Stats 120B Homework 2: Due Jan. 25

1. [15 pts] Let $I(\cdot)$ be the indicator function. Prove $I_{A\cap B} = I_A \times I_B$ for any two sets A and B.

2. Let X and Y follow a bivariate normal distribution with means $(3, 2)$, variances $(1, 4)$ and covariance c. In other words,

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & c \\ c & 4 \end{pmatrix}\right)$$

   (a) [10 pts] What's the marginal distribution of X?

   (b) [10 pts] If $c = 0$, what's the conditional variance of $W = Y + 999$ given $X = 999$? Explain.

   (c) [10 pts] If $c = 1$, find the covariance between $2X$ and $X + Y$.

   (d) [10 pts] If $c = .5$. Consider two conditional expectations $\mathrm{E}(Y|X = 1)$ and $\mathrm{E}(Y|X = 2)$. Which one is bigger? Explain.

3. [15 pts] Suppose that two random variables X_1 and X_2 have a bivariate normal distribution, and $\mathrm{Var}(X_1) = \mathrm{Var}(X_2)$. Show that the sum $X_1 + X_2$ and the difference $X_1 - X_2$ are independent.

4. Let X_1 and X_2 be two random variables following Binomial distribution $\mathrm{Bin}(n_1, p)$ and $\mathrm{Bin}(n_2, p)$, respectively. Assume that X_1 and X_2 are independent.

   (a) [10 pts] The mgf of binomial distribution $\mathrm{Bin}(n, p)$ is $(1 - p + pe^t)^n$. Use this fact to obtain the distribution of $X_1 + X_2$.

   (b) [20 pts] Find the probability $P(X_1 + X_2 = 1|X_2 = k)$ for $k = 0$ and 1. Then use the law of total probability to find $P(X_1 + X_2 = 1)$.