Part I: Mathematical Solutions

Instructions: Manually solve the problems listed, clearly showing all steps and explanations for your process. Use a clean and readable format for your written solutions.

1. (a) Given two vectors $\mathbf{v} = [2, 4, 6, 8, 10]$ and $\mathbf{u} = [1, 3, 5, 7, 9]$, compute the result of the following

   - component-wise division $\dfrac{\mathbf{v}}{\mathbf{u}}$.

   - $3\mathbf{v} - 2\mathbf{u} + \frac{1}{2}\mathbf{v}$

   (b) Compute the Euclidean norm (magnitude) of the vector $\mathbf{v} = [8, 0, 5, 2, 1]$.

2. Compute the determinant of the matrix $A = \begin{pmatrix} 2 & 3 & 7 \\ -4 & 0 & 6 \\ 1 & 5 & 0 \end{pmatrix}$.

3. (a) Given a square matrix $B = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$, compute the eigenvalues and eigenvectors.

   (b) Verify whether the following vectors are eigenvectors of the matrix $D = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}$:

   $\mathbf{v}_1 = [1, 1, 1]$ and $\mathbf{v}_2 = [-1, 1, -1]$.

4. (a) Given a dataset with three features $X = \{(4, 2, 3), (3, 13, 4), (9, 4, 5), (0, 5, 7)\}$, perform PCA to reduce the dimensionality of the dataset to two dimensions.

   (b) Compute the percentage of variance explained by each principal component in the PCA transformation.

Part II: Python Script in Jupyter Notebook

Instructions: Implement the solutions using Python in a Jupyter Notebook, applying the concepts taught in your first lecture. Ensure your code is well-commented to explain the implementation.

Submission Instructions

File Formats to Submit:

1. PDF Document: Submit a PDF that includes both your written solutions and the exported PDF version of your Jupyter Notebook.

2. Jupyter Notebook (.ipynb): Also submit the original Jupyter Notebook file.

How to Merge Documents:

- Export your Jupyter Notebook to a PDF and combine/merge it with your manual solutions using any standard PDF editing tool.