Forecasting of Meteorological Elements Time Series and Pricing of Weather Multi-factor Options: Integrating BP Neural Network and SARIMA Model

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Abstract

With the increasing frequency of abnormal climate change and weather catastrophes in recent years, weather risk management has played a more and more important role in economic development. Weather derivatives are the most effective tool for weather risk management, and the pricing of weather derivatives is one of the core topics in this field. Current studies aim to forecast meteorological elements time series and the price of weather multi-factor options by integrating BP neural network and SARIMA model. Through analyzing the meteorological data from 1951 to 2013 in Beijing, including the monthly average temperature and the dynamic change of the average monthly rainfall, we calculate four weather indices: CDD (cooling degree days), HDD (heating degree days), EDD (energy degree days), CRI (cumulative rainfall index), and price some weather multi-factor options by using the estimates of meteorological elements time series of the underlying indices. We find that integrating the BP neural network and SARIMA model has a strong nonlinear mapping ability, and its forecast and valuation result is better than a single model. Weather multi-factor options can be applied to analyzing the influence of different elements on weather, thus avoiding the weather risk, and gaining profits effectively.

Key Words Weather multi-factor option; SARIMA model; BP neural network

1 Introduction

The weather derivatives market has its origins in the liberal reform of the energy industry in the United States. In 1996, U.S. energy companies created a new type of risk management tool, weather derivatives contracts linked to weather risk management, which companies or agricultural producers can use to transfer weather risk to third parties who have the will and ability to deal with the risk. tripartite. In 1997, the first real weather derivatives contract trading was officially born. In 1999, the main types of weather derivatives traded on the US Financial Futures Exchange were weather index futures and weather futures options. At the same time, the Chicago Mercantile Exchange lists dozens of weather futures contracts, all of which are based on changes in weather elements in numerous cities, traded like stocks, and rise and fall daily under the influence of weather forecasts.

At present, our understanding of weather derivatives is still in its infancy, and the weather derivatives market has not yet been established in China. The economic value of China affected by weather accounts for about 45% of the total GDP (released by the Weather Bill Company of the United States). At the same time, the weather market represents a new area not covered by domestic insurance and financial markets. The experience of using weather derivatives abroad shows that the weather derivatives market has the functions of optimizing resource allocation, increasing company value, hedging losses due to natural disasters, cooling the market's hype atmosphere, smoothing out the income fluctuations of weather-affected industries, and reducing operators' income. Concerns, to a certain extent, improve the enthusiasm of production so that more resources can play their maximum functions smoothly in a relatively stable operating environment, which is of great benefit to the national economy and people's livelihood. Moreover, the weather financial derivatives market has a relatively wide range of trading subjects, mainly including industries that are greatly affected by weather disasters, such as agricultural companies, energy companies, tourism companies, construction companies, transportation companies, etc., as well as various financial institutions.

2 Literature Review

Existing related research mainly involves two types of literature. The first type focuses on the direct valuation of options based on the time series forecast of meteorological elements based on the basic definition of weather options, and mainly adopts two methods: one is the linear autoregressive model; the other is the mean reversion model. Sean D. Campbell and Francis X. Diebold (2005) took an unstructured time-series approach to model and forecast the daily average temperature in 10 U.S. cities and systematically studied whether it would Derivatives market participants play a role in their vantage point. The time series model revealed strong conditional mean dynamics and conditional variance dynamics at daily mean temperature. Tol RSJ (1996) used the GARCH(1, 1) model to analyze the volatility of Dutch weather processes, and used different models to simulate winter and summer, and found that there was a significant clustering phenomenon in the volatility of weekly data. Fred Espen Benth and Jurate Saltyte Benth (2010, 2015) et al. Continuous-time-based autoregressive and Levy processes for pricing and valuation of weather derivatives such as futures and options, with an in-depth look at CME Exchange CDD and HDD futures analysis. The second category of literature focuses on exploring the feasibility of existing option valuation methods in the application of weather option valuation. Equilibrium pricing models, such as the Monte Carlo method, and semi-martingale pricing method, are considered feasible. In addition, Ahmet Goncu (2011) also proposed a seasonal fluctuation model based on the Ornstein-Uhlenbeck mean regression process, and used the Monte Carlo method to approximate the heating index and cooling index of these cities. Garman et al. (2010) proposed that the underlying assets of weather derivatives—HDD/CDD have mean reversion characteristics, the basic framework of the BS model is not applicable to weather derivatives, and the returns of weather derivatives are dynamically related to the underlying asset flow. Stevenson and Saltyte Benth (2012) et al. (2012) et al. found that the temperature data in Norway were characterized by semi-thick tails and skewness, so they advocated using the O-U process and the residual part using the Levy process. Furthermore, they argue that fixed volatility leads to underestimating the price of weather risk, so they propose the use of truncated Fourier series to model seasonal fluctuations. Anandadeep Mandal (2010) only focuses on exploring a new pricing method, which uses 30 years of daily data from 4 UK cities to capture daily temperature fluctuations including seasonal effects and annual trend characteristics, using the Ornstein-Uhlenbeck model, without arbitrage Option pricing and comparison with

Monte Carlo simulation results. However, the application of these models to predict a variety of different changing characteristics of meteorological elements may have large deviations. Therefore, these special structural characteristics will lead to limitations in the application of these models.

Compared with the continuous deepening of the research on temperature derivatives, the financial derivatives of rainfall index face many difficulties in both research and financial practice. The concept of rainfall index derivatives is too innovative, and rainfall is more difficult to accurately predict than temperature, but the international academic community also has some references for the time series changes and fluctuations of rainfall. Brenda López Cabrera (2013) et al. proposed to model the precipitation index through a series of different probability distributions, namely the normal inverse Gaussian distribution, capturing its asymmetry and rear-tail behavior. They calculated the price of the precipitation index futures by using the Esscher transform. O.Kisi and J.Shiri (2011) applied Genetic Algorithm (GP) to analyze daily rainfall data, but it performed poorly by itself, even though they used wavelet theory to improve the accuracy. The pricing of weather derivatives is challenging because there is no arbitrage assumption to ensure the existence of a risk-neutral measure in contrast to a perfect market. Although some of the second category literature has overcome this problem, the evaluation of the value of meteorological elements should fully consider the particularity of meteorological elements and requires in-depth analysis of the characteristics of meteorological elements.

Research on weather derivatives in China mainly focuses on qualitative research, and quantitative research on the price of weather derivatives is not sufficient. On the basis of the O-U mean regression model, Liu Guoguang (2006) used the mean regression method to estimate the volatility of temperature and conducted an empirical study on the temperature in Beijing from 1980 to 1999, and found that it can adapt to temperature changes. Similarly, Li Yong (2011) also obtained relevant parameters of temperature change through regression analysis and Fourier transform and studied the temperature in Shanghai from 1951 to 2008, and found that the relative error of the predicted value was less than 5%. Huang Jianfeng (2016) used wavelet-NAR neural network meteorology to predict temperature changes in Sydney and obtained the valuation of weather rainbow options.

In the above literature, we find that most of the academic literature is designed for weather futures or weather selection for specific meteorological elements. They can only deal with the uncertainty risk caused by the weather risk caused by individual weather elements, but the real weather risk is usually caused by the uncertainty of various meteorological elements, which means that the participation of weather elements in existing weather derivatives does not whole. As a result, the hedging effect of weather derivatives may be weakened or even lose their practical application value. At present, China's weather derivatives market is facing new development opportunities. China's vast territory is affected by the monsoon climate characteristics, and major weather indexes such as temperature and rainfall in the country change greatly, and the uncertainty is higher. This paper applies the classical SARIMA time series analysis model and BP neural network forecasting technology to establish the meteorological time series forecasting and the weather option valuation model, so as to reveal the basic principle of the weather multi-factor option, and discuss the implied option value of each weather factor and economic impact.

3 Methodology Analysis

3.1 Time Series Forecasting of Meteorological Elements

Time series forecasting of meteorological elements is the basis for the valuation of multi-factor options on weather indices. There exists complicated interaction amongst seasonal effects of meteorological element time series, the long-term trend effects, and random fluctuations. A simple ARMA model is not enough to extract the correlation. The SARIMA model can make up for it well. The SARIMA model (Seasonal Autoregressive Integrated Moving Average) comes from autoregression Single Integer Moving Average Seasonal Models (ARIMA), capable of employing

Box-Jenkins' model identification, estimation, and forecasting procedures, allowing for real-time adjustments to the model as historical data becomes available. The general form of the SARIMA model is:

$$\phi(\mathrm{B})\Phi(B^s)(1-B)^d(1-B^S)^D y_t = c + \theta(\mathrm{B})\Theta(B^S)\varepsilon_t \quad (1)$$

In this formula, S and D represent the exponent number of length and season difference of seasonal cycle separately, p, q respectively be the exponent number of auto-regression and moving mean, P, Q respectively be season auto-regression and seasonal movement average exponent number. B^S is the backward shift operator, $\Phi(B^s) = 1 - \Phi_1 B^S - \Phi_2 B^S - \cdots \Phi_p B^S$; $\Theta(B^S) = 1 - \Theta_1 B^S - \Theta_2 B^S - \cdots \Theta_p B^S$. The SARIMA model was recorded as the SARIMA (p, d, q) (P, D, Q) S model. Based on observations of meteorological elements, we separately carry on d step trend difference and take D step cyclical S as the seasonal difference of length of stride, after a time sequence to determine the difference of time series by the unit root test steady characteristics, according to the characteristics of the autocorrelation coefficient and partial autocorrelation coefficient to determine short-term correlation model. Finally, we determined by residual test sequence diagram under the trend of steady model.

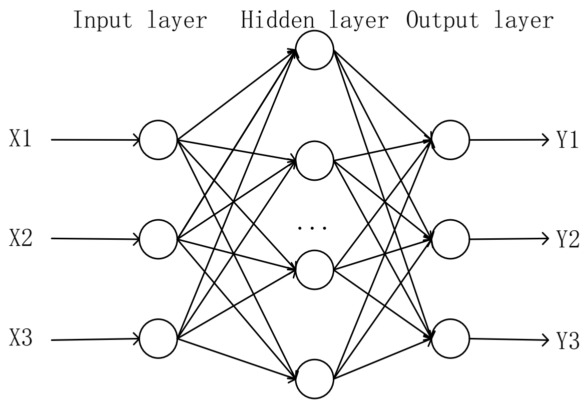

Figure 1 The BP neural network diagram

BP neural network, also known as a back-propagation neural network, is a multi-layer forward neural network (Figure 1). By using the back-propagation algorithm, the train network has strong non-linearity mapping ability and predictive ability on the data fitting and approximation of a function. In the BP neural network, the signal is the forward dissemination, while the error is the counter propagation. The BP neural network usually has one or more sigmoid implicit strata and linear output level and can approach the limited branch point function.

In the model, the relationship of the output y_t and input x_{t-1}, x_{t-2}, …, x_{t-p} is as follows:

$$y_t = \omega_0 + \sum_{j=1}^{Q} \omega_j \, g\left(\omega_{0j} + \sum_{i=1}^{P} \omega_{ij} \, x_{t-i}\right) + \varepsilon_t \qquad (2)$$

ω_{ij}, $(\mathrm{i} = 0,1,2 \ldots, \mathrm{P}; \mathrm{j} = 0,1,2 \ldots \mathrm{Q})$ is the connection weight vector, $\omega_j(j = 0,1,2,,\ldots,Q)$ is the threshold vector, Input layer and output layer nodes are P and Q. g (·) is the activation function.

Meteorological element time series data can be classified into the training data and testing data. The original weight and threshold value use the random number, the activation function for the logarithmic tangent S form function and linear function, the training function uses the Fletcher-Reeves conjugate gradient method. The method of determining the number of hidden layer unit: $\sqrt{n+m}+a$, n is the input layer number of units, m is the output layer number of units, a is constant between [1,10]. We selected 1951-1980 monthly mean temperature and month precipitation as the training input, 1981 meteorological element data as the target training output. After the iterative time achieves the hidden-layer and error of the actual output and expected value of various output neurons, selected 1982 - 2012 meteorological element data as testing set to input, 2013 meteorological element data as the target testing set to output.

It optimizes the BP neural network and SARIMA model composition with the optimal weight combined forecasting method. It may make the model have bigger reliability and objectivity. The combined forecasting method is a non-linear system integration of technologies method, namely conducts the synthetic study to the results of many models, hoping to improve the reliability and objectivity of forecasting results. The main purpose of an integrated model is to take advantage of relatively elaborate information which provided by the comprehensive utilization of a variety of models. The model can enhance the accuracy of forecast and its tenet is more comprehensive than any single model system. The determination of combined forecast weight proportion: it takes the combined prediction error sum of squares to be the smallest as the optimal criterion, and extracts the result from linear equation. Therefore, the predicted change of weather system in the BP neural network and SARIMA model would be established. Furthermore, accuracy will be enhanced by integrated information from varied prediction models. Assuming B and S separately are the predicted value of BP neural network and SARIMA model, Y is the combination forecast predicted value, the prediction error respectively is E_b, E_s, E_y, takes the prediction error sum of squares to be smallest for the optimal principle: take the $\mathrm{E} = \sum {E_y}^2$ is smallest, K_b and K_s for corresponding weight proportion ($K_b + K_s = 1$), then we have

$$E_y = K_b * E_b + K_s * E_s \qquad (3)$$

$$K_b = 1 - K_s \qquad (4)$$

$$\mathrm{ks} = -\frac{\sum(E_s - E_b)E_b}{\sum(E_s - E_b)^2} \qquad (5)$$

$$\mathrm{RMSE} = (\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2)^{\frac{1}{2}} \qquad (6)$$

Through determining the weight ratio of the SARIMA model and BP neural network in an optimal combination forecast model, the predicted value of intEgrated model meteorological element time series will be obtained. To compare the combination model with a single model meteorological time series quantitatively, we use error indicators - root mean square error (RMSE) to test the valuation results.

3.2 Weather Index Multi-factor Options Valuation

In the computation of the weather index, if the monthly mean temperature and monthly rainfall amount use the actual value, computation obtains according to the (7)-(10) is the actual value of weather index computation; if we use the predicted value to obtain the weather exponential model estimated value. Absolute deviation of the weather index computation between the predicted value and actual value is smaller, then the prediction is more precise, otherwise, the prediction result error is bigger. We calculate monthly

HDD (heating degree days), CDD (cooling degree days), CRI (cumulative rainfall index) and annual EDD (energy degree days) as follows according to the Chicago mercantile exchange weather derivatives of the underlying index calculation method:

$$\mathrm{HDD} = \sum_{t=1}^{n} \{[\max(T_0 - T_t, \ 0)] * 30\} \quad (7)$$

$$\mathrm{CDD} = \sum_{t=1}^{n} \{[\max\,(T_t - T_0, \ 0)] * 30\} \quad (8)$$

$$\mathrm{CRI} = \sum_{t=1}^{n} R_t \quad (9)$$

$$\mathrm{EDD} = \mathrm{HDD} + \mathrm{CDD} \quad (10)$$

T_t, R_t respectively is the weather multi-factor option the monthly average temperature and month precipitation of t-th months; T_0 is temperature threshold value; n expressed the contract deadline of weather multifactor option. The Energy values of EDD avoid weather risk mainly comes from the energy department's demand. In warm winter or cool summer, people would reduce their energy consumption, which may plunge the sales volume of energy providers. Hence, the profit margins of these energy corporations will be affected negatively. Under this circumstance, EDD was designed to avoid energy suppliers' weather risks. EDD is a cumulative total of HDD and CDD. The contract is usually traded during the year. Formula (7) - (10) can be seen that the value of HDD and CDD, CRI, EDD are not negative, and in the extension of maturities, the weather multi-factor options each target index value present a positive cumulative effect.

In the ordinary weather index call/put option, the call option will enable to gain the revenue which is used to compensate for some, or all of the economic losses caused by extreme weather factors. Within the contract period, when the cumulative value of the index exceeds a predetermined level, options would have exercise value.

In other words, when the cumulative value of the index does not exceed the level of agreement, the value of the option is not executed. In this way, the buyer of a call option will give up exercising. Loss is only the premium which paid to the seller. In the weather multi-factor option valuation, firstly, we design multi-factor underlying weather index for each index (CDD, HDD, CRI, EDD) respectively, then, we set the appropriate level of implementation, combined with weather multi-factor option valuation basic definition, giving the corresponding single weather index options valuation of each factor.

Assume that weather index execution for K_i, upper / lower underlying weather index for L_i.Maturity weather underlying index for x_i. D_i for Weather monetary value of the underlying index, i is the weather multi-factor option i-th weather index.

The payoff function of call option buyer is:

$$\mathrm{C}(x_i) = D_i \times \mathrm{Min}\left(\mathrm{Max}\big((x_i - K_i), 0\big), (L_i - K_i)\right) \tag{10}$$

The payoff function of put option buyer is:

$$\mathrm{P}(x_i) = D_i \times \mathrm{Min}\left(\mathrm{Max}\big((K_i - x_i), 0\big), (K_i - L_i)\right) \tag{11}$$

The payment characteristics of weather index multifactor option depend on the maximum value or the minimum value in two or more weather index option value. So we get the maximum multi-factor call option ($call_{max}$), multi-factor minimum call option ($call_{min}$), multi-factor max put (put_{max}), multi-factor minimum value put (put_{min}). The four kinds of weather index multi-factor option valuation expressions are as follows:

$$\begin{aligned} call_{max} &= \max\left(\mathrm{C}(x_1), \mathrm{C}(x_2), \mathrm{C}(x_3), \ldots, \mathrm{C}(x_r)\right) \\ &= \max\left(D_1 \times \mathrm{Min}\left(\mathrm{Max}\big((x_1 - K_1), 0\big), (L_1 - K_1)\right), D_2 \times \mathrm{Min}\left(\mathrm{Max}\big((x_2 - K_2), 0\big), (L_2 - K_2)\right), \ldots, D_r \times \mathrm{Min}\left(\mathrm{Max}\big((x_r - K_r), 0\big), (L_r - K_r)\right)\right) \\ &= \max\left(D_1\left(\mathrm{Max}\big((x_1 - K_1), 0\big)\right), D_2\left(\mathrm{Max}\big((x_2 - K_2), 0\big)\right), \ldots, D_r\left(\mathrm{Max}\big((x_r - K_r), 0\big)\right)\right). \end{aligned} \tag{12}$$

$$\begin{aligned} call_{min} &= \max\left(\mathrm{C}(x_1), \mathrm{C}(x_2), \mathrm{C}(x_3), \ldots, \mathrm{C}(x_r)\right) \\ &= \max\left(D_1\left(\mathrm{Min}\big((x_1 - K_1), 0\big)\right), D_2\left(\mathrm{Min}\big((x_2 - K_2), 0\big)\right), \ldots, D_r\left(\mathrm{Min}\big((x_r - K_r), 0\big)\right)\right) \end{aligned} \tag{13}$$

$$put_{max} = \max\left(\mathrm{C}(x_1), \mathrm{C}(x_2), \mathrm{C}(x_3), \ldots, \mathrm{C}(x_r)\right)$$

$$= \max\left(D_1\left(\mathrm{Max}\left((K_1 - x_1), 0\right)\right),\ D_2\left(\mathrm{Max}\left((K_2 - x_2), 0\right)\right),\ \ldots, D_r\left(\mathrm{Max}\left((K_r - x_r), 0\right)\right)\right). \qquad (14)$$

$$put_{min} = \max\left(\mathrm{C}(x_1),\ \mathrm{C}(x_2),\ \mathrm{C}(x_3),\ \ldots,\ \mathrm{C}(x_r)\right)$$

$$= \max\left(D_1\left(\mathrm{Min}\left((K_1 - x_1), 0\right)\right),\ D_2\left(\mathrm{Min}\left((K_2 - x_2), 0\right)\right),\ \ldots, D_r\left(\mathrm{Min}\left((K_r - x_r), 0\right)\right)\right) \quad (15)$$

The above multi-factor option valuation expressions (except the last one of the 0) indicate the pay structure for the weather multi-factor option buyer.

In the options valuation process, if the weather prediction model uses the prediction index value, the weather index for the long-term multi-factor option valuation of the option which is determined within the options contract reflects the value of weather uncertainty. By using the actual index value, the weather index for the long-term multi-factor option valuation is the actual value of the option. By comparison of the relative error of the actual value and predicted value of the model, the accuracy of different options valuation models can be compared. If the relative error is smaller, the option valuation will be more accurate, namely, the weather risk decision-making by meteorological prediction model based on the weather multi-factor options is more effective, and vice versa.

4 Empirical Analysis

The theoretical analysis of the time series forecasting principle of meteorological elements (monthly average temperature and monthly rainfall) is detailed above, and the time series prediction data using the SARIMA model, BP neural network, and BP neural network-SARIMA combined model are described respectively. Calculate the weather option underlying index and the weather index multi-factor option valuation method. The following will analyze 28 weather index multi-factor options consisting of HDD, CDD, CRI, and EDD, which are the weather options underlying indexes constructed by the two meteorological elements of temperature and rainfall. The data in this paper comes from the China Meteorological Statistical Yearbook, covering the two meteorological elements of Beijing's monthly average temperature and monthly rainfall from 1951 to 2013, with a sample size of 1512. The overall sample is divided into two sub-samples, taking the period from 1951 to 2012. The annual meteorological data was used for model prediction, and the meteorological data in 2013 was taken as the reference standard for model prediction.

4.1 Meteorological Element Time Series Forecasting Analysis

Under the sole SARIMA model analysis, we discover that Beijing 1951-2012 years of each month average temperature and month rainfall amount which have the regular changing (as shown in Fig. 2) through the factor decomposition fitting synthesis output. It further illustrates the changes in temperature and rainfall in Beijing by the long-term effects of linear trend, seasonal effect and the influence of random effects, the general rule is more complicated. Apparently, it is a non-stationary sequence, first we have seasonal difference (Figure 3), it is clear that its cycle S = 12, the form of its season difference is: $Y_k = X_{k+12} - X_k$, $k = 1,2, \dots \dots ,744$

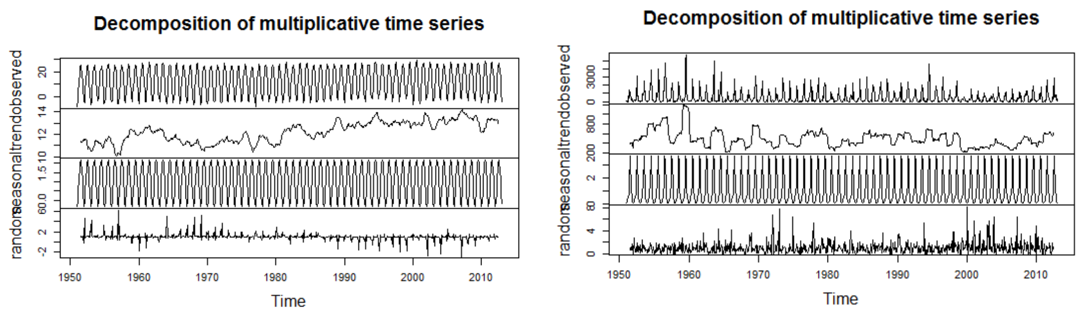

Figure 2 Beijing 1951-2012 monthly mean temperature (left) and monthly rainfall (right) factorization fitting comprehensive output figure

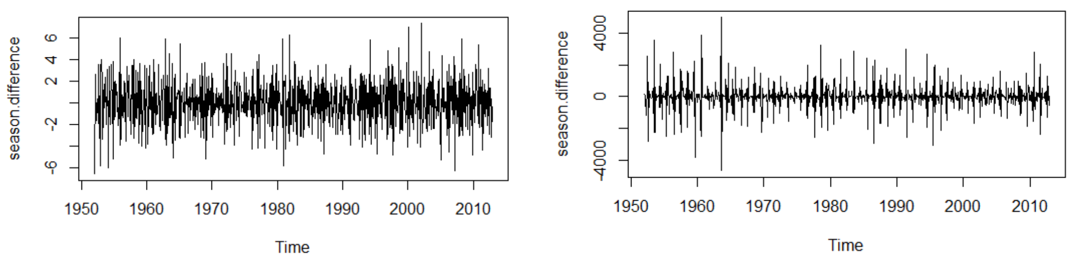

Figure 3 Beijing 1951-2012 years of monthly mean temperature (left) and month rainfall amount (right) first-order 12 season difference chart

By observing the graph, we can carry on the stationarity test. The steady sequence usually has short-term relevance. This nature is described with the autocorrelation with increase of detention exponent number k, the steady sequence autocorrelation coefficient quickly weakens to 0. Otherwise, the non-stationary series autocorrelation coefficient weaken to 0 which speeds is quite usually slow. Under the ADF examination, it is demonstrated the P value is smaller than 0.05, its first-order 12 difference sequences are steady. We separately observe monthly mean temperature and autocorrelation coefficient of month rainfall amount first-order seasonal difference sequence chart and autocorrelation coefficient chart is trailing. Through the monthly mean temperature and month rainfall amount of observation the QQ chart of the first-order seasonal difference sequence, we can basically conform to the normal distribution.

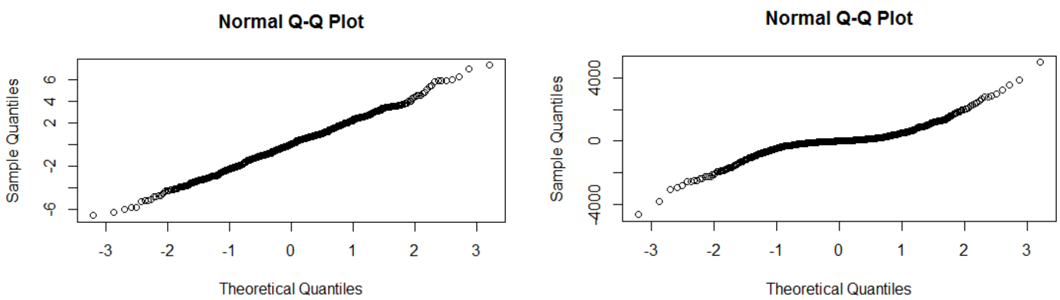

Figure 4 Beijing 1951-2012 monthly mean temperature (left) and monthly rainfall (r) standardized residuals QQ figure

According to the minimum information standards, we establish a SARIMA (p, 0, q) (p, 1, q) model for monthly average temperature in Beijing, finally select SARIMA (4, 0, 0), (2, 1, 0), the parameter estimation results in table 1; similarly, we select SARIMA (5, 0, 0) (2, 1, 0) for monthly rainfall, the parameter estimation results in table 2.

Table 1 The monthly average temperature model parameter estimation results

	AR1	AR2	AR3	AR4	SAR1	SAR2
Value	0.2192	0.0403	0.0596	-0.0142	-0.6430	-0.3655
s.e.	0.0374	0.0380	0.0380	0.0372	0.0352	0.0354

Table 2 The monthly rainfall model parameter estimation results

	AR1	AR2	AR3	AR4	AR5	SAR1	SAR2
Value	0.0839	0.0372	-0.0063	0.0261	0.0183	-0.6242	-0.3189
s.e.	0.0370	0.0373	0.0371	0.0371	0.0370	0.0356	0.0358

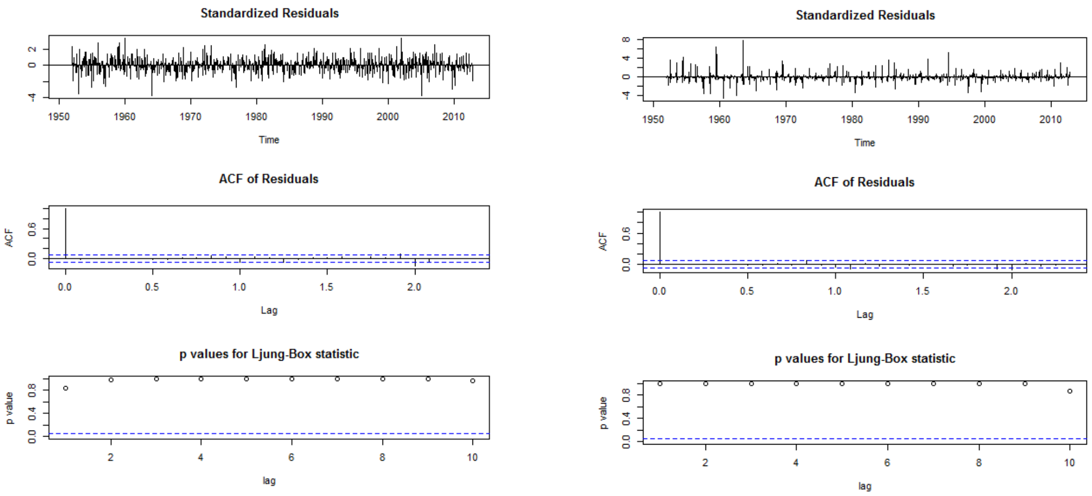

Figure 5 Beijing 1951--2012 years diagnostic tests for average temperature (left) and monthly rainfall (right) Prediction Model

The model test chart provides the standard residual error sequence diagram, samples of residual ACF, and from 0 to 10 Ljung - Box test statistics P values, is a comprehensive analysis of the time series model diagnostic tool. The ACF examination showed that residual is not obvious autocorrelation. The Ljung-Box test demonstrated: All P-values >0.5, the showing residual is the white noise. Thus, it is explained that we obtained good forecasting results.

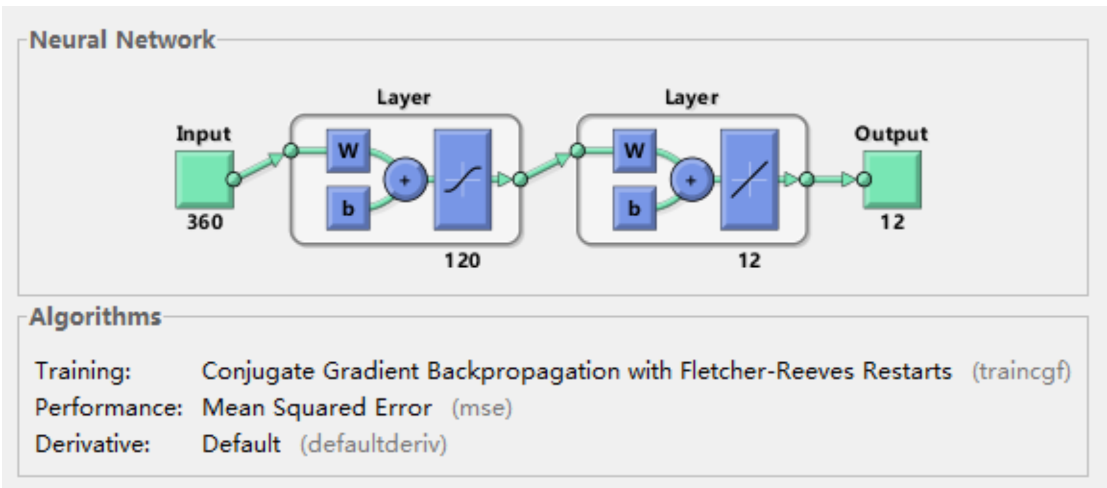

Figure 6 Beijing monthly mean temperature and month rainfall amount BP prediction mode of neural network schematic drawing

We based on the meteorological data of 1951 ~ 1980 as a training set, meteorological data in 1981 as a training set output, using meteorological data from 1982 to 2012 as the test set, and meteorological data in 2013 as a test set output, using the neural networks form as shown in figure 6. Categorizing meteorological time series data for training input data, variables data, test data, the initial weights and thresholds which adopt random number, the activation function is logarithmic tangent S function and linear function, training function the Fletcher - Reeves conjugate gradient method.According to neural network output, comparison between prediction chart and actual chart as shown in Fig. 7. Moreover, Fig 8 demonstrates that achieves the error goal after 249 iterations in backpropagation 250th time.

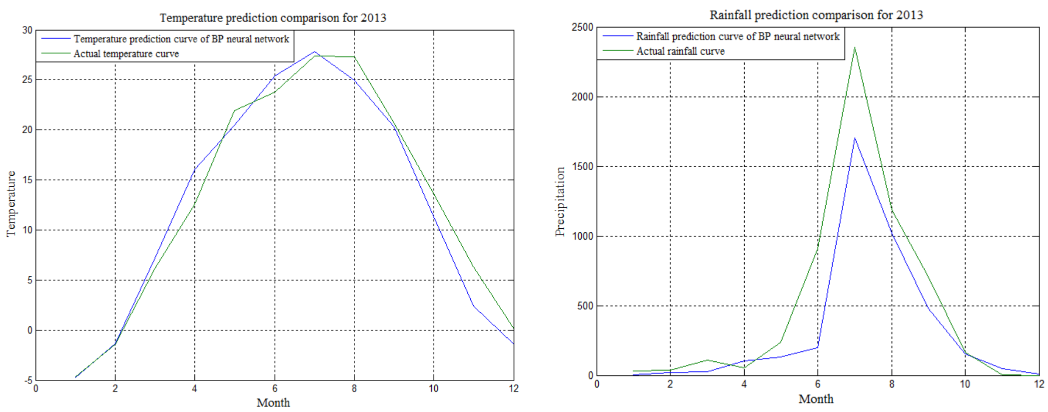

Figure 7 Beijing monthly mean temperature and month rainfall amount BP neural network output chart

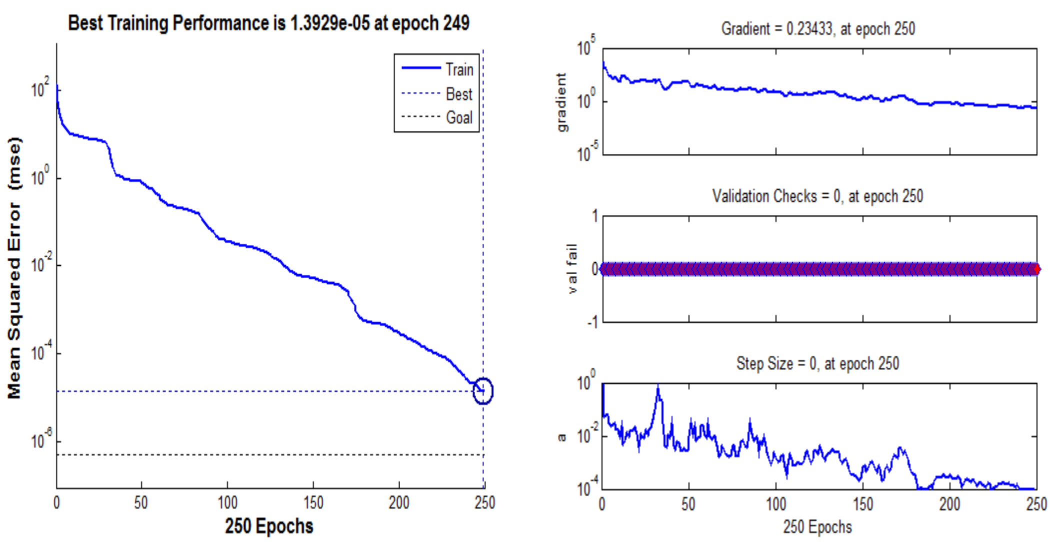

Figure 8 BP neural network optimal training iteration error convergence chart

Table 3 The monthly mean temperature and rainfall forecast results compared with the actual value under two kinds of single model

Model	SARIMA model		BP neural network model		Actual value	
month	monthly mean temperature	monthly rainfall	monthly mean temperature	monthly rainfall	monthly mean temperature	monthly rainfall
Jan-13	-4.91	55.91	-4.80	8.01	-4.7	30
Feb-13	-1.14	48.20	-1.40	23.01	-1.4	34
Mar-13	5.61	121.23	7.20	32.98	6.2	107
Apr-13	14.03	318.44	16.00	105.01	12.6	55
May-13	21.95	281.86	20.50	140.05	21.9	236
Jun-13	25.26	1032.95	25.40	194.06	23.8	910
Jul-13	27.86	1987.38	27.80	1708.95	27.4	2356
Aug-13	26.29	1315.80	24.80	1024.94	27.3	1186
Sep-13	20.92	758.67	20.30	481.99	20.7	711
Oct-13	14.12	356.36	11.30	150.01	13.6	164
Nov-13	5.62	369.57	2.30	52.03	6.3	2
Dec-13	-2.14	37.45	-1.40	10.98	0.1	0

On the principle of optimal linear combination model, using the fitted values and actual values of the individual samples of linear regression prediction model. In order to determine each single forecasting model weights, we take the fitting value about BP neural network model and SARIMA model as the independent variable, select the corresponding actual value as the dependent variable, and eventually establish a linear regression model and determine the weight of combination model:

Table 4 the weight of different model under integrated model

weight	SARIMA model	BP neural network model
monthly mean temperature	1.0779	-0.0779
monthly rainfall amount	0.764	0.236

The integrated model expression is:

Monthly average temperature model: $y_{\widehat{combine}} = 1.0779 y_{\widehat{SARIMA}} - 0.0779 \widehat{y_{BP}}$

Monthly rainfall model： $y_{\widehat{Combine}} = 0.7640 y_{\widehat{SARIMA}} + 0.2360 \widehat{y_{BP}}$

Table 5 illustrates predict outcomes that comparing with the single SARIMA model and

BP neural network, combined model computation is more precise: the root mean square errors of combined model under different time span are minimum.

Table 5 the temperature and rainfall forecasts of Beijing under portfolio model

Month	monthly mean temperature	month rainfall amount
Jan-13	-4.91	44.60
Feb-13	-1.12	42.26
Mar-13	5.49	100.41
Apr-13	13.88	268.07
May-13	22.06	248.39
Jun-13	25.25	834.97
Jul-13	27.87	1921.67
Aug-13	26.41	1247.16
Sep-13	20.97	693.37
Oct-13	14.34	307.66
Nov-13	5.88	294.63
Dec-13	-2.20	31.21

Table 6 The forecast appraisal of Beijing monthly mean temperature and precipitation: RMSE computed result in 2013

Meteorological elements	Time	SARIMA model	BP neural network model	Integrating model
monthly mean temperature	first quarter	0.7913	1.7727	0.7525
	half of year	0.8793	1.6877	0.8557
	One year	0.9861	1.9832	0.9786
month rainfall amount	first quarter	132.74	46.32	106.92
	half of year	120.89	297.31	92.65
	One year	186.24	293.01	171.13

4.2 Weather Multi-factor Option Evaluation Analysis

Table 7 the weather index calculation results of Beijing in 2013

Meteorological elements	Time	Actual value	SARIMA model	BP neural network model	Integrating model

CDD/℃	first quarter	0	0	0	0
	half of year	291	336.3	297	339.3
	One year	933	968.4	864	976.8
HDD/℃	first quarter	1617	1633.2	1590	1636.2
	half of year	1779	1752.3	1650	1759.8
	One year	2799	2871	3006.3	2839.2
EDD/℃	first quarter	1617	1633.2	1590	1636.2
	half of year	2070	2088.6	1947	2099.1
	One year	3732	3839.4	3870.3	3816
CRI/mm	first quarter	17.1	22.5	6.4	18.7
	half of year	137.2	185.9	50.3	153.9
	One year	579.1	668.4	393.2	603.4

In accordance with the estimations of mentioned model and different calculated weather indexes, the valuation of the weather multi-factor option could be deduced. Because there exist no listing trades between weather index multiple factors options in the current market. In this article, the monetary value of different weather indexes and unit weather indexes, which would be involved in an underlying index options trading, will be referred to as the parameters-setting method of CME current transaction about temperature and rainfall index options. Executive levels of weather multi-factor options (CDD, HDD, EDD, CRI) were set up to (0, 1600, 1600, 50) quarterly, (300, 1700, 1900, 100) semi-annually, and (800, 2500, 3600, 500) annually. Unit index of monetary value was set to (20, 20, 50,500). The unit value is denominated in US dollars. As shown in table 8, the option valuation model based on integrating BP neural network and SARIMA model is more accurate than a single model. That is to say, the combined model could capture the implied economic value behind quarterly semi-annual and annual weather information of Beijing in 2013. Thus, relatively reasonable estimations would be helpful to take rational actions to avoid weather risks.

Table 8 Beijing weather index multifactor option estimation result in 2013 ($)

weather index multifactor option	Underlying index	Time	Actual value	SARIMA model	BP neural network model	Integrating model
Multi-factor maximum call options	CDD-CRI	first quarter	0	0(0)	0(0)	0(0)
		half of year	18600	42950(131%)	0(∞)	26950（45%）
		One year	39550	84200（112%）	1280（97%）	51700（31%）
	HDD-CRI	first quarter	340	664（95%）	0（100%）	724（112%）
		half of year	18600	42950(131%)	0(∞)	26950（45%）
		One year	39550	84200（112%）	1280（97%）	51700（31%）
	EDD-CRI	One year	39550	84200（112%）	13515（66%）	51700（31%）
Multi-factor maximum put options	CDD-CRI	first quarter	16450	13750（17%）	21800（33%）	15650（5%）
		half of year	180	0（100%）	24850（13705%）	0（100%）
		One year	0	0（0）	53400（∞）	0（0）
	HDD-CRI	first quarter	16450	13750（17%）	21800（33%）	15650（5%）
		half of year	0	0（0）	24850（∞）	0（0）
		One year	0	0（0）	53400（∞）	0（0）
	EDD-CRI	One year	0	0（0）	53400（∞）	0（0）
Multi-factor minimum call options	CDD-CRI	first quarter	0	0(0)	0(0)	0(0)
		half of year	0	736(∞)	0(0)	786(∞)
		One year	2660	3368（27%）	1280（52%）	3536（32%）
	HDD-CRI	first quarter	0	0(0)	0(0)	0(0)
		half of year	1580	1046（34%）	0（∞）	1196（24%）
		One year	14950	18550（25%）	0（100%）	16960（13%）
	EDD-CRI	One year	6600	11970（82%）	0（100%）	10800（63%）
Multi-factor minimum put	CDD-CRI	first quarter	0	0(0)	0(0)	0(0)
		half of year	0	0（0）	0（0）	0（0）
		One year	0	0（0）	0（0）	0（0）
	HDD-CRI	first quarter	0	0（0）	200（∞）	0（0）

options		half of year	0	0（0）	1000（∞）	0（0）
		One year	0	0（0）	0（0）	0（0）
	EDD-CRI	One year	0	0（0）	0（0）	0（0）

Note: In the parenthesis is the relative error of weather multifactor option and actual value under the different models.

5 Conclusion

Weather derivatives, as a relatively cutting-edge risk management tool, have an increasingly urgent domestic demand for hedging weather risks, but no related

Table 8 Beijing weather index multifactor option estimation result in 2013 ($)

products have been launched so far. Among them, product pricing and valuation is the core issue of research in this field. Most researchers use a single model to predict a single weather element (such as a single temperature option, or a single rainfall option) for the pricing and valuation of weather derivatives, while this paper uses the BP neural network-SARIMA combination model to predict multiple weather elements (monthly and monthly). The dynamic change trend of average temperature and monthly rainfall) makes the meteorological forecast more accurate. The forecast value is used to calculate the four different underlying weather index values under the weather option, and a comparative analysis is made based on the valuation of weather multi-factor options under different models. In addition, through the integration of BP neural network and SARIMA model, the change characteristics of meteorological elements can be well reflected. The expected effect is better than the single BP neural network model and the single SARIMA model. The comprehensive model has good relative error estimation results for the weather index and weather option valuation results, so we have reason to believe that the model is reasonable for the valuation and pricing of weather multi-factor options. In this paper, the valuation of weather multi-factor options involves the transformation process of "change of meteorological elements-calculation of weather index-estimation of option value". It reflects the economic value of future weather uncertainty. Precise weather forecasts provide more accurate estimates of weather options. It is helpful for weather risk bearers to improve the accuracy of weather risk decision-making and make reasonable insurance decisions.

This paper is mainly based on the combined model of SAIMA model and BP neural network attempts, but the SAIMA forecasting method is linear, completely data-driven, and has certain limitations. Although the prediction result of the combined model is better than that of any single model, considering the nonlinear mapping ability, generalization ability, and fault tolerance ability of BP neural network, we still need to increase the number of samples and the selection of input and output layers to do further research. In addition, considering the impact of changes in the weather system and the monthly statistics of ground observations on the monthly average temperature and monthly rainfall, we hope that the follow-up research will introduce more analysis factors through the combined model to study its internal change law and improve the weather option estimates and the accuracy of the value.

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基于BP神经网络-SARIMA组合模型对气象要素预测与天气多因子期权的估值

黄豪南　郑　祥　韦勇凤

摘要：本文主要以BP神经网络-SARIMA组合模型为基础，通过分析北京市1951-2013年气象数据，包括月平均气温，月平均降雨量的动态变化，分别计算天气期权中的标的天气指数：CDD（制冷指数），HDD（取暖指数），EDD(能源温值)，CRI（降雨指数），进而由气象要素的估计进行天气标的指数的估计和天气多因子期权的估值。研究表明天气多因子期权可以分析不同天气因素对天气影响的影响，同时可以有效地避免天气风险，获得收益。

关键词：天气指数多因子期权；SARIMA模型；BP神经网络
JEL分类号：C32,C45,G12

一、引言

天气衍生品市场起源于美国对能源行业的自由改革。1996年，美国能源公司创立了一种新型风险管理工具，即与天气风险管理挂钩的天气衍生品合约，企业或者农业生产者可以利用这些工具把天气风险转移到有意向和有能力处理风险的第三方。1997年，第一份真正意义上的天气衍生品合约交易正式诞生。1999年，在美国金融期货交易所开始交易的天气衍生品交易品种主要有天气指数期货和天气期货期权。与此同时，芝加哥商品交易所挂牌了几十种天气期货合约，它们都基于众多城市的气象要素变化情况，可以像股票一样交易，而且每天都会在天气预报的影响下出现涨跌。

目前，我国对天气衍生品的了解尚处于起步阶段，而天气衍生品市场也尚未建立起来。中国受天气影响的经济价值占GDP总量的45%左右（美国Weather Bill公司公布）。同时，天气市场代表了国内保险市场和金融市场未覆盖的新领域。国外利用天气衍生品的经验表明，天气衍生品市场具有优化资源配置、增加公司价值、对冲或有天灾损失、冷却市场的炒作氛围等功效，抹平受天气影响行业的收入波动，减少经营者的顾虑，在一定程度上提高了生产的积极性，使更多的资源在较稳定的经营环境下平稳地发挥最大功用，对国计民生大有裨益。而且，天气金融衍生品市场交易主体相对广泛，主要包括受天气灾害影响较大的行业，比如农业企业、能源企业、旅游企业、建筑企业、交通运输企业等，还包括各类金融机构等。

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二、文献综述

已有相关研究主要涉及两大类文献。第一类侧重于在气象要素时间序列预测基础上依据天气期权基本定义式对期权进行直接估值，主要采取两种方法：一是线性自回归模型；二是均值回复模型。Sean D. Campbell and Francis X. Diebold (2005) 采取了一种非结构性的时间序列方法，对美国10个城市的日平均气温进行了建模和预测，并系统地研究了它是否会在天气衍生品市场的参与者的有利位置上发挥作用。时间序列模型揭示了在日平均温度下的强条件均值动态和条件方差动态。Tol RSJ(1996)运用GARCH(1, 1)模型分析荷兰天气过程的波动性，并运用不同的模型来模拟冬季和夏季，发现周数据的波动性方面存在显著的集聚现象。Fred Espen Benth and Jurate Saltyte Benth (2010, 2015) et al. 基于连续时间的自回归过程和Levy过程对天气衍生品如期货和期权进行定价估值，并对CME交易所CDD和HDD期货进行了深入的分析。第二类文献则着力探索已有的期权估值方法在天气期权估值应用上的可行性。如蒙特卡洛法，半鞅定价法，均衡定价模型被认为是可行的。此外，Ahmet Goncu(2011)基于Ornstein-Uhlenbeck均值回归过程还提出了一种季节性的波动模型，利用蒙特卡罗方法对这些城市的制热指数和制冷指数进行近似分析。Garman(2010)等人提出了天气衍生品的标的资产—HDD/CDD有均值回复特性，BS模型的基本框架不适用于天气衍生品，而且天气衍生品的收益是与标的资产流量动态相关的。Stevenson and Saltyte Benth (2012) et al. 分析发现温度数据在挪威呈现半厚尾和偏斜的特点，因此他们主张利用O-U过程，而残差部分使用Levy过程。此外，他们认为固定波动率会导致低估天气风险的价格,因此他们建议使用截断傅里叶序列来模拟季节性波动。Anandadeep Mandal (2010) 则只关注探索一种新的定价方法，其使用英国4个城市30年的日数据，捕捉包括季节效应和年趋势特征的日气温波动，使用Ornstein-Uhlenbeck模型，进行无套利期权定价，并将其与蒙特卡洛模拟结果进行对比。然而，应用这些模型预测多种不同变化特征的气象要素可能具有较大的偏差，因此，这些特殊的结构性特征，会导致这些模型应用上的局限性。

相比较气温衍生品研究的不断深入，降雨指数金融衍生品无论在研究和金融实践中都面临着诸多困难。关于降雨指数衍生品的概念太过创新，而且降雨量比温度更加难以准确预测，但国际学术界针对降雨量的时间序列变化及波动也有部分文献可以参考。Brenda López Cabrera(2013) et al. 提出通过一系列不同的概率分布来模拟降水指数，即正态逆高斯分布，捕获其不对称和后尾行为。他们通过使用Esscher变换计算降水指数期货的价格。O.Kisi and J.Shiri (2011) 应用遗传算法(GP)来分析每天的降雨量数据，但是它本身的表现很差，即使它们利用小波理论提高了准确性。天气衍生品的定价颇具挑战性，因为不存在套利的假设，无法确保与完全市场形成对比的风险中性测度的存在。尽管部分第二类文献已经克服了这一问题，但对气象要素的价值的评估应充分考虑气象要素的特殊性，并需要深入分析气象要素的特征。

国内关于天气衍生品的研究主要集中在定性研究上，对天气衍生品价格的定量研究不够充分。在O-U均值回归模型的基础上，刘国光(2006)运用了均值回归方法估计温度的波动率,并对1980-1999年北京市气温进行了实证研究，发现它可以适应温度的变化。类似地，李勇(2011)等也通过回归分析和傅里叶变换获得温度变化的相关参数，并对1951-2008年上海的温度进行了研究，发现预测值的相对误差小于5%。黄健风(2016)使用了小波-NAR神经网络气象预测悉尼的温度变化，并得到了天气彩虹期权的估值。

在上面的文献中，我们发现大多数学术文献都是为特定气象要素的天气期货或天气选择而设计的。他们只能处理个别天气要素引起的天气风险造成的不确定性风险，但真正的天气风险通常是由各种气象要素的不确定性引起的，这意味着天气元素参与现有天气衍生品并不完整。结果可能导致天气衍生品套期保值效果的减弱甚至丧失其实际应用价值。目前，中国的天气衍生品市场正面临着新的发展机遇。中国辽阔的国土受到季风气候特征的影响，国内的气温和降雨量等主要天气指数变化较大，变化的不确定性

较高。本文应用经典的SARIMA时间序列分析模型和BP神经网络预测技术来建立气象时间序列预测和天气期权估值模型，从而揭示天气多因子期权的基本原理，并讨论每个天气因子的隐含期权价值的经济影响。

三、理论分析

（一）气象要素时间序列预测原理

气象要素时间序列预测是天气指数多因子期权估值的基础。本文中气象要素时间序列的季节效应，长期趋势效应和随机波动之间存在着复杂的交互影响关系，简单的ARMA模型并不足以提取其中的相关关系，SARIMA模型可以很好的弥补，SARIMA模型(Seasonal Autoregressive Integrated Moving Average)来源于自回归单整移动平均季节模型（ARIMA），能够采用Box-Jenkins的模型识别、估计和预测程序，可以随着历史数据的获得而对模型进行实时调整。SARIMA模型的一般形式为：

$$\phi(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D y_t = c + \theta(B)\Theta_Q(B^s)\varepsilon_t \quad (1)$$

其中d和D分别表示逐期差分和季节差分的阶数，p,q分别为自回归和移动平均的阶数，P,Q分别为季节自回归和季节移动平均的阶数；S为季节周期，B^S 表示季节后移算子；$\Phi_P(B^s)=1-\Phi_1 B^S-\Phi_2 B^{2S}-\ldots\Phi_p B^{PS}$；$\Theta_Q(B^S)=1-\Theta_1 B^S-\Theta_2 B^{2S}-\ldots\Theta_Q B^{QS}$；$\phi(B)=1-\phi_1 B^1-\phi_2 B^2-\ldots\phi_p B^p$；$\theta(B)=1-\theta_1 B^S-\theta_2 B^2-\ldots\theta_q B^q$；$\varepsilon_t$ 为残差，是一个高斯白噪声的随机过程。式(1)所表示的SARIMA模型被记为SARIMA(p, d, q)(P, D, Q)S模型。通过对观测得到的气象要素时间序列数据分别进行d阶趋势差分和D阶以周期S为步长的季节差分，确定差分后的时间序列通过单位根检验呈现平稳特征，根据自相关系数和偏自相关系数的特征确定短期相关模型，通过残差检验时序图确定在趋势平稳下的最终模型。

BP神经网络又称为误差反向传播(Back Propagation)神经网络，它是一个多层前向型神经网络(如图1)。利用误差反向传播算法对网络进行训练，具有很强的非线性映射能力，在数据拟合和函数逼近上具有很强的预测能力。在BP神经网络中，信号是前向传播，而误差是反向传播的。BP神经网络通常具有一个或者多个sigmoid隐层和线性输出层，能够对有限个不连续点的函数进行逼近。

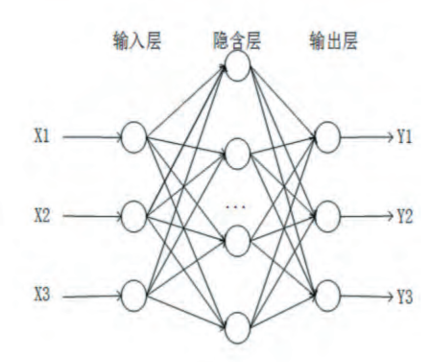

图1 BP神经网络示意图

在模型中，输出 y_t 和输入 x_{t-1}，x_{t-2}，…，x_{t-p} 的关系可用下式表达：

$$y_t = \omega_0 + \sum_{j=1}^{Q} \omega_j g\left(\omega_{0j} + \sum_{i=1}^{P} \omega_{ij} x_{t-i}\right) + \varepsilon_t \tag{2}$$

式(2)中 ω_{ij},$(i=0,1,2\ldots,P;j=0,1,2\ldots Q)$ 为连接权值向量，$\omega_j(j=0,1,2,,\ldots,Q)$ 为阈值向量，输入层和输出层的节点数分别为P和Q。 $g(\cdot)$ 为激活函数。上述表达式反映出BP神经网络能学习和存储大量输入 x_{t-1}， x_{t-2}， …， x_{t-p} 与输出 y_t 之间的映射关系，而又无需事前揭示描述这种映射关系的数学方程。它的学习规则是使用最速下降法，通过反向传播来不断调整网络的权值和阈值，使网络的误差平方和最小。

因此本文将气象要素时间序列数据分类为训练输入数据、测试数据，初始权值和阈值采用随机数，激活函数为对数正切S型函数和线性函数，训练函数采用Fletcher-Reeves共轭梯度法。隐含层单元数的确定方法为：$\sqrt{n+m}+a$，其中n为输入单元数，m为输出单元数，a为[1,10]之间的常数。BP神经网络下的学习过程分别由以下几个步骤逐次进行：首先设置所有连接权重值和阈值为最小随机数，其次根据所提供的训练数据给与网络，再计算隐含层，输出层以及各个神经元的实际输出，通过比较实际输出与期望值的相对误差修正网络连接权重值和阈值，从而再继续计算隐含层，输出层和各个神经元的实际输出，计算输出和期望值的误差，反复迭代下，直至误差满足要求为止。本文采用的数据为1951年-2013年每年1-12月份的月平均气温和当月降雨量，即为矩阵数据756×2，我们选取1951年-1980年的月平均气温和月降水量作为训练集输入，即矩阵数据360×2，1981年的气象要素数据作为当期目标训练集输出。将该训练集输出结果与1981年实际样本数据进行误差分析，修改网络的连接权重值和阈值，直至当迭代次数达到隐含层与输出层各神经元的实际输出与期望值的误差后，选择1952年-1981年的月平均气温和月降水量作为第二次迭代训练集输入，即矩阵数据360×2，1982年的气象要素数据作为当期目标训练集输出，将该训练集输出结果与1982年实际样本数据进行误差分析，修改网络的连接权重值和阈值，直至当迭代次数达到隐含层与输出层各神经元的实际输出与期望值的误差后，以此类推，不断迭代下去，最终根据训练数据下的实际网络结构选取1982年-2012年气象要素数据作为测试集输入，2013年的气象要素数据作为目标测试集输出。

采用最优加权组合预测方法将BP神经网络和SARIMA模型组合优化，可使模型具有更大的可靠性和客观性。组合预测方法是一种非线性系统综合技术方法，即对多种模型的结果进行综合研究，以期提高预测结果的可靠性和客观性。组合模型的主要目的是综合利用多种模型所提供的信息，尽可能地提高预测准确率，其比单独模型考虑问题更全面系统。组合预测权系数的确定的方法：以组合预测的预测误差平方和最小为最优准则，用线性方程的形式求出结果。由此分别建立BP神经网络和SARIMA模型预测天气系统的变化，可以综合利用多种模型所提供的信息，提高预测准确率。

设B和S分别是BP神经网络和SARIMA模型的预测值，Y为最优组合预测值，预测误差分别为 E_b，E_s，E_y，以预测误差平方和最小为最优原则即 $E=\sum E_y^{\,2}$ 取最小，K_b 和 K_s 为相应的权系数（$K_b+K_s=1$），则有

$$E_y = K_b{*}E_b + K_s{*}E_s \tag{3}$$

$$Y = K_b B + K_s S \tag{4}$$

K_b 和 K_s 的计算方程为：

$$K_b = 1 - K_s \tag{5}$$

$$K_s = -\frac{\sum(E_s - E_b)E_b}{\sum(E_s - E_b)^2} \tag{6}$$

$$RMSE = \left(\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2\right)^{\frac{1}{2}} \tag{7}$$

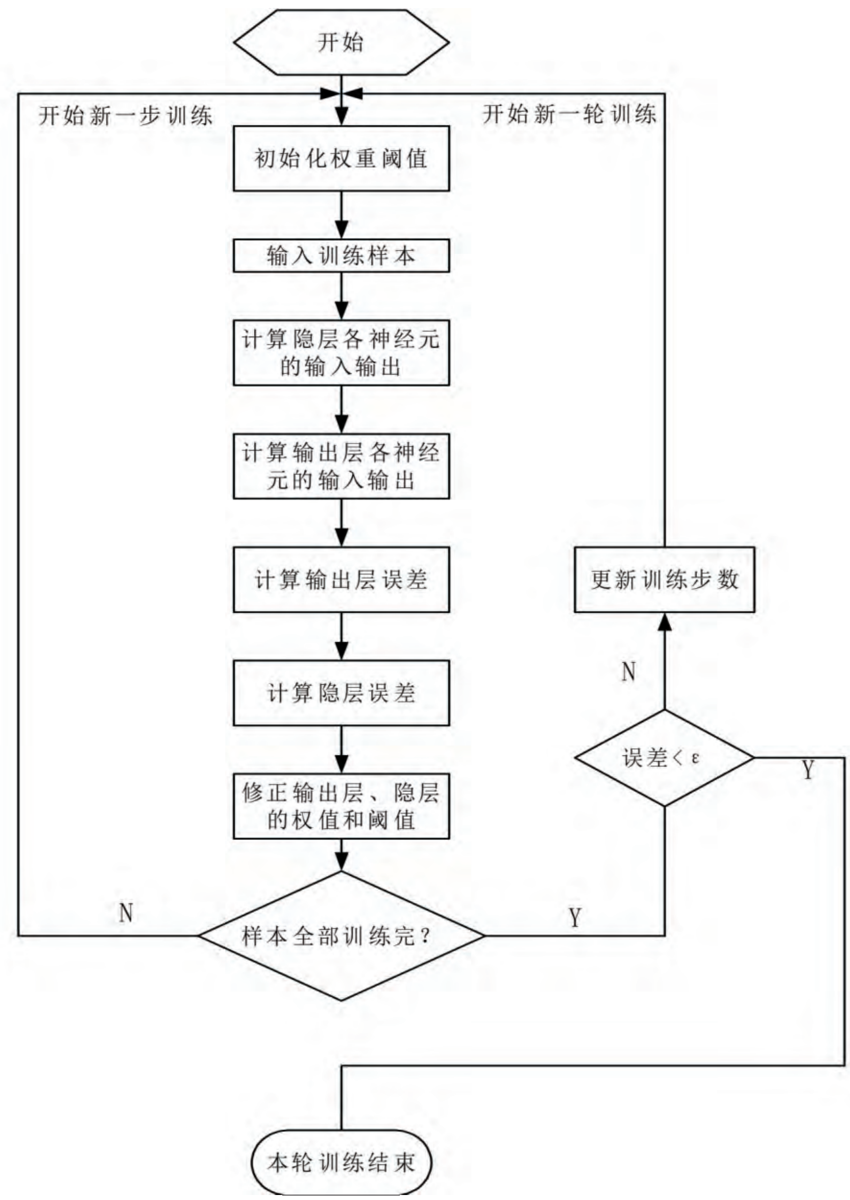

图2　BP算法程序流程图

通过确定SARIMA模型和BP神经网络在最优组合模型中的权重比率，得到组合模型气象要素时间序列

的预测值。为了定量比较组合模型与各单一模型的气象要素时间序列估值结果，使用误差指标—均方根误差(RMSE)来检验估值效果。

(二)天气指数多因子期权估值原理

在天气指数的计算中，若月平均气温和月降雨量采用实际值，则依据式(8)-(11)计算所得到的是天气指数的实际值；若采用预测值则将得到天气指数模型估计值。天气指数的预测值与实际值绝对偏差越小则模型预测越精确，反之，则模型预测结果谬误越大。根据芝加哥商业交易所天气衍生品的标的指数计算方法，计算月度HDD(取暖指数)，CDD(制冷指数)，CRI(累积降雨指数)以及年度EDD(能源温值)的表达式如下：

$$HDD=\sum_{t=1}^{n}\{[\max(T_0-T_t,\ 0)]*30\} \tag{8}$$

$$CDD=\sum_{t=1}^{n}\{[\max(T_t-T_0,\ 0)]*30\} \tag{9}$$

$$CRI=\sum_{t=1}^{n}R_t \tag{10}$$

$$EDD=HDD+CDD \tag{11}$$

其中T_t，R_t分别为天气多因子期权生效日起第t个月的月度平均气温和月降水量；T_0为气温阈值；n表示天气多因子期权的合约期限。能源温值EDD主要来源于能源部门规避天气风险的需求。在暖冬或者凉夏，人们所需的能源减少，使得能源供应商减少其销售量，收入降低。因此设计了EDD来对能源供应商全年的天气风险进行保护。EDD是HDD和CDD的累计总数，其合同期间通常是全年。从式(7)-(10)可以看出HDD,CDD,CRI,EDD均为非负值，并且在随着天气多因子期权到期期限的延长，各标的指数值呈现正向累加效应(或无累加效应)[①] 在普通的天气指数看涨/看跌期权中，看涨期权可以使风险暴露者将买入期权而获得的收入用于部分或全部弥补极端气象因素而造成的经济损失。在合约期限内，当指数累积值超出了预先协定的水平时，期权具有执行价值。而当指数累积值没有超出协定的水平时，期权没有执行价值，看涨期权的买方将放弃行权，损失的只有支付给卖方的期权费；看跌期权亦是如此。在天气指数多因子期权估值中，我们对天气指数多因子期权标的中的每个指数(CDD,HDD,CRI,EDD)分别设定相应的执行水平，结合天气多因子期权估值的基本定义式，得到各个因子对应的单一天气指数期权估值。

值得一提的是，与传统的Black-Scholes-Merton期权定价模型和二叉树定价模型不同，天气类衍生品对应的标的资产是一个物理量(如温度，降雨量等)，而不是一个可交易的资产。天气多因子期权之所以可以用于现实天气风险决策，缘于他兼具天气期权和多因子期权的特征值，可以较好的补偿未来各种不利天气对某些天气敏感企业造成的损失，而这一损失的程度可以通过天气指数来确定。那么如何通过购买或者沽出天气多因子期权并期望从中获益以补偿不利天气造成的经济损失，企业决策者依据在于该期权到期价值的事先预估结果。因此本文引入天气多因子期权估算不同标的下的期权期望收益，也即天气多因子期权到期价值。

假设执行天气标的指数为K_i，上限/下限天气标的指数为L_i，到期天气标的指数为x_i，D_i为天气标的指数的货币值，i为天气多因子期权的第i个天气指数。

看涨期权买方的支付函数为：

$$C(x_i)=D_i\times Min\Big(Max\big((x_i-K_i),0\big),(L_i-K_i)\Big) \tag{12}$$

①天气期权是一种期货期权，表示投资者在未来某一日期有权买卖一个天气指数期货合同，分为看涨期权和看跌期权。当指定场所的测定的天气指数的累计数量超出(低于)预先商定的水平时，看涨(跌)期权的买方通过支付期权费来执行此权利，即按预定的价格买入(卖出)天气指数期货合约来获取利润。

看跌期权买方的支付函数为：

$$P(x_i)=D_i\times \mathrm{Min}\Big(\mathrm{Max}\big((K_i-x_i),0\big),(K_i-L_i)\Big) \tag{13}$$

天气指数多因子期权的支付特点是取决于两种或者多种单一天气指数期权价值中的最大值或者最小值。因此我们得到多因子最大值看涨期权（$cal\ l_{max}$），多因子最小值看涨期权（$cal\ l_{min}$），多因子最大值看跌期权（put_{max}），多因子最小值看跌期权（put_{min}）。该四种天气指数多因子期权估值表达式如下：

$$\begin{aligned} cal\ l_{max} &= \max\big(C(x_1),\ C(x_2),\ C(x_3),\ \ldots,\ C(x_r)\big) \\ &= \max\Big(D_1\times \mathrm{Min}\Big(\mathrm{Max}\big((x_1-K_1),0\big),(L_1-K_1)\Big),\ D_2\times \mathrm{Min}\Big(\mathrm{Max}\big((x_2-K_2),0\big),(L_2-K_2)\Big), \\ &\quad \ldots D_r\times \mathrm{Min}\Big(\mathrm{Max}\big((x_r-K_r),0\big),(L_r-K_r)\Big) \\ &= \max\Big(D_1\Big(\mathrm{Max}\big((x_1-K_1),0\big)\Big),\ D_2,\ldots,D_r\Big(\mathrm{Max}\big((x_r-K_r),0\big)\Big) \end{aligned} \tag{14}$$

$$\begin{aligned} cal\ l_{min} &= \max\big(C(x_1),\ C(x_2),\ C(x_3),\ \ldots,\ C(x_r)\big) \\ &= \max\Big(D_1\Big(\mathrm{Min}\big((x_1-K_1),0\big)\Big),D_2\Big(\mathrm{Min}\big((x_2-K_2),0\big)\Big),\ldots,D_r\Big(\mathrm{Min}\big((x_r-K_r),0\big)\Big) \end{aligned} \tag{15}$$

$$\begin{aligned} put_{max} &= \max\big(C(x_1),\ C(x_2),\ C(x_3),\ \ldots,\ C(x_r)\big) \\ &= \max\Big(D_1\Big(\mathrm{Max}\big((K_1-x_1),0\big)\Big),D_2\Big(\mathrm{Max}\big((K_2-x_2),0\big)\Big),\ldots,D_r\Big(\mathrm{Max}\big((K_r-x_r),0\big)\Big)\Big) \end{aligned} \tag{16}$$

$$\begin{aligned} put_{min} &= \max\big(C(x_1),\ C(x_2),\ C(x_3),\ \ldots,\ C(x_r)\big) \\ &= \max\Big(D_1\Big(\mathrm{Min}\big((K_1-x_1),0\big)\Big),D_2\Big(\mathrm{Min}\big((K_2-x_2),0\big)\Big),\ldots,D_r\Big(\mathrm{Min}\big((K_r-x_r),0\big)\Big)\Big) \end{aligned} \tag{17}$$

上述多因子期权估值表达式括号中各子项(除最后一项0外)表示该天气多因子期权购买者对标的中各天气指数对应分量的支付结构。在上述期权估值过程中,若天气指数采用模型预测值,则该天气指数多因子期权估值为期权的远期价值判断,反映了该期权合约期限内天气不确定性的评估价值;若天气指数采用实际值,则该天气指数多因子期权估值为期权的远期实际价值。通过比较远期实际价值和远期模型预测价值的相对误差,则可以比较不同模型下期权估值的准确度。若相对误差越小,则期权估值越准确,也即通过气象预测模型下的天气多因子期权为依据作出的天气风险决策更加有效,反之亦然。

四、实证分析

上文中详述了关于气象要素(月平均气温和月降雨量)时间序列预测原理的理论分析,并阐述了分别利用SARIMA模型、BP神经网络、BP神经网络—SARIMA组合模型下的时间序列预测数据计算天气期权标的指数和天气指数多因子期权估值方法。下文将以气温和降雨这两个气象要素构建的天气期权标的指数HDD,CDD,CRI,EDD组成的28种天气指数多因子期权进行分析。本文数据来源于中国气象统计年鉴,涵盖了北京市1951年-2013年的月平均气温,月降雨量这两个气象要素,样本量达1512.总体样本分为两个子样本,取1951年至2012年的气象数据用于模型预测,取2013年的气象数据作为模型预测的参照标准。

(一)单一模型与组合模型下气象要素时间序列预测分析

在单一SARIMA模型分析下,首先通过因素分解拟合综合输出图我们发现北京市1951-2012年每月的平均气温和月降雨量随变动有非常规律的变化(如图2)。进一步说明了北京市的气温和降雨变化受季节效应、长期线性趋势效应及随机效应的影响,其总体规律较为复杂。显然序列是非平稳的,首先进行季节性差分(如图3),很显然其周期S=12,其季节差分的形式为

$$Y_k = X_{k+12}-X_k\ ,\quad k=1,2,\ldots\ldots,744 \tag{18}$$

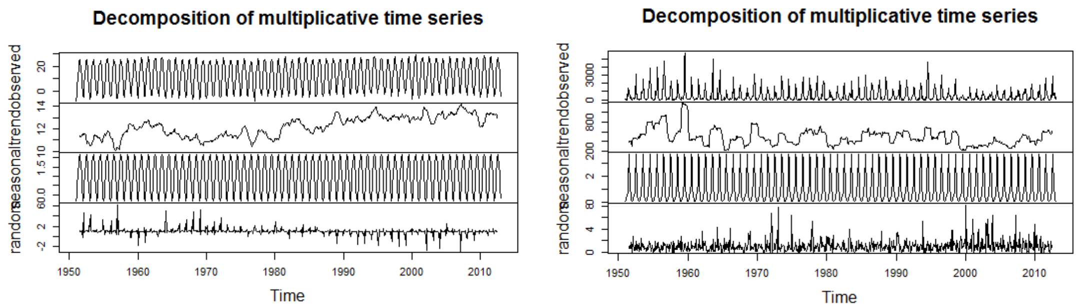

图3　北京市1951–2012年月平均气温(左)与月降雨量(右)因素分解拟合综合输出图

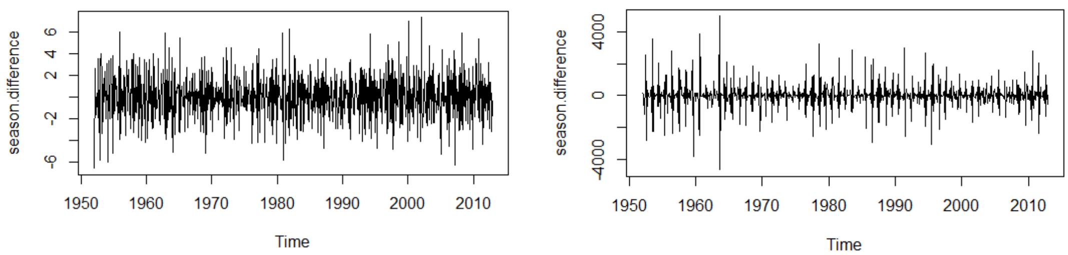

图4　北京市1951–2012年月平均气温(左)与月降雨量(右)1阶12步季节差分图

通过观察图形，我们可以进行平稳性的检验。平稳序列通常具有短期相关性。该性质用自相关来描述就是随着延迟阶数k的增加，平稳序列的自相关系数 $\widehat{\rho_k}$ 会很快的衰减为0。反之，非平稳序列的自相关系数 $\widehat{\rho_k}$ 衰减为0的速度通常比较慢。在ADF检验下显示其中P值小于0.05，可以认为其一阶12步差分序列平稳。观察可见月平均温度和月降雨量一阶季节性差分序列图的自相关系数和偏自相关系数图均是拖尾。通过观察月平均气温和月降雨量的一阶季节性差分序列的QQ图，可以看出基本符合正态分布。

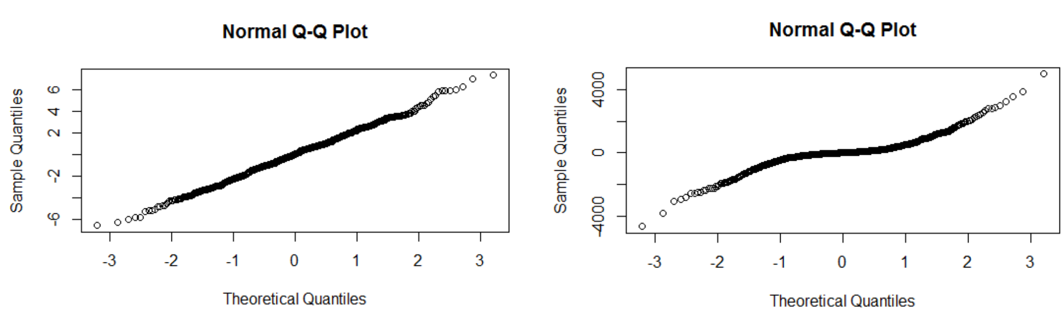

图5　北京市1951–2012年月平均气温(左)与月降雨量(右)标准化的残差QQ图

根据最小信息准侧，对北京市月平均气温建立SARIMA(p,0,q)(P,1,Q)模型，最终选取SARIMA()(4,0,0)(2,1,0)，参数估计结果如表1；类似的，对北京市月降雨量建立SARIMA(p,0,q)(P,1,Q)模型，最终选取SARIMA()(5,0,0)(2,1,0)，参数估计结果如表2.

表1　月平均气温模型参数估计结果

	AR1	AR2	AR3	AR4	SAR1	SAR2
Value	0.2192	0.0403	0.0596	-0.0142	-0.6430	-0.3655
s.e.	0.0374	0.0380	0.0380	0.0372	0.0352	0.0354

表2　北京市月降雨量模型参数估计结果

	AR1	AR2	AR3	AR4	AR5	SAR1	SAR2
Value	0.0839	0.0372	-0.0063	0.0261	0.0183	-0.6242	-0.3189
s.e.	0.0370	0.0373	0.0371	0.0371	0.0370	0.0356	0.0358

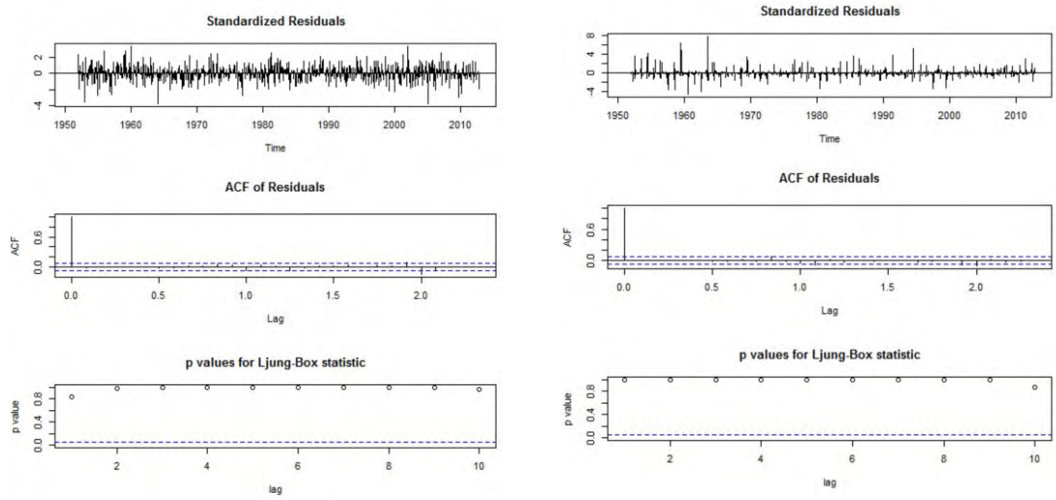

图6　北京市1951–2012年月平均气温(左)与月降雨量(右)预测模型的诊断检验

上述模型检验图提供了标准残差的序列图、残差的样本ACF和从0-10的Ljung-Box检验统计量的P值，是一个综合分析诊断时间序列模型的工具。ACF检验说明：残差没有明显的自相关性。Ljung-Box测试显示：所有的P-value>0.5，说明残差为白噪声。从而说明得到了较好的预测结果。

本文采用1951-1980年的气象数据作为训练集，而1981年一年的气象数据作为训练集输出，将该训练集输出结果与1981年实际样本数据进行误差分析，通过修改网络的连接权重值和阈值达到误差要求，之后不断迭代，直至当迭代次数达到隐含层与输出层各神经元的实际输出与期望值的误差后，采用最优的网络的结构选取1982-2012年的气象数据作为测试集，而2013年的气象数据作为测试集输出，采用神经网络形式如图6所示。将气象要素时间序列数据分类为训练输入数据、测试数据，初始权值和阈值采用随机数，激活函数为对数正切S型函数和线性函数，训练函数采用Fletcher-Reeves共轭梯度法

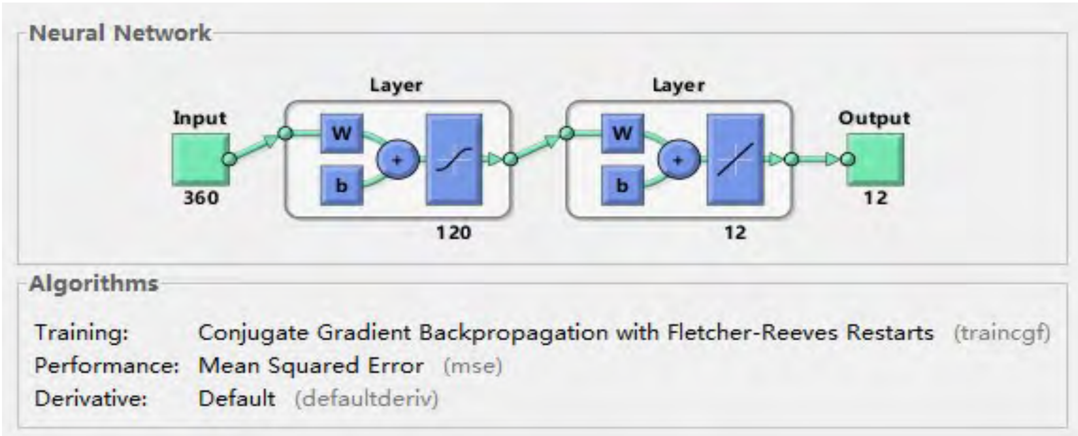

图7　北京市月平均气温和月降雨量BP神经网络预测模型示意图

根据神经网络输出，预测图与实际图相比较如图7所示。图8显示在经过250次迭代后在误差反向传播的第249次达到预设误差目标。

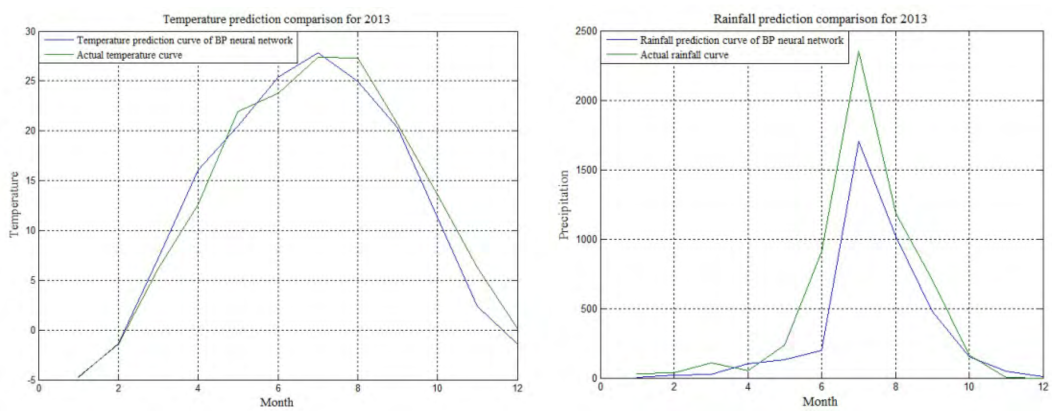

图8　北京市月平均气温和月降雨量BP神经网络输出图

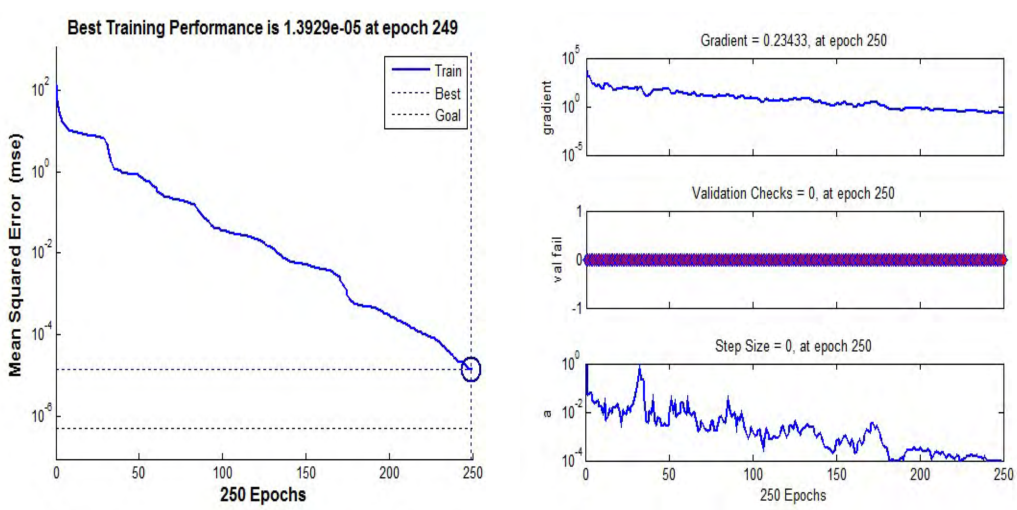

图9　BP神经网络最优训练迭代误差收敛图

表3　两种单一模型下对北京市月平均气温和月降雨量的预测结果与实际值对比

模型比较	SARIMA 模型估计值		BP 神经网络估计值		实际值	
年份	月平均温度	月降水量	月平均温度	月降水量	月平均温度	月降水量
Jan-13	-4.91	55.91	-4.80	8.01	-4.7	30
Feb-13	-1.14	48.20	-1.40	23.01	-1.4	34
Mar-13	5.61	121.23	7.20	32.98	6.2	107
Apr-13	14.03	318.44	16.00	105.01	12.6	55
May-13	21.95	281.86	20.50	140.05	21.9	236
Jun-13	25.26	1032.95	25.40	194.06	23.8	910
Jul-13	27.86	1987.38	27.80	1708.95	27.4	2356
Aug-13	26.29	1315.80	24.80	1024.94	27.3	1186
Sep-13	20.92	758.67	20.30	481.99	20.7	711
Oct-13	14.12	356.36	11.30	150.01	13.6	164
Nov-13	5.62	369.57	2.30	52.03	6.3	2
Dec-13	-2.14	37.45	-1.40	10.98	0.1	0

采用最优线性组合模型原理，利用样本期实际值和各单项预测模型的拟合值进行线性回归，可以确定各单项预测模型的权重，分别选取BP神经网络模型和SARIMA模型样本期内的拟合值作为自变量，选取对应的实际值作为因变量，最终建立线性回归模型，确定组合模型的权重：

表4　组合模型下不同模型的权重比率

权重比率	SARIMA 模型	BP 神经网络模型
月平均气温	1.0779	-0.0779
月降水量	0.764	0.236

组合模型表达式如下：

月平均气温模型：

$$\widehat{y_{\mathrm{combine}}} = 1.0779\,\widehat{y_{\mathrm{SARIMA}}} - 0.0779\,\widehat{y_{\mathrm{BP}}} \tag{19}$$

月降水量模型：

$$\widehat{y_{\mathrm{Combine}}} = 0.7640\,\widehat{y_{\mathrm{SARIMA}}} + 0.2360\,\widehat{y_{\mathrm{BP}}} \tag{20}$$

预测结果如表5。通过计算结果可以发现组合模型的均方根误差在不同时间跨度下均为最小值，其相对于单一的SARIMA模型和BP神经网络而言，预测更加精确(如表6)。

表5　组合模型下北京市气温与降雨量的预测值

月份	月平均温度	月降水量
Jan-13	-4.91	44.60
Feb-13	-1.12	42.26
Mar-13	5.49	100.41
Apr-13	13.88	268.07
May-13	22.06	248.39
Jun-13	25.25	834.97
Jul-13	27.87	1921.67
Aug-13	26.41	1247.16
Sep-13	20.97	693.37
Oct-13	14.34	307.66
Nov-13	5.88	294.63
Dec-13	-2.20	31.21

表6　2013年北京市月平均气温和降水量的预测评价:RMSE计算结果

气象要素	时间跨度	SARIMA模型	BP神经网络	组合模型
月平均气温	一季度	0.7913	1.7727	0.7525
	上半年	0.8793	1.6877	0.8557
	全年	0.9861	1.9832	0.9786
月降水量	一季度	132.74	46.32	106.92
	上半年	120.89	297.31	92.65
	全年	186.24	293.01	171.13

（二）单一模型与组合模型下天气多因子期权估值结果分析

表7　2013年北京市天气指数计算结果

气象要素	时间跨度	实际值	SARIMA模型	BP神经网络	组合模型
CDD/℃	一季度	0	0	0	0
	上半年	291	336.3	297	339.3
	全年	933	968.4	864	976.8

	一季度	1617	1633.2	1590	1636.2
HDD/℃	上半年	1779	1752.3	1650	1759.8
	全年	2799	2871	3006.3	2839.2
	一季度	1617	1633.2	1590	1636.2
EDD/℃	上半年	2070	2088.6	1947	2099.1
	全年	3732	3839.4	3870.3	3816
	一季度	17.1	22.5	6.4	18.7
CRI/mm	上半年	137.2	185.9	50.3	153.9
	全年	579.1	668.4	393.2	603.4

根据前文中单一模型与组合模型下的估计结果，根据式(8)—(11)的计算方法，可以得出的不同天气标的指数的估计，也即CDD,HDD,EDD,CRI在不同时间跨度下的估计值，由此计算天气多因子期权的估值，如表7所示。由于当前市场上并无天气指数多因子期权的挂牌交易。在文章中，期权交易涉及到不同天气标的指数的执行指数和单位天气标的指数的货币值。我们参照CME时下交易气温指数期权和降雨指数期权的参数设置方法。本文一季度、上半年、全年的天气多因子期权（CDD,HDD,EDD,CRI）的执行水平分别取（0，1600，1600，50）、（300，1700，1900，100）、（800，2500，3600，500），其单位标的天气指数的货币值设置为（20，20，50，500），单位以美元计价。如表8所示.基于BP神经网络—SARIMA模型下的期权估值较单一BP神经网络和单一SARIMA模型准确，意味着基于BP神经网络-SARIMA组合模型下的期权估值能够更好的估算出2013年北京市一季度、上半年、全年的天气隐含的经济价值，有助于决策者做出正确的规避天气风险的决策。

如表8所示,如果决策者进入多因子最大值看涨期权的多头头寸,当期权的存续期结束时该期权将被执行,在到期支付最大化的原理基础上,这意味着天气风险决策是完全正确的,期权的购买者将收到由于2013年上半年或一年因为高温,低温或降雨造成实体经济损失的补偿。此外，从表8可以看出，基于BP神经网络和SARIMA模型的期权估值模型比单一的SARIMA模型或BP神经网络更准确。也就是说，最优权重下的模型能够捕捉到2013年北京季度半年度和年度天气信息背后隐含的经济价值。此外，单一SARIMA模型或BP神经网络由于预测天气要素的误差出入较大，导致天气期权的估值可能造成更大的决策偏差或决策失误。因此，相对合理的天气期权估值将有助于采取合理的措施避免天气风险造成的损失。

五、结论及启示

天气衍生品作为较为前沿的风险管理工具，国内对这种对冲天气风险的需求日益迫切，却至今没有相关产品推出。而其中产品定价估值又是该领域研究的核心问题。大多数研究者通过单一模型预测单一天气要素（如单一气温期权，单一降雨期权）来进行天气衍生品的定价与估值，而本文采用BP神经网络—SARIMA组合模型来预测多种天气要素（月平均温度和月降雨量）的动态变化趋势，使得气象预测更加精确，利用预测值计算天气期权下四种不同标的天气指数值，并基于不同模型下天气多因子期权估值作出对比分析。此外，通过对BP神经网络和SARIMA模型的整合，可以很好地反映气象要素变化特征。预期效果优于单一的BP神经网络模型和单一的SARIMA模型。综合模型对天气指数和天气期权估值的计算结果有较好的相对误差估值结果，因此我们有理由认为，该模型对天气多因子期权的估值与定价是合理的。在本文中，天气多因子期权估值涉及“气象要素的变化-天气指数的计算-期权价值的估计”的转型过程。它反映了未来天气不确定性的经济价值。精确的天气预报提供了更加准确的天气期权的估值，有助于天

气风险承受者提高应对天气风险决策的准确性,作出合理的保险决策。

本文主要基于SAIMA模型和BP神经网络的组合模型,是对天气系统研究与天气衍生品估值的一种新的尝试,但SAIMA预测方法是线性的,完全数据驱动的,具有一定的局限性。虽然组合模型预测结果优于任一个单一模型,但考虑到BP神经网络的非线性映射能力,泛化能力和容错能力,我们仍需要增大样本和输入输出层的选择上做继续深入研究。此外,考虑到天气系统变化和地面观测月统计资料对月均温度与月降雨量的影响,我们希望后续研究通过组合模型来引入更多的分析因子,来研究其内在变化规律,提高天气期权估值的准确性。

表8　2013年北京市天气指数多因子期权估值结果(单位:$)

天气指数多因子期权	标的	时间跨度	实际值	SARIMA模型	BP神经网络	组合模型
多因子最大值看涨期权	CDD-CRI	一季度	0	0(0)	0(0)	0(0)
		上半年	18600	42950(131%)	0(∞)	26950(45%)
		全年	39550	84200(112%)	1280(97%)	51700(31%)
	HDD-CRI	一季度	340	664(95%)	0(100%)	724(112%)
		上半年	18600	42950(131%)	0(∞)	26950(45%)
		全年	39550	84200(112%)	1280(97%)	51700(31%)
	EDD-CRI	全年	39550	84200(112%)	13515(66%)	51700(31%)
多因子最大值看跌期权	CDD-CRI	一季度	16450	13750(17%)	21800(33%)	15650(5%)
		上半年	180	0(100%)	24850(13705%)	0(100%)
		全年	0	0(0)	53400(∞)	0(0)
	HDD-CRI	一季度	16450	13750(17%)	21800(33%)	15650(5%)
		上半年	0	0(0)	24850(∞)	0(0)
		全年	0	0(0)	53400(∞)	0(0)
	EDD-CRI	全年	0	0(0)	53400(∞)	0(0)
多因子最小值看涨期权	CDD-CRI	一季度	0	0(0)	0(0)	0(0)
		上半年	0	736(∞)	0(0)	786(∞)
		全年	2660	3368(27%)	1280(52%)	3536(32%)
	HDD-CRI	一季度	0	0(0)	0(0)	0(0)
		上半年	1580	1046(34%)	0(∞)	1196(24%)
		全年	14950	18550(25%)	0(100%)	16960(13%)
	EDD-CRI	全年	6600	11970(82%)	0(100%)	10800(63%)
多因子最小值看跌期权	CDD-CRI	一季度	0	0(0)	0(0)	0(0)
		上半年	0	0(0)	0(0)	0(0)
		全年	0	0(0)	0(0)	0(0)
	HDD-CRI	一季度	0	0(0)	200(∞)	0(0)
		上半年	0	0(0)	1000(∞)	0(0)
		全年	0	0(0)	0(0)	0(0)
	EDD-CRI	全年	0	0(0)	0(0)	0(0)

注:括号内为不同模型下天气多因子期权与实际值的相对误差。

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Abstract: With the increasing frequency of abnormal climate change and weather catastrophe in recent years, weather risk management has played a more and more important role in economic development. Weather derivatives are the most effective tool for weather risk management, and the pricing of weather derivatives is one of the core topics in this field. Current studies aim to forecast meteorological elements time series and the price of weather multi factor options with the integrating BP neural network and SARIMA model. Through analyzing the meteorological data from 1951 to 2013 in Beijing, including the monthly average temperature and the dynamic change of the average monthly rainfall, we calculate four weather indices: CDD (cooling degree days), HDD (heating degree days), EDD (energy degree days), CRI (cumulative rainfall index), and price some weather multi-factor options by using the estimates of meteorological elements time series of the underlying indices. We find that integrating the BP neural network and SARIMA model has strong nonlinear mapping ability, and its forecast and valuation result is better than single model. Weather multi-factor options can be applied to analyzing the influence of different elements on weather, thus avoid the weather risk and gain profits effectively.

Key Words: Weather multi-factor option; SARIMA model; BP neural network