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Game Theory in Multi-Agent Systems

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Abstract

Multi-Agent Systems domain is full of challenges, with enthusiasts still doing a lot of research uncovering many new theories. The Game Theory is rather an old theoretical work, which still proves itself worthy nowadays. In my report I present some insights on decision theory, to make the reader capable of fully understanding the game theory and furthermore there are few words on Evolutionary Game Theory, to make the image even greater. A specific application of game theory in Multi-Agent Systems is discussed in the last part of my report, with a cooperative approach being used.

Keywords:
Multi-Agent Systems; Game Theory; Evolutionary Game Theory; Rational Agents; Strategies; Decision Theory; Decision Tree; Nash Equilibrium; Bayesian Network; Negotiations Agents; Distributed AI

Contents

1. Background

The strategic interaction between rational decision-makers can be mathematically modeled. The *Game Theory* studies such models [4]. The basics of Game Theory appeared as a result of the analysis of games, particularly chess and chess players. Usually, back then, zero-sum games were considered, i.e., the balance between every success or loss is maintained among opponents.

Nowadays, there is a wide range of applications, from the traditional field of logic and computer science to social sciences where game theory has been used to model behavioral links. A strong bond characterizes the game theory and the decision theory for the simple fact that the first one has become an umbrella term for the study of processes implied when human beings or computers are reasoning to take a decision.

1.1. Briefing on Decision Theory

A number of new-age theories about decision theory have been sighted in the last years, but originally it was meant to offer a collection of mathematically designed techniques utilized in decision-making processes, with the mention that the outcome of different operations is not acquainted. We choose to talk here from a theoretical point of view, mainly because this theory of decision making has been mature enough when the idea of intelligent agents was born, but this kind of agents are one canonical example in which it is easy to observe how agents can apply the traditional theory of decision making.

Let us consider an agent that operates in an elaborated environment, inevitably the agent cannot be certain about the environment, as it doesn't hold enough information to process in order to find out which is the actual state or how it'll evolve in the next stages. So, every stage of the environment can be encapsulated with the help of some variables, W_i. The interesting part, is that typically speaking, an agent is aware that the eventual values w_i, of the variables W_i, have a probability $P(w_{ij})$ of being the actual state of W_i. Thus, we consider Z to be the set of all the w_{ij} have:

$$P : w \in w_{ij} \mapsto [0, 1] \textit{ and } \sum_j P(w_{ij}) = 1;$$

We must bring to attention two special cases in which the probabilities of two variables, W_1 and W_2, can be. If they don't relate to one another, then W_1 and W_2 are *independent*, furthermore for all the possible values w_{1i} and w_{2j} the relation holds:

$$P(w_{1i} \wedge w_{2j}) = P(w_{1i})P(w_{2j});$$

Consequently, if these two variables are not independent and we consider $P(w_{1i}|w_{2j})$ to be the corresponding probability of W_1 when it has the value w_{1i}, knowing that W_2 took the value w_{2j}, the next result is true:

$$P(w_{1i} \wedge w_{2j}) = P(w_{1i}|w_{2j})P(w_{2j});$$

These are known as *conditional likelihoods*, and they properly secure the relationship between the variables W_1 and W_2, expressing, let's say, the idea of w_{1i} (for example, the value *flood* of the variable *state of homeland* being more probable if w_{2j} (the value *cloudy* corresponding to the variable *sky status*) is an acquainted authentic fact.

Let us denote with W the collection of variables W_i of which our agent may be aware; we are capable of building a graph (known in the literature as a Bayesian network) [5] where there is a link between a node with its corresponding variable from W and another node, if the two implied variables are not characterized by an independence relationship. This gives the agent the proper computational framework used to compute the likelihoods of interest.

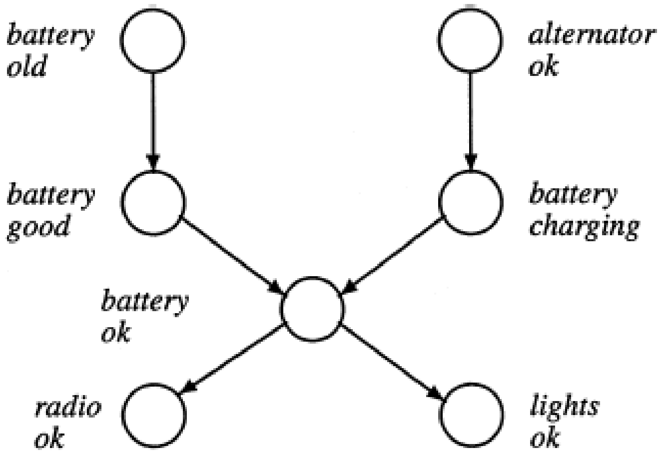

Fig. 1. Bayesian network to identify car failures

In the Figure 1 we can see a small Bayesian network. The idea behind it, is very simple, which makes it a desirable example. Basically, it's expressing that an older battery (see the node *battery old*) may be good or not based on a probabilistic influence (*battery good*), which further affects the functionality of the battery (*battery ok*). The back-most being also influenced by the functionality of the alternator which triggers the battery to charge when the car is on the move. The quality and the availability for use of the battery ultimately has an influence on the operational state of the radio and the lights outside (and inside) the car.

The whole philosophy is that the bottom variables are used to gain information about the ones situated on the top, which are the most important, as they're the one related to fixing the faulties. And we are aware, that an agent must show interest in the variables related to its goals, to its purpose. Thus, because the agent would want to obtain success in achieving the goal, it might show interest in picking an action that will maximize the chance of success. In the scenario in which the agent is driven by the will to achieve multiple goals, we can extend the strategy so that the agent would pick to perform the actions needed in order to obtain the goal with the highest probability of being reached.

Designing an agent in the previous scenario, is still somehow shortsighted mainly because achieving an easier (*worthless*) goal would be a preferred choice, over performing actions in order to obtain more difficult goals and therefore more valuable.

The *utility* concept was introduced in decision theory, with the purpose of addressing these exact issues. It's nothing but a value in direct relationship with a state of the environment, with the agent putting this value on the state. The strategy which takes into account utility is providing us with proper a way of preference-encoding of agents, so there are utility functions that are capable of encoding the preference and making the agent to choose a particular state over another, related to which one has the greater assigned utility [9].

2. (Evolutionary) Game Theory

The beauty of Game Theory comes from the early work of von Neumann [9] and it's sharing multiple ideas with the decision theory. It is often interpreted from an economical point of view, modeling strategies performed interactively by intelligent agents, into games.

2.1. History

The fundamentals of choices are the preferences of the agent which also acknowledges the preferences of other agents. Because distinct scenarios are leading the agent to choose differently based on other preferences than its own, there are two important factors which trigger the choices:

- the agent is continuously concerned about the relation between its priorities and the eventual outcome determined by its decision,
- the agent acts driven by strategical thinking, aware of the existent link between decision made by other agents and its choice.

Still, these things which von Neumann and his colleague successfully achieved can be used in defining an equilibrium idea in the case of two players zero-sum games. As I previously explained, zero-sum games are the scenarios that imply an equality between wins and losses.

2.2. Conceptual Framework

John Nash was the first to address the scenario of competitive agents driven by ideas as mutual gain, and in order to do that he defined best-response functions through Theorem 2.1. His most important legacy include *Nash Equilibrium* and *Nash Bargaining Solution*.

Theorem 2.1 (Kakutani's fixed-point theorem)**.** *Let us consider G to be a set characterized by non-emptiness and compactness. Then we consider $\beta : G \to 2^G$ a set-valued function. If all the next three properties hold:*

1. *G is convex,*
2. *for all $g \in G$, $\beta(g)$ is both convex and non-empty,*
3. *β is continuous,*

then β has a fixed point.

Proof. Performing all the computations stated in the theorem and implicitly verifying the given conditions to particular best-reply functions the reality of the so called *Nash equilibrium* is proved to exist. □

Definition (Nash Equilibrium). *A set of rewards together with the strategies that provided them in the first place, in the situation that makes both the next two game properties to be simultaneously met:*

1. *a change of strategy cannot trigger an increasing reward for any player, while*
2. *all the (other) participants in the game are keeping their initial strategies.*

is called the Nash Equilibrium.

There is a formal way, we can utilize it to express Nash Equilibrium:

When 2 players play the strategy profile

$$s = (s_i, s_j)$$

belonging to the product set $S_1 \times S_2$ then s is a *Nash equilibrium* if:

$$P_1(s_i, s_j) \geq P_1(s_x, s_j), \forall x \in \{1, \dots, n\}$$

and

$$P_2(s_i, s_j) \geq P_2(s_i, s_x), \forall x \in \{1, \dots, m\}.$$

Fig. 2. Nash Equilibrium Formally Expressed

Although, the classical Game Theory is feasible when it comes to understanding the theoretical concepts and how it all started, nowadays it is no longer realistic. On the other hand, *Evolutionary Game Theory* is more bounded to the reality, the participants playing the game are doing it by other rules, they do not play it only once and they do not hold knowledge of each other preferences.

Evolutionary Game Theory is describing ways in which a rational individual takes decisions and how those affect the outcomes and the strategical interaction between the players, all these while providing the fundamentals of issuing rationally acceptable decisions in a tricky environment from the real – unsafe, doubtful – world.

So it is obvious to believe that the process of modeling such learning agents, beside being tricky sometimes, it's also requiring a shrewd mind to design strategies affecting the behaviors between existent agents in the system and toward the environment. Participants in a standard game are often used to theoretically model the agents involved from a Multi-Agent Systems perspective. There are three main characteristics that need addressing here:

- the hyper-rationality of the agents must be considered, as they must be capable of properly anticipating the other moves,
- agents are assuming 100 % environment related familiarity,
- the agents need to be aware of the fact that optimum strategy is being equivalent all the time, implying the existence of static Nash equilibrium.

Bearing in mind these insights, and knowing for sure that in the reality we people do not hold all the details about one another so the equilibrium is dynamic, continuously changing, made the Evolutionary Game Theory to be a growing, evolving field. Next, to clear the water even more, we will brief on three widely renowned instances of strategical interactions.

2.3. MAS Strategy Explained through Examples

2.3.1. The Prisoner's Dilemma

We begin with a game known in the literature as problem of the two prisoners; its premises say that these two have committed a criminal offense together [2] [10]. They are presented with a set of two choices:

1. collaboration with the detectives on their case – *defection* (D),
2. denying their implication, matching their stories as a team – *cooperation* (C).

The A and B tables from below offer a view of every reward and payoff received by the two agents, based on their own choices, as well as the other player decisions. The table A must be observed from left to right with the rows being the most important, while the table B has to be seen from top to bottom with the columns being the important ones.

A=

D	1	5
C	0	3

B=

D	C
1	0
5	3

As an example, let's consider the first player – row – has decided to cooperate, its reward can be found on the second row, but the final reward is also depending on the strategy of the second player – column – if he chooses to not cooperate, then the payoff for our first player will be 3 and the second will receive the reward of 1.

2.3.2. The Matching Coins

The second example [2] [10], a children game, two kids are both considered to have one coin. There is an independence relationship between their decisions; there are two possible choices *head* or *tails*. If both coins are returning the same side, then the player 1 has won, if this is not the case, the second player is the winner.

This game is a remarkable instance of a game in which the equality between wins and losses always holds – in other words it's a zero-sum game.

The rewards are expressed in the next tables. The table A provides the payoff related to the agent who plays the rows, while the second table, B, offers an insight on what the payoff of the second agent will be, regarding their both decisions.

A=

H	1	-1
T	-1	1

B=

H	T
-1	1
1	-1

As it can be seen, there are two strategies that can be decided upon, the one that makes the agent go with the tails, T, and the other which is the head, H.

2.3.3. *The Battle of the Spouses*

The last but not the least example considered here, is pretty much alike the previous ones, an debates the battle of the spouses [2] [10]. For this game, all is needed are two players which give live to a couple of humans from reality. They are so in love with each other,thus they desire to spend as much time together as it's possible. For their holiday, one wants to go see the mountains in Fiji, and the other wants to swim in the Ocean.

The next tables, show us their payoff based on their decision.

A=

F	2	0
O	0	1

B=

F	O
1	0
0	2

If both the players decide to listen their own preferences and follow their desires, they obtain the most unsatisfying reward.

2.4. *Game Theory Applications in MAS*

Considering the principles of Game Theory, people are capable of modeling cooperative or competitive MAS. These efforts are correlated with the fact that particular strategies are suited for particular games:

- the cooperative MAS assumes that all the agents act on behalf of the general interest of the agent society, yielding the best possible performance,
- the competitive MAS is fitted for situations where there is no strategy which acquires dominance over the others, with a high probability that each agent would perform better following its own strategy.

2.4.1. *Cooperative MAS – Wireless Networks Dynamically Allocated Channels*

The problem of dynamically allocated channels problem is widely acquainted by scientists in their work for more than 20 years already. The availability of all kinds of strategies utilized to models the allocation of dynamic channels in multi-agent systems [3] has begun in 1996. Nowadays, we are aware that this problem is NP-difficult [7]. Something of a great importance is the idea that the process of arriving calls asking for a channel is a stochastic one, this process being also characterized by a set of constraints that limit the use of alike frequencies, minimizing the interference among calls; these constraints are known as CRC, meaning Channel Reuse Constraints [6].

In the literature two different approaches to solve our allocation problem can be identified:

- FCA – fixed channel allocation tries to avoid CRC violations by appointing the same number of channels for all cells

- DCA – dynamic channel allocation purpose is the minimizing of the call drops, by sharing the necessary channels through the environment using negotiation techniques in the framework of multi-agent systems.

It is trivial to observe in Figure 4 that the fixed channel allocation is nothing else but the Nash Equilibrium expressed in terms of the channel allocation problem.

In designing a cooperative Multi-Agent Systems in the framework of Dynamic Channel Allocation problem, there is considered an agent x being the cell tower of transmission which keeps track of the current situation of the environment, by maintaining an inventory of collections:

- A_x – describing the number of channels being *available* at the time,
- U_x – describing the number of channels being not available, *unavailable*,
- B_x – representing the number of channels being *busy* at a given time,
- R_x – describing the number of channels being *reserved*.

In addition to that, in the network, let us consider the total number of available channels to be denoted by ac and the total number of cells to be denoted by c.

Anytime, in our network, a channel can simultaneously belong only to one collection of channels. Furthermore, it is mathematically proved to be true that any ac channel can exist precisely in one single collection. Below can be observed a more formal way of saying these, with two other incidental conditions:

$$\forall x \in \{1, \ldots, c\};\ A_x \cap\ U_x \cap B_x \cap R_x = \emptyset,$$

$$\forall x \in \{1, \ldots, c\};\ A_x \cup U_x \cup B_x \cap R_x = \{1, 2, \ldots, c\}.$$

The CRC requires that for two different adjacent neighboring cells x and y, so

$$B_x \cap A_y = B_y \cap A_x = \emptyset,$$

additionally, reserved channels in one cell are not available in the adjacent neighboring cells, i.e.,

$$R_y \cup A_y = R_y \cap A_x = \emptyset.$$

Fig. 3. Formal Properties – MAS DCA

The initial circumstances must presume that each agent has all the channels in the A_x collection, while the remaining collections must be empty. Once the arriving of calls begins, the agent x will remove that particular channels from its collection of available channels, A_x and it will be added to the collection of channels already being in use B_x, the back-most action also trigger the agent to notify the adjacent neighbors that the channel is unreachable and they need to move it into the collection of unavailable channels, U_x. All these because the CRC (the channel reuse constraints) need to be met.

Furthermore, to assure the consistence of CRC, when the call to agent x is done and the corresponding channel is no longer busy, all the interested agents are notified of this fact, so they can proceed to move it into the available channels collection. A handoff call is usually handled as a new call. Any agent involved in this network is always aware of the rate of its call drops, and moreover it will compute the call drop likelihood.

What's really interesting, is a discussion about the reserved channels, because the agents need to negotiate about how to divide them among themselves. The exclusivity of these channels makes them of great interest, and they are basically a collection which contains channels only available to specific cells, based on a previously expressed period known as *negotiation tenure*. Obviously, these special channels are to be interpreted as unavailable ones by the adjacent neighbors, the constraints still being in place.

The philosophy of this thinking, is that of allowing the cells experimenting a higher demand of calls than they're capable of handling, to anticipate times with a great request of calls, desiring the minimizing of the corresponding call drop likelihood.

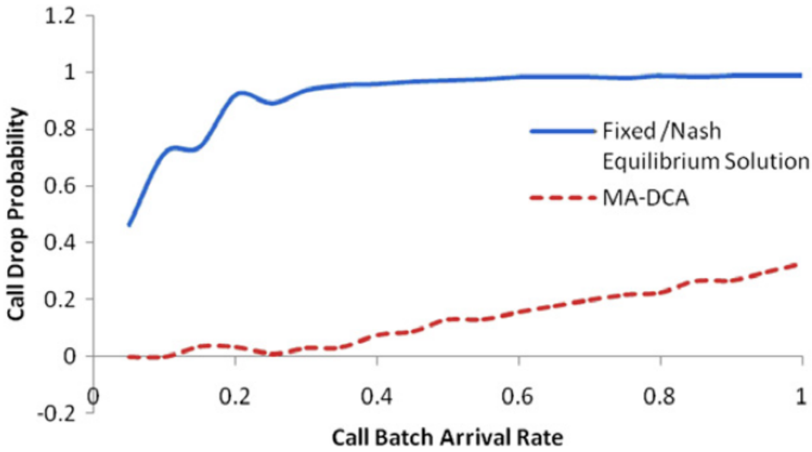

Fig. 4. FCA vs DCA

The Figure 4, shows a comparison between the likelihood of call drop, from two perspectives, the first being expressed for Fixed Channel Allocation - the Nash Equilibrium Solution and the second one for our Multi-Agent Distributed Channel Allocation. These results are proving the superiority of the distributed manner, as it possess the lowest likelihood of dropping calls.

In this attempt of offering an idea on call dropout probabilities, a 14-cell network was utilized, see Figure 5

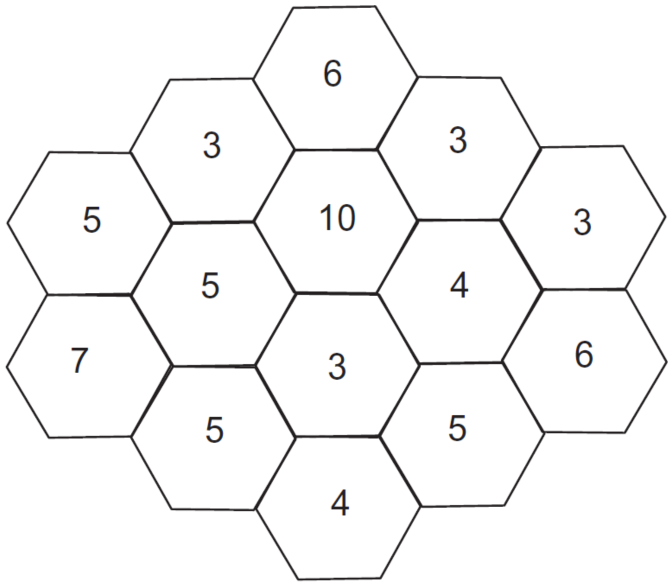

Fig. 5. The considered network

Further discussion on negotiations between agents

The negotiations are possible only between adjacent neighbors, so it's safe to affirm that a cell with a number of n neighbors, will participate in $n + 1$ such processes. There can be identified two particular situations, the cell stars the negotiations, that's the *auctioneer* in the literature, or it participates in the auction process as a common *participant*.

The first step in the negotiation process, is identifying the extremes of the call drop out likelihoods and their corresponding cells. If the maximum and the minimum do not belong to the same cell, the reserved channels held by the cell with the minimum likelihood will be released as available, the neighbors being updated too. The expected step, is that the cell with the highest likelihood of call dropout will move channels from A_x to R_x, keeping its adjacent neighbors updated. To preserve the convexity characteristics of the preference, a new constraint must be added on the collection R_x, the cardinality.

3. Final Remarks

There is safety when concluding that Game Theory and especially Evolutionary Game Theory are offering the proper guidelines when it comes to designing Multi-Agent Systems, in the framework of games approaches. Even if in my example application I gave only details about a cooperative approach, in the literature numerous cases of non-cooperative, competitive designs are debated upon. The most important fact that needs to be addressed here, is that particular approaches are fitted for particular games.

3.1. Future Work

Nowadays, the public good games are a trend in multi-agent systems. The cooperation of agents, instead of the so called self-interested agents is considered to be a better approach in the literature of public good games [1] [8]. It is to be believed that in the next few years, these domains will know an exponentially growing interest, because the rational agents seems to be a good choice when it comes to playing games, acting in cooperative environments, following the best reward strategies and so on.

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