Two-Stream Radiative Transfer Model of ED2.2

Emma Van de Walle

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1 Introduction

The two-stream radiative transfer model (RTM) of the Ecosystem Demography model version 2.2, ED2.2, provides a framework to calculate surface albedo and light absorption in a canopy, while allowing for multiple reflections by leaves and branches. It reduces the multidirectional scattering into 2 discrete fluxes of radiation in the upward and downward directions.

2 Advantages and disadvantages of the two-stream approximation

On wikipedia, a large list of (almost all) atmospheric RTMs can be found. Each of them have their own benefits and limitations and are used in a broad spectrum of applications ([1]).
There exist several applications of the two-stream model. Such an example is the Kubelka-Munk application. It is the simplest approach that can be used to explain general observations that cannot be explained by single-scattering arguments. It concerns, for example, the brightness and color of the sky, the brightness of clouds, the whiteness of a glass of milk and the darkening of sand when it becomes wet. Apart from the Kubelka-Munk application, the two-stream approximation is also a frequently used method for parameterizing radiative transport in global circulation models and weather forecasting models.

In addition to various applications, there are different model variants of the two-stream model. These include the Eddington approximation, the modified Eddington approximation, the Quadrature approximation, Hemispheric constant approximation and the Delta funciton approximation. The model variants differ in the way the scattering, absorption, and transmission of diffuse and direct sunlight are parameterized. ED2.2 uses a two-stream RTM based on the RTM from the 'CLM-CN' (Community Land Model - Carbon

Nitrogen). The Community Land Model is a component of the Community Earth System Model (CESM), which is a widely used earth system model for studying climate and biogechemical cycles. The CLM-CN utilizes the Eddington approximation in modeling radiative transfer within vegetation ([2], [3]). The goal is to model the distribution of radiation within the medium, taking into account reflection, absorption and transmission of radiation by the various components of the medium. The approximation is often expressed in terms of an Eddington factor, which indicates the fraction of scattered radiation relative to the total radiation. This factor is frequently adjusted based on the characteristics of the medium and the desired level of accuracy in the simulation.

Advantages

- Radiation is modeled in only two directions (up and down) ⇒ computationally less expensive
- Relatively easy to implement compared to more complex radiation transport models* ⇒ accessible for a wide range of applications
- It is highly suitable for climate models due to the compromise between accuracy and computational efficiency ⇒ suitable for large-scale climate modeling
- It can help in understanding the fundamental processes of radiation transport without the full complexity of the problem ⇒ fundamental insight into how radiation is absorbed, scattered and transmitted

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More complex models generally have the advantage of greater accuracy and a more detailed representation of atmospheric processes. However, they are often computationally more intensive and require more advanced computer systems. For example, there is DISORT (Discrete Ordinate RT), which takes into account different discrete radiation directions and considers various atmospheric parameters such as aerosols and clouds. In addition, DART (Discrete Anisotropic RT) is a model that also considers various discrete radiation directions and, moreover, radiation intensity is direction-dependent (anisotropic). In general, the RTMs we are interested in for vegetation modeling are models that simulate what happens with radiation within the canopy layers. Hence, the quantity and quality of radiation that the forest receives at the top of the canopy is considered as an input for our vegetation models. As a consequence, we are not aiming to improve radiative transfer above the canopy layers, in the atmosphere.

Disadvantages

- It does not consider a detailed radiation distribution (only 2 discrete ordinates & isotropy vs. anisotropy) ⇒ less accurate
- It does not consider a detailed spectral distribution (wavelengths), e.g. ED2.2 uses an average wavelength for both VIS and NIR and like this, runs ED-RTM just twice ⇒ less accurate

- It heavily relies on assumptions (parametrization) ⇒ can become unreliable
- It can sometimes overestimate reflected light ⇒ overestimation of albedo

3 Forest structure

ED2.2 is characterized by a hierarchical structure, where the fundamental unit of analysis is a plant cohort. A plant cohort refers to a collective of individual plants sharing similar characteristics such as size, age and species, categorized into plant functional types (PFTs). Patches, defined by a fraction of the landscape with competing cohorts and with a shared disturbance history (time since the last disturbance), form larger units. Sites are constituted by co-located patches experiencing the same meteorological and soil conditions.

The default ED2.2 theory treats the canopy as an infinitely long, plane-parallel and horizontally homogeneous layer of leaves randomly distributed in space or as multiple such layers distributed vertically through the canopy. The nonrandom distribution of foliage can be accounted for with a clumping factor q_i where i refers to a cohort type. In addition to this flat topped crown approach, there are other approaches like the perfect plasticity approximation (PPA) in which canopies are given a shape and depth ([4]). The PPA is a model of forest dynamics that is characterized by parameters for individual trees, encompassing aspects such as allometry, growth, and mortality. In the context of perfect plasticity, each tree has the flexibility to position its crown area at any location in the horizontal plane, allowing for adjustments to its crown shape or even the fragmentation of its crown into discontinuous segments ([5]). It takes into account the growth of individual tree specific crown shapes, phototropism and lets all tree crowns intersect at a certain height $Z^*(t)$ as shown in figure 1. Like this, the canopy structure output from a PPA model could be used as a starting point to run the ED-RTM instead of assuming a flat-topped canopy.
The default submodel configurations of ED2.2 are given in tabel 1 ([6]). It is the two-stream RTM that we will further discuss in this report. Apart from papers from Shiklomanov et al., the book of Gordon Bonan, 'Climate Change and Terrestrial Ecosystem Modeling' ([7]), was an interesting source to further delve into the theory explained in the Shiklomanov papers.

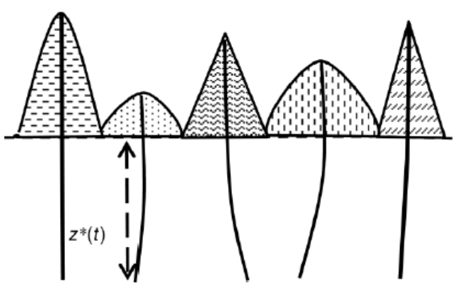

Figure 1: Example of the canopy structure, modeled by a Perfect Plasticity Approximation model [8].

Figure 2 shows such a default patch. Especially the crown model can be recognized. This fully closed crown model ensures that cohorts with their canopy layer below other cohorts

Table 1: Default ED2.2 configurations

Model	Model type	Description
Radiative transfer	two-stream	two-stream approximation
Crown	fully closed	cohort crowns take up the entire patch area, competition for light is purely based on height
Trait plasticity	static	specific leaf area (SLA) and maximum rate of RuBisCO carboxylase activity at 15°C ($V_{c,max}$) are constant

are fully shaded, i.e., only the highest tree in the patch receives full sunlight and is shading all smaller trees in the patch. In this figure, the highest cohort is labeled as 'N' and the lowest as '1'. In this report, a cohort with a value greater than the value of another cohort is positioned above the latter. Therefore, in the code itself, when simulating the radiative transfer from the upper canopy layer to the lower canopy layer, the iteration happens from N to 1.

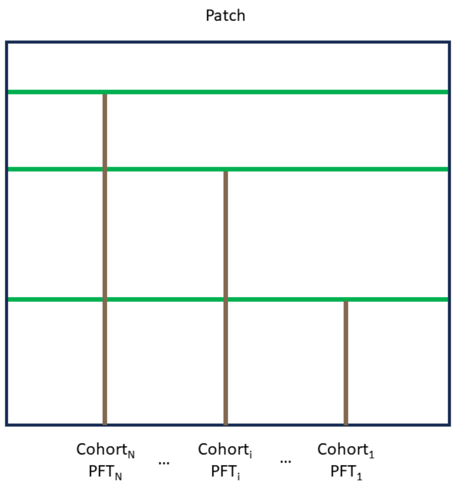

Figure 2: Default patch structure in ED2.2

Other possible submodel configurations are given in table 2.

Table 2: Other ED2.2 configurations

Model	Model type	Description
Radiative transfer	multi-scatter	multi-scatter approximation
Crown	finite	cohort crown area is proportional to diameter at breast height according to PFT-specific allometry
Trait plasticity	plastic	SLA increases and $V_{c,max}$ decreases with light level

4 Radiation types and input parameters

In the two-stream RTM, both diffuse radiation $I^{\downarrow}_{sky}$ and direct beam radiation $I^{\downarrow}_{sky,b}$ hit the upper canopy layer from the sky. The RTM resolves the full vertical radiation profile within a patch as a function of canopy structure and incident solar radiation. More specifically, in ED2.2, optical properties of a single canopy layer (i.e., a single cohort belonging to a PFT-type) depend on leaf and wood optical properties (reflectance R_{leaf} & R_{wood} and transmittance T_{leaf}; $T_{wood} = 0$ since wood is assumed not to transmit any light), leaf orientation factor χ_l (related to leaf angle distribution G), canopy clumping q, solar zenit angle z, leaf area index (LAI), wood area index (WAI) and canopy area index (CAI). These parameters, together with soil saturation and the fraction of direct beam radiation (wrt total radiation), are used as input parameters to solve for the vertical radiation profile.
In ED2.2, 17 PFT can be chosen from (see table 3), each of them having more than 100 parameters. Every submodel uses several of these parameters.

Table 3: 17 PFT types of ED2.2

PFT type	PFT name	PFT type	PFT name
1	C4 grass	10	Mid hardwood
2	Early tropical	11	Late hardwood
3	Mid tropical	12	Early savannah
4	Late tropical	13	Mid savannah
5	Temperate c3 grass	14	Late savannah
6	Northern pines	15	Araucaria
7	Southern pines	16	Tropical C3 grass
8	Late conifers	17	Liana
9	Early hardwood		

5 Radiative transfer

Direct beam radiation is represented by an exponentially decaying process** and the RTM solves for the diffuse (isotropic) radiation at each canopy layer using a linear matrix equation. The solution of the matrix equation results in the distribution of light intensity over the several depth layers of the forest. This is a discrete approximation (cohort layer after cohort layer) of the original differential equations and can be solved using linear algebra techniques.

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Every time direct sunlight passes a cohort layer (with thickness dx), it undergoes a reduction determined by the extinction coefficient K_b. We can represent this using the following formula:

$$\frac{dI_b^{\downarrow}}{dx} = -K_b I_b^{\downarrow}$$

If we integrate this from the upper cohort layer, where $I_b^{\downarrow} = I_{sky,b}^{\downarrow}$, to a certain layer where the cumulative leaf area index is X, we get:

$$I_b^{\downarrow}(X) = I_{sky,b}^{\downarrow} e^{-K_b X}$$

This is the fraction of the direct beam that still reaches the depth at which the cumulative leaf area index is X. In ED2.2, X is actually defined as the Total Area Index (TAI), the sum of the Leaf Area Index (LAI) and the Wood Area Index (WAI). In literature, this TAI is usually called PAI (Plant Area Index).

We will now derive the differential equations that are used in the two-stream RTM of ED2.2. These equations are equations (1) and (2) in [9]. Eventually, we will discretize the equations and show how to translate this into a linear matrix equation. Before we derive the equations, we define following symbols:

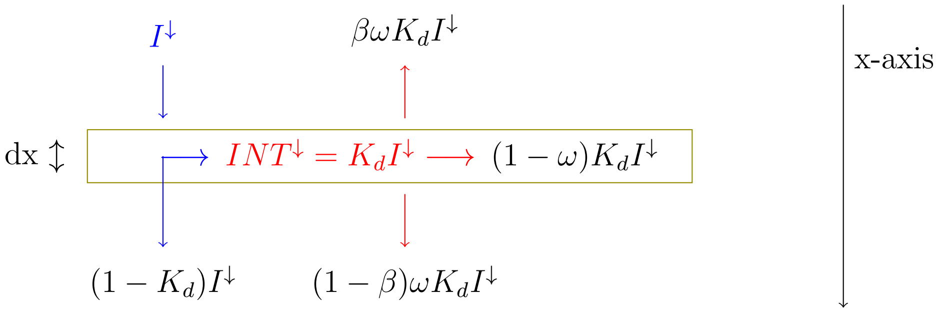
$I^{\downarrow}$ = downward diffuse radiation
$I^{\uparrow}$ = upward diffuse radiation
$I_{sky,b}^{\downarrow}$ = downward direct beam radiation at the top of the upper canopy layer
$I_{sky,b}^{\downarrow} e^{-K_b X}$ = direct beam radiation at the cumulative total area index X
ω = scattering coefficient (related to the probability of scattering)
β = fraction of the scattered diffuse radiation in the backward direction
$(1-\beta)$ = fraction of the scattered diffuse radiation in the forward direction
    $\Rightarrow \omega\beta$ = fraction of the diffuse light that is scattered in the backward direction due to scattering
β_0 = fraction of the scattered direct beam radiation in the backward direction
$(1-\beta_0)$ = fraction of the scattered direct beam radiation in the forward direction
K_d = diffuse extinction coefficient = the optical depth per unit leaf area. An extinction coefficient represents a measure of the extent to which light is absorbed and scattered when it passes a particular substance. The definition of this coefficient will later be explained.
K_b = direct beam extinction coefficient

We will now look at the downward diffuse radiation $I^{\downarrow}$, the upward diffuse radiation $I^{\uparrow}$ and the direct beam radiation $I_b^{\downarrow}(X)$ at a certain canopy layer with a cumulative total area index X. The light interactions within and around the canopy layer (olive green rectangulars) can be investigated in the 3 figures below.

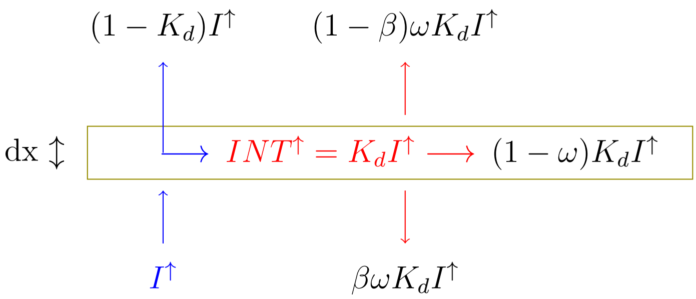
Focussing on the downward diffuse radiation $I^{\downarrow}$, we can see that an amount $INT^{\downarrow} = K_d I^{\downarrow}$ of $I^{\downarrow}$ is intercepted and depends on the diffuse extinction coefficient K_d. Another part $(1-K_d)I^{\downarrow}$ is not intercepted, i.e., this part of $I^{\downarrow}$ does not interact with any leaves or wood segments of the cohort layer. From the intercepted part, an amount $\beta\omega K_d I^{\downarrow}$ is

backscattered, an amount $(1-\beta)\omega K_d I^{\downarrow}$ is scattered in the forward direction and an amount $(1-\omega)K_d I^{\downarrow}$ is absorbed. In other words, $(1-\omega)$ can be defined as the absorption coefficient a. Assuming no light is lost, the absorption coefficient a, reflection coefficient R and transmission coefficient T sum up to 1: $a+R+T=1$.
In an analogous way, one can interpret the interactions of the upward diffuse radiation $I^{\uparrow}$ and the direct beam radiation $I_b^{\downarrow}(x)$.

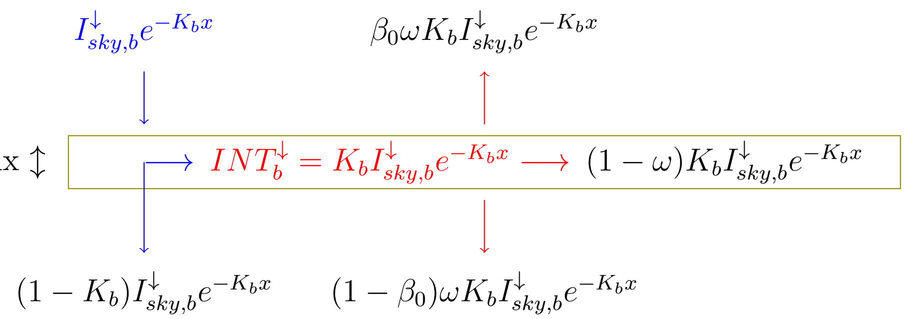
Downward diffuse



Upward diffuse



Direct beam



Formulae

If we investigate the figures and look at the downward and upward diffuse radiation, we can see that they change through the canopy layer, along the **direction** of the x-axis as follows:

$$I^{\downarrow} \longrightarrow (1-K_d)I^{\downarrow} + (1-\beta)\omega K_d I^{\downarrow} + \beta\omega K_d I^{\uparrow} + (1-\beta_0)\omega K_b I_{sky,b}^{\downarrow}e^{-K_b x}$$

$$I^{\uparrow} \longrightarrow (1-K_d)I^{\uparrow} + (1-\beta)\omega K_d I^{\uparrow} + \beta\omega K_d I^{\downarrow} + \beta_0 \omega K_b I^{\downarrow}_{sky,b} e^{-K_b x}$$

This results in following differential equations for te downward and upward diffuse radiation, along the **sense** (= orientation) of the x-axis:

$$\frac{dI^{\downarrow}}{dx} = -K_d I^{\downarrow} + (1-\beta)\omega K_d I^{\downarrow} + \beta\omega K_d I^{\uparrow} + (1-\beta_0)\omega K_b I^{\downarrow}_{sky,b} e^{-K_b x} \tag{1}$$

$$\frac{-dI^{\uparrow}}{dx} = -K_d I^{\uparrow} + (1-\beta)\omega K_d I^{\uparrow} + \beta\omega K_d I^{\downarrow} + \beta_0 \omega K_b I^{\downarrow}_{sky,b} e^{-K_b x} \tag{2}$$

The minus sign in (2) is a result of the sense of the x-asis. We can simplify equations (1) and (2) to get the following equations:

$$\frac{dI^{\downarrow}}{dx} = -[1-(1-\beta)\omega]K_d I^{\downarrow} + \beta\omega K_d I^{\uparrow} + (1-\beta_0)\omega K_b I^{\downarrow}_{sky,b} e^{-K_b x} \tag{3}$$

$$\frac{dI^{\uparrow}}{dx} = [1-(1-\beta)\omega]K_d I^{\uparrow} - \beta\omega K_d I^{\downarrow} - \beta_0 \omega K_b I^{\downarrow}_{sky,b} e^{-K_b x} \tag{4}$$

Equations (1) and (2) in [9] still look a bit different, but let's define the following symbols for a cohort i:

$F_i^{\downarrow} = I_i^{\downarrow}$, the downward diffuse radiation below canopy layer i
$F_i^{\uparrow} = I_i^{\uparrow}$, the upward diffuse radiation below canopy layer i
$F_i^{\odot} = I^{\downarrow}_{sky,b} e^{-K_{bi} x_i}$
$a_i = (1-\omega_i)K_{di}$, the absorption coefficient
$\gamma_i = \beta_i \omega_i K_{di}$
$s_i = \beta_{0i} \omega_i K_{bi}$
$s'_i = (1-\beta_{0i})\omega_i K_{bi}$
$K_{di} = \frac{1}{\bar{\mu}_i}$, μ is named the inverse of the optical depth per unit plant area index. In a simpler RTM model, it could be defined as $K_{di} = K_d \cdot TAI_i$.
$K_{bi} = \frac{1}{\mu_i^{\odot}}$

Indeed, with these newly defined symbols, we can rewrite (3) and (4) as:

$$\frac{dF_i^{\downarrow}}{dx} = -(a_i + \gamma_i)F_i^{\downarrow} + \gamma_i F_i^{\uparrow} + s'_i F_i^{\odot} \tag{5}$$

$$\frac{-dF_i^{\uparrow}}{dx} = -(a_i + \gamma_i)F_i^{\uparrow} + \gamma_i F_i^{\downarrow} + s_i F_i^{\odot} \tag{6}$$

And these equtions are the same ones as defined in [9].
Before we can solve (5) and (6) by integrating over all canopy cohorts i, we have to define and determine several parameters. Within the equations (5) and (6), one distinguishes the parameters a, γ, s' and s. On their term, they are dependent on ω, β, β_0, K_d and K_b. We will now further define these latter parameters, starting with the extinction coefficients K_d and K_b.

Diffuse extinction coefficient K_d

As mentioned before, the diffuse extinction coefficient represents a measure of the extent to which light is absorbed and scattered when it passes a particular substance. Let's now define that 'particular substance' as the canopy of a forest and let's consider a specific ray of diffuse radiation coming from the zenit angle θ ($\theta = 0$ refers to radiation coming directly from above) and let's assume a leaf orientation angle ϕ. Like this, we can define the diffuse extinction coefficient as:

$$K_d = \frac{1}{\bar{\mu}} = \frac{G(\theta, \phi)}{cos(\theta)}$$

with $G(\theta)$ the simplified Goudrian leaf angle distribution, the relative projected area of leaf elements in the direction of θ, defined as:

$$G(\theta, \chi_l) = \phi_1 + \phi_2 cos(\theta)$$

with ϕ_1 and ϕ_2, parameters defined as:

$$\phi_1 = 0.5 - 0.633\chi_l - 0.33\chi_l^2$$

$$\phi_2 = 0.877 \cdot (1 - 2\phi_1)$$

In the equations of ϕ_1 and ϕ_2, χ_l is known as the Ross Index or the leaf orientation factor and takes a value between -0.4 and 0.6. The Ross Index quantifies the departure of leaf angles from a spherical distribution, i.e., the deviation of the leaf angles from a spherical pattern. χ_l is PFT-specific and theoretically ranges from -1 (vertical leaves) to 0 (randomly distributed leaves) to 1 (horizontal leaves), but in practice, it is limited to $-0.4 < \chi_l < 0.6$.
As follows from the definition of diffuse radiation, it comes from various directions, in contrast with the direct solar beam radiation. For this, we have to account for all possible zenit angles θ from which diffuse radiation might come, i.e., θ ranging from 0 till $\frac{\pi}{2}$ (upper hemisphere). Like this, we can integrate over θ to get the total diffuse extinction coefficient:

$$\frac{1}{K_d} = \int_0^{\frac{\pi}{2}} \frac{cos(\theta)}{G(\theta, \chi_l)} d\theta = \frac{1}{\phi_2} - \frac{\phi_1}{\phi_2^2} ln(\frac{\phi_1 + \phi_2}{\phi_1})$$

Because the Ross Index is a PFT-specific parameter, this parameter is set fixed at the beginning of the model run, and is said to be an input parameter.

Direct beam extinction coefficient K_b

In an analogous manner, for a given solar zenith angle (θ_s), K_b is defined as:

$$K_b = \frac{1}{\mu^{\odot}} = \frac{G(\theta_s, \chi_l)}{cos(\theta_s)} = \frac{\phi_1 + \phi_2 cos(\theta_s)}{cos(\theta_s)}$$

Remark:
The provided definitions for the extinction coefficients assumed a CAI (Canopy Area Index) of 1 for all cohorts, i.e., dense vegetation where the vegetation completely covers the ground surface. If CAI is not equal to 1, the definitions for the extinction coefficients

(or, consequently, the inverse optical depths) need to be adjusted. They then take the following, more general form:

$$\mu_{new}^{\odot} = \frac{-eTAI}{ln[(1-CAI)+CAI.exp[\frac{-eTAI}{\mu_{old}^{\odot}CAI}]]}$$

$$\overline{\mu}_{new} = \frac{-eTAI}{ln[(1-CAI)+CAI.exp[\frac{-eTAI}{\overline{\mu}_{old}}]]}$$

And in these equations, $eTAI$ is the effective total area index.

Scattering coefficient ω and fraction of the scattered diffuse radiation in the backward direction β

For each cohort i, both ω_i and β_i are calculated independently for leaves and wood and then averaged based on the relative effective area of leaves L_i (see next paragraph for the definition of the effective leaf area) and wood W_i within a canopy layer. Their definition is as follows:

$$\omega_i = \omega_{i,leaf}\frac{L_i}{L_i+W_i} + \omega_{i,wood}(1-\frac{L_i}{L_i+W_i}) \tag{7}$$

$$\beta_i = \beta_{i,leaf}\frac{L_i}{L_i+W_i} + \beta_{i,wood}(1-\frac{L_i}{L_i+W_i}) \tag{8}$$

To account for a non-uniform distribution of leaves in the canopy, ED defines a PFT-specific clumping factor q_i that rescales the LAI_i from the actual LAI_i to an effective LAI_i, L_i:

$$L_i = LAI_i.q_i$$

The clumping factor takes a value between 0 and 1 and so, from the equation above, it follows that an increasing clumping factor results in less leave clumping and vice versa. The LAI_i of a cohort is calculated as a function of leaf biomass ($B_{leaf,i}$; $\frac{kgC}{plant}$), specific leaf area (SLA_i ; $\frac{m^2}{kgC}$) and stem density ($n_{plant,i}$; $\frac{plants}{m^2}$):

$$LAI_i = n_{plant,i}B_{leaf,i}SLA_i$$

On its term, $B_{leaf,i}$ is calculated via the cohort diameter at breast hight (DBH_i) according to following allometric equation:

$$B_{leaf,i} = b_1B_{l,i}DBH_i^{b_2B_{l,i}}$$

In this equation, $b_1B_{l,i}$ and $b_2B_{l,i}$ are PFT-specific input parameters. In a complete analogous way, one defines WAI_i:

$$WAI_i = n_{plant,i}b_1B_{w,i}DBH_i^{b_2B_{w,i}}$$

And again, $b_1B_{w,i}$ and $b_2B_{w,i}$ are PFT-specific input parameters.
Parametres that define LAI and WAI, such as SLA, DBH and B_{leaf}, are determined from submodels of ED2.2, such as the carbon allocation and photosynthesis model.

Diffuse scattering (ω) and backscattering (β) coefficients of canopy elements are defined as a function of those elements' reflectance (R) and transmittance (T). For this, $\omega_{i,leaf}$,

$\omega_{i,wood}$, $\beta_{i,leaf}$ and $\beta_{i,wood}$ in equations (7) and (8) are defined by the input spectra (reflectance R and transmittance T) (both for VIS and NIR) for every single PFT. The coefficients take following format:

$$\omega_{i,leaf} = R_{i,leaf} + T_{i,leaf}$$

$$\omega_{i,wood} = R_{i,wood}; \ (T_{i,wood} = 0)$$

$$\beta_{i,leaf} = \frac{1}{2\omega_{i,leaf}}[R_{i,leaf} + T_{i,leaf} + (R_{i,leaf} - T_{i,leaf})J^2(\chi_i)]$$

$$\beta_{i,wood} = \frac{1}{2\omega_{i,wood}}[R_{i,wood} + R_{i,wood}J^2(\chi_i)]$$

in which $J(\chi_i)$ represents the leaf and branch inclination and is approximated by:

$$J(\chi_i) = \frac{1+\chi_i}{2}$$

χ_i is the leaf AND wood orientation factor, previously simplified as χ_l, the leaf orientation factor.

Fraction of the scattered direct beam radiation in the backward direction β_0

If equations (5) and (6) are solved for the limit $\omega \to 0$ (no factorisation in backscatter and transmission, but only the single scatter approximation applicable), $F^{\uparrow}$ at the top of the canopy may be taken as equal to the single scattering albedo ([10]). For this, the scattering coefficient β_0 is defined as a function of the single scattering albedo α_s (in the scattering direction θ_s):

$$\beta_{0i} = \frac{\overline{\mu}_i + \mu_i^{\odot}}{\overline{\mu}_i} . \frac{\alpha_{s,i}(\theta_s)}{\omega_i}$$

The single scatter albedo is defined as an integral over all illumination angles θ:

$$\alpha_s(\theta_s) = \omega \int_0^{\frac{\pi}{2}} \frac{\Gamma(\theta_s,\theta)cos(\theta)}{G(\theta_s,\phi)cos(\theta_s) + G(\theta,\phi)cos(\theta)} sin(\theta)d\theta$$

with

$$\Gamma(\theta_s,\theta) = G(\theta_s,\phi)G(\theta,\phi)P(\theta_s,\theta)$$

with $P(\theta_s,\theta)$ the scattering phase function that defines the relative fraction of scattered flux in any direction relative to the projected leaf area in that direction. It is defined as:

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} P(\theta_s,\theta)G(\theta,\phi)sin(\theta)d\theta = 1$$

If we use the simplified Goudrian for the (leaf and wood) angle distribution G, we get:

$$G_i(\theta_s,\phi) = \phi_{1,i} + \phi_{2,i}cos(\theta_s)$$

Assuming a uniform scattering, we get:

$$P(\theta_s,\theta) = \frac{1}{4\pi}$$

And so:

$$\Gamma(\theta_s,\phi) = \frac{G(\theta_s,\phi)}{2}$$

Using these difinitions for P and G, we get following analytical solution for $\alpha_{s,i}(\theta_s)$:

$$\alpha_{s,i}(\theta_s) = \frac{\omega_i}{2(1+\phi_{2,i}\mu_i^{\odot})} .[1 - \frac{\phi_{1,i}\mu_i^{\odot}}{1+\phi_{2,i}\mu_i^{\odot}} .ln(\frac{1+(\phi_{1,i}+\phi_{2,i})\mu_i^{\odot}}{\phi_{1,i}\mu_i^{\odot}})]$$

Solving the downward and upward diffuse radiation vertical profile

Now we have determined the five parameters ω, β, β_0, K_d and K_b for every cohort i, we can solve equations (3) and (4) and finally get following analytical solutions:

$$F_i^{\downarrow} = x_{(2i-1)}\gamma_i^{+}.exp[-\lambda_i.TAI_i] + x_{(2i)}\gamma_i^{-}.exp[+\lambda_i.TAI_i] + \delta_i^{+}.exp[\frac{-TAI_i}{\mu_i^{\odot}}] \tag{9}$$

$$F_i^{\uparrow} = x_{(2i-1)}\gamma_i^{-}.exp[-\lambda_i.TAI_i] + x_{(2i)}\gamma_i^{+}.exp[+\lambda_i.TAI_i] + \delta_i^{-}.exp[\frac{-TAI_i}{\mu_i^{\odot}}] \tag{10}$$

In equations (9) and (10), x_i is a vector with cohort specific unknowns and $\gamma_i^{\pm}$, $\delta_i^{\pm}$ and λ_i are defined as:

$$\gamma_i^{\pm} = \frac{1}{2}(1 \pm \sqrt{\frac{1-\omega_i}{1-(1-2\beta_i)\omega_i}})$$

$$\delta_i^{\pm} = \frac{(\kappa_i^{+} \pm \kappa_i^{-})\mu_i^{\odot 2}}{2(1-\lambda_i^2\mu_i^{\odot 2})}$$

$$\lambda_i^2 = \frac{[1-(1-2\beta_i)\omega_i](1-\omega_i)}{\overline{\mu}_i^{\,2}}$$

And in the equation for $\delta_i^{\pm}$, $\kappa_i^{\pm}$ are defined as:

$$\kappa_i^{+} = -[\frac{1-(1-2\beta_i)\omega_i}{\overline{\mu}_i} + \frac{1-2\beta_{0i}}{\mu_i^{\odot}}]\frac{\omega_i F_{(i+1)}^{\odot}}{\mu_i^{\odot}}$$

$$\kappa_i^{-} = -[\frac{(1-2\beta_{0i})(1-\omega_i)}{\overline{\mu}_i} + \frac{1}{\mu_i^{\odot}}]\frac{\omega_i F_{(i+1)}^{\odot}}{\mu_i^{\odot}}$$

For n cohorts, the full diffuse canopy radiation profile in ED2.2 is defined by a vector A of size 2n + 2 with $F_i^{\downarrow}$ and $F_i^{\uparrow}$ radiation fluxes for every interface immediately below cohort i. This means the following for the canopy and ground fluxes:

$F_{n+1}^{\uparrow}$ = upward diffuse radiation flux from the canopy top into the atmosphere. It is related to the canopy albedo ρ. $F_{n+1}^{\downarrow}$ = downward diffuse radiation flux from the atmosphere into the canopy top. $F_1^{\uparrow}$ = upward diffuse radiation flux from the ground to the canopy layer of the shortest cohort. $F_1^{\downarrow}$ = downward diffuse radiation flux from the shortest cohort towards the ground.

The canopy albedo is defined as:

$$\rho = \frac{F_{n+1}^{\uparrow}}{F_{sky}^{\odot} + F_{sky}^{\downarrow}}$$

Equations (9) and (10) can be rewritten in matrix format to solve for x_i:

$$Sx = y \tag{11}$$

with S a (2n + 2) x (2n + 2) pentadiagonal matrix and

$$x = (x_1, x_2 ... x_{2n+1}, x_{2n+2})$$

$$y = (y_1, y_2 ... y_{2n+1}, y_{2n+2})$$

To solve the matrix equation, ED2.2 defines following boundary conditions for the top of the canopy:

$F^{\downarrow}_{n+1} = F^{\downarrow}_{sky}$, a prescribed input
$TAI_{n+1} = 0$
$\overline{\mu}_{n+1} = \mu^{\odot}_{n+1} = 1$
$\omega_{n+1} = 1$, no absorption, only scattering (in forward direction)
$\beta_{n+1} = 0$, no backward scattering, all radiance is transmitted (i.e., scattered forwards)
$\beta^{\odot}_{n+1} = 2$
$\gamma^{+}_{n+1} = 1$
$\gamma^{-}_{n+1} = 0$
$\kappa^{+}_{n+1} = 3F^{\odot}_{sky}$
$\kappa^{-}_{n+1} = -F^{\odot}_{sky}$
$\delta^{+}_{n+1} = \frac{1}{2} \cdot (\kappa^{+}_{n+1} + \kappa^{-}_{n+1})$
$\delta^{-}_{n+1} = \frac{1}{2} \cdot (\kappa^{+}_{n+1} - \kappa^{-}_{n+1})$

In equation (11), the elements of S take the following format:

$$S_{1,1} = (\gamma^{-}_1 - \omega_g \gamma^{+}_1) exp[-\lambda_1 TAI_1] \tag{12}$$

$$S_{1,2} = (\gamma^{+}_1 - \omega_g \gamma^{-}_1) exp[+\lambda_1 TAI_1] \tag{13}$$

$$S_{2i,2i-1} = \gamma^{+}_i$$

$$S_{2i,2i} = \gamma^{-}_i$$

$$S_{2i,2i+1} = -\gamma^{+}_{i+1} exp[-\lambda_{i+1} TAI_{i+1}]$$

$$S_{2i,2i+2} = -\gamma^{-}_{i+1} exp[+\lambda_{i+1} TAI_{i+1}]$$

$$S_{2i+1,2i-1} = \gamma^{-}_i$$

$$S_{2i+1,2i} = \gamma^{+}_i$$

$$S_{2i+1,2i+1} = -\gamma^{+}_{i+1} exp[-\lambda_{i+1} TAI_{i+1}]$$

$$S_{2i+1,2i+2} = -\gamma^{-}_{i+1} exp[+\lambda_{i+1} TAI_{i+1}]$$

$$S_{2n+2,2n+1} = \gamma^{+}_{n+1}$$

$$S_{2n+2,2n+2} = \gamma^{-}_{n+1}$$

In equations (12) and (13), ω_g is the ground scattering (one assumes no ground transmittance, only ground scattering and ground absorption). In ED2.2, the ground scattering is defined by an input parameter 'soil saturation', i.e., the soil reflectance is defined as follows:

$$R_{soil} = R_{wet\ soil}.(soil\ moisture) + R_{dry\ soil}.(1 - soil\ moisture)$$

And soil saturation equals 0 for a fully dry soil and equals 1 for a fully wet soil. $R_{wet\ soil}$ and $R_{dry\ soil}$ are prescribed input spectra.
In addition, in equation 11, the elements of y take the following form:

$$y_1 = \omega_g F_1^{\odot} - (\delta_1^- - \omega_g \delta_1^+) exp[\frac{-TAI_1}{\mu_1^{\odot}}]$$

$$y_{2i} = \delta_{i+1}^+ exp[\frac{-TAI_{i+1}}{\mu_{i+1}^{\odot}}] - \delta_i^+$$

$$y_{2i+1} = \delta_{i+1}^- exp[\frac{-TAI_{i+1}}{\mu_{i+1}^{\odot}}] - \delta_i^-$$

$$y_{2n+2} = F_{sky}^{\downarrow} - \delta_{n+1}^+$$

These radiative transfer calculations are solved in 2 broad spectral bands: VIS ($\lambda = 400 - 700nm$) and NIR ($\lambda = 700 - 2500nm$). As mentioned before, for both bands, ED2.2 uses PFT-specific reflection and transmission spectra, both for leaves and wood, as input parameters.
Because most of the parameters are independent of λ (expect for R and T) the choice of using two bands is a choice to make the model fast and the above theory is valid for every wavelength. That is why in ED-RTM, many different wavelengths can be investigated.

6 Translation of the ED2.2 two-stream RTM into the programming language R

The entire ED2.2 is written in Fortran, but there already exists an R version of the ED-RTM. Its R-package is called RRTM and can be found on Github ([11]).
In RRTM, ED-RTM is coupled with the PROSPECT-5 leaf radiative transfer model to simulate R_{leaf} and T_{leaf}. For this purpose, 5 additional parameters are required. You can find them in the box below.

N = effective number of leaf mesophyl layers, unitless, >= 1
C_{ab} = total chlorophyl content, $\frac{\mu g}{cm^2}$
C_{ar} = total carotenoid content, $\frac{\mu g}{cm^2}$
C_w = water content, $\frac{g}{cm^2}$
C_m = dry matter content, $\frac{g}{cm^2}$

Wood is assumed not to transmit any radiance, so T_{wood} equals 0. Regarding R_{wood}, RRTM uses a prescribed input spectrum.
In comparison with ED2.2, RRTM uses the radiative transfer model in a single wavelength range, from 400 to 2500 nm, with increments of 1 nm and not in 2 spectral bands, as is the case for ED2.2. This means that ED2.2 runs the RTM simulation twice, once for an average VIS value and once for an average NIR value, whereas RRTM runs the RTM 2101 times, for each wavelength between 400 and 2500 nm. Hence, the runtime of the ED-RTM is shorter, but RRTM considers a more detailed spectral distribution.

Some examples of RRTM

The orientation factor of leaves and branches χ and the clumping factor q of leaves in the canopy layer of a certain PFT are the two most undetermined model input parameters and, nevertheless, have a significant impact on the output if changed. For this, with changing χ and q, examples of the albedo output of RRTM are given in the figures below. While changing χ and q, other input parameters of RRTM were kept fixed. These fixed parameters are given in table 4. In the simulations, we choose 1 cohort of PFT 1 and 2 cohorts of PFT 2. The cohort of PFT 1 is the tallest, followed by two shorter cohorts of PFT 2.

Table 4: Fixed RRTM input parameters

	PFT 1	1^{st} cohort of PFT 2	2^{nd} cohort of PFT 2
Cohort state variables			
PFT	1	2	2
LAI	2	1	0.8
WAI	0.01	0.01	0.01
CAI	1	1	1
PFT parameters PROSPECT5			
N	1.4	1.5	
C_{ab}	40	30	
C_{ar}	8	8	
C_w	0.01	0.01	
C_m	0.008	0.01	
Global parameters			
Soil saturation	0.5		
Direct sky fraction	0.9		
$cos(z)$, z = solar zenit	1		

χ and q are PFT parameters and in the figures below, these parameters are indexed by their PFT. Hence, in the 'reference' simulation of figure 3, PFT 1 has a clumping factor of 0.51, while the two cohorts of PFT 2 have a clumping factor of 0.49. In addition, as 'reference' orientation factors, we choose for both PFTs a value of 0.1. This means that the leaves are roughly spherically distributed ($q = 0$ refers to randomly oriented leaves).

From figure 4, we notice that decreasing the clumping factors, increases the overall albedo of the forest quite strongly, while increasing clumping factors result in only a very small increase in albedo. At a lower clumping factor, leaves are more clumped, resulting in a lower effective LAI. This results in more radiation being captured and reflected by the soil wrt the portion reflected and absorbed by the leaves. In this case, the role of the soil in light reflection is enhanced and this effect particularly contributes to increased reflection in longer wavelengths (NIR).
When we set the clumping factors to roughly 0, the effective LAIs become almost 0, resulting in an albedo equivalent to the ground albedo. (In RRTM, clumping factors have to be set > 0.) This is illustrated in figure 5.

A more clear relation between the forest albedo and the orientation factor can be derived from figure 6. A decreasing orientation factor (more vertically oriented leaves), results in

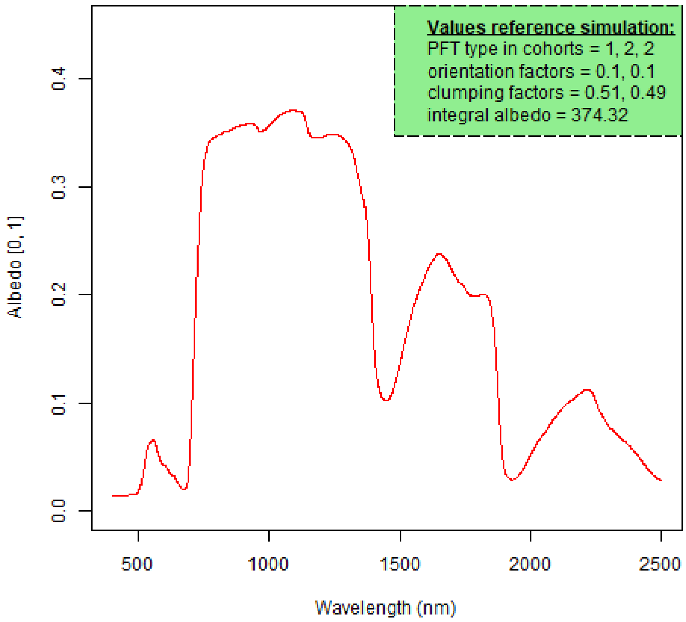
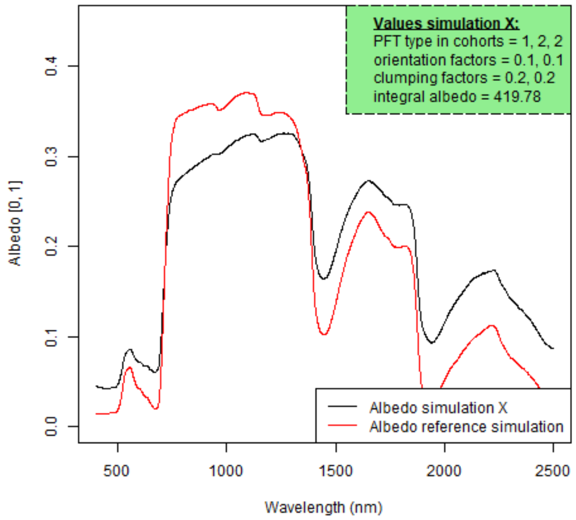


Figure 3: Reference RRTM simulation

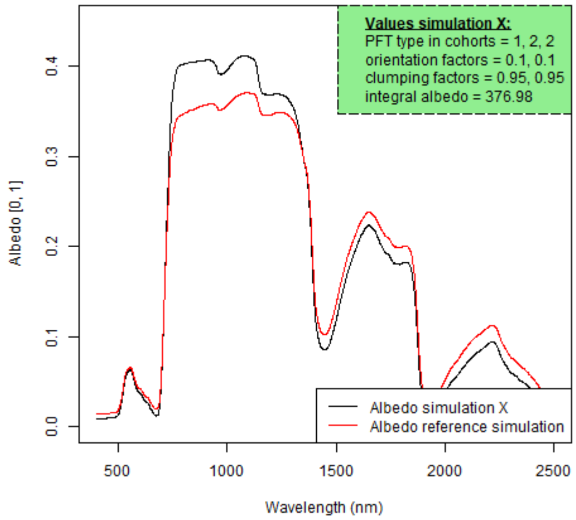
a decreasing forest albedo. An increasing orientation factor (more horizontally oriented leaves), increases the forest albedo. This can be explained as follows. Vertically oriented leaves generally have a lower albedo than horizontally oriented leaves because they can more directly absorb sunlight. When sunlight comes from above, vertically oriented leaves capture more light on their surface. As a result, a greater portion of the sunlight is absorbed for photosynthesis and other processes.
Horizontally oriented leaves, on the other hand, will reflect more sunlight because they present a larger portion of their surface perpendicular to the sun's rays. This can assist in reducing the amount of absorbed heat, which can be particularly beneficial in warm climates.

This can also be explained in more physical terms. When light strikes nearly perpendicular to a surface, a large portion of the light is typically reflected and only a small portion is absorbed. This is because the light has less chance of being absorbed into the surface and a greater chance of being reflected. On the other hand, at a high angle of incidence (almost parallel to the surface), the light has a greater chance of being absorbed and a lesser chance of being reflected.

However, the angle of incidence for maximum reflection also depends on the surface properties and may not solely be determined by the incident angle due to several factors. Microscopic structures such as hairs and waxy layers, the optical properties varying with light wavelength, the chemical composition influencing pigments and the inherent variability in leaf structures can all play roles in affecting reflectance. These surface characteristics interact with the angle of incident light, contributing to a nuanced relationship between solar angle and reflectance in leaves.



(a) Low clumping factors



(b) High clumping factors

Figure 4: Changing clumping factors wrt the 'reference' simulation

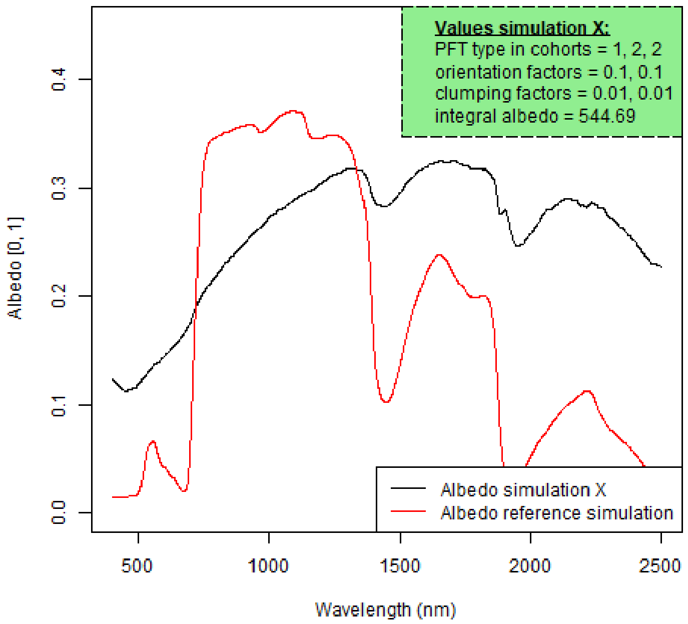
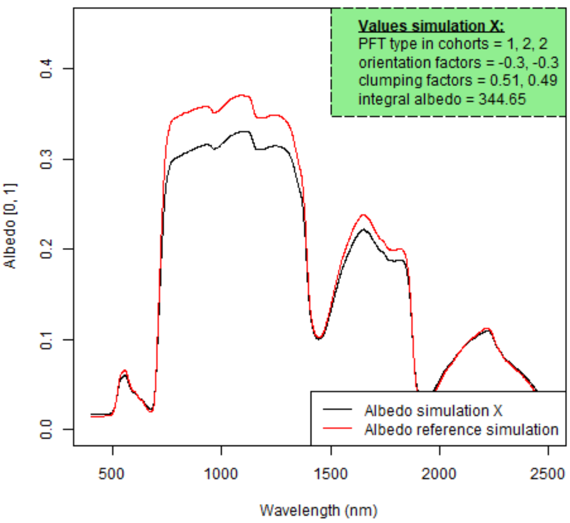
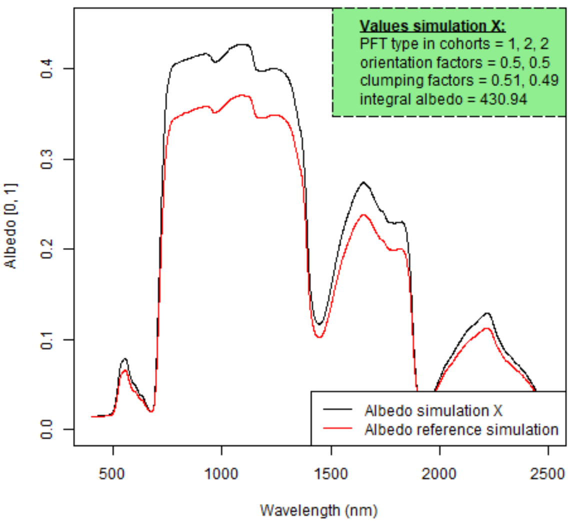


Figure 5: RRTM simulation with clumping factors of 0.01, resulting in roughly the ground albedo



(a) Low orientation factors



(b) High orientation factors

Figure 6: Changing orientation factors wrt the 'reference' simulation

7 Translation of the ED2.2 two-stream RTM into the programming language julia

At CAVElab of Ghent University, we also developed a Julia version of this two-stream RTM. We called it JRTM. Julia is a high-level, open-source, high-performance, and dynamic programming language. It utilizes a just-in-time (JIT) compiler, allowing it to achieve speeds comparable to low-level languages like C. This makes Julia suitable for intensive computational tasks, such as numerical calculations and data analysis. Other advantages of JRTM compared to RRTM include the fact that in JRTM, it is also possible and straight forward to run the transfer model for a single cohort, something that is not inherent in RRTM. Additionally, the entire JRTM model is written in the Julia programming language, whereas RRTM also relies on functions from C++. In creating JRTM, we mainly relied on RRTM. The input parameters for JRTM are presented in table 5.

Table 5: Input parameters JRTM

Parameter	Explanation
Cohort state variables	
PFT	PFT-types (integer values)
LAI	leaf area index values
WAI	wood area index values
CAI	canopy area index values
PFT parameters	
orientation factor	leaf (and wood) orientation values
clumping factor	leaf (and wood) clumping values
T_{leaf}	leaf transmittance values
R_{leaf}	leaf reflectance values
R_{wood}	wood reflectance values
Global parameters	
R_{soil}	dry and wet soil reflectance values
soil saturation	fraction of wet soil
direct sky fraction	fraction of direct beam radiation
$cos(z)$	z = solar zenit

T_{leaf}, R_{leaf} and R_{wood} are dataframes consisting of rows representing wavelength values and columns representing PFT types. For R_{soil}, rows represent wavelength values and columns respectively dry and wet soil reflectance values. Transmittance and reflectance values are floats between 0 and 1.

The input parameters can be attained using Terrestrial Laser Scanning (TLS), a spectroradiometer and light sensors. Structural data like LAI, WAI, CAI, orientation factor and clumping factor can be achieved using TLS. T_{leaf}, R_{leaf}, R_{wood}, R_{soil} and soil saturation can be determined using a spectroradiometer. And direct sky fraction and solar zenit can be measured with light sensors.

JRTM returns a list with several radiative values for each wavelength. These are presented in table 6. The radiative values are relative values, i.e. in JRTM, the incoming radiation (direct beam + diffuse) is set to 100% (flat solar spectrum, i.e. every wavelength has the same intensity). This means that resulting radiative values will be fractions of the 100% incoming radiation.

Table 6: Output values JRTM

Radiative value	Explanation
albedo	upwelling radiation at the top of the canopy
upward flux	upward flux under every cohort layer and above the canopy top
downward flux	downward flux under every cohort layer and above the canopy top
light diffuse level	diffuse light level at every cohort layer
light beam level	beam light level at every cohort layer
light level	total light level at every cohort layer

In this way, the two-stream RTM, JRTM, can be executed for a specific region, allowing verification of whether, using the collected input data, the modeled output values align with collected measurements of the output values (using light sensors). Like this, we can assess the performance of our model and determine which parameters it is most sensitive to. We can also evaluate the accuracy required for the input data to obtain meaningful output. Additionally, by comparing the model output to observed data, we can optimize the physical equations within our model.

A first comparison between RRTM and JRTM

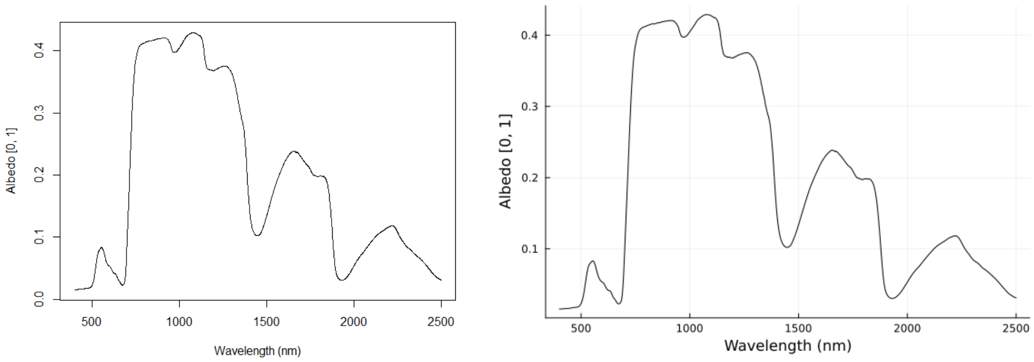
An initial test run of JRTM was conducted with three cohorts of different trees. The same set of input parameters was provided to both RRTM and JRTM, and the output radiative values were compared. Upon summing across all wavelengths for each output variable (see also table 6) in both models, it was found that they yielded exactly the same values. For one of the output variables, namely albedo, a graph was generated in both models. This can be found in figure 7. Indeed, an identical curve is observed in both models.

Using JRTM, apart from albedo, also the other radiative output values were plotted. Upward diffuse and downward diffuse fluxes under every cohort layer and above the canopy can be found in figure 8. The curves labeled 'Cohort 4' represent the light above cohort 3, and the curves labeled 'Cohort 3' represent the light below cohort 3. Similarly, the curves 'Cohort 2' represent the light below cohort 2, and so on. In those graphs, cohort layer 1 refers to the lowest canopy layer, cohort 3 to the highest. Cohort 4 refers to the radiation above the canopy. This means that the upward flux of cohort 4 is equal to the JRTM albedo from figure 7.

As can be seen from figure 8a, it is evident that the uppermost canopy layer, cohort 3, reflects the most light. This is to be expected, as the uppermost canopy layer also receives the most sunlight.

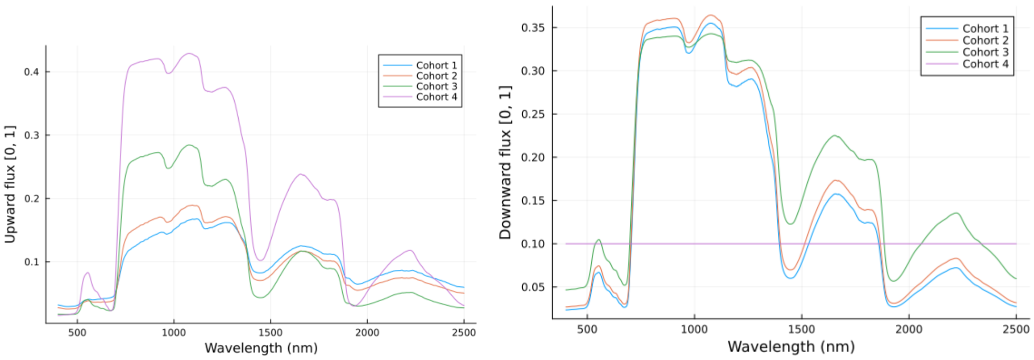
Figure 8b is consistent with the results from figure 8a. The diffuse light above cohort 3, the highest cohort, is indicated by 'Cohort 4'. It can be observed that 10% of the incoming light is diffuse, and its intensity is uniform across all wavelengths (flat solar

spectrum). The diffuse light below cohorts 3, 2, and 1 is, on average, higher than the incoming diffuse light. This is because during the radiative transport process, a portion of the direct sunlight (90% of incoming radiation) is converted into diffuse light due to scattering. Additionally, it is noted that the diffuse light below cohort 3 (indicated by the curve 'Cohort 3') is generally higher than the diffuse light below, for example, cohort 2, especially in the longer wavelengths. This is logical, as deeper penetration of light into the canopy structure increases the likelihood of reflection or absorption, resulting in less transmitted light.
The upward flux and downward flux indicated by 'Cohort 1' are located beneath cohort 1 and thus represent, respectively, ground reflection and the light reaching the ground.



(a) Albedo RRTM (b) Albedo JRTM

Figure 7: Comparison albedo RRTM *vs* JRTM for 3 cohorts.



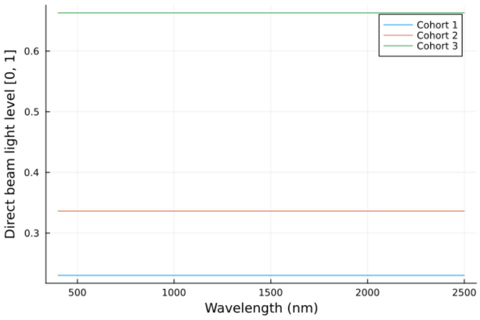
(a) Upward fluxes (b) Downward fluxes

Figure 8: Upward diffuse and downward diffuse fluxes under every cohort layer (cohort 1, 2 & 3) and above the canopy (cohort 4).

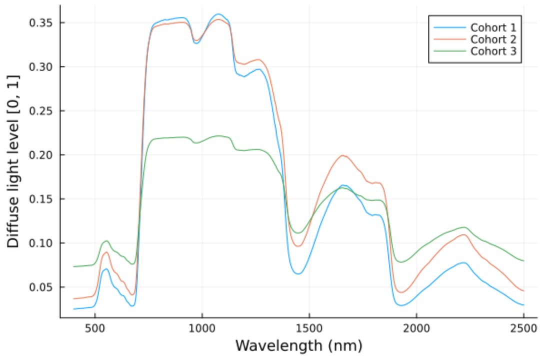
In addition, light levels can be found in figure 9. 'Light level' typically refers to the amount of sunlight reaching a specific layer within the canopy. In JRTM, it is defined as the average value of the light above and below a specific cohort. Therefore, the diffuse light level of e.g. cohort 2 is the average value of the diffuse light above cohort 2 and below cohort 2.

In 9a, we observe that approximate 70% of direct sunlight penetrates below cohort 3, and almost no direct sunlight reaches the ground.

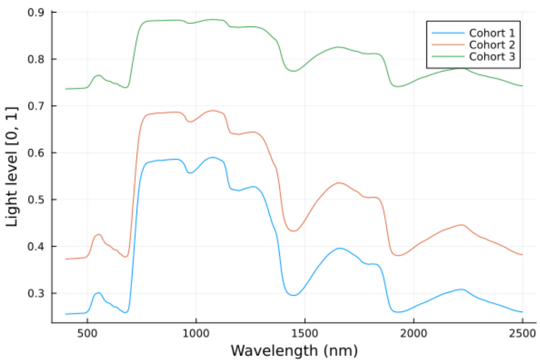
From figure 9b, we observe that, averaged across wavelengths, the diffuse light levels of cohorts 1 and 2 are higher than that of cohort 3. This is particularly noticeable in wavelengths around 1000 nm. This implies that the average of the diffuse light both above and below cohort 3 is lower than the average of the diffuse light both above and below cohort 2 and the same is true for cohort 3 w.r.t. cohort 1. Therefore, there is more diffuse light present around the lower cohorts than around the highest one. This is logical because there is almost no diffuse light above canopy layer 3, so its average of diffuse light both above and below the layer will not be as substantial. When examining the summed values from under each curve, it appears that the diffuse light level around cohort 2 (358 units) is greater than that around cohort 1 (318 units) (cohort 3 counts 299 units). This is because cohort 2, being positioned above cohort 1, will absorb a portion of the diffuse light and, consequently, transmit a smaller amount to cohort 1.
Figure 9c then displays the sum of figures 9a and 9b, representing the total light level. We indeed observe the 'direct beam level' base of approximately 70% for cohort 3.



(a) Direct beam light level



(b) Diffuse light level



(c) Total light level

Figure 9: Direct beam, diffuse and total light level at every cohort layer.

Further testing of JRTM

In a subsequent test comparing RRTM and JRTM, we simulated radiative transfer for a single cohort (after the RRTM code was modified so it could run for a single cohort). Once again, both models yielded precisely identical output values. Following this, we employed JRTM to simulate an extreme scenario with only one cohort. We set the leaf reflection and leaf transmission spectrum to 0.5 for each wavelength. For the soil, we opted for a reflectance value of 1 across all wavelengths. Additionally, we assumed an LAI of 1, a WAI

of almost 0 an orientation factor of 0, and a clumping factor of 1. The latter two values signify a perfectly spherical leaf orientation distribution and a perfectly homogeneous leaf spatial distribution within the canopy, respectively. Consequently, absorption should be 0 and this configuration should lead to an albedo of 1. Upon computing the average albedo over all wavelengths, we obtain a value of 0.99998. So, we see that the modeled albedo is almost exactly the same as what we theoretically expect. However, when we calculate the average light level (between ground and canopy) across all wavelengths, we obtain a value of 1.09556. This is more than 100% and seems strange at first glance. But this is likely explained by the fact that light penetrating the canopy to the ground is reflected back 100%, and a part of it is reflected back again by the canopy above. Due to the multiple reflections of light from the canopy to the ground, we ultimately retain more light than what originally passed through the canopy.

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