Analyze S&P 500 returns by LSTM.

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0.1 Problem

Use LSTM to analyze S&P 500 returns over the period 2004-2006.

The data file "stock-treasury-2004_2006.csv", to be found in the "Data" folder, contains the following:
+ TREAS_3M: the yield of the 3-month treasury note in percent (i.e 2.1 means 2.1%) + Adjusted close price of ten major stocks: GM, F, UTX, CAT, MRK, PFE, IBM, MSFT, C, XOM + SP: The S&P 500 equity index level at the close of the trading day

Do the following:

Use the pandas read_csv function to read the Date and SP columns in a data frame called "sp_df".
Rename the "SP" column into "ClosePx" in the same read_csv call.
Compute the close-to-close index returns as: $r_t = P_{t+1}/P_t - 1$ and add them as a new column "DailyRet".
It is recommended to express all daily returns in basis points (10,000 bps = 100% = 1)

We want to train an RNN that looks back M days and forecasts forward N days.
Therefore the RNN will use return sequences of size M, and targets of size N.

Reformat the return data suitable for RNN processing as follows.
From the "DailyRet" column of "sp_df", create a data input matrix X containing rows as below:

$$r_0, r_1, r_2, \ldots, r_{M-1}$$

$$r_1, r_2, r_3, \ldots, r_M$$

$$r_2, r_3, r_4, \ldots, r_{M+1}$$

$$\ldots$$

From the "DailyRet" column of "sp_df", create also a target matrix y containing rows as below:

$$r_M, r_{M+1}, \ldots, r_{M+N-1}$$

$$r_{M+1}, r_{M+2}, \ldots, r_{M+N}$$

$$r_{M+2}, r_{M+3}, \ldots, r_{M+N+1}$$

$$\ldots$$

Set $M = 16$ and $N = 4$.
Build an RNN with two LSTM cells and train it on the first 607 sequences.
This means that the training set contains returns with the latest date of 2016-05-31.
Use the remaining returns for out-of-sample testing.
This is a regression task, so train the network using mean_squared_error loss.
When connecting the two LSTMs, make sure you set the parameter return_sequences=True on the first LSTM, so that the second can see the sequences.

Compute the out-of-sample actual and predicted 2-day, 3-day, ..., N-day return, by summing 1-day forward returns up to this horizon.
N-day return is the return from today's close to the close of the N-th day forward from today.
Calculate and report the RMSE and the correlation between actual and predicted 1-day, 2-day, ..., N-day returns.
Plot the actual and predicted returns in the out-of-sample part.
What do you conclude regarding the quality of the forecasts?

0.2 Solution

```
In [1]: %matplotlib inline
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        plt.style.use('ggplot')
        np.random.seed(42)

        from keras.datasets import mnist
        from keras.utils import to_categorical
        from keras.models import Sequential
        from keras.layers import Dense, Dropout, Activation
        from keras.layers import Conv2D, MaxPooling2D, Flatten, SimpleRNN, LSTM
        from keras.callbacks import EarlyStopping

        from sklearn.metrics import accuracy_score
        from sklearn.linear_model import LogisticRegression
        from sklearn.preprocessing import MinMaxScaler
```

```
Using TensorFlow backend.
```

0.3 import data

```
In [2]: plt.style.use('ggplot')
        M=16
        N=4
```

```
sp_df=pd.read_csv("stock-treasury-2004_2006.csv",
                  usecols=[0,12],parse_dates=['Date'],header=0,names=['Date','ClosePx']

sp_df.head()
```

```
Out[2]:         Date  ClosePx
        0 2004-01-02  1108.48
        1 2004-01-05  1122.22
        2 2004-01-06  1123.67
        3 2004-01-07  1126.33
        4 2004-01-08  1131.92
```

```
In [3]: sp_df = sp_df.assign(DailyRet=10000 * (sp_df.ClosePx.shift(periods=-1)/ sp_df.ClosePx
        sp_df.head()
```

```
Out[3]:         Date  ClosePx   DailyRet
        0 2004-01-02  1108.48 123.953522
        1 2004-01-05  1122.22  12.920818
        2 2004-01-06  1123.67  23.672431
        3 2004-01-07  1126.33  49.630215
        4 2004-01-08  1131.92 -88.875539
```

```
In [4]: sp_df.tail()
```

```
Out[4]:           Date  ClosePx   DailyRet
        668 2006-08-28  1301.78  19.204474
        669 2006-08-29  1304.28   8.357101
        670 2006-08-30  1305.37 -11.874028
        671 2006-08-31  1303.82  55.145649
        672 2006-09-01  1311.01        NaN
```

0.4 rescale train data

```
In [6]: traindata=np.array(sp_df.DailyRet)[0:607].reshape(-1,1)
        scaler = MinMaxScaler(feature_range=(-1, 1))
        traindata = scaler.fit_transform(traindata)
        traindata=pd.Series(np.squeeze(traindata))
        X_trainoriginal=traindata
```

```
In [336]: X_train = np.array([X_trainoriginal[0:M]])
          for i in range(1,(len(X_trainoriginal)-M+1-N)):
              xadd = np.array([X_trainoriginal[i:i+M]])
              X_train=np.vstack((X_train,xadd))
          X_train = X_train.reshape((-1, M, 1))
          X_train.shape
```

```
Out[336]: (588, 16, 1)
```

```
In [337]: y_train = np.array([X_trainoriginal[M:M+N]])
          for i in range(M+1,(len(X_trainoriginal)-N+1)):
              xadd = np.array([X_trainoriginal[i:i+N]])
              y_train=np.vstack((y_train,xadd))
          y_train.shape

Out[337]: (588, 4)
```

```
In [338]: y_train

Out[338]: array([[-0.75215881,  0.22510419, -0.17511201,  0.15480958],
                 [ 0.22510419, -0.17511201,  0.15480958, -0.00140475],
                 [-0.17511201,  0.15480958, -0.00140475, -0.47691249],
                 ...,
                 [ 0.04616983,  0.56039631,  0.26347798, -0.86986174],
                 [ 0.56039631,  0.26347798, -0.86986174,  0.38919919],
                 [ 0.26347798, -0.86986174,  0.38919919,  0.60917362]])
```

```
In [339]: X_train = X_train.reshape((-1, M, 1))
          y_train = y_train.reshape((-1, N))
```

0.5 construct model

```
In [340]: model = Sequential()
          model.add(LSTM(50, input_shape=(X_train.shape[1], X_train.shape[2]),return_sequences=
          model.add(LSTM(50, input_shape=(X_train.shape[1], X_train.shape[2])))
          model.add(Dense(N))
```

```
In [341]: model.summary()

_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
lstm_13 (LSTM)               (None, 16, 50)            10400
_________________________________________________________________
lstm_14 (LSTM)               (None, 50)                20200
_________________________________________________________________
dense_7 (Dense)              (None, 4)                 204
=================================================================
Total params: 30,804
Trainable params: 30,804
Non-trainable params: 0
_________________________________________________________________
```

```
In [342]: model.compile(loss='mean_squared_error', optimizer='nadam', metrics=['accuracy'])
```

```
In [343]: history = model.fit(X_train, y_train, epochs=100, batch_size=72, validation_split=0.
```

```
Train on 529 samples, validate on 59 samples
Epoch 1/100
529/529 [==============================] - 4s 7ms/step - loss: 0.1240 - acc: 0.2476 - val_loss
Epoch 2/100
529/529 [==============================] - 0s 358us/step - loss: 0.1236 - acc: 0.2571 - val_los
Epoch 3/100
529/529 [==============================] - 0s 364us/step - loss: 0.1236 - acc: 0.2552 - val_los
Epoch 4/100
529/529 [==============================] - 0s 371us/step - loss: 0.1234 - acc: 0.2439 - val_los
Epoch 5/100
529/529 [==============================] - 0s 366us/step - loss: 0.1235 - acc: 0.2231 - val_los
Epoch 6/100
529/529 [==============================] - 0s 368us/step - loss: 0.1234 - acc: 0.2363 - val_los
Epoch 7/100
529/529 [==============================] - 0s 370us/step - loss: 0.1233 - acc: 0.2382 - val_los
Epoch 8/100
529/529 [==============================] - 0s 377us/step - loss: 0.1233 - acc: 0.2382 - val_los
Epoch 9/100
529/529 [==============================] - 0s 368us/step - loss: 0.1232 - acc: 0.2325 - val_los
Epoch 10/100
529/529 [==============================] - 0s 368us/step - loss: 0.1233 - acc: 0.2439 - val_los
Epoch 11/100
529/529 [==============================] - 0s 358us/step - loss: 0.1233 - acc: 0.2401 - val_los
Epoch 12/100
529/529 [==============================] - 0s 349us/step - loss: 0.1233 - acc: 0.2193 - val_los
Epoch 13/100
529/529 [==============================] - 0s 368us/step - loss: 0.1233 - acc: 0.2401 - val_los
Epoch 14/100
529/529 [==============================] - 0s 371us/step - loss: 0.1232 - acc: 0.2439 - val_los
Epoch 15/100
529/529 [==============================] - 0s 381us/step - loss: 0.1232 - acc: 0.2363 - val_los
Epoch 16/100
529/529 [==============================] - 0s 375us/step - loss: 0.1233 - acc: 0.2420 - val_los
Epoch 17/100
529/529 [==============================] - 0s 375us/step - loss: 0.1231 - acc: 0.2325 - val_los
Epoch 18/100
529/529 [==============================] - 0s 388us/step - loss: 0.1231 - acc: 0.2514 - val_los
Epoch 19/100
529/529 [==============================] - 0s 371us/step - loss: 0.1231 - acc: 0.2533 - val_los
Epoch 20/100
529/529 [==============================] - 0s 371us/step - loss: 0.1230 - acc: 0.2420 - val_los
Epoch 21/100
529/529 [==============================] - 0s 359us/step - loss: 0.1231 - acc: 0.2647 - val_los
Epoch 22/100
529/529 [==============================] - 0s 364us/step - loss: 0.1231 - acc: 0.2533 - val_los
Epoch 23/100
529/529 [==============================] - 0s 377us/step - loss: 0.1229 - acc: 0.2609 - val_los
Epoch 24/100
```

```
529/529 [==============================] - 0s 377us/step - loss: 0.1231 - acc: 0.2250 - val_los
Epoch 25/100
529/529 [==============================] - 0s 379us/step - loss: 0.1230 - acc: 0.2495 - val_los
Epoch 26/100
529/529 [==============================] - 0s 371us/step - loss: 0.1228 - acc: 0.2628 - val_los
Epoch 27/100
529/529 [==============================] - 0s 386us/step - loss: 0.1229 - acc: 0.2628 - val_los
Epoch 28/100
529/529 [==============================] - 0s 379us/step - loss: 0.1226 - acc: 0.2363 - val_los
Epoch 29/100
529/529 [==============================] - 0s 379us/step - loss: 0.1233 - acc: 0.2533 - val_los
Epoch 30/100
529/529 [==============================] - 0s 375us/step - loss: 0.1229 - acc: 0.2571 - val_los
Epoch 31/100
529/529 [==============================] - 0s 368us/step - loss: 0.1228 - acc: 0.2609 - val_los
Epoch 32/100
529/529 [==============================] - 0s 375us/step - loss: 0.1226 - acc: 0.2684 - val_los
Epoch 33/100
529/529 [==============================] - 0s 375us/step - loss: 0.1227 - acc: 0.2439 - val_los
Epoch 34/100
529/529 [==============================] - 0s 394us/step - loss: 0.1231 - acc: 0.2382 - val_los
Epoch 35/100
529/529 [==============================] - 0s 377us/step - loss: 0.1229 - acc: 0.2647 - val_los
Epoch 36/100
529/529 [==============================] - 0s 377us/step - loss: 0.1227 - acc: 0.2552 - val_los
Epoch 37/100
529/529 [==============================] - 0s 403us/step - loss: 0.1226 - acc: 0.2268 - val_los
Epoch 38/100
529/529 [==============================] - 0s 388us/step - loss: 0.1225 - acc: 0.2344 - val_los
Epoch 39/100
529/529 [==============================] - 0s 383us/step - loss: 0.1225 - acc: 0.2798 - val_los
Epoch 40/100
529/529 [==============================] - 0s 397us/step - loss: 0.1224 - acc: 0.2382 - val_los
Epoch 41/100
529/529 [==============================] - 0s 377us/step - loss: 0.1220 - acc: 0.2344 - val_los
Epoch 42/100
529/529 [==============================] - 0s 369us/step - loss: 0.1219 - acc: 0.2552 - val_los
Epoch 43/100
529/529 [==============================] - 0s 396us/step - loss: 0.1218 - acc: 0.2571 - val_los
Epoch 44/100
529/529 [==============================] - 0s 390us/step - loss: 0.1218 - acc: 0.2590 - val_los
Epoch 45/100
529/529 [==============================] - 0s 392us/step - loss: 0.1216 - acc: 0.2647 - val_los
Epoch 46/100
529/529 [==============================] - 0s 387us/step - loss: 0.1212 - acc: 0.2552 - val_los
Epoch 47/100
529/529 [==============================] - 0s 392us/step - loss: 0.1219 - acc: 0.2533 - val_los
Epoch 48/100
```

```
529/529 [==============================] - 0s 407us/step - loss: 0.1211 - acc: 0.2665 - val_los
Epoch 49/100
529/529 [==============================] - 0s 436us/step - loss: 0.1214 - acc: 0.2590 - val_los
Epoch 50/100
529/529 [==============================] - 0s 567us/step - loss: 0.1209 - acc: 0.2514 - val_los
Epoch 51/100
529/529 [==============================] - 0s 615us/step - loss: 0.1207 - acc: 0.2665 - val_los
Epoch 52/100
529/529 [==============================] - 0s 583us/step - loss: 0.1204 - acc: 0.2268 - val_los
Epoch 53/100
529/529 [==============================] - 0s 577us/step - loss: 0.1200 - acc: 0.2590 - val_los
Epoch 54/100
529/529 [==============================] - 0s 581us/step - loss: 0.1202 - acc: 0.2628 - val_los
Epoch 55/100
529/529 [==============================] - 0s 598us/step - loss: 0.1198 - acc: 0.2665 - val_los
Epoch 56/100
529/529 [==============================] - 0s 573us/step - loss: 0.1194 - acc: 0.2628 - val_los
Epoch 57/100
529/529 [==============================] - 0s 579us/step - loss: 0.1188 - acc: 0.2836 - val_los
Epoch 58/100
529/529 [==============================] - 0s 679us/step - loss: 0.1184 - acc: 0.2665 - val_los
Epoch 59/100
529/529 [==============================] - 0s 813us/step - loss: 0.1177 - acc: 0.2892 - val_los
Epoch 60/100
529/529 [==============================] - 0s 866us/step - loss: 0.1172 - acc: 0.2892 - val_los
Epoch 61/100
529/529 [==============================] - 0s 848us/step - loss: 0.1167 - acc: 0.3006 - val_los
Epoch 62/100
529/529 [==============================] - 0s 670us/step - loss: 0.1151 - acc: 0.3214 - val_los
Epoch 63/100
529/529 [==============================] - 0s 784us/step - loss: 0.1156 - acc: 0.3176 - val_los
Epoch 64/100
529/529 [==============================] - 0s 795us/step - loss: 0.1142 - acc: 0.3119 - val_los
Epoch 65/100
529/529 [==============================] - 0s 865us/step - loss: 0.1137 - acc: 0.3327 - val_los
Epoch 66/100
529/529 [==============================] - 0s 873us/step - loss: 0.1119 - acc: 0.3308 - val_los
Epoch 67/100
529/529 [==============================] - 0s 625us/step - loss: 0.1118 - acc: 0.3308 - val_los
Epoch 68/100
529/529 [==============================] - 0s 639us/step - loss: 0.1107 - acc: 0.3611 - val_los
Epoch 69/100
529/529 [==============================] - 0s 638us/step - loss: 0.1110 - acc: 0.3403 - val_los
Epoch 70/100
529/529 [==============================] - 0s 646us/step - loss: 0.1085 - acc: 0.3440 - val_los
Epoch 71/100
529/529 [==============================] - 0s 650us/step - loss: 0.1084 - acc: 0.3516 - val_los
Epoch 72/100
```

```
529/529 [==============================] - 0s 605us/step - loss: 0.1062 - acc: 0.3724 - val_los
Epoch 73/100
529/529 [==============================] - 0s 583us/step - loss: 0.1117 - acc: 0.3629 - val_los
Epoch 74/100
529/529 [==============================] - 0s 657us/step - loss: 0.1077 - acc: 0.3611 - val_los
Epoch 75/100
529/529 [==============================] - 0s 600us/step - loss: 0.1057 - acc: 0.3573 - val_los
Epoch 76/100
529/529 [==============================] - 0s 600us/step - loss: 0.1071 - acc: 0.3403 - val_los
Epoch 77/100
529/529 [==============================] - 0s 582us/step - loss: 0.1039 - acc: 0.3629 - val_los
Epoch 78/100
529/529 [==============================] - 0s 588us/step - loss: 0.1036 - acc: 0.3800 - val_los
Epoch 79/100
529/529 [==============================] - 0s 554us/step - loss: 0.1020 - acc: 0.3705 - val_los
Epoch 80/100
529/529 [==============================] - 0s 583us/step - loss: 0.1008 - acc: 0.3648 - val_los
Epoch 81/100
529/529 [==============================] - 0s 594us/step - loss: 0.1011 - acc: 0.3592 - val_los
Epoch 82/100
529/529 [==============================] - 0s 569us/step - loss: 0.0998 - acc: 0.3686 - val_los
Epoch 83/100
529/529 [==============================] - 0s 579us/step - loss: 0.0977 - acc: 0.4008 - val_los
Epoch 84/100
529/529 [==============================] - ETA: 0s - loss: 0.0975 - acc: 0.390 - 0s 594us/step
Epoch 85/100
529/529 [==============================] - 0s 551us/step - loss: 0.0956 - acc: 0.4064 - val_los
Epoch 86/100
529/529 [==============================] - 0s 603us/step - loss: 0.0957 - acc: 0.3913 - val_los
Epoch 87/100
529/529 [==============================] - 0s 588us/step - loss: 0.0937 - acc: 0.4216 - val_los
Epoch 88/100
529/529 [==============================] - 0s 588us/step - loss: 0.0927 - acc: 0.4197 - val_los
Epoch 89/100
529/529 [==============================] - 0s 607us/step - loss: 0.0906 - acc: 0.4197 - val_los
Epoch 90/100
529/529 [==============================] - 0s 594us/step - loss: 0.0901 - acc: 0.4253 - val_los
Epoch 91/100
529/529 [==============================] - 0s 589us/step - loss: 0.0921 - acc: 0.4386 - val_los
Epoch 92/100
529/529 [==============================] - 0s 563us/step - loss: 0.0870 - acc: 0.4612 - val_los
Epoch 93/100
529/529 [==============================] - 0s 613us/step - loss: 0.0858 - acc: 0.4631 - val_los
Epoch 94/100
529/529 [==============================] - 0s 644us/step - loss: 0.0840 - acc: 0.4802 - val_los
Epoch 95/100
529/529 [==============================] - 0s 652us/step - loss: 0.0831 - acc: 0.4745 - val_los
Epoch 96/100
```

```
529/529 [==============================] - 0s 659us/step - loss: 0.0828 - acc: 0.4802 - val_los
Epoch 97/100
529/529 [==============================] - 0s 605us/step - loss: 0.0814 - acc: 0.4745 - val_los
Epoch 98/100
529/529 [==============================] - 0s 573us/step - loss: 0.0814 - acc: 0.4877 - val_los
Epoch 99/100
529/529 [==============================] - 0s 622us/step - loss: 0.0781 - acc: 0.4839 - val_los
Epoch 100/100
529/529 [==============================] - 0s 606us/step - loss: 0.0789 - acc: 0.5123 - val_los
```

0.6 rescale test data

In [344]:
```
testdata=np.array(sp_df.DailyRet)[607:len(sp_df)-1].reshape(-1,1)
scaler = MinMaxScaler(feature_range=(-1, 1))
testdata = scaler.fit_transform(testdata)
testdata=pd.Series(np.squeeze(testdata))
```

In [345]:
```
X_testoriginal=testdata
X_testoriginal.reset_index(inplace=True,drop=True)
X_testoriginal.head()
```

Out[345]:
```
0    0.003536
1   -1.000000
2   -0.153473
3   -0.405191
4   -0.023656
dtype: float64
```

In [346]:
```
X_test = np.array([X_testoriginal[0:M]])
for i in range(1,(len(X_testoriginal)-M-N)):
    xadd = np.array([X_testoriginal[i:i+M]])
    X_test=np.vstack((X_test,xadd))
X_test = X_test.reshape((-1, M, 1))
X_test.shape
```

Out[346]:
```
(45, 16, 1)
```

In [347]:
```
y_test = np.array([X_testoriginal[M:M+N]])
for i in range(M+1,(len(X_testoriginal)-N)):
    xadd = np.array([X_testoriginal[i:i+N]])
    y_test=np.vstack((y_test,xadd))
y_test.shape
```

Out[347]:
```
(45, 4)
```

0.7 test score and loss

In [348]:
```
test_val_score = model.evaluate(X_test, y_test, verbose=0)
print('Test loss score: {0:.4f}'.format(test_val_score[0]))
print('Test accuracy:   {0:.4f}'.format(test_val_score[1]))
```

```
Test loss score: 0.2315
Test accuracy:   0.2222
```

0.8 make predictions

```
In [349]: y_pred = model.predict(X_test)
          y_pred.shape

Out[349]: (45, 4)
```

0.9 transform scaled test data to original state

```
In [350]: y_pred = scaler.inverse_transform(y_pred)
          y_test = scaler.inverse_transform(y_test)
```

0.10 Calculate 1 day, 2 day, 3 day, 4 day returns

```
In [351]: y_predcumsum=np.cumsum(y_pred,axis=1)
          y_testcumsum=np.cumsum(y_test,axis=1)
```

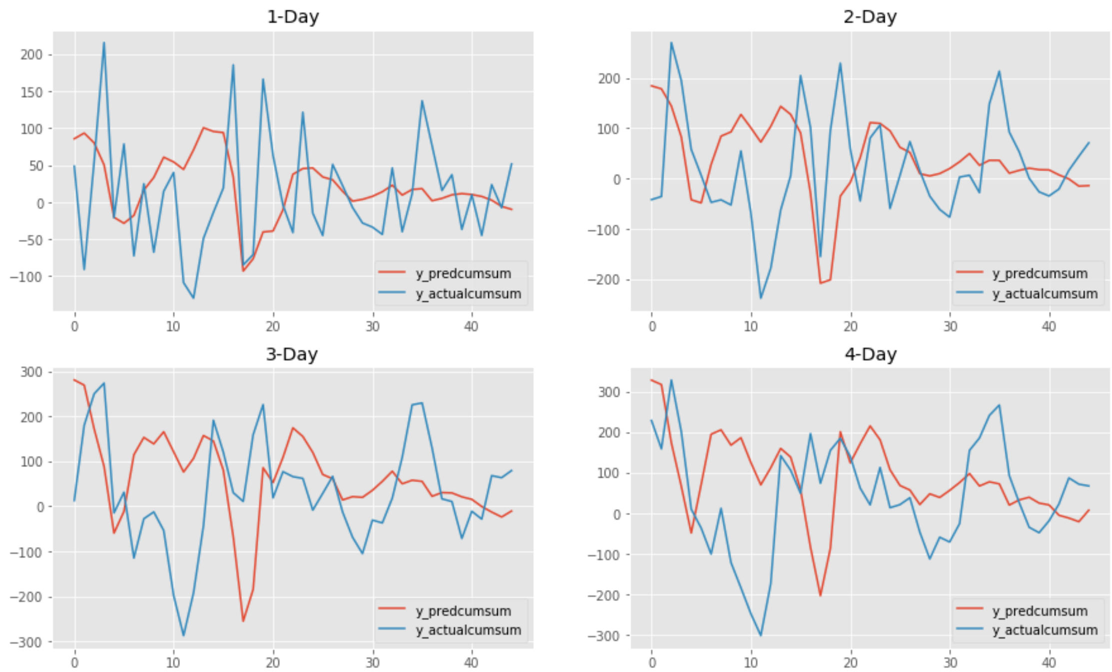
0.11 RMSE and Correlation

```
In [352]: from sklearn import metrics
          rmselist=[]
          corrlist=[]
          for i in range(N):
              rmse=np.sqrt(metrics.mean_squared_error(y_predcumsum[:,i], y_testcumsum[:,i]))
              rmselist.append(rmse)
              corr=np.corrcoef(y_predcumsum[:,i], y_testcumsum[:,i])[0,1]
              corrlist.append(corr)
              print("The %d day rmse is is %f, corr is %f" % (i+1,rmse,corr))

The 1 day rmse is is 99.155141, corr is -0.021979
The 2 day rmse is is 145.391972, corr is 0.067523
The 3 day rmse is is 167.667202, corr is -0.111708
The 4 day rmse is is 180.300575, corr is -0.143005
```

0.12 plot of predict and actual data

```
In [329]: plt.figure(1,figsize=(15,9))
          for i in range(N):
              plt.subplot(2,2,i+1) #plt.subplot('','','')
              plt.plot(y_predcumsum[:,i])
              plt.plot(y_testcumsum[:,i])
              plt.legend(["y_predcumsum","y_actualcumsum"],loc =4)
              titlename=''.join([str(i+1),"-Day"])
              plt.title(titlename)
          plt.show()
```



0.13 What do you conclude regarding the quality of the forecasts?

0.13.1 According to correlation, the prediction and the actual data don't have strong linear correlation. According to rmse, the prediciton errors are a little big. According to plot above, the predction have the similar trend with actual data.