We can never observe both *potential outcomes.*

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The Fundamental Problem of Causal Inference

We can never observe more than one *potential outcome* for a given unit.

So, how can we get a *causal* estimate?

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We infer missing potential outcomes.

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The problem with our naive estimate of effect:

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Knowing whether would enroll under canvass *predicts* whether canvassed!

When comparing two groups **does** recover truth

Here, potential outcomes do **not** help predict treatment:

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Good!

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That's the goal!

- Aspirin $\implies$ headache!
- Canvass $\implies$ turnout!
- Insurance $\implies$ health spending!
- CBT $\implies$ remission!

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How to ensure potential outcomes won't predict treatment?

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How to *assign* treatment so it won't predict pot. outcomes?

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- (You'll estimate a huge effect)
- What if you *randomly select whom gets Canvass?*
- ("Citizen got Canvass" won't help guess pot. out.)

The Potential Outcomes Model

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  The objective is to determine for some population of units ... the 'typical' causal effect of the [treatment vs. control conditions] on a dependent variable Y.
  —Rubin (1974)

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- One study, one causal effect

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- *Assignment mechanism*: means by which units come to be sorted into conditions

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Stable Unit Treatment Value Assumption (SUTVA)

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The assignment mechanism *selects* which potential outcome we observe.

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- *estimand*: the parameter an estimator attempts to estimate
- *estimate*: application of an estimator func to some obs data

The Potential Outcomes Model: Estimands

Statistical language

- *parameter*: unknown numeric value characterizing feature of prob model (Greek; θ, β)
- *statistic*: quantity calculable from observed data. A function. (Roman)
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"The sample statistic $\bar{x}$ is an estimator of true mean param μ".

The Potential Outcomes Model: Estimands

Statistical language

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"The sample statistic $\bar{x}$ is an estimator of true mean param μ".

μ is my estimand. 5.1 is my estimate.

The Potential Outcomes Model: Estimands

For i, individual treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

- True treatment effect?

The Potential Outcomes Model: Estimands

For i, individual treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

- True treatment effect?
- Estimate?

The Potential Outcomes Model: Estimands

For i, individual treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

- True treatment effect?
- Estimate?
- Observable?

The Potential Outcomes Model: Estimands

For i, individual treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

- True treatment effect?
- Estimate?
- Observable?

The Potential Outcomes Model: Estimands

For i, individual treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

- True treatment effect?
- Estimate?
- Observable?

We can never know.

Random variables and the Expectation

Random variable X is a function mapping sample space to set of reals:

$$X : \Omega \to \mathbb{R}$$

Random variables

- summarize outcome of probabilistic/stochastic trial

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E.g.,

- Let X = number of heads in 3 coin flips. Is X a random variable?

Random variables and the Expectation

Random variable X is a function mapping sample space to set of reals:

$$X : \Omega \to \mathbb{R}$$

Random variables

- summarize outcome of probabilistic/stochastic trial
- take numerical values.

E.g.,

- Let $X =$ number of heads in 3 coin flips. Is X a random variable?
- Let Y be outcome of two coin flips, $Y \in \{HH, TH, HT, TT\}$. Is Y a random variable?

Expectation of a Random Variable

The *expected value* or *expectation* of a random variable is mean of its outcomes, weighted by their probabilities. For a discrete random variable,

$$E(X) = \sum_{i=1}^{n} x_i p(x_i)$$

For continuous random variable,

$$E(X) = \int_{-\infty}^{\infty} x p(x) dx$$

Expectation is **not** the sample mean from particular instantiation, a particular data set. We use sample mean $\bar{x}$ to *estimate* the expected value.

Variance of a Random Variable

The *variance* of a random variable is mean of outcomes' squared deviations from expectation, weighted by their probabilities:

$$
\begin{aligned}
V(X) &= E[(X - E(X))^2] \\
&= E(X^2) - (E(X))^2
\end{aligned}
$$

Properties of Expectation

- $E(c) = c$
- $E(a + bX) = a + bE(X)$
- $E(X + Y) = EX + EY$
- If X and Y indep., then $E(XY) = E(X)E(Y)$

Properties of Variance

- $Var(X) = E(X - EX)^2$
- $Var(X) = E(X^2) - (EX)^2$
- $Var(c) = 0$
- $Var(Y|X) = E(Y^2|X) - (E(Y|X))^2$
- If X and Y indep., then $Var(X + Y) = Var(X) + Var(Y)$
- If X and Y indep, then $Var(X - Y) = Var(X) + Var(Y)$

The Potential Outcomes Model: Estimands

- Average treatment effect

$$ATE = E(Y_1 - Y_0) = E(Y_1) - E(Y_0)$$

The Potential Outcomes Model: Estimands

- Average treatment effect

$$ATE = E(Y_1 - Y_0) = E(Y_1) - E(Y_0)$$

- Average treatment effect for the treated

$$ATT = E(Y_1|T = 1) - E(Y_0|T = 1)$$

The Potential Outcomes Model: Estimands

The average treatment effect (ATE):

$$\overline{\tau} = \overline{TE} = ATE \quad \equiv \quad E(Y_1 - Y_0) = \frac{1}{2n} \sum_{i=1}^{2n} (Y_{i1} - Y_{i0})$$

The Potential Outcomes Model: Estimands

The average treatment effect (ATE):

$$\begin{aligned}\overline{\tau} = \overline{TE} = ATE &\equiv E(Y_1 - Y_0) = \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i1} - Y_{i0}) \\ &= E(Y_1) - E(Y_0)\end{aligned}$$

The Potential Outcomes Model: Estimands

The average treatment effect (ATE):

$$\begin{aligned} \overline{\tau} = \overline{TE} = ATE &\equiv E(Y_1 - Y_0) = \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i1} - Y_{i0}) \\ &= E(Y_1) - E(Y_0) \end{aligned}$$

- True?

The Potential Outcomes Model: Estimands

The average treatment effect (ATE):

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- True?
- Estimated?

The Potential Outcomes Model: Estimands

The average treatment effect (ATE):

$$\begin{aligned}\bar{\tau} = \overline{TE} = ATE &\equiv E(Y_1 - Y_0) = \frac{1}{2n}\sum_{i=1}^{2n}(Y_{i1} - Y_{i0}) \\ &= E(Y_1) - E(Y_0)\end{aligned}$$

- True?
- Estimated?
- Observable?

The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T

The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T
- Then,

The Potential Outcomes Model: Estimands

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- If we know (Y_1, Y_0) indep of T
- Then,

The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T
- Then,

$$\begin{aligned} E\left(Y_1\right) &= E(Y_1|T=1) \\ E\left(Y_0\right) &= E(Y_0|T=0) \end{aligned}$$

The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T
- Then,

$$
\begin{aligned}
E\left(Y_1\right) &= E(Y_1|T=1) \\
E\left(Y_0\right) &= E(Y_0|T=0)
\end{aligned}
$$

- Then, can substitute

$$
\begin{aligned}
ATE &= E(Y_1) - E(Y_0) \\
&= E(Y_1|T=1) - E(Y_0|T=0)
\end{aligned}
$$

The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T
- Then,

$$\begin{aligned} E\left(Y_1\right) &= E(Y_1|T=1) \\ E\left(Y_0\right) &= E(Y_0|T=0) \end{aligned}$$

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The Potential Outcomes Model: Estimands

Something we can calculate:

- If we know (Y_1, Y_0) indep of T
- Then,

$$
\begin{aligned}
E\left(Y_1\right) &= E(Y_1|T=1) \\
E\left(Y_0\right) &= E(Y_0|T=0)
\end{aligned}
$$

- Then, can substitute

$$
\begin{aligned}
ATE &= E(Y_1) - E(Y_0) \\
&= E(Y_1|T=1) - E(Y_0|T=0)
\end{aligned}
$$

Observed diff in Tr and Co group means is unbiased est. of $\overline{TE}$!

Common Assumptions, Null Hyp's in Causal Inference

- Constant effect:

$$\tau_i = Y_{i1} - Y_{i0} = \tau \quad \forall i$$

Common Assumptions, Null Hyp's in Causal Inference

- Constant effect:

$$\tau_i = Y_{i1} - Y_{i0} = \tau \quad \forall i$$

- Null hypothesis of no average effect:

$$ATE = \overline{\tau} = 0$$

Common Assumptions, Null Hyp's in Causal Inference

- Constant effect:

$$\tau_i = Y_{i1} - Y_{i0} = \tau \quad \forall i$$

- Null hypothesis of no average effect:

$$ATE = \overline{\tau} = 0$$

- Sharp null hypothesis of no effect:

$$\tau_i = 0$$

Compliance with Treatment Assignment

Sometimes, units don't follow assignment!

Assigned Tr	Assigned Co	Type
Tr	Co	Complier

Compliance with Treatment Assignment

Sometimes, units don't follow assignment!

Assigned Tr	Assigned Co	Type
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Co	Tr	Defier

Common Estimates under Noncompliance

Let T_i be treatment *assigned*, D_i be treatment *received*.

Common Estimates under Noncompliance

Let T_i be treatment *assigned*, D_i be treatment *received*.

- Intent-to-treat effect

$$\begin{aligned} ITT &= E(Y_1|T=1) - E(Y_0|T=0) \\ &= E(Y_1|T=1, D(T=1)) - E(Y_0|T=0, D(T=0)) \end{aligned}$$

Common Estimates under Noncompliance

Let T_i be treatment *assigned*, D_i be treatment *received*.

- Intent-to-treat effect

$$\begin{aligned} ITT &= E(Y_1|T=1) - E(Y_0|T=0) \\ &= E(Y_1|T=1, D(T=1)) - E(Y_0|T=0, D(T=0)) \end{aligned}$$

- As-treated effect

$$ASTRE = E(Y_1|D=1) - E(Y_0|D=0)$$

Computing

Computing

- R for simulations, estimation, graphics

Computing

- R for simulations, estimation, graphics
- RStudio, an IDE for R

R Primer

Functions

```
function(arg1, arg2, ...){
  <the function code here...>
}
```

Functions

```
function(arg1, arg2, ...){
  <the function code here...>
}
```

```
sum(5, 2)
```

```
## [1] 7
```

Functions

```
function(arg1, arg2, ...){
  <the function code here...>
}
```

```
sum(5, 2)
```

```
## [1] 7
```

```
mean(1:4)
```

```
## [1] 2.5
```

Functions

```r
nchar("greetings")
```

Functions

```r
nchar("greetings")
```

```
## [1] 9
```

Functions

```r
nchar("greetings")
```

```
## [1] 9
```

```r
ls()
```

Functions

```r
nchar("greetings")
```

```
## [1] 9
```

```r
ls()
```

```
## character(0)
```

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

```r
c(1, 3, 8, 20)
```

```
## [1]  1  3  8 20
```

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

```
c(1, 3, 8, 20)
```

```
## [1]  1  3  8 20
```

```
c("a", "merican", "u")
```

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

```r
c(1, 3, 8, 20)
```

```
## [1]  1  3  8 20
```

```r
c("a", "merican", "u")
```

```
## [1] "a"        "merican" "u"
```

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

```
c(1, 3, 8, 20)
```

```
## [1]  1  3  8 20
```

```
c("a", "merican", "u")
```

```
## [1] "a"       "merican" "u"
```

```
c(1, 2, "hello")
```

A Useful Function: `c()`

To concatenate objects into a vector, use `c()`:

```
c(1, 3, 8, 20)
```

```
## [1]  1  3  8 20
```

```
c("a", "merican", "u")
```

```
## [1] "a"       "merican" "u"
```

```
c(1, 2, "hello")
```

```
## [1] "1"     "2"     "hello"
```

What arguments does a function have?

Functions' Arguments

What arguments does a function have?

```r
help(median)
args(median)
```

Functions' Arguments

What arguments does a function have?

```
help(median)
args(median)
```

```
## function (x, na.rm = FALSE, ...)
## NULL
```

Functions' Arguments

```r
median(1:3)
```

```
## [1] 2
```

Functions' Arguments

```r
median(1:3)
```

```
## [1] 2
```

```r
x <- c(1, 2, 3, NA)
median(x)
```

Functions' Arguments

```r
median(1:3)
```

```
## [1] 2
```

```r
x <- c(1, 2, 3, NA)
median(x)
```

```
## [1] NA
```

Functions' Arguments

```r
median(1:3)
```

```
## [1] 2
```

```r
x <- c(1, 2, 3, NA)
median(x)
```

```
## [1] NA
```

```r
median(x, na.rm = TRUE)
```

Functions' Arguments

```r
median(1:3)
```

```
## [1] 2
```

```r
x <- c(1, 2, 3, NA)
median(x)
```

```
## [1] NA
```

```r
median(x, na.rm = TRUE)
```

```
## [1] 2
```

Functions' Arguments

You can specify arguments in order or by name:

Functions' Arguments

You can specify arguments in order or by name:

```r
median(x, TRUE)
```

```
## [1] 2
```

Functions' Arguments

You can specify arguments in order or by name:

```r
median(x, TRUE)
```

```
## [1] 2
```

```r
median(na.rm = TRUE, x)
```

```
## [1] 2
```

Functions' Arguments

You can specify arguments in order or by name:

```r
median(x, TRUE)
```

```
## [1] 2
```

```r
median(na.rm = TRUE, x)
```

```
## [1] 2
```

```r
median(TRUE, x)
```

```
## [1] TRUE
```

Some Useful Functions

Managing the workspace:

```r
# Get the working directory ("Where am I?"):
getwd()
```

```
## [1] "/Users/rtm/Documents/github/ci-exp-essex-2019/r
```

Some Useful Functions

Managing the workspace:

```
# Get the working directory ("Where am I?"):
getwd()
```

```
## [1] "/Users/rtm/Documents/github/ci-exp-essex-2019/r
```

```
# Set the working directory:
setwd("~/Desktop/")
```

Some Useful Functions

Managing the workspace:

```r
# Get the working directory ("Where am I?"):
getwd()
```

```
## [1] "/Users/rtm/Documents/github/ci-exp-essex-2019/r
```

```r
# Set the working directory:
setwd("~/Desktop/")
```

```r
# List objects in working dir:
ls()
```

```
## [1] "x"
```

```r
# Remove `x' from working dir:
rm(x)
# Remove everything from working dir:
rm(list = ls())
```

Some Useful Functions

Making vectors:

```r
c(1, 2, 10)
```

```
## [1]  1  2 10
```

Some Useful Functions

Making vectors:

```r
c(1, 2, 10)
```

```
## [1]  1  2 10
```

```r
1:4
```

```
## [1] 1 2 3 4
```

```r
6:3
```

```
## [1] 6 5 4 3
```

Some Useful Functions

```r
seq(from = 5, to = 30, by = 5)
```

```
## [1]  5 10 15 20 25 30
```

Some Useful Functions

```r
seq(from = 5, to = 30, by = 5)
```

```
## [1]  5 10 15 20 25 30
```

```r
rep(c("a", "b"), 2)
```

```
## [1] "a" "b" "a" "b"
```

Some Useful Functions

```r
seq(from = 5, to = 30, by = 5)
```

```
## [1]  5 10 15 20 25 30
```

```r
rep(c("a", "b"), 2)
```

```
## [1] "a" "b" "a" "b"
```

```r
rep(c("a", "b"), each = 2)
```

```
## [1] "a" "a" "b" "b"
```

Extracting Elements from Vectors with [

```
x <- sample(0:1, size = 10, replace = TRUE)
```

Extracting Elements from Vectors with [

```r
x <- sample(0:1, size = 10, replace = TRUE)
```

```r
x
```

```
##  [1] 0 0 0 0 1 0 1 0 0 1
```

Extracting Elements from Vectors with [

```r
x <- sample(0:1, size = 10, replace = TRUE)
```

```r
x
```

```
##  [1] 0 0 0 0 1 0 1 0 0 1
```

```r
x[3]
```

```
## [1] 0
```

Extracting Elements from Vectors with [

```r
x <- sample(0:1, size = 10, replace = TRUE)
```

```r
x
```

```
##  [1] 0 0 0 0 1 0 1 0 0 1
```

```r
x[3]
```

```
## [1] 0
```

```r
x[3] <- 999
```

Extracting Elements from Vectors with [

```
x <- sample(0:1, size = 10, replace = TRUE)
```

```
x
```

```
##  [1] 0 0 0 0 1 0 1 0 0 1
```

```
x[3]
```

```
## [1] 0
```

```
x[3] <- 999
```

```
x
```

```
##  [1]   0   0 999   0   1   0   1   0   0
```

Some Useful Mathematical Functions

```r
5 + 2
```

```
## [1] 7
```

```r
5 - 2
```

```
## [1] 3
```

```r
5 * 2
```

```
## [1] 10
```

```r
5 / 2
```

```
## [1] 2.5
```

Some Useful Mathematical Functions

```r
5 ^ 2
```

```
## [1] 25
```

```r
sqrt(25)
```

```
## [1] 5
```

```r
20 %% 3
```

```
## [1] 2
```

Some Useful Mathematical Functions and Values

```r
pi
```

```
## [1] 3.141593
```

```r
abs(-3)
```

```
## [1] 3
```

```r
exp(1)
```

```
## [1] 2.718282
```

```r
log(exp(2))
```

```
## [1] 2
```

```r
sin(pi / 2)
```

```
## [1] 1
```

Some Useful Mathematical Functions and Values

```r
pi
```

```
## [1] 3.141593
```

```r
abs(-3)
```

```
## [1] 3
```

```r
exp(1)
```

```
## [1] 2.718282
```

```r
log(exp(2))
```

```
## [1] 2
```

```r
sin(pi / 2)
```

```
## [1] 1
```

(See R Short Ref Card . . .)

Logicals

```r
TRUE
```

```
## [1] TRUE
```

```r
FALSE
```

```
## [1] FALSE
```

Logicals

```r
TRUE
```

```
## [1] TRUE
```

```r
FALSE
```

```
## [1] FALSE
```

```r
TRUE == FALSE
```

Logicals

```r
TRUE
```

```
## [1] TRUE
```

```r
FALSE
```

```
## [1] FALSE
```

```r
TRUE == FALSE
```

```
## [1] FALSE
```

Logicals

```r
c(1, 2) == c(1, 3)
```

Logicals

```r
c(1, 2) == c(1, 3)
```

```
## [1]  TRUE FALSE
```

Logicals

```r
c(1, 2) == c(1, 3)
```

```
## [1]  TRUE FALSE
```

```r
c(1, 2) != c(1, 3)
```

Logicals

```r
c(1, 2) == c(1, 3)
```

```
## [1]  TRUE FALSE
```

```r
c(1, 2) != c(1, 3)
```

```
## [1] FALSE  TRUE
```

Logicals

```
c(1, 2) == c(1, 3)
```

```
## [1]  TRUE FALSE
```

```
c(1, 2) != c(1, 3)
```

```
## [1] FALSE  TRUE
```

```
c(1, 2) < c(1, 3)
```

Logicals

```
c(1, 2) == c(1, 3)
```

```
## [1]  TRUE FALSE
```

```
c(1, 2) != c(1, 3)
```

```
## [1] FALSE  TRUE
```

```
c(1, 2) < c(1, 3)
```

```
## [1] FALSE  TRUE
```

Logicals

```r
c(1, 2) > c(1, 3)
```

```
## [1] FALSE FALSE
```

```r
c(1, 2) <= c(1, 3)
```

```
## [1] TRUE TRUE
```

```r
c(1, 2) >= c(1, 3)
```

```
## [1]  TRUE FALSE
```

How to Write a New Function

```
sumDiff <- function(num1 = 3, num2 = 5){

  sum <- num1 + num2

  diff <- num1 - num2

  return(c(sum, diff))
}
```

How to Write a New Function

```r
sumDiff <- function(num1 = 3, num2 = 5){

  sum <- num1 + num2

  diff <- num1 - num2

  return(c(sum, diff))
}
```

Now, cut and paste function into R prompt.

How to Write a New Function

```r
sumDiff <- function(num1 = 3, num2 = 5){

  sum <- num1 + num2

  diff <- num1 - num2

  return(c(sum, diff))
}
```

Now, cut and paste function into R prompt.

(R will tell you if syntax error.)

My New Function

```
sumDiff()
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

```
## [1]  8 -2
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

```
## [1]  8 -2
```

```r
sumDiff(num2 = 5, num1 = 3)
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

```
## [1]  8 -2
```

```r
sumDiff(num2 = 5, num1 = 3)
```

```
## [1]  8 -2
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

```
## [1]  8 -2
```

```r
sumDiff(num2 = 5, num1 = 3)
```

```
## [1]  8 -2
```

```r
sumDiff(5, 3)
```

My New Function

```r
sumDiff()
```

```
## [1]  8 -2
```

```r
sumDiff(3, 5)
```

```
## [1]  8 -2
```

```r
sumDiff(num2 = 5, num1 = 3)
```

```
## [1]  8 -2
```

```r
sumDiff(5, 3)
```

```
## [1] 8 2
```

My New Function

```r
sumDiff(2, 20)
```

My New Function

```r
sumDiff(2, 20)
```

```
## [1]  22 -18
```

My New Function

```r
sumDiff(2, 20)
```

```
## [1]  22 -18
```

```r
sumDiff(1, "yes")
```

My New Function

```r
sumDiff(2, 20)
```

```
## [1]  22 -18
```

```r
sumDiff(1, "yes")
```

```
## Error in num1 + num2: non-numeric argument
```

Next:
Randomized Experiments,
Estimation, and Inference