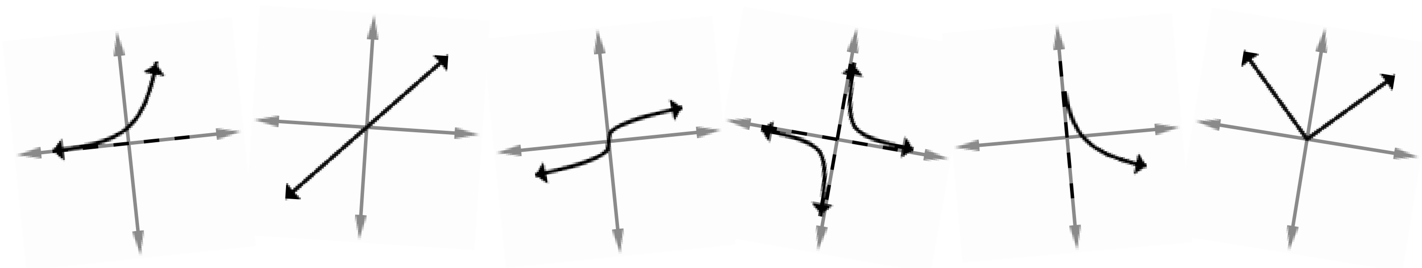
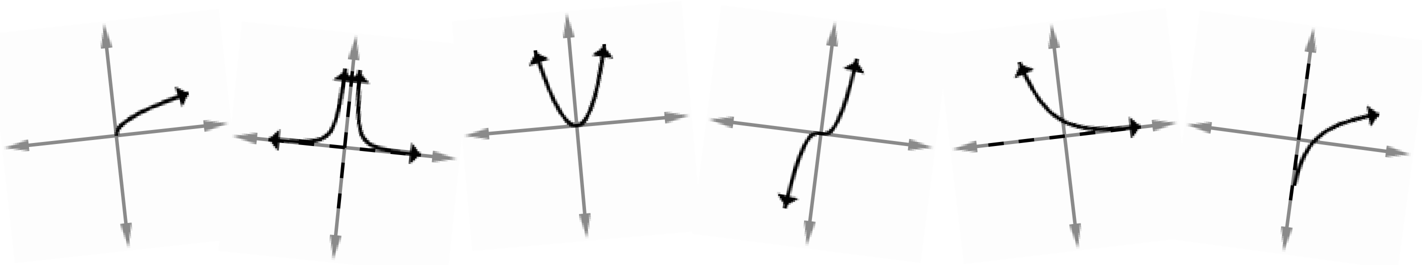


MTH 111 – College Algebra
Course Notebook



Name: ____

Authors: Sara Clark, Elizabeth Jones, Lyn Riverstone, Katy Williams, Scott Peterson, Dan Rockwell, Dave Wing

Version 4. Updated Fall 2018.

Course Information

Class meetings: ____

Instructor(s): ____

Office: ____

Office hours: ____

Exam 1: ____

Exam 2: ____

Final Exam: ____

******** Always log into ALEKS via our Canvas course site. ********

Getting Help Outside of Class

	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
8am							
9am		MSLC	MSLC	MSLC	MSLC	MSLC	
10am		MSLC	MSLC	MSLC	MSLC	MSLC	
11am		MSLC	MSLC	MSLC	MSLC	MSLC	
12pm		MSLC	MSLC	MSLC	MSLC	MSLC	
1pm		MSLC	MSLC	MSLC	MSLC	MSLC	
2pm		MSLC	MSLC	MSLC	MSLC	MSLC	
3pm		MSLC	MSLC	MSLC	MSLC	MSLC	
4pm		MSLC	MSLC	MSLC	MSLC		
5pm							
6pm							
7pm							
8pm							
9pm							

- Add your Instructor's and your TA's office hours to this weekly schedule!
- MSLC: math tutoring in the Math & Stats Learning Center, Kidder Hall 108

Table of Contents

Preface

Welcome to College Algebra! This course is the study of functions and their properties. With just a handful of function types, we can develop models of many real-world situations (such as population growth or the path of a rocket). We can then formulate questions about these situations that can be answered mathematically, solve problems related to these questions, interpret the solutions in context, and validate our results. With the goal of making connections between function properties and the situations they model, we will focus on analyzing functions using different representations--symbolic, numerical, graphical, and verbal.

College Algebra satisfies the Baccalaureate Core Mathematics requirement here at Oregon State University. The rationale for this requirement is:

Everyone needs to manipulate numbers, evaluate variability and bias in data (as in advertising claims), and interpret data presented both in numerical and graphical form. Mathematics provides the basis for understanding and analyzing problems of this kind. Mathematics requires careful organization and precise reasoning. It helps develop and strengthen critical thinking skills.

Function Family Approach:
This College Algebra course is taught using the *Function Family Approach*. Studying functions may seem overwhelming at first. There are an infinite number of them after all! However, we will be systematic in our approach, first carefully examining some basic functions, and categorizing them according to their common characteristics. These categories we call *families* of functions. We study seven different families in this course:

1. Polynomial (including Linear and Quadratic)
2. Radical
3. Piecewise
4. Absolute value
5. Rational
6. Exponential
7. Logarithmic

Within each family there are sub-families (linear functions are a sub-family of polynomials), and each sub-family has a parent function. All functions in a family are built from the parent functions. A deep understanding of the ten parent functions studied in this course allows us to more fully understand *all* functions in a family or sub-family. For example, we can build any quadratic function as a transformation of the parent function $y = x^2$, and we can apply our knowledge of this parent function to situations involving other quadratic functions.

After establishing the foundation of this course with our ten parent functions, we are ready to study the properties of functions including end behavior, intercepts, asymptotes, domain and range, increasing/decreasing behavior, and multiplicity. Some of these properties help us answer questions about the overall shape of the graph, or how the function is changing in the long run. Other properties help us determine key points on the graph in the context of a situation. When did the baseball hit the ground? What happened to the population of deer over time?

In addition to studying specific families of functions you will gain broader mathematical skills. The tasks and assessments in this course are designed to help you meet the Baccalaureate Core Learning Outcomes:

1. Identify situations that can be modeled mathematically.
2. Calculate and/or estimate the relevant variables and relations in a mathematical setting.
3. Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.

Problem Solving:
As you move through this class, you will build skills in mathematical reasoning and problem solving and you will begin to see how mathematicians think. Mathematicians use what they know to build new understanding.

In this course, you will build new functions from the ten basic parent functions. You will use what you know about the simpler parent function to analyze the new function. Because you are thinking about function families instead of individual functions, there will be a lot less memorization than you may have used in previous math classes. You will need to know one big idea about the family instead of a lot of little thing about each function.

Mathematicians do not spend their days working solely with symbols. They use graphs, pictures, tables, verbal discussions and symbols to understand and communicate about a concept. You will choose and make use of an appropriate function representation (graphical, numerical, symbolic, and verbal) to model and analyze real-world situations. You will decide which representation will best help you find a solution and you will use other representations to check your answers and verify your thinking.

Focus on Mathematical Modeling:
As you gain confidence in using different representations of a function, you will start to realize that visual representations, even those that are drawn quickly and imperfectly, can help you to understand a situation, come up with a solution to a problem, and correct any errors. You will evaluate whether a graph is "good enough" for your purposes, and you will create and use "good enough" graphs as a tool, not just an end result.

You will develop a toolbox consisting of the ten parent functions, their families and subfamilies; the methods for building new functions from those you know (transformations, function algebra); and the different representations of a function (graphical, numerical, symbolic, and verbal). You will use these tools to determine which function family is a more appropriate model for a given situation. Is the Gender Wage Gap modeled best by a linear, exponential or quadratic function? How can you justify the choice of model used?

Once you have a function that gives a mathematical model for a situation, you will analyze the function and determine its properties to answer questions. You will ask and answer big picture questions such as how is this situation changing over time? And you will determine the different points on the graph that give you information that is most relevant to the situation.

Features of this Course Notebook

Learning Objectives:
Each chapter in this Course Notebook is divided into multiple Lessons, each of which is organized around a set of learning objectives. The learning objectives are listed at the start of each Lesson.

Team Learning:
In class you will work with your team to solve the problems and answer the questions posed in the notebook. As you work through this course, there are four main types of activities you and your team will be doing: 1) Warm-ups; 2) Lessons; 3) Wrap-ups; and 4) ALEKS.

- ✓ **Warm-up:**
  Most Lessons in the notebook begin with a Warm-Up activity, designed to elicit prerequisite knowledge needed for success on the upcoming Lesson. These Warm-ups are to be done BEFORE class, along with the ALEKS Prep assignment.
- ✓ **Lesson:**
  As you work with your team to solve the problems presented in each Lesson, you will learn new concepts, build understanding and connections, and practice your mathematical reasoning and communication skills.
- ✓ **Wrap-up:**
  Finally, to check your understanding of the material studied in the Lesson, you'll work with your team to complete a Wrap-up activity, to be handed in at the end of class so you can get feedback from your instructor or GTA. Many of problems on these Wrap-up assignments provide an opportunity to apply what you have learned in the lesson to real-world contexts, and put together the pieces of the lesson to answer larger questions.

ALEKS:
In addition to these in-class activities, you will also complete assignments in ALEKS both *before* and *after* class. The preparation assignments are designed to make sure you have the pre-requisite knowledge needed for success on the in-class activities. These ALEKS topics are introductory, and may be things you've seen before, but you may have forgotten. After class, you'll work on the more challenging ALEKS topics, to reinforce your in-class learning, practice mathematical procedures, and gauge your understanding to focus your studying efforts.

Let's roll up our sleeves and get started. You've got this!

More on Bacc Core Learning Outcomes: https://main.oregonstate.edu/baccalaureate-core/current-students/bacc-core-learning-outcomes-criteria-and-rationale

Chapter Learning Objectives

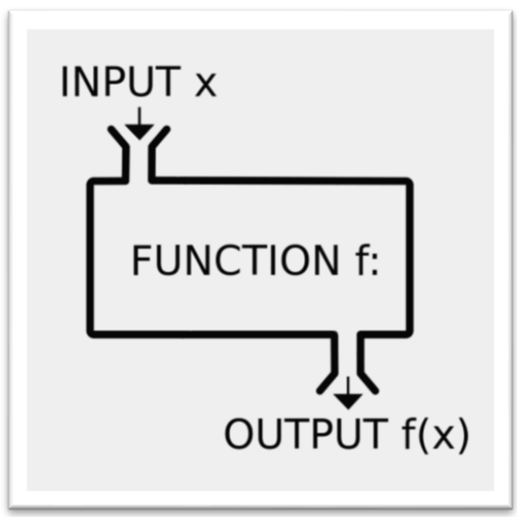
1. Identify the parent functions studied in this course, given their graphical, symbolic, or numerical representations.
2. State the domain and range of the parent functions using their graph.
3. Determine whether a given relation is a function from a verbal description, table, or list of ordered pairs.
4. Given various representations of a function, identify inputs and outputs and interpret these in the context of a situation.
5. Use a table or equation to evaluate a function for a given input and express the value using function notation (for example, find $f(8)$ for $f(x) = \log_2 x$).
6. Use a table to calculate the average rate of change of a function over a specified interval and express the value using function notation.
7. Use tables, equations, and graphs to explore polynomial, rational, exponential, and logarithmic parent functions.
8. Identify functions in the polynomial and rational families, given their symbolic representations.
9. Identify exponential and logarithmic functions, given their graphical, symbolic, or numerical representations.

Chapter 1 What are Some Fundamental Functions?

Chapter Overview

In Chapter 1, we concern ourselves with the concept of function. Various families of functions form the basis of this College Algebra course: polynomial (including linear and quadratic), radical, piecewise, absolute value, rational, exponential, and logarithmic.

This Chapter introduces our function families, their symbolic, graphical, and numerical forms, as well as their domains and ranges. We will continue to study these functions throughout the course.



Chapter 1 Contents

1.1: Reference Guide: Families of Functions

Learning Objectives
Together with your team: • Identify the parent functions studied in this course, given their graphical, symbolic, or numerical representations. • State the domain and range of these parent functions using their graphs.

Our study of functions in this class will include these Function Families:

- Polynomial
- Rational
- Exponential
- Logarithmic
- Radical
- Absolute Value

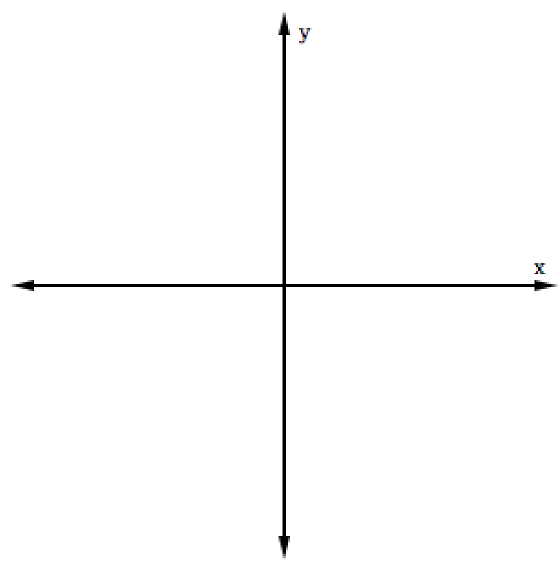
Some families have "sub-families"; for instance, ***quadratic functions*** *are a sub-family of the polynomials.*

Each function family (or sub-family) has a Parent Function. We will study these ten parent functions in order to help us better understand the function families. They are:

$y = x$ $\qquad y = x^2$ $\qquad y = x^3$ $\qquad y = \frac{1}{x}$ $\qquad y = \frac{1}{x^2}$

$y = b^x$ $\qquad y = \log_b(x)$ $\qquad y = \sqrt{x}$ $\qquad y = \sqrt[3]{x}$ $\qquad y = |x|$

In this class, we will sketch what we like to call "good enough" graphs. What do we mean by a graph that's *"good enough"*?



Recall,

✓ To find the **domain** of a function using a graph,

✓ To find the **range** of a function using a graph, Let's explore these families and parent functions!

Directions: Go to the website www.desmos.com and click "Start Graphing." Graph each of the following parent functions in Desmos and record a sketch of a "good enough" graph of each. Then, describe the shape of the graph, and state the domain and range of the parent function. Finally, write a verbal description of the function that gives the value of y in terms of x.

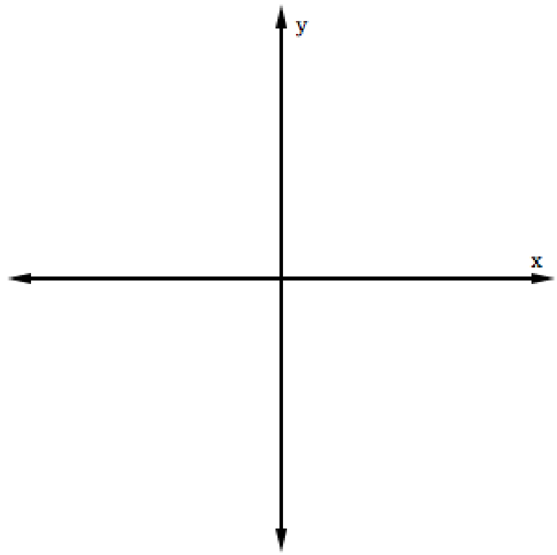
Function Family: Polynomial

Sub-family: Linear

Parent Function: $y = x$

- Shape: ______________________
- Domain of $y = x$: _________________
- Range of $y = x$: ___________________
- Table of values:

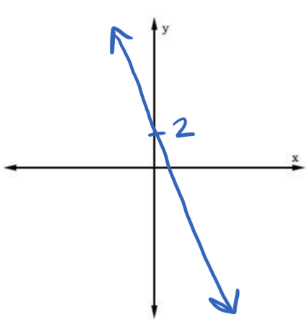
x	$y = x$
-2	-2
-1	-1
0	0
1	1
2	2



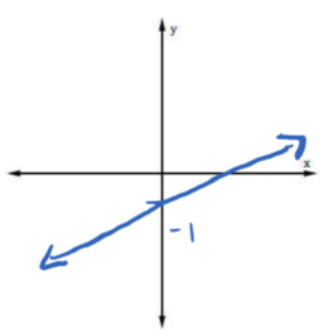
Verbal description:

Examples of Linear Functions

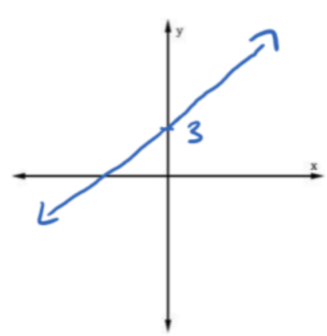
1. $y = -3x + 2$



2. $y = \frac{1}{4}x - 1$



3. $y = x + 3$



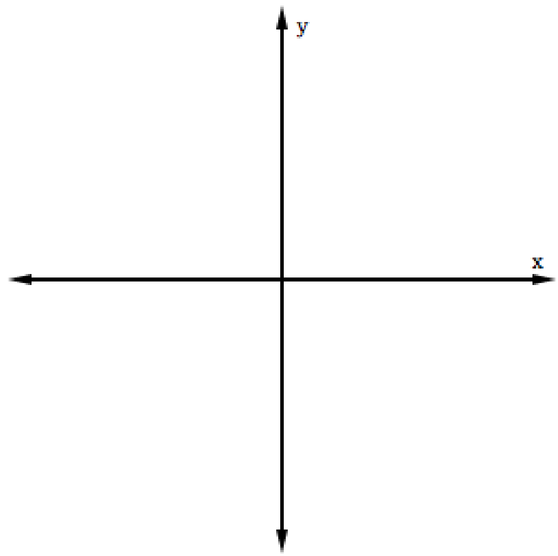
Function Family: Polynomial

Sub-family: Quadratic

Parent Function: $y = x^2$

- Shape: ________________________
- Domain of $y = x^2$: _________________
- Range of $y = x^2$: _________________
- Table of values:

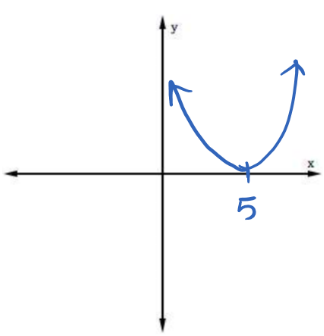
x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4



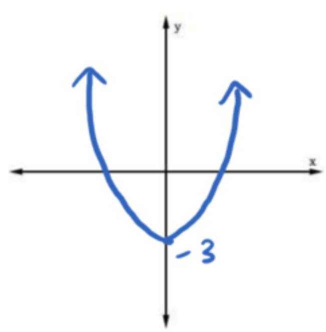
Verbal description:

Examples of Quadratic Functions

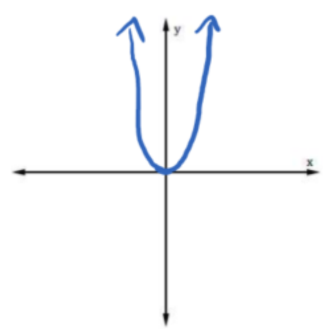
1. $y = (x - 5)^2$



2. $y = x^2 - 3$



3. $y = 3x^2$



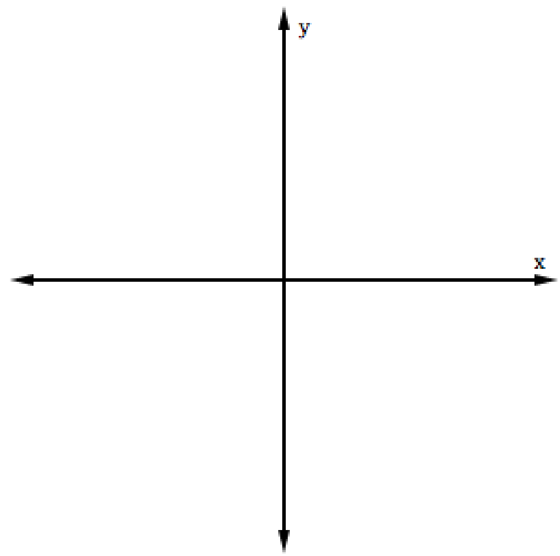
Function Family: Polynomial

Sub-family: Cubic

Parent Function: $y = x^3$

- Shape: ______________________
- Domain of $y = x^3$: ________________
- Range of $y = x^3$: ________________
- Table of values:

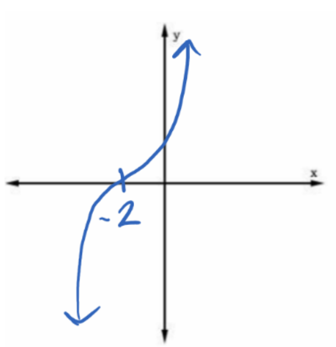
x	$y = x^3$
-2	-8
-1	-1
0	0
1	1
2	8



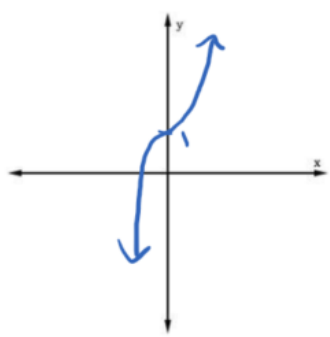
Verbal description:

Examples of Cubic Functions

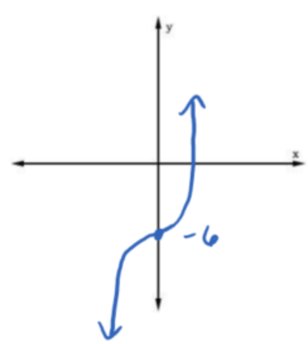
1. $y = (x + 2)^3$



2. $y = x^3 + 1$



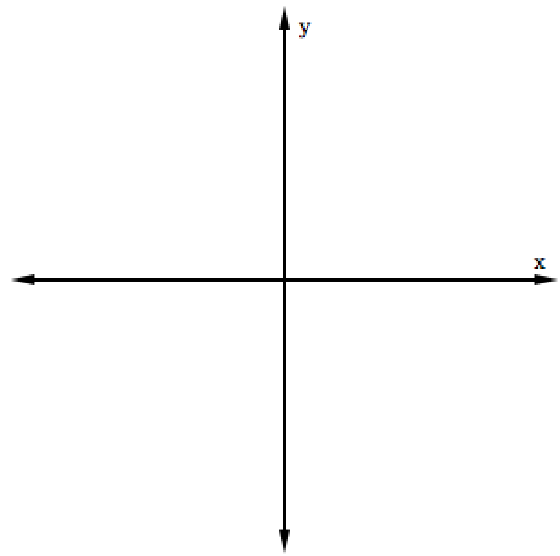
3. $y = x^3 - 6$



Function Family: Absolute Value

Parent Function: $y = |x|$

- Shape: ______________________
- Domain of $y = |x|$: ________________
- Range of $y = |x|$: ________________
- Table of values:

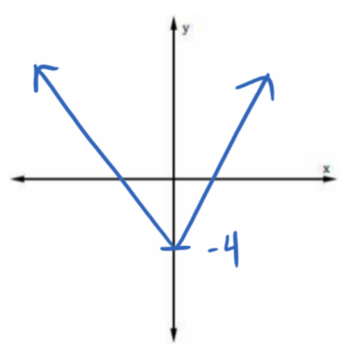


x	$y = \|x\|$
-2	2
-1	1
0	0
1	1
2	2

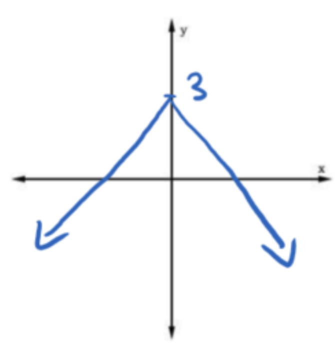
Verbal description:

Examples of Absolute Value Functions

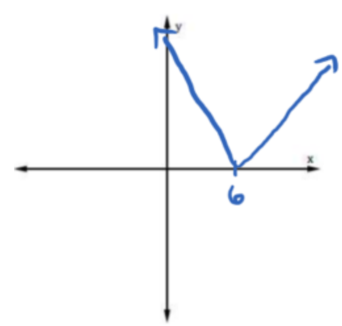
1. $y = |x| - 4$



2. $y = -|x| + 3$



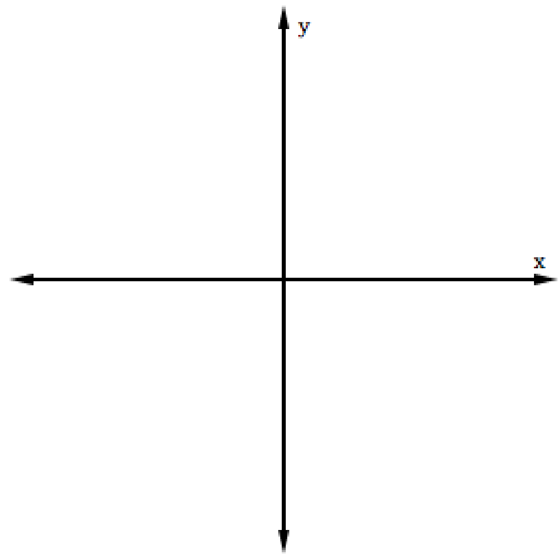
3. $y = |x - 6|$



Function Family: Radical
Sub-family: Square Root

Parent Function: $y = \sqrt{x}$

- Shape: ______________________
- Domain of $y = \sqrt{x}$: ________________
- Range of $y = \sqrt{x}$: ________________
- Table of values:

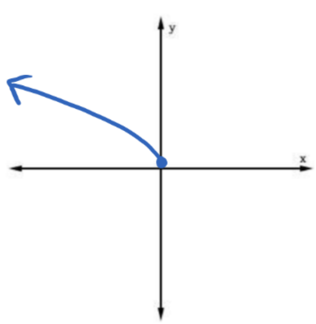


x	$y = \sqrt{x}$
-2	undefined
-1	undefined
0	0
1	1
2	$\sqrt{2} \approx 1.41$
3	$\sqrt{3} \approx$
4	2

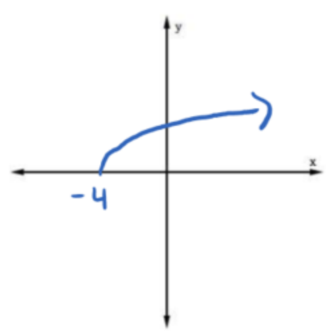
Verbal description:

Examples of Square Root Functions

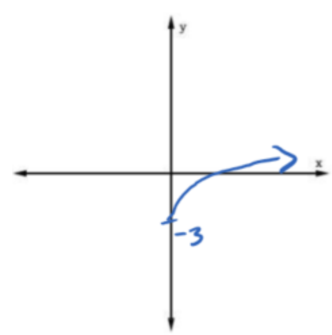
1. $y = \sqrt{-x}$



2. $y = \sqrt{x+4}$



3. $y = \sqrt{x} - 3$



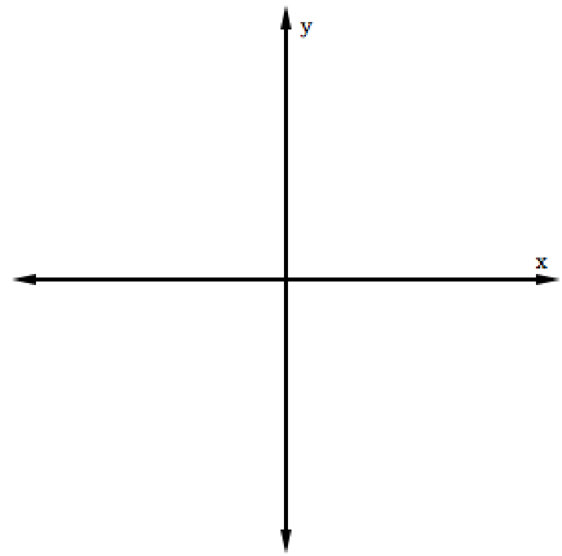
Function Family: Radical

Sub-family: Cube Root

Parent Function: $y = \sqrt[3]{x}$

- Shape: ________________________
- Domain of $y = \sqrt[3]{x}$: _________________
- Range of $y = \sqrt[3]{x}$: _________________
- Table of values:

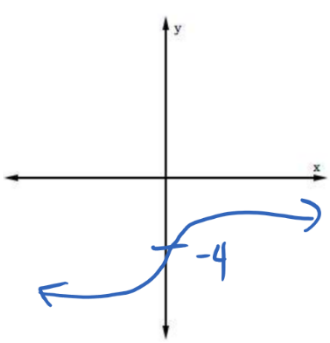
x	$y = \sqrt[3]{x}$
-2	$\sqrt[3]{-2} \approx$
-1	-1
0	0
1	1
2	$\sqrt[3]{2} \approx$



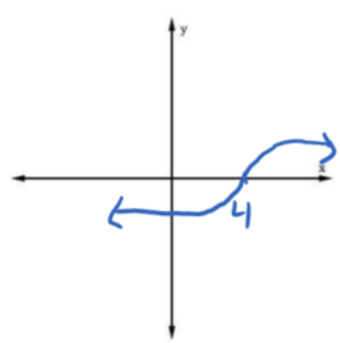
Verbal description:

Examples of Cube Root Functions

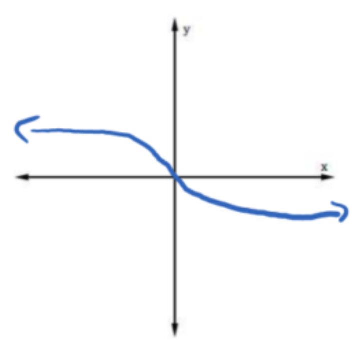
1. $y = \sqrt[3]{x} - 4$



2. $y = \sqrt[3]{x - 4}$



3. $y = -\sqrt[3]{x}$

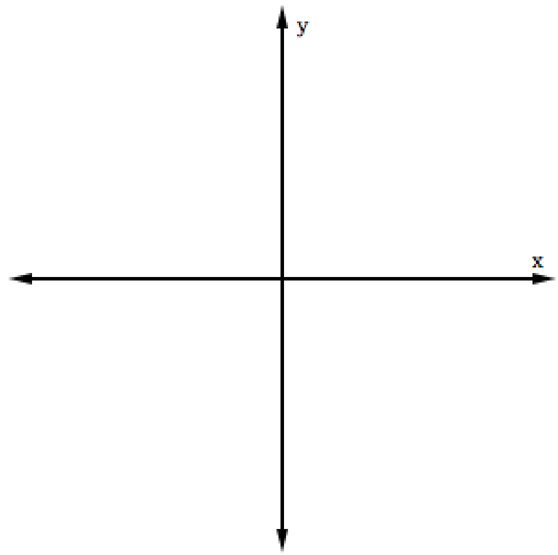


Function Family: Exponential

Sub-family: $y = b^x$ with base b, $b > 1$

Parent Function: $y = 2^x$, with $b > 0$ and $b \neq 1$

- Shape: ____________________
- Domain of $y = 2^x$: _________________
- Range of $y = 2^x$: _________________
- Equation of asymptote: ________________
- Table of values for $y = 2^x$:

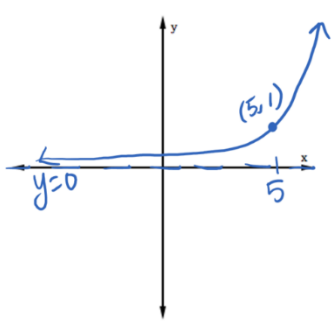


x	$y = 2^x$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

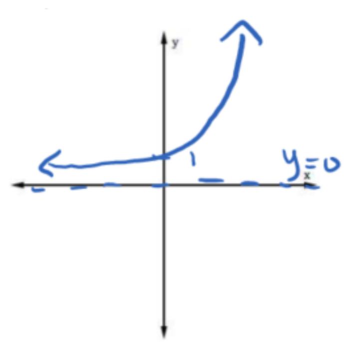
Verbal description:

Examples of Exponential Functions with base b, $b > 1$

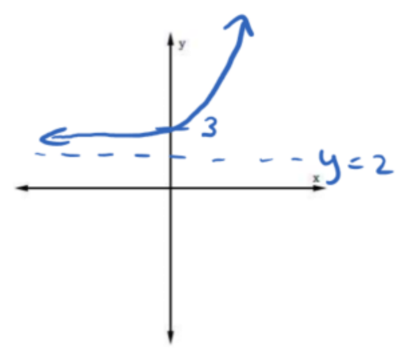
1. $y = 2^{x-5}$



2. $y = e^x$



3. $y = 5^x + 2$

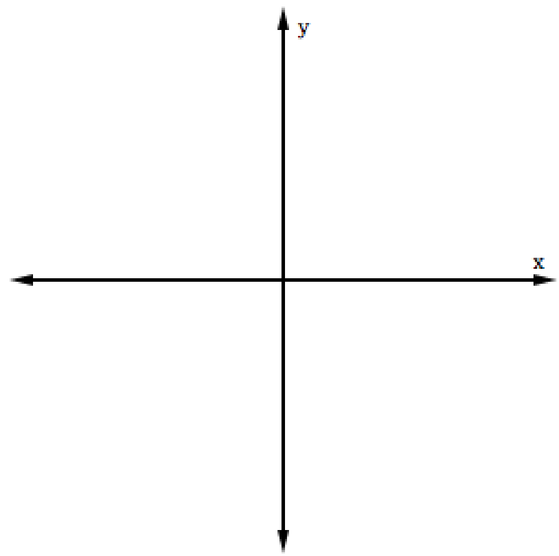


Function Family: Exponential

Sub-family: $y = b^x$ with base b, $0 < b < 1$

Parent Function: $y = \left(\frac{1}{2}\right)^x$, with $b > 0$ and $b \neq 1$

- Shape: ____________________
- Domain of $y = \left(\frac{1}{2}\right)^x$: _________________
- Range of $y = \left(\frac{1}{2}\right)^x$: _________________
- Equation of asymptote: ________________
- Table of values for $y = \left(\frac{1}{2}\right)^x$:

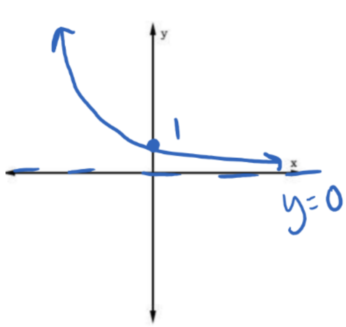


x	$y = \left(\frac{1}{2}\right)^x$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

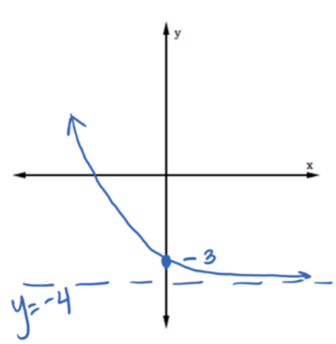
Verbal description:

Examples of Exponential Functions with base b, $0 < b < 1$

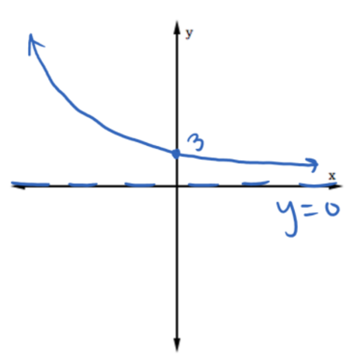
1. $y = \left(\frac{1}{3}\right)^x$



2. $y = \left(\frac{1}{2}\right)^x - 4$



3. $y = 3(0.298)^x$

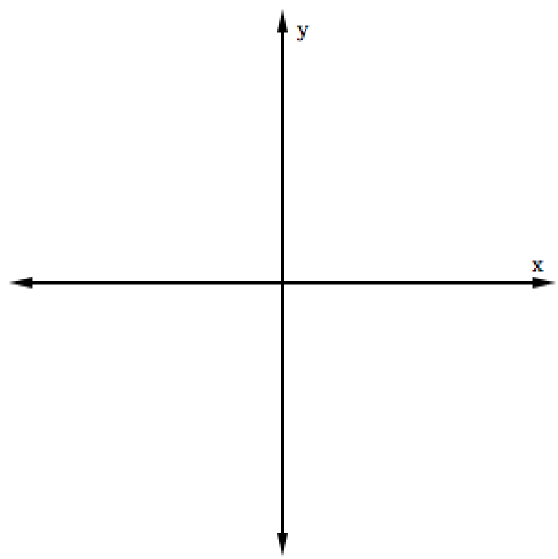


Function Family: Logarithmic

Sub-family: $y = \log_2(x)$

Parent Function: $y = \log_b(x)$, with $b > 0$ and $b \neq 1$

- Shape: ____________________
- Domain of $y = \log_2(x)$: ________________
- Range of $y = \log_2(x)$: ________________
- Equation of asymptote: ________________
- Table of values for $y = \log_2(x)$:

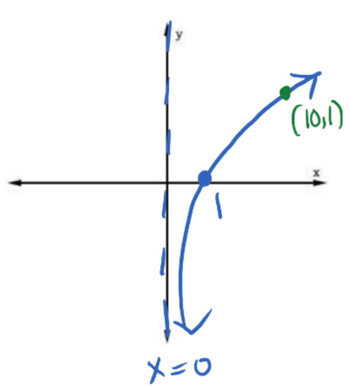


x	$y = \log_2(x)$
-1	undefined
0	undefined
1	0
2	1
4	2
8	3
16	4

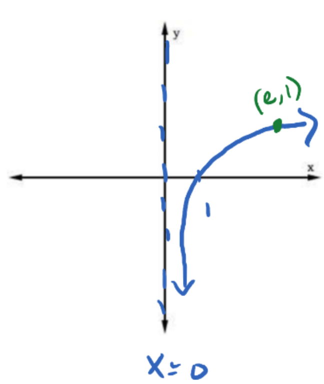
Verbal description:

Examples of Logarithmic Functions with base b, $b > 1$

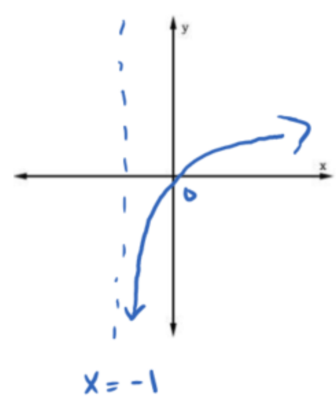
1. $y = \log_{10}(x) = \log x$



2. $y = \ln(x) = \log_e(x)$



3. $y = \log_3(x + 1)$

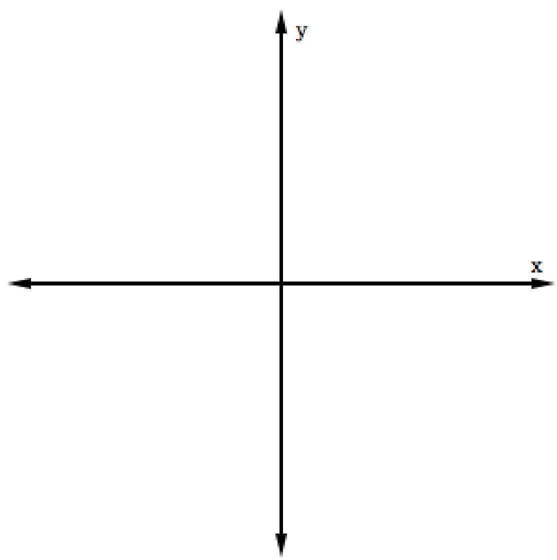


Function Family: Logarithmic

Sub-family: $y = \log_{\frac{1}{2}}(x)$

Parent Function: $y = \log_b(x)$, with $b > 0$ and $b \neq 1$

- Shape: ____________________
- Domain of $y = \log_{\frac{1}{2}}(x)$: _________________
- Range of $y = \log_{\frac{1}{2}}(x)$: _________________
- Equation of asymptote: ________________
- Table of values for $y = \log_{\frac{1}{2}}(x)$:

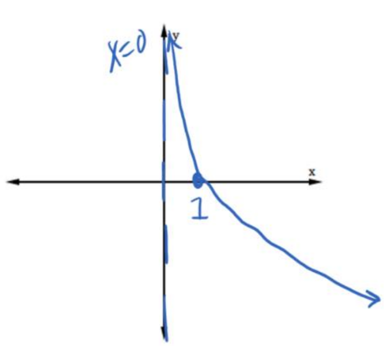


x	$y = \log_{\frac{1}{2}}(x)$
-1	undefined
0	undefined
1	0
2	-1
4	-2
8	-3
16	-4

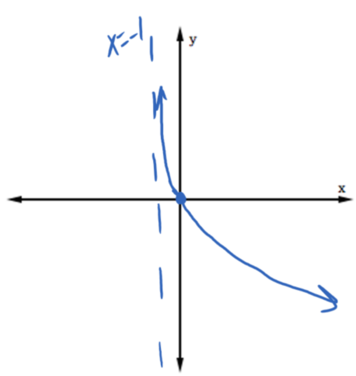
Verbal description:

Examples of Logarithmic Functions with base b, $0 < b < 1$

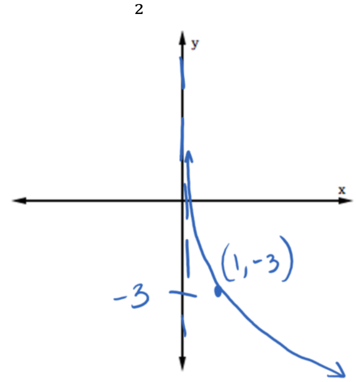
1. $y = \log_{\frac{1}{3}}(x)$



2. $y = \log_{\frac{1}{2}}(x+1)$



3. $\log_{\frac{1}{2}}(x) - 3$

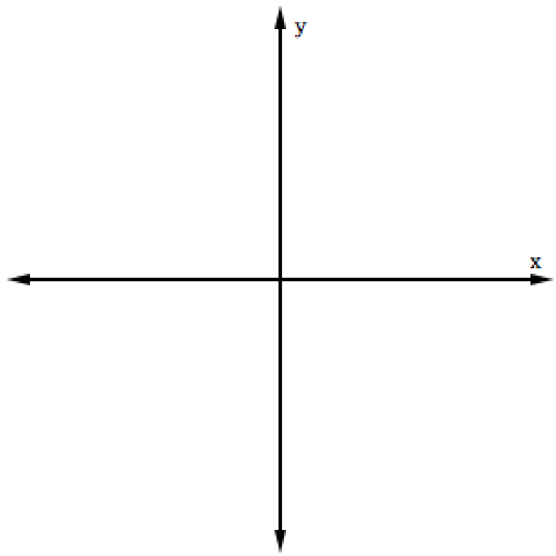


Function Family: Rational

Sub-family: Reciprocal

Parent Function: $y = \frac{1}{x}$

- Shape: ______________________
- Domain of $y = \frac{1}{x}$: _________________
- Range of $y = \frac{1}{x}$: _________________
- Equations of asymptotes: _______________________
- Table of values:

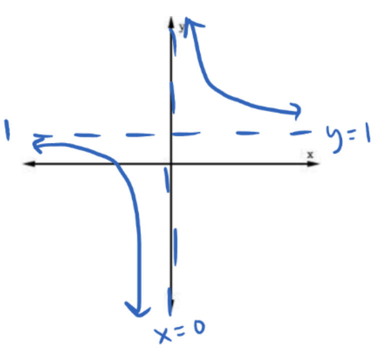


x	$y = \frac{1}{x}$
-2	$-\frac{1}{2}$
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$

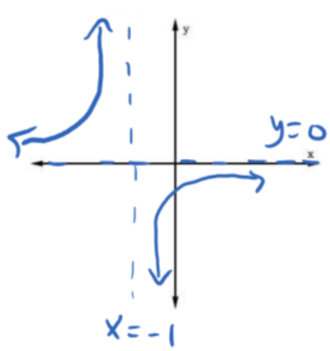
Verbal description:

Examples of Reciprocal Functions

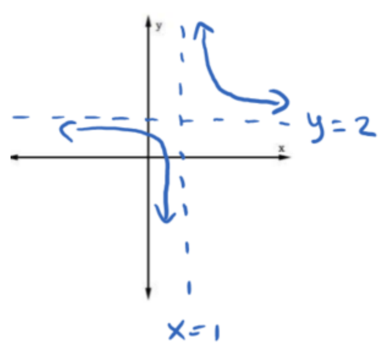
1. $y = \frac{1}{x} + 1$



2. $y = \frac{1}{x+1}$



3. $y = \frac{1}{x-1} + 2$



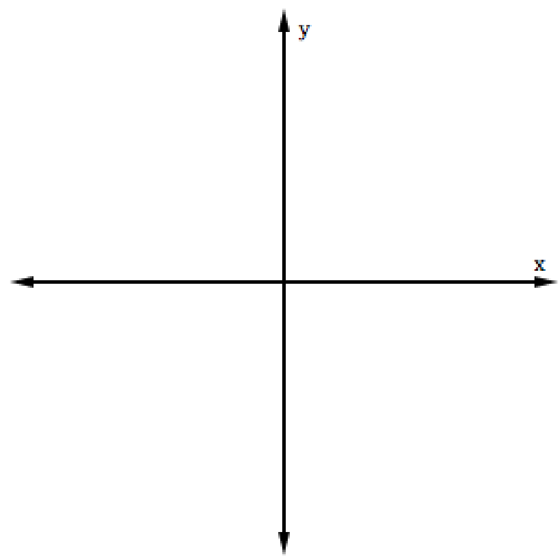
Function Family: Rational

Sub-family: Squared Reciprocal

Parent Function: $y = \frac{1}{x^2}$

- Shape: ____________________
- Domain of $y = \frac{1}{x^2}$: __________________
- Range of $y = \frac{1}{x^2}$: __________________
- Equations of asymptotes: ________________________
- Table of values:

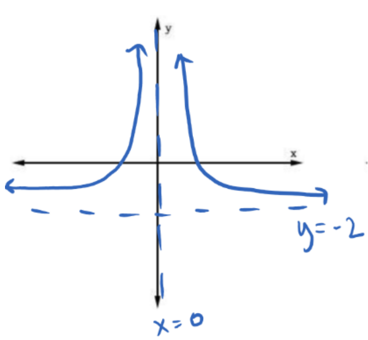
x	$y = \frac{1}{x^2}$
-2	$\frac{1}{4}$
-1	1
0	undefined
1	1
2	$\frac{1}{4}$



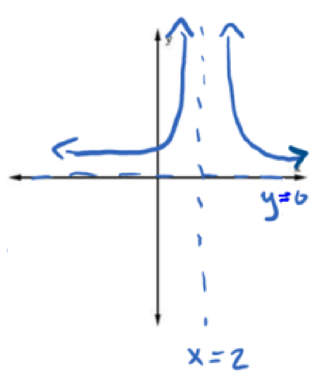
Verbal description:

Examples of Squared Reciprocal Functions

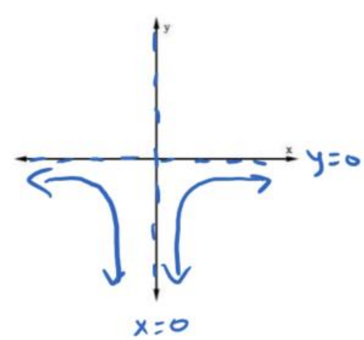
1. $y = \frac{1}{x^2} - 2$



2. $y = \frac{1}{(x-2)^2}$



3. $y = -\frac{1}{x^2}$



1.2: Functions, Domain and Range

Learning Objectives
Together with your team: • Determine whether a given relation is a function from a verbal description, table, or list of ordered pairs. • Given various representations of a function, determine its domain and range.

Definitions: Relation and Function

- A **relation** is a set of ordered pairs: (input, output).

  The set consisting of the first components of each ordered pair is called the **domain** and the second set consisting of the second components of each ordered pair is called the **range**.

  The **input** values make up the **domain**, and the **output** values make up the **range**.

- A **function** is a relation in which each possible input value leads to exactly one output value. We say "the output is a function of the input."

For each relation given in **1) – 6)** below, determine if it is a function and state its domain and range.

1) $\{(1,3),(2,6),(3,9),(4,12),(5,15)\}$

Function: Yes No

Domain: Range:

2) The relation that pairs *each letter in the alphabet* with *its ordinal number* (Table 1 shows an <u>example</u>.)

Function: Yes No

Domain: Range:

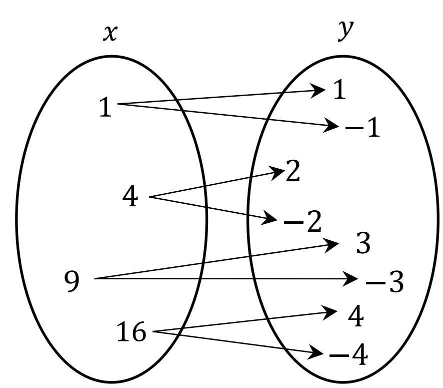
Table 1

Input	Output
e	5
g	7
g	7

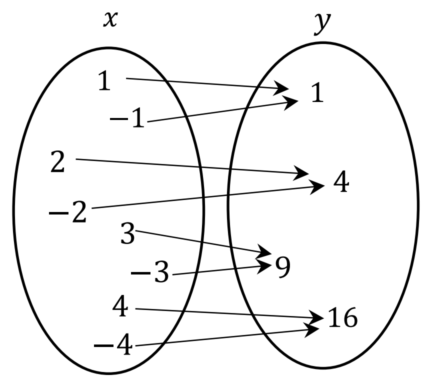
3) Does the arrow diagram represent y as a function of x?

Function: Yes No

Domain: Range:



4) Does the arrow diagram represent y as a function of x?



Function: Yes No

Domain: Range:

5) Is the price of a stamped letter a function of its weight? (See the table *2017 First-class Mail*.)

Function: Yes No

Domain: Range:

2017 First-class Mail	
Weight in oz.	*Price of a Stamped Letter*
$0 < w \leq 1$	\$0.49
$1 < w \leq 2$	\$0.70
$2 < w \leq 3$	\$0.91
$3 < w \leq 3.5$	\$1.12

6) A relation that pairs an Oregon State University student's name to an OSU ID card number.

Function: Yes No

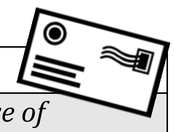
Domain: Range:

<u>Summary:</u> Determining Domain and Range from a Table or Set of Ordered Pairs

1.3: Function Notation and Average Rate of Change

Learning Objectives
Together with your team: • Given various representations of a function, identify inputs and outputs and interpret these in the context of a situation. • Use a table or equation to evaluate a function for a given input and express the value using function notation. • Use a table to calculate the average rate of change of a function over a specified interval and express the value using function notation.

1) The table *2017 First-class Mail* represents the price, p, of a stamped letter as a function of weight, w. Using function notation, we could represent this relationship symbolically as price $= p(w)$.



2017 First-class Mail	
Weight in oz. w	*Price of a Stamped Letter, p*
$0 < w \leq 1$	\$0.49
$1 < w \leq 2$	\$0.70
$2 < w \leq 3$	\$0.91
$3 < w \leq 3.5$	\$1.12

a) What is the input variable? Describe what it represents.

b) What is the output variable? Describe what it represents.

c) Evaluate $p(0.91) =$ _______

d) Interpret your answer to part (c) in a complete sentence in the context of this situation.

e) Solve the following equation for w.

$$p(w) = 1.12$$

f) Challenge: try to represent the piecewise-defined function p given in the table above using an equation (hint: think about your ALEKS Prep assignment).

Summary: Function Notation

2) The following question was on a College Algebra exam:

$$\text{Let } f(x) = \begin{cases} 2x+1 & \text{if } \quad x \le 2 \\ 3x & \text{if } \quad 2 < x < 5 \\ x-1 & \text{if } \quad 5 \le x < 9 \end{cases}.$$

Evaluate $f(9)$ and explain your reasoning.

Here are five different student responses and their explanations. Decide which student(s) should receive full credit. Select all that apply.

A. $f(9) = 8$. "I evaluated the third piece of f at $x = 9$."

B. $f(9) = 8$. "I plugged 9 into $x - 1$ because $x = 9$ is between 5 and 9."

C. "I think $f(9)$ is undefined because 9 is not in the domain of f."

D. "$f(9)$ is undefined."

E. $f(9) = 19, 27, 8$. "I plugged $x = 9$ into all three pieces of the function."

3) Let $E(t)$ be the function that gives the enrollment at OSU in year t. For example, $E(2008) = 20{,}167$.

Year, t	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
OSU Enrollment, E	20,167	27,040	22,364	24,114	25,372	26,628	28,196	29,084	29,911	31,904

a) Describe how OSU enrollment has changed over time.

b) *How fast* did the university enrollment grow, on average, over the years 2008 to 2017?

c) In b) you calculated the Average Rate of Change (ARC) of OSU Enrollment between 2008 and 2017. Use *function notation* to write an expression for the ARC of OSU enrollment between 2008 and 2017.

d) Use *function notation* to write a formula for the ARC of OSU enrollment between any two years, t_1 and t_2.

e) What are the units of the ARC?

f) How do these units relate to the input units and the output units of the enrollment function?

Summary: Average Rate of Change

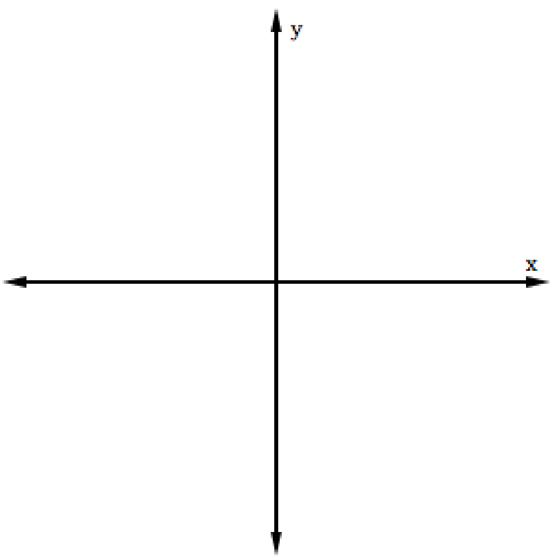
1.4: Warm Up

Student Name:

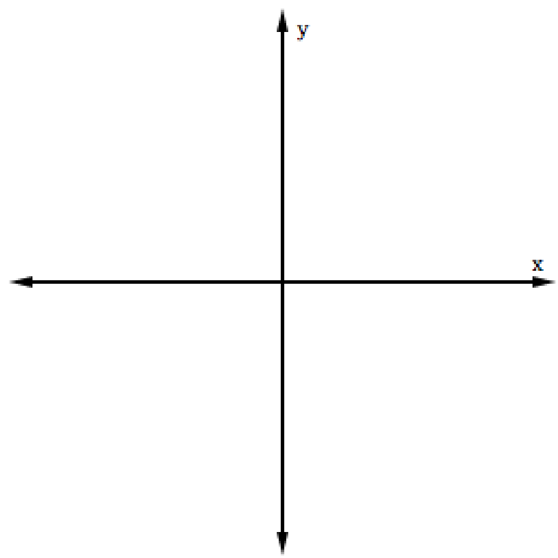
1) The Exponential and Logarithmic Functions make up two more of the families we will study throughout this course. Recall from your Families of Functions Reference Guide in Lesson 1.1, our Exponential *parent* function is $f(x) = b^x$, and the Logarithmic parent function is $h(x) = \log_b(x)$.

a) Sketch "good enough" graphs of the base 2 and base $\frac{1}{2}$ exponential functions f and g.

$f(x) = 2^x$



$g(x) = \left(\frac{1}{2}\right)^x$



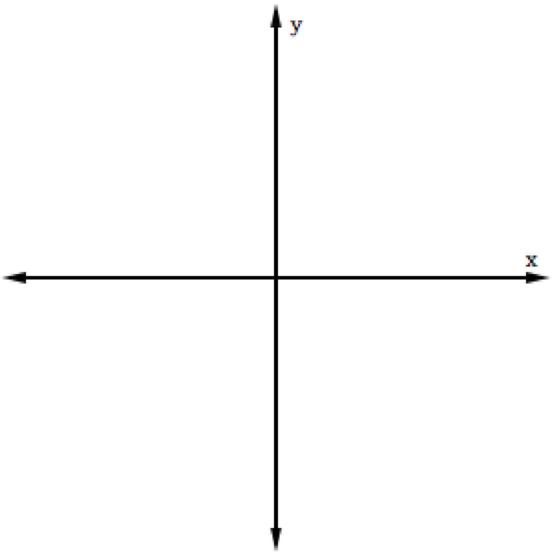
b) Describe how the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ are similar to each other. What do you think <u>explains</u> the similarities?

c) Describe how the graphs of the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ are different from one another. What do you think <u>explains</u> the differences?

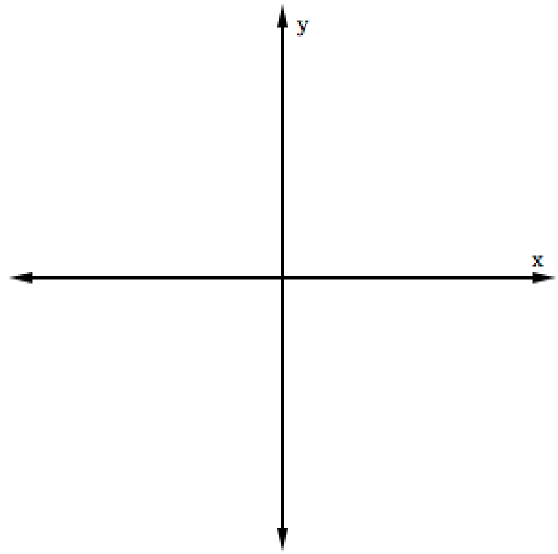
2) Next, consider the base 2 and base $\frac{1}{2}$ logarithmic functions, $h(x) = \log_2(x)$ and $k(x) = \log_{\frac{1}{2}}(x)$

a) Sketch "good enough" graphs of h and k.

$h(x) = \log_2(x)$



$k(x) = \log_{\frac{1}{2}}(x)$



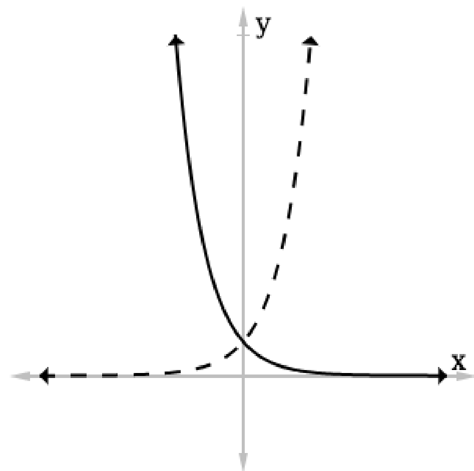
b) Describe how the functions $h(x) = \log_2(x)$ and $k(x) = \log_{\frac{1}{2}}(x)$ are similar to each other. What do you think <u>explains</u> the similarities?

c) Describe how the graphs of the functions $h(x) = \log_2(x)$ and $k(x) = \log_{\frac{1}{2}}(x)$ are different from one another. What do you think <u>explains</u> the differences?

1.4: Introduction to Exponential and Logarithmic Functions

Learning Objectives
Together with your team: • Use tables, equations, and graphs to explore the exponential and logarithmic parent functions. • Identify exponential and logarithmic functions, given their graphical, symbolic, or numerical representations. • For exponential and logarithmic functions, use a table or equation to evaluate the function for a given input, and express the value using function notation (for example, find $f(8)$ for $f(x) = \log_2(x)$).

1) Using graphs and tables can help us further understand the end behavior of the Exponential Family and the Logarithmic Family.



a) First, click the Desmos link below and open the folder for each function, $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Examine the graphs and tables.

https://www.desmos.com/calculator/vwzxrln9v0

b) From the Desmos tables, record the output values for the inputs given in the table below.

x	-10	-5	-4	-3	-2	-1	0	1	2	3	4	5	10
2^x													
$\left(\frac{1}{2}\right)^x$													

c) How do the function values $f(x) = 2^x$ change, as x becomes large and positive (i.e. increases without any upper limit)?

d) How do the function values $f(x) = 2^x$ change, as x becomes large and negative (i.e. decreases without any lower limit)?

e) How do the function values $g(x) = \left(\frac{1}{2}\right)^x$ change, as x becomes large and positive (i.e. increases without any upper limit)?

f) How do the function values $g(x) = \left(\frac{1}{2}\right)^x$ change, as x becomes large and negative (i.e. decreases without any lower limit)?

g) The end behavior of the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ creates a horizontal asymptote. What is the equation of the asymptote for both of these graphs? Explain.

Summary: $f(x) = b^x$

	$b > 1$	$0 < b < 1$
End behavior:		
Equation of asymptote:		
Increasing/decreasing:		
Positive/negative function values:		
x-intercept:		
y-intercept:		

2) In the previous question, we saw that, *as we increase the input values x* of an exponential function, the output values $f(x) = b^x$:

- Increase, if $b > 1$ and
- Decrease, if $0 < b < 1$

We can also describe *how fast* a function increases or decreases (as we did when we computed the ARC of the OSU Enrollment function). For the sake of comparison, let's also include a linear function here.

a) Fill out the table at the right with the outputs for the function l.

b) Describe how the outputs of $f(x)$ change, as x increases by 1 unit.

x	$f(x) = 2^x$	$g(x) = \left(\frac{1}{2}\right)^x$	$l(x) = 2x$
-1	$\frac{1}{2}$	2	
0	1	1	
1	2	$\frac{1}{2}$	
2	4	$\frac{1}{4}$	
3	8	$\frac{1}{8}$	
4	16	$\frac{1}{16}$	

c) Describe how the output values of $g(x)$ change, as x increases by 1 unit.

d) Describe how the output values of $l(x)$ change, as x increases by 1 unit.

e) How might you use a table to identify an exponential function?

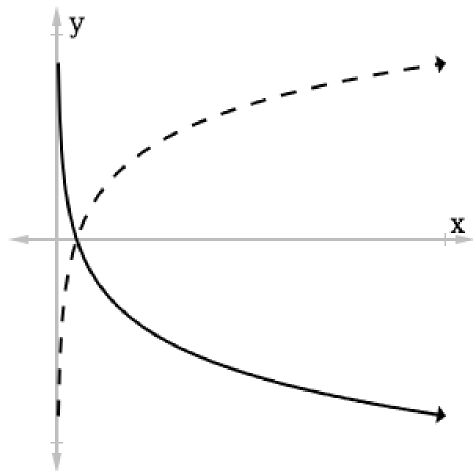
f) How might you use a table to identify a linear function?

3) Two functions in the Logarithmic Family are the base 2 and base $\frac{1}{2}$ logarithmic functions.

$$h(x) = \log_2(x) \text{ and } k(x) = \log_{\frac{1}{2}}(x)$$

Consider their tables in Desmos: https://www.desmos.com/calculator/d2ckalocvu

a) Record the output values of $h(x) = \log_2(x)$ and $k(x) = \log_{\frac{1}{2}}(x)$ for the specified inputs in the following table.



x	-2	-1	0	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
$\log_2(x)$														
$\log_{\frac{1}{2}}(x)$														

b) How do the function values $h(x) = \log_2(x)$ change, as x becomes large and positive (i.e. increases without any upper limit)?

c) How do the function values $h(x) = \log_2(x)$ change, as x becomes closer to 0?

d) How do the function values $k(x) = \log_{\frac{1}{2}}(x)$ change, as x becomes large and positive (i.e. increases without any upper limit)?

e) How do the function values $k(x) = \log_{\frac{1}{2}}(x)$ change, as x becomes closer to 0?

Summary: $f(x) = \log_b(x)$

	$b > 1$	$0 < b < 1$
End behavior:		
Equation of Asymptote:		
Increasing/decreasing:		
Positive/negative function values:		
x-intercept:		
y-intercept:		

Definitions: The Logarithmic Functions: Base b, Base 10, and Base e	
The Base b Logarithmic Function	
The Base 10 Logarithmic Function (Common Log)	The Common Log Function is used to model the Richter scale and Decibel.
The Base e Logarithmic Function (Natural Log)	Recall: e is an irrational number (like π) that is used in real-world applications. For example, e shows up in calculating the amount of money in a savings account with continuously compounded interest. $e \approx$____________________

4) Some students evaluated the following logarithmic expressions. Write a mathematical equation to justify their answers.

a) $\log_7(49) = 2$

b) $\log_8(1) = 0$

5) Let $f(x) = \log_4(x)$. Evaluate $f(1024)$.

6) Let $g(x) = \log_6(x)$. Evaluate $g(216)$ and show your reasoning.

7) Write a mathematical equation to justify the statement: $\ln(17) = 2.833$

1.5: Introduction to Polynomial Functions

Learning Objectives
Together with your team: • Use tables, equations, and graphs to explore the polynomial parent functions. • Identify polynomial functions given their symbolic representations.

Polynomials are one of the families of functions we'll study throughout this course. Recall from Section 1.1, our polynomial *parent* functions are: linear ($y = x$); quadratic ($y = x^2$); and cubic ($y = x^3$).

As we will see, *the family of polynomial functions is built by adding or subtracting constant multiples of basic parent functions like these.*

1) First, go to www.Student.Desmos.com and enter the class code: ______________________________

a) **Screen 2:** Record your two groups of functions. Sketch "good enough" graphs below.

Group 1 Group 2

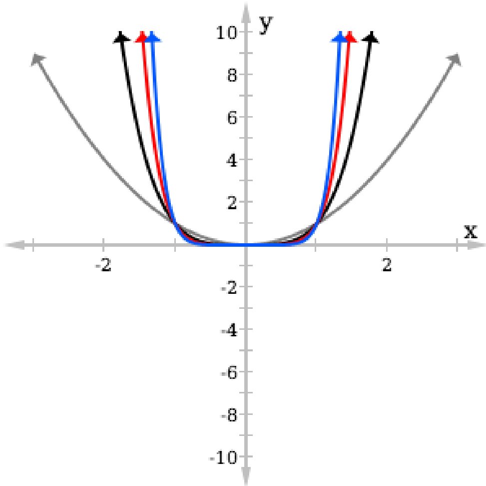
b) Give your reasoning for grouping the functions the way you did. Write complete sentences.

2) Compare the equations, graphs, and tables of the functions shown in Group 1 below with those in Group 2.

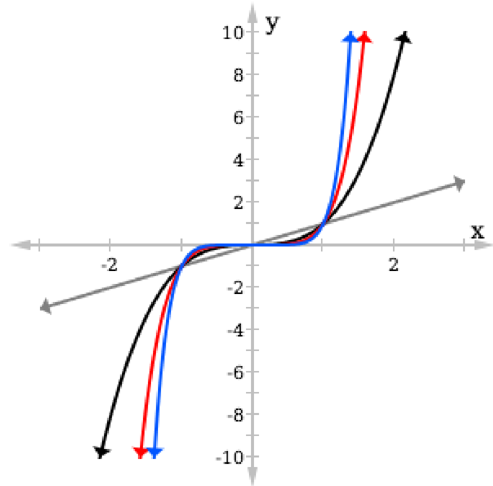
<u>Group 1</u>: $y = x^2, y = x^4, y = x^6, y = x^8, \ldots$

<u>Group 2</u>: $y = x,\ \ y = x^3, y = x^5, y = x^7, \ldots$

Group 1



Group 2



x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

x	$y = x^4$
-2	16
-1	1
0	0
1	1
2	16

x	$y = x$
-2	-2
-1	-1
0	0
1	1
2	2

x	$y = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

x	$y = x^6$
-2	64
-1	1
0	0
1	1
2	64

x	$y = x^8$
-2	256
-1	1
0	0
1	1
2	256

x	$y = x^5$
-2	-32
-1	-1
0	0
1	1
2	32

x	$y = x^7$
-2	-128
-1	-1
0	0
1	1
2	128

a) List at least three differences between the functions in these two groups.

I.

II.

III.

b) Using the equations justify *why* the differences you listed in a) exist.

Summary: Power functions of the form $y = x^n$, where $n = 0, 1, 2, 3, 4, 5, 6, \ldots$

	n even	n odd
End behavior:		
Increasing/decreasing:		
Positive/negative function values:		
x-intercept:		
y-intercept:		

By adding constant multiples of these basic polynomial functions (called monomials), we build the Polynomial Family of functions.

3) For example, adding $7x^2$, $\frac{1}{3}x^4$, and -5, gives us the polynomial function:

$$f(x) = 7x^2 + \frac{1}{3}x^4 - 5$$

a) $7x^2$ is a *term* of the polynomial function f. What are its other terms? ______________________

b) The leading term of the polynomial function f is $\frac{1}{3}x^4$. What do you think is meant by "leading term" of a polynomial?

c) The *degree* of the polynomial function f is 4. What do you think is meant by the "degree" of a polynomial?

d) The leading coefficient of the polynomial function f is $\frac{1}{3}$. What do you think is meant by "leading coefficient" of a polynomial?

Summary: Polynomial Function

A polynomial function consists of the sum of a finite number of *terms*, each of which is a product of a real number *coefficient*, and a variable raised to a non-negative integer power; in other words, a polynomial function is a sum of power functions of the form ax^n where $n = 0, 1, 2, 3, \ldots$ and a is any real number.

Note: When $n = 0$, we have: ______________________________

4) Next, identify the Polynomial Functions and Non-Polynomial Functions listed here. Sort them in the table below. Be prepared to give reasons for your choices.

$w(x) = \frac{5}{3}x^4 - 7x + \frac{3}{x}$	$h(x) = 7$	$d(x) = 3^x$	$l(x) = -x(x^2 - 4)$	$g(x) = x^{-1}$
$a(x) = \frac{9x^3 + 4x - 11}{2}$	$s(x) = x^8$	$j(x) = x + \sqrt{2}$	$n(x) = \sqrt{x^2 + 4x + 1}$	$i(x) = 6x^{\frac{1}{3}} + x$

Examples of Polynomial Functions	Examples of Non-Polynomial Functions

How would you explain the difference between a polynomial function and a non-polynomial function to a friend who missed class today?

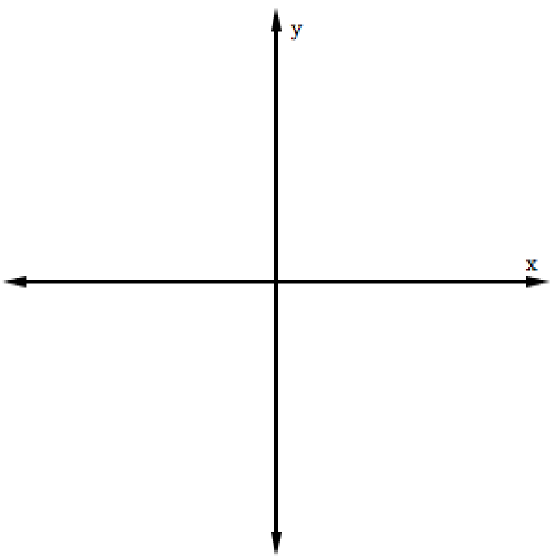
1.6: Warm Up

Student Name:

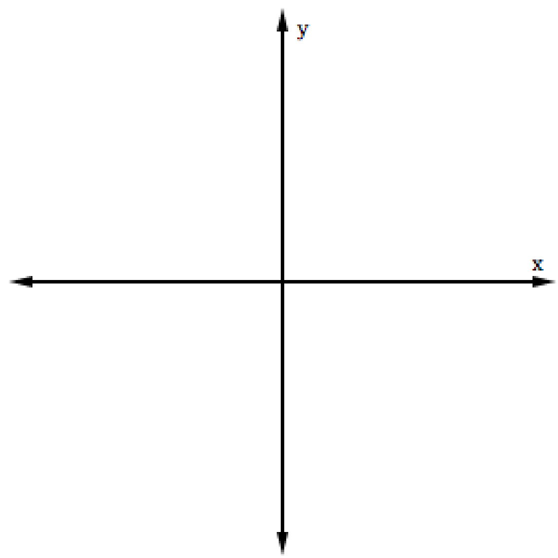
1) The Rational Functions make up the final family we'll consider in this chapter and continue to study throughout this course. Recall from your Families of Functions Reference Guide in Section 1.1, our rational *parent* functions are the reciprocal function, $f(x) = \frac{1}{x}$, and the squared reciprocal function, $g(x) = \frac{1}{x^2}$.

a) Sketch "good enough" graphs of the rational functions f and g.

$f(x) = \frac{1}{x}$



$g(x) = \frac{1}{x^2}$



b) Describe how the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ are similar to each other. What do you think <u>explains</u> the similarities?

c) Describe how the graphs of the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ are different from one another. What do you think <u>explains</u> the differences?

1.6: Introduction to Rational Functions

Learning Objectives
Together with your team: • Use tables, equations, and graphs to explore the rational parent functions. • Identify rational functions given their symbolic representations.

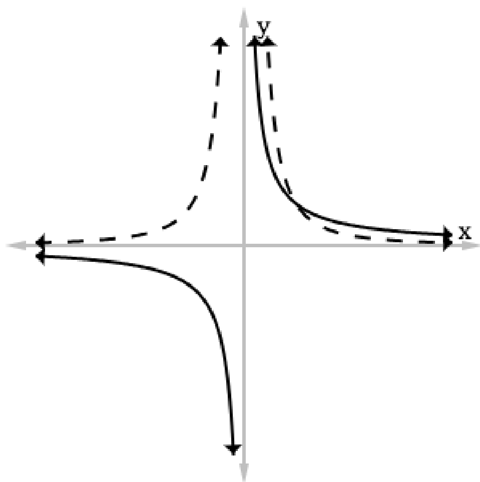
1) Using graphs and tables can help us further understand the asymptotic behavior of the reciprocal function and the squared reciprocal function.

a) First, click the Desmos link below and open the folder for each function, $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Examine the graphs and tables. Go ahead and play around, try more values in the table, zoom in and out. You can always refresh the page to get back to the original graphs and tables.

https://www.desmos.com/calculator/wkx6dbij7g

b) From the Desmos tables, record the output values for the inputs given in the tables below.

x	-1000	-100	-10	-1
$\frac{1}{x}$				
$\frac{1}{x^2}$				



x	1	10	100	1000
$\frac{1}{x}$				
$\frac{1}{x^2}$				

c) Explain *why* the output values of $f(x) = \frac{1}{x}$ are both positive and negative, while the outputs of $g(x) = \frac{1}{x^2}$ are only positive.

d) How do the function values $f(x) = \frac{1}{x}$ change, as x becomes large and positive (i.e. increases without any upper limit)?

e) How do the function values $f(x) = \frac{1}{x}$ change, as x becomes large and negative (i.e. decreases without any lower limit)?

f) How do the function values $g(x) = \frac{1}{x^2}$ change, as x becomes large and positive (i.e. increases without any upper limit)?

g) How do the function values $g(x) = \frac{1}{x^2}$ change, as x becomes large and negative (i.e. decreases without any lower limit)?

h) Write what you think it means to say, "$f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ each have a horizontal asymptote of $y = 0$."

2) In **1)**, we considered the *end behavior* of the rational functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. End behavior refers to what happens on the far left or far right side of a graph.

Here, let's turn our attention to the *local behavior* of these functions. Local behavior refers to what happens near the origin of the graph.

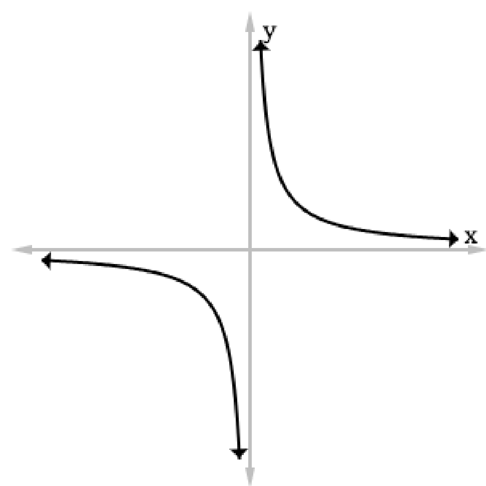
a) Click the Desmos link below and open the folder for each function, $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Examine the graphs and tables.

https://www.desmos.com/calculator/oa6sqychy1

b) From the Desmos tables, record the input values for the outputs given in the tables below.

x	$\frac{1}{x}$
	1000
	100
	10
	1

c) How does the input value x change, as $f(x) = \frac{1}{x}$ becomes large and positive (i.e. increases without any upper limit)?



d) How does the input value x change, as $f(x) = \frac{1}{x}$ becomes large and negative (i.e. decreases without any lower limit)?

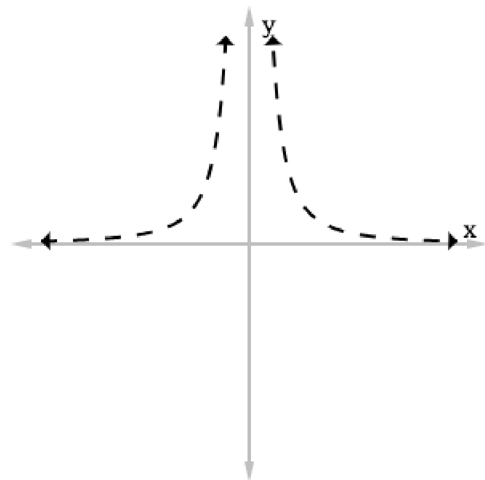
x	$\frac{1}{x}$
	-1
	-10
	-100
	-1000

e) From the Desmos tables, record the input values for the outputs given in the table below.

f) How does the input value x change, as $g(x) = \frac{1}{x^2}$ becomes large and positive (i.e. increases without any upper limit)?

x_1	x_2	$\frac{1}{x^2}$
		1,000,000
		10,000
		100
		1

g) Why can we not ask the question, "How does the input value x change, as $g(x) = \frac{1}{x^2}$ becomes large and *negative*?"



h) Write what you think it means to say, "$f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ each have a vertical asymptote of $x = 0$."

Summary: Rational Parent Functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$

	$f(x) = \dfrac{1}{x}$	$g(x) = \dfrac{1}{x^2}$
End behavior:		
Local Behavior:		
Equations of Asymptotes:		
Increasing/decreasing:		
Positive/negative function values:		
x-intercept:		
y-intercept:		

3) So far, we have been considering only rational parent functions. The table below shows functions that are in the Family of Rational Functions and some that are not in that family.

Examples of Rational Functions	Examples of NON-Rational Functions
○ $h(x) = \frac{x^2}{7+x^4}$ ○ $y = \frac{x^2+x-9}{7}$ ○ $y = 4x - 12$ ○ $j(x) = \frac{(6x-2)(x-1)}{(x+5)^2}$ ○	○ $m(x) = \frac{\sqrt{8x}}{9x-1}$ ○ $h(x) = \frac{5^x}{x^3}$ ○ $y = \frac{x^{\frac{2}{3}}+8}{x}$ ○ $n(x) = \frac{5}{\lvert x \rvert}$

Talk with your team and write down what you notice about rational functions. Give an example of a rational function.

The **definition** of <u>rational function</u> we will use in this class:

Chapter Learning Objectives

1. Given the *graph* of a relation, determine whether the relation is a function.
2. Given the *graph* of a function, determine or estimate:
    - Inputs and outputs and interpret these in the context of a situation.
    - The domain and range of the function.
    - The intervals of the domain where the function is increasing or decreasing.
    - The intervals of the domain where the function is positive or negative.
    - Any extreme values, absolute and local, of the function.
    - The x-intercepts (zeros, roots), and their respective multiplicities for polynomial functions.
    - The average rate of change of a function over a given interval and interpret in context, specifying appropriate units.
3. Model a given situation graphically.
4. Write a narrative for a situation that could be represented by a given graph.
5. Give the equation of all vertical and horizontal asymptotes of a graph.
6. Use arrow notation to describe the end behavior of a function, given its graph.

Chapter 2
What Can We Learn from a Graph?

Chapter Overview

In Chapter 2, we will see there are many questions we can answer about a function from its graphical representation, such as:

- What is its domain and range?
- What are the intervals of the domain where the function is increasing, decreasing, or constant?
- What are the x- and y-intercepts, if any?
- What are intervals of the domain where the function is positive or negative?
- What are the maximum and minimum values of the function?
- What is the end behavior of the function?

Chapter 2 Contents

2.1: Interpreting Graphs

Learning Objectives
Together with your team, given the graph of a function: • Model a given situation graphically. • Write a narrative for a given graph. • Evaluate the function for a given input and interpret in the context of a situation. • Represent and calculate the average rate of change of a function over a given interval and interpret in context, specifying appropriate units.

1) This graph models the amount of money in a particular automatic teller machine (ATM) over a one-hour period.

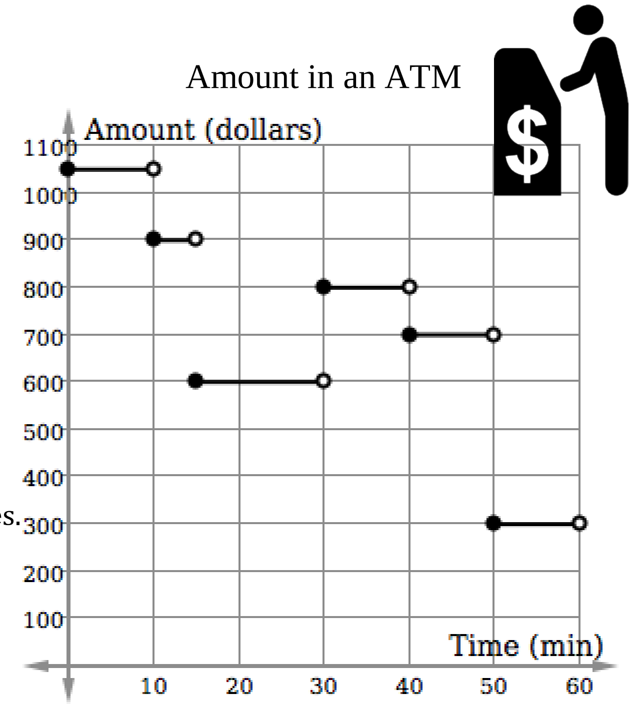
a) What was the initial amount of money in the ATM?

b) How many withdrawals occurred?

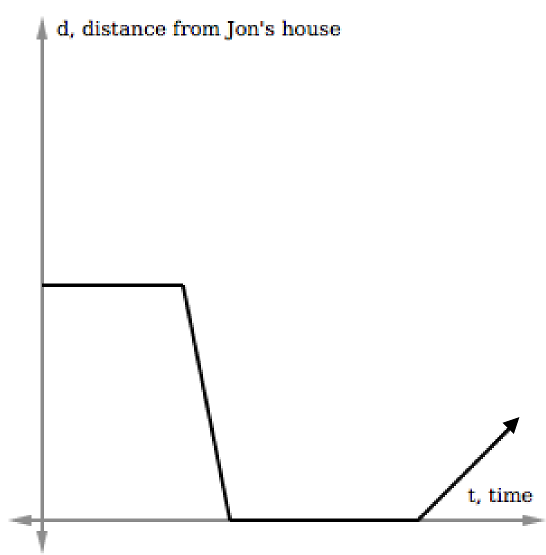
c) How is a withdrawal represented on the graph?

d) Describe what happened in the situation at 30 minutes.

e) What is the domain of this function?



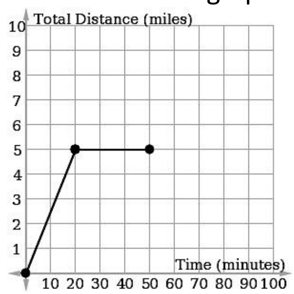
2) This graph models Jon's distance from his house as a function of time. Write a story that fits the graph.



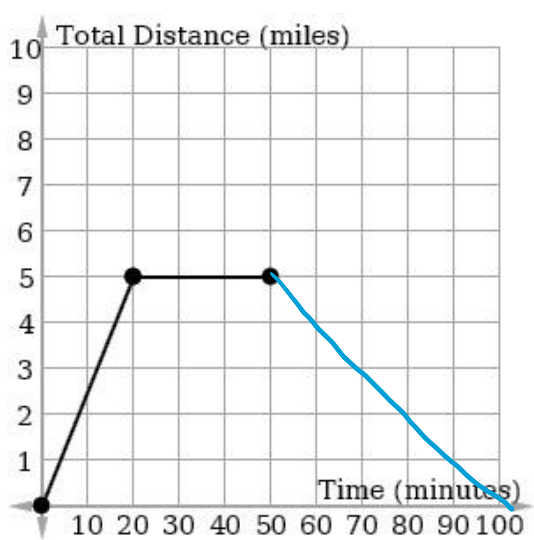
3) Sasha was given the following problem by her College Algebra instructor:

PROBLEM: One Saturday, Josef realizes he left his laptop at work and decides to make a trip to his office to retrieve the computer. The function graphed here models Josef's *total distance traveled* over time.

Sketch what the graph may look like if Josef headed for home at 50 minutes.

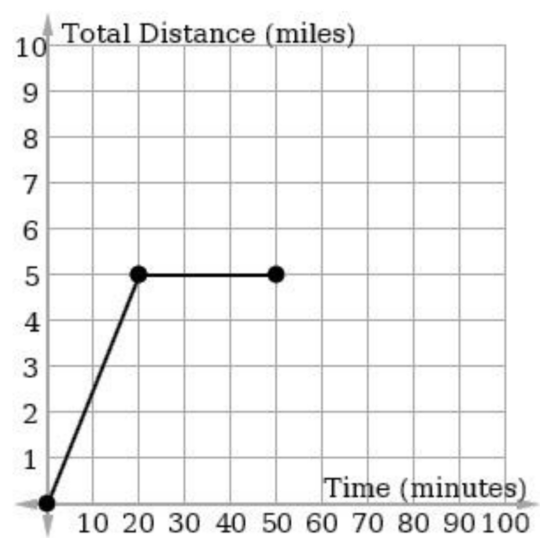


a) Here is the graph Sasha drew:



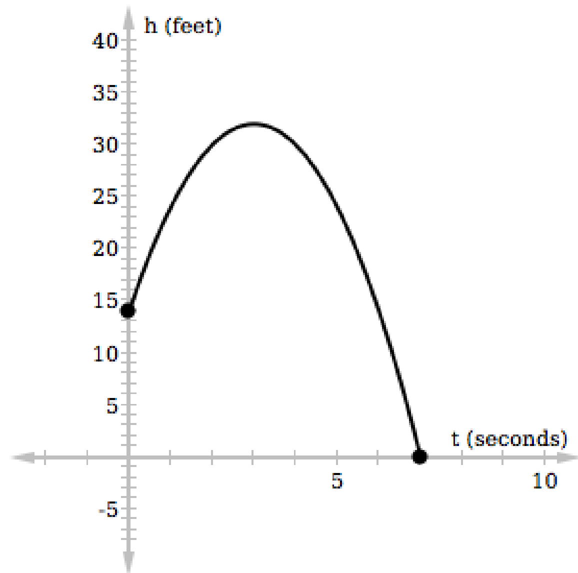
Do you agree with Sasha's answer? Why or why not?

b) If not, what would your graph look like to answer this question?



Explain your reasoning for the graph you drew.

4) A ball is thrown straight up in the air over the edge of a hotel balcony. The height in feet of the ball after t seconds is given by the function $h(t)$.



a) Draw the path that the ball takes.

b) Explain why the picture of the path of the ball is different from the graph of h.

c) Mark the point on the graph that shows the maximum height of the ball. Label this point A.

d) Mark the point on the graph that shows where the ball hit the ground. Label this point B.

e) Mark the point on the graph that shows the initial height of the ball, $h(0)$. Label this point C.

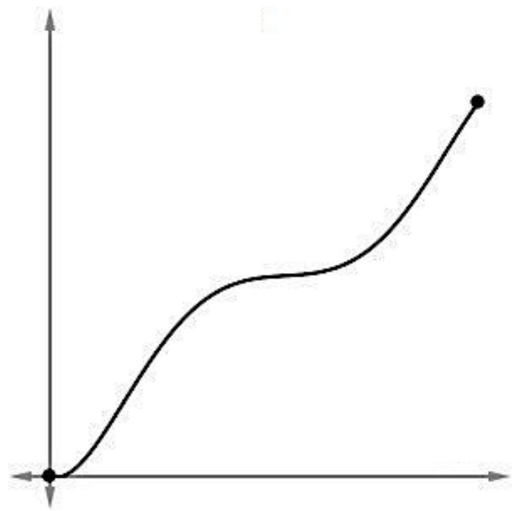
f) How many times will the ball be 20 feet above the ground ($h(t) = 20$)? Mark those points on the graph.

g) Estimate for how long the ball will be more than 20 feet above the ground.

h) How many times will the ball be 40 feet above the ground? Mark those points on the graph.

i) Explain why this parabola is graphed with endpoints, rather than arrows on each end.

5) Michael took a trip to visit his stepdad in Portland (110 miles away from his home). The graph below shows the trip where his distance from home is a function of time. The total trip took him 2.25 hours.



a) Label the axes, including units, and the two coordinates given.

b) Give the graph a title.

c) Describe what is happening to Michael's speed over time.

d) Determine Michael's *average speed* for the whole trip.

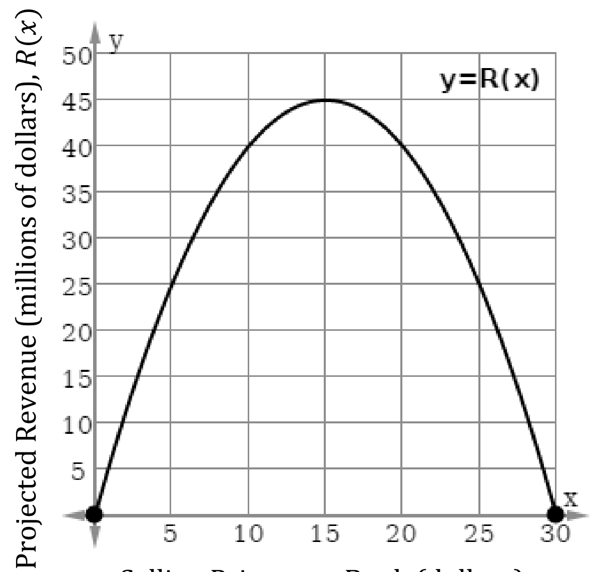
e) How could you represent his average speed on the graph?

6) The Math Books Inc. publishing company is conducting a marketing analysis for releasing their titles as eTextbooks. The graph of $R(x)$ below gives the projected revenue, R, as a function of the selling price, x, for the eBook.

a) Approximate the value of $R(5)$. Include units.

$R(5) =$____________________

Projected Revenue from eTextbooks



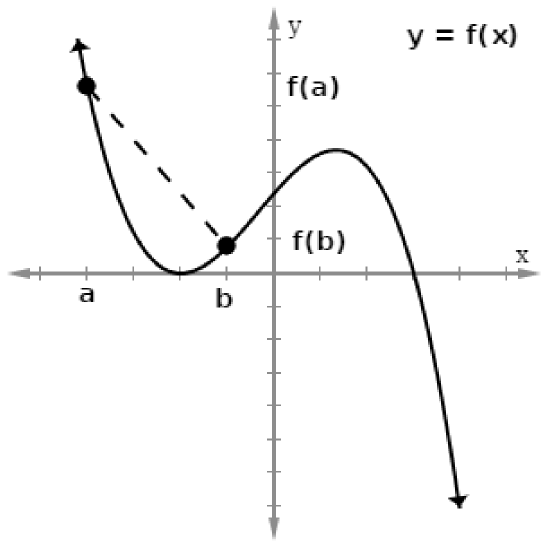
b) *Interpret* $R(5)$ in the context of the situation.

c) What is the company's maximum projected revenue? Include units.

d) Find the *average rate of change* of the projected revenue, $R(x)$, on the price interval [20, 25].

e) Interpret the average rate of change you calculated in d) in the context of this situation.

Summary: Average Rate of Change and Interpretation

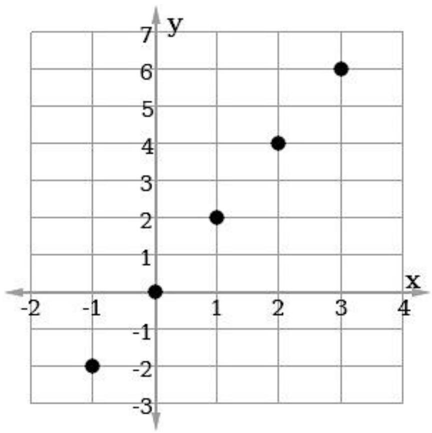


2.2: Warm Up

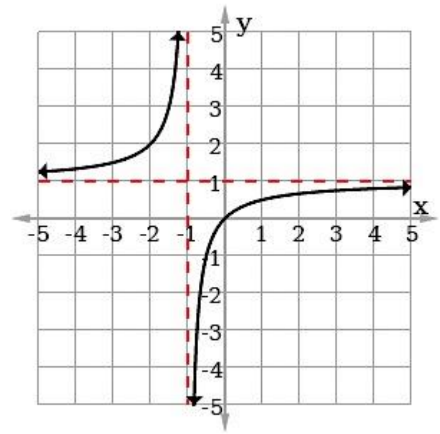
Student Name:

1) For each relation graphed below, determine if it is a function.

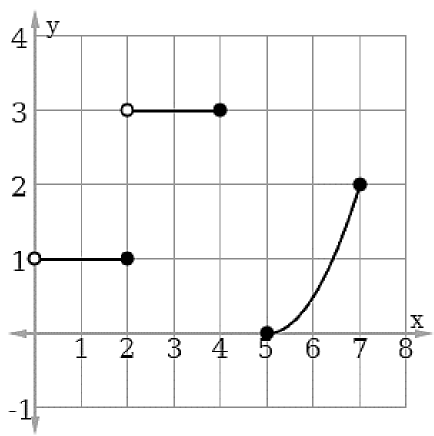
a) Function: Yes No



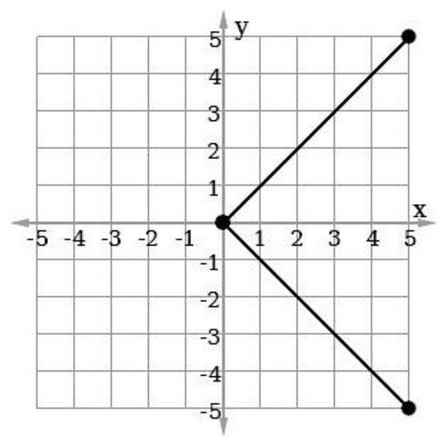
b) Function: Yes No



c) Function: Yes No



d) Function: Yes No



2) In your own words, describe how to determine from a graph of a relation whether it is a function.

3) The water temperature in a particular outdoor swimming pool changes over time. The function $y = C(t)$ models the temperature in degrees Celsius of the pool t hours after midnight on Sunday morning.

a) College Algebra students were asked to do the following on a test:

Interpret the y-intercept of the function in the context of this situation.

Which of the answers below should earn full credit?

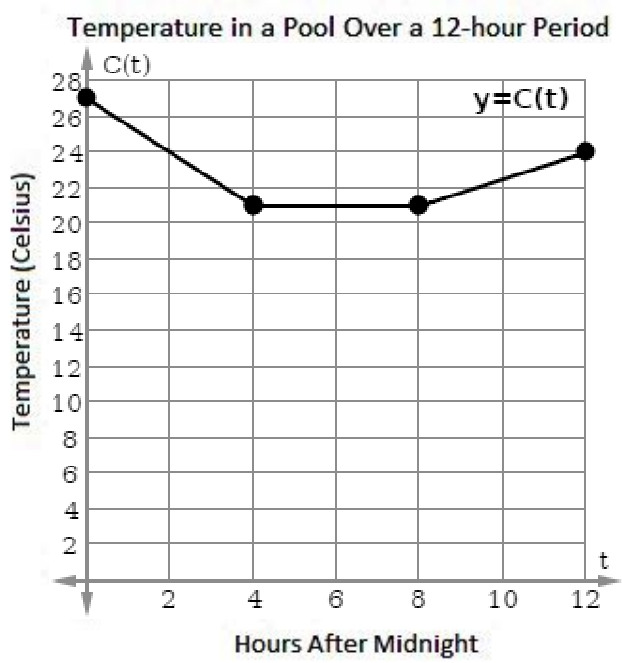
A. The y-intercept is 27.

B. At hour zero, the temperature was 27° Celsius.

C. The y-intercept is $(0,27)$.

D. The temperature was 27° Celsius at midnight.

E. There is no y-intercept since the temperature never hits 0° Celsius.

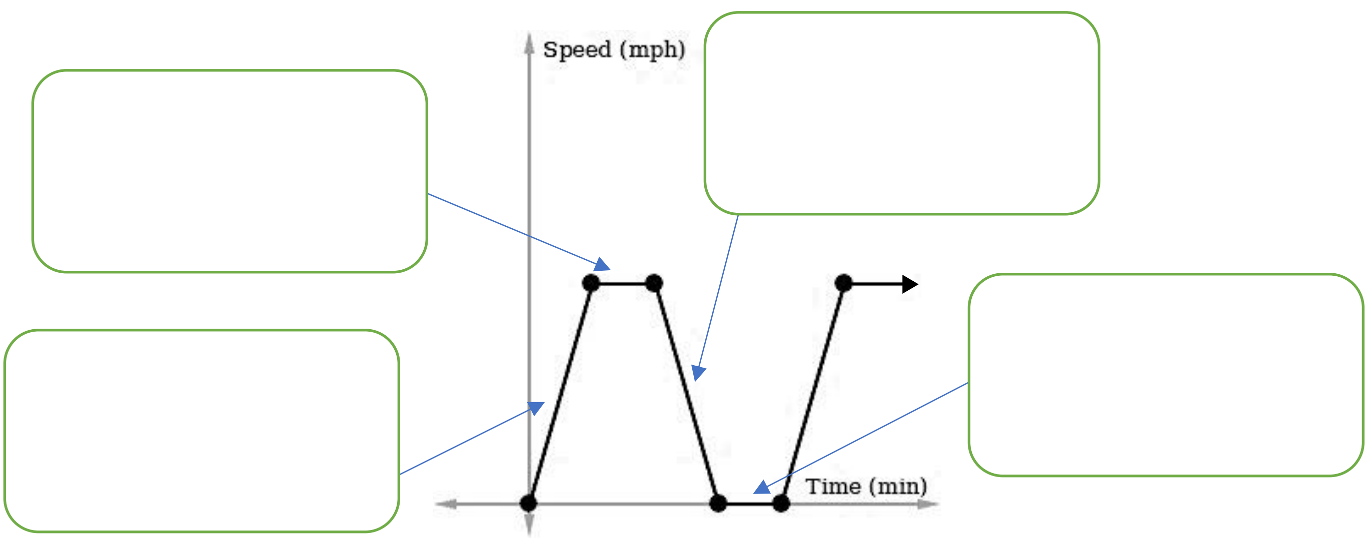


b) Determine over what time interval is the temperature in the pool getting warmer.

c) Describe what is happening with the pool's temperature between 4am and 8am.

4) This graph models the speed of a school bus traveling along a portion of its afternoon route.

Write a narrative that fits the graph. Record each part of your story in the box for the corresponding segment of the graph.



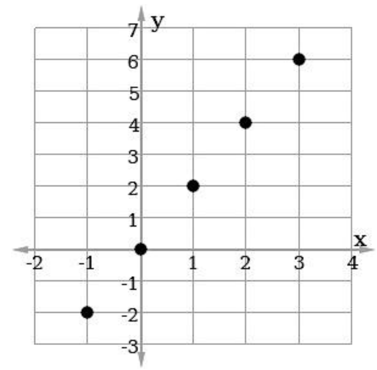
2.2: Features of a Graph

Learning Objectives
Together with your team, given the graph of a function: • State the domain and range of the function. • Determine the intervals of the domain where the function is increasing, decreasing, or constant. • Determine the intervals of the domain where the function is positive or negative. • Determine any extreme values, absolute and local, of the function.

1) For each function graphed here, state its domain and range.

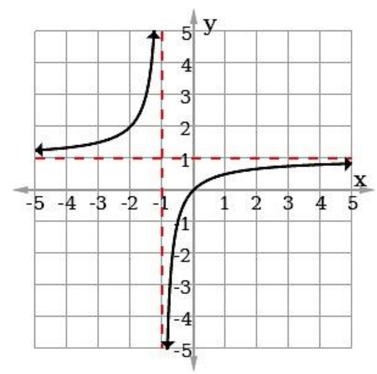
a) Domain:

Range:



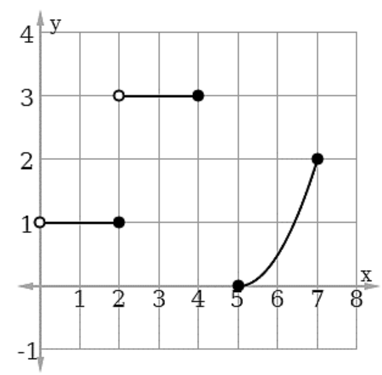
b) Domain:

Range:



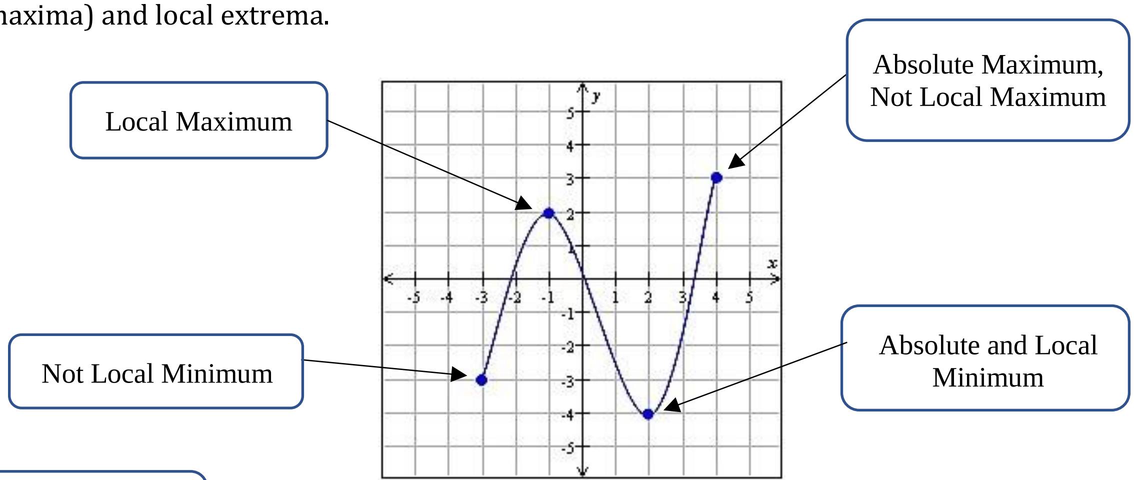
c) Domain:

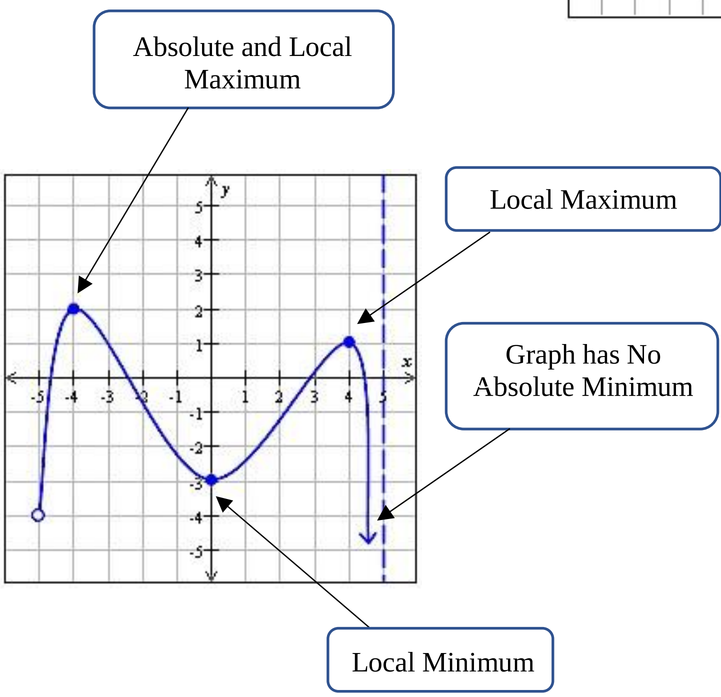
Range:



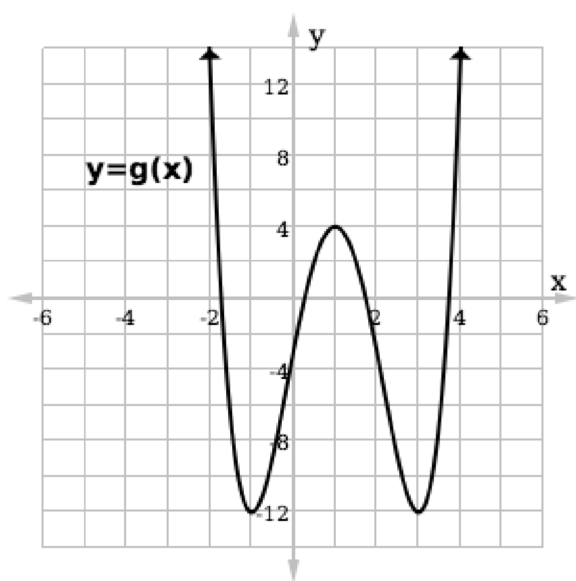
2) With your team, explain how to determine the domain and range of a function from its graph.

3) Together with your team, use the graphs below to determine the difference between absolute extrema (minima and maxima) and local extrema.





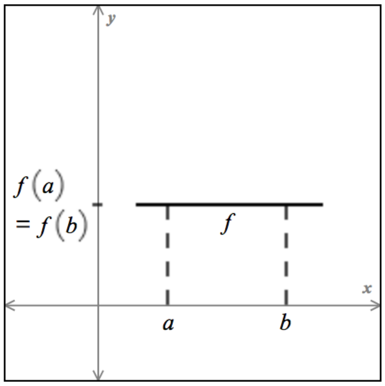
4) Use the graph of $g(x)$ to determine the following features of the graph.



a) Estimate all the values at which $g(x)$ has a local minimum.

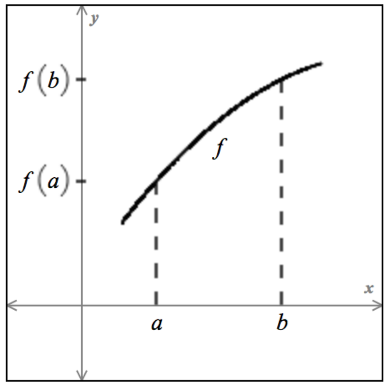
b) Estimate all the local minimum values of $g(x)$.

5) Let f be a continuous function. For each graph shown below, insert the appropriate symbol (>, <, or =) in the blanks. Then, *in your own words* (using a single word or short phrase) describe the function on the interval (a, b).



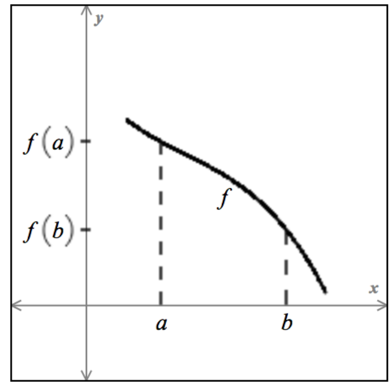
When $b > a$, $f(b)$_____ $f(a)$

Word/Phrase:



When $b > a$, $f(b)$_____ $f(a)$

Word/Phrase:



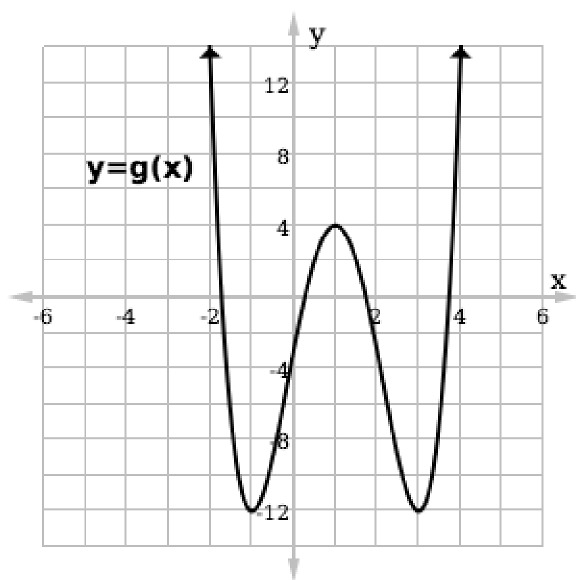
When $b > a$, $f(b)$_____ $f(a)$

Word/Phrase:

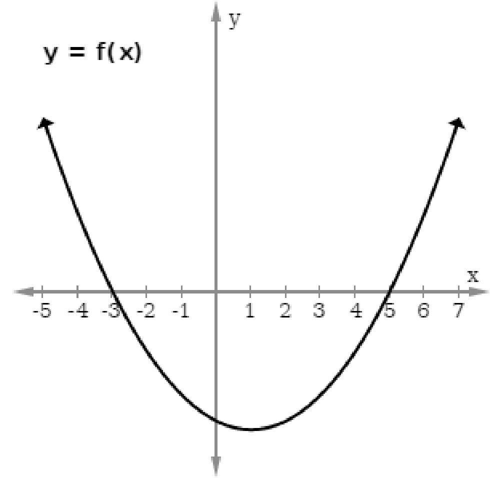
6) Use the graph of $g(x)$ to determine the following features of the graph.

a) Determine intervals of the domain where $g(x)$ is increasing.

b) Determine intervals of the domain where $g(x)$ is decreasing.



7) Use the graph of $f(x)$ to answer the following questions.



a) Is the output of $f(x)$ positive or negative at $x = 0$? In other words, is $f(0)$ positive or negative?

b) Is $f(7)$ positive or negative?

c) Is $f(-2)$ positive or negative?

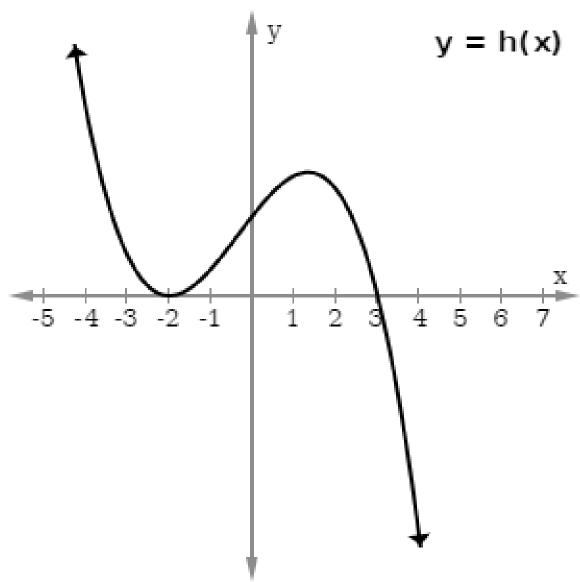
d) Is $f(-9)$ positive or negative?

e) Write a sentence stating what it means to say "$f(x) \geq 0$", in terms of the graph.

f) Using the function f graphed above, determine the intervals of the domain where $f(x) \geq 0$.

g) Explain how you used the graph of $f(x)$ to determine your answer to the previous question.

8) Use the graph of $h(x)$ to answer the following questions.



a) Determine the intervals of the domain where $h(x) \geq 0$.

b) Determine the intervals of the domain where $h(x) > 0$.

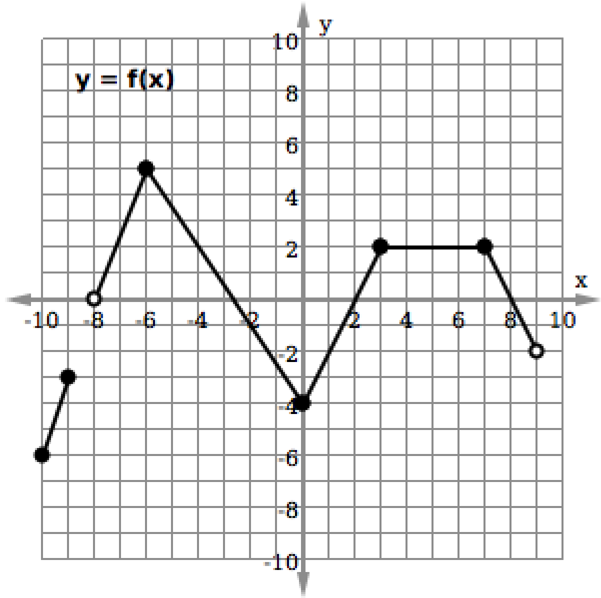
c) Summarize what you learned in a) and b).

d) Determine the intervals of the domain where $h(x) \leq 0$.

Reference Guide: Features of a Function from a Graph

<table>
<tr><th>Feature of
$y = f(x)$</th><th colspan="6">How to Determine from a Graph of $y = f(x)$</th></tr>
<tr><td rowspan="2">Domain & Range</td><td colspan="3">Domain</td><td colspan="3">Range</td></tr>
<tr><td colspan="3">Find the set of all possible x-coordinates of points on the graph (look from left to right)</td><td colspan="3">Find the set of all possible y-coordinates of points on the graph (look from bottom to top)</td></tr>
<tr><td rowspan="2">Zeros &
Positive/Negative</td><td colspan="2">Zeros</td><td colspan="2">Positive</td><td colspan="2">Negative</td></tr>
<tr><td colspan="2">Find the x-values where the graph crosses or touches the x-axis ($f(x) = 0$)</td><td colspan="2">Find the x-values where the graph is above the x-axis ($f(x) > 0$)</td><td colspan="2">Find the x-values where the graph is below the x-axis ($f(x) < 0$)</td></tr>
<tr><td rowspan="2">Intervals where f is Increasing/Decreasing/ Constant</td><td colspan="2">Increasing</td><td colspan="2">Decreasing</td><td colspan="2">Constant</td></tr>
<tr><td colspan="2">Find the x-values where, as you move left to right, the graph rises</td><td colspan="2">Find the x-values where, as you move left to right, the graph falls</td><td colspan="2">Find the x-values where, as you move left to right, the graph is flat</td></tr>
<tr><td rowspan="4">Local
Extreme Values</td><td colspan="3">Values at which f has an
Local Minimum</td><td colspan="3">Values at which f has an
Local Maximum</td></tr>
<tr><td colspan="3">Find the x-value(s) where f changes from decreasing to increasing
(as you move left to right)</td><td colspan="3">Find the x-value(s) where f changes from increasing to decreasing
(as you move left to right)</td></tr>
<tr><td colspan="3">A Local Minimum Value of f</td><td colspan="3">A Local Maximum Value of f</td></tr>
<tr><td colspan="3">Find the y-values where f changes from decreasing to increasing
(as you move left to right)</td><td colspan="3">Find the y-values where f changes from increasing to decreasing
(as you move left to right)</td></tr>
<tr><td rowspan="4">Absolute
Extreme Values</td><td colspan="3">A Value at which f has an
Absolute Minimum</td><td colspan="3">A Value at which f has an
Absolute Maximum</td></tr>
<tr><td colspan="3">Find the x-coordinate of the lowest point on the graph of f (if any)</td><td colspan="3">Find the x-coordinate of the highest point on the graph of f (if any)</td></tr>
<tr><td colspan="3">An Absolute Minimum Value of f</td><td colspan="3">An Absolute Maximum Value of f</td></tr>
<tr><td colspan="3">Find the y-coordinate of the lowest point, if any, on the graph of f</td><td colspan="3">Find the y-coordinate of the highest point, if any, on the graph of f</td></tr>
</table>

Reference Guide: Features of a Function from a Graph Example

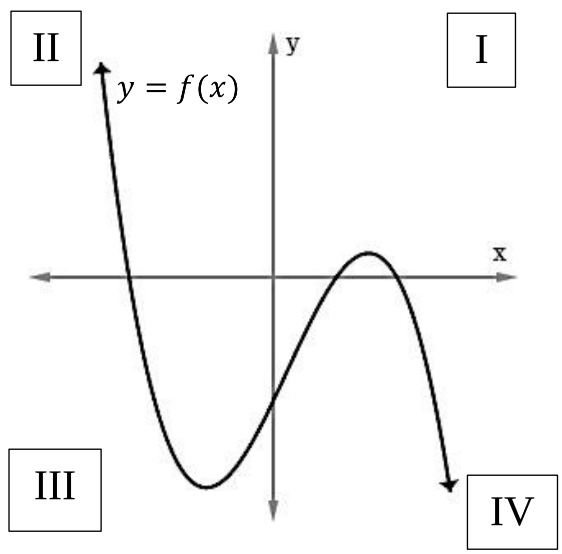


Feature of $y = f(x)$	**Feature on Graph of $y = f(x)$**
Domain	$[-10,-9] \cup (-8,9)$
Range	$[-6,5]$
y-intercept	$(0,-4)$
Zeros of the function	$-\frac{8}{3}, 2, 8$
Intervals of the domain where the function values are positive	$\left(-8,-\frac{8}{3}\right) \cup (2,8)$
Intervals of the domain where the function values are negative	$[-10,-9] \cup \left(-\frac{8}{3},2\right) \cup (8,9)$
Intervals of the domain where the function values are increasing	$(-10,-9) \cup (-8,-6) \cup (0,3)$
Intervals of the domain where the function values are decreasing	$(-6,0) \cup (7,9)$
Intervals of the domain where the function values are constant	$(3,7)$
Local maximum	The function has a local max **of** $y = 5$ **at** $x = -6$
Local minimum	The function has a local min **of** $y = -4$ **at** $x = 0$
Absolute maximum	The function has an absolute max **of** $y = 5$ **at** $x = -6$
Absolute minimum	The function has an absolute min **of** $y = 6$ **at** $x = -10$

2.3: Warm Up

Student Name:

1) Use the graph of f to answer the following questions.



a) Which area on the graph represents, "as $x \to \infty$, $f(x) \to -\infty$"?

I II III IV

b) Translate the mathematical statement given in part a) into an English sentence.

c) Use the graph of f to complete the mathematical statement: As $x \to -\infty$, $f(x) \to$ ______

d) Which area on the graph of f represents the statement you wrote in c)?

I II III IV

2.3: End Behavior and Zeros of Polynomial Functions

Learning Objectives
Together with your team, given the graph of a polynomial function, • Determine its end behavior and represent it using arrow notation. • Determine the x-intercepts (zeros, roots) and their respective multiplicities.

1) Use the graph to answer the following questions.

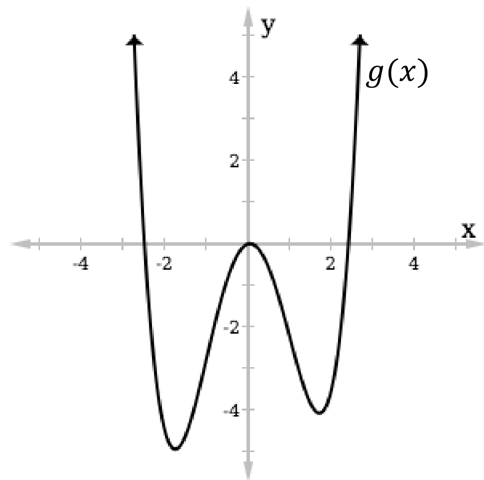
Which of the following correctly describes the end behavior of the graph of $g(x)$?

A. As $x \to -\infty$, $g(x) \to -\infty$ and as $x \to \infty$, $g(x) \to -\infty$

B. As $x \to -\infty$, $g(x) \to -\infty$ and as $x \to \infty$, $g(x) \to \infty$

C. As $x \to -\infty$, $g(x) \to \infty$ and as $x \to \infty$, $g(x) \to -\infty$

D. As $x \to -\infty$, $g(x) \to \infty$ and as $x \to \infty$, $g(x) \to \infty$



2) The remainder of this lesson involves completing a Desmos Activity that will help you explore the different possible end behavior that can occur with polynomial functions.

Go to https://student.desmos.com/ and enter the Class Code: ______________________________

a) **Screen 2:** Record *one* example from each of your two groups of functions. Sketch "good enough" graphs below.

Group 1 Group 2

b) **Screen 3:** Give your reasoning for grouping the functions the way you did. Write complete sentences.

c) **Screen 4:** <u>Record *one* example from each of your four groups of functions</u>. Sketch "good enough" graphs below.

Group 1

Group 2

Group 3

Group 4

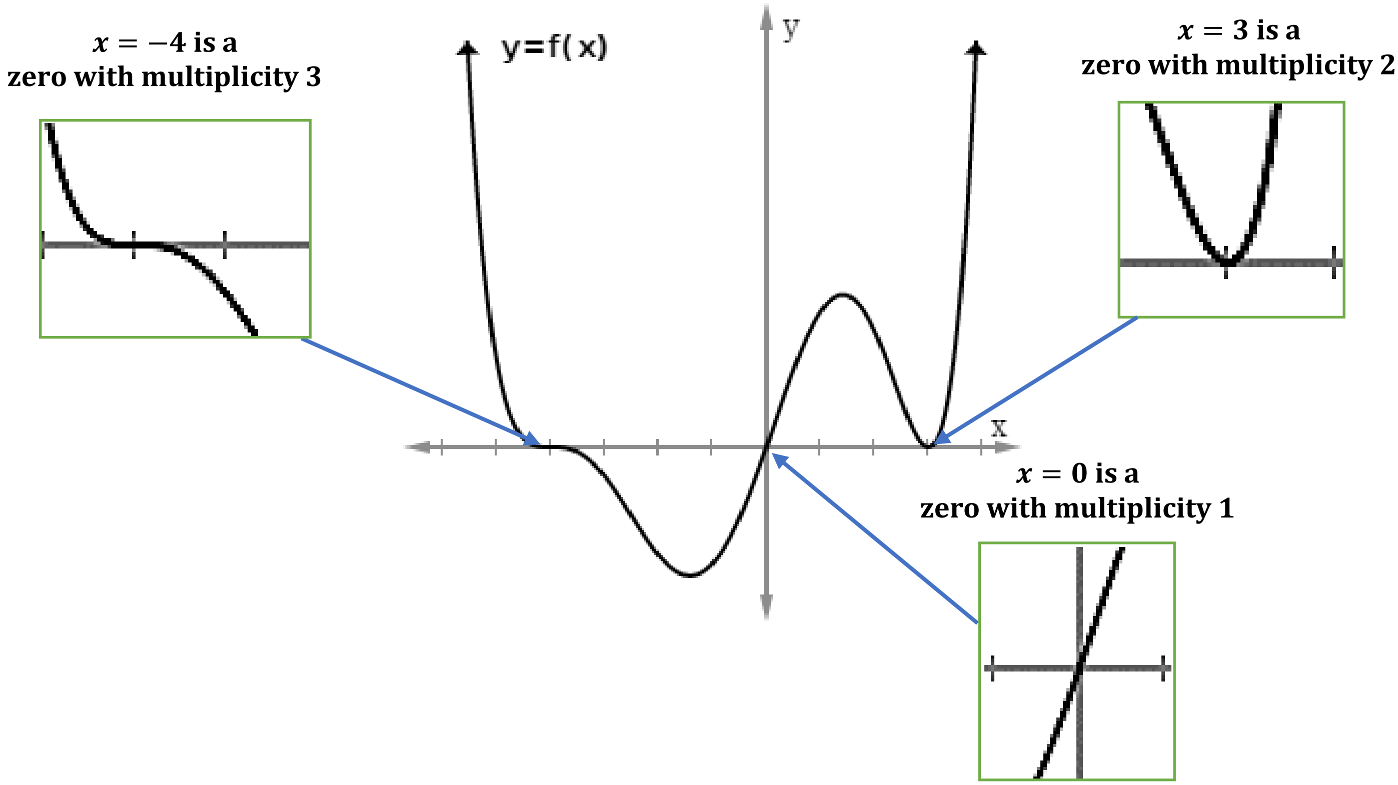
d) **Screen 5:** Give your reasoning for grouping the functions the way you did. Write complete sentences.

<u>Summary</u>: End Behavior of a Polynomial Graph

As $x \to -\infty$, $f(x) \to -\infty$ and, as $x \to \infty$, $f(x) \to -\infty$	As $x \to -\infty$, $f(x) \to \infty$ and, as $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$ and, as $x \to \infty$, $f(x) \to -\infty$	As $x \to -\infty$, $f(x) \to -\infty$ and, as $x \to \infty$, $f(x) \to \infty$

Next, let's consider the "local behavior" of the graph of a polynomial function, which includes the zeros of the function.

3) The graph of the polynomial function $f(x) = x(x-3)^2(x+4)^3$ is given here. The function has three types of ***zeros***.



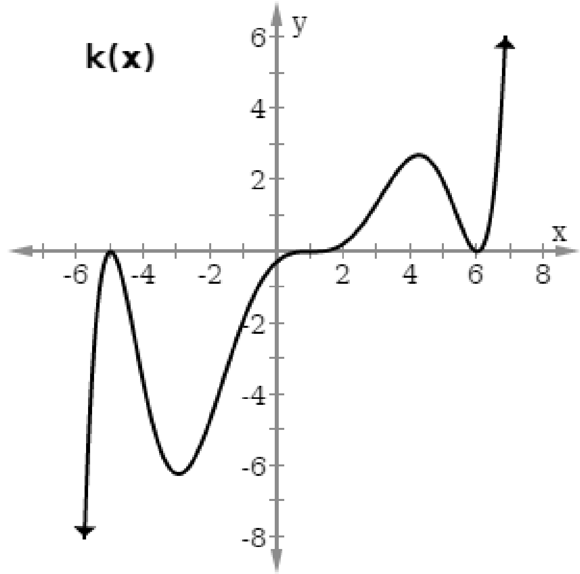
Describe the differences you see in the shape of the graph at the three zeros on the graph of f.

4) In the table below, for each type of zero, write a word or short phrase **YOU** would use to describe/remember (say, for an exam) the different types of multiplicities.

Multiplicity of Zeros		
A Zero with Multiplicity 1	A Zero with Even Multiplicity	A Zero with Odd Multiplicity > 1

5) The polynomial function, $k(x)$, is graphed here. List all the real zeros of the function, then circle whether the multiplicity of each zero is 1, even, or odd > 1. Finally, state the least possible multiplicity of each zero.

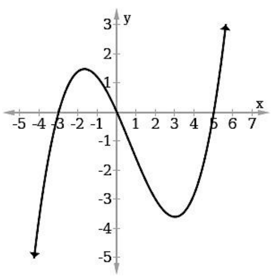
Zero	Multiplicity (circle)	Least Possible Multiplicity
	1 Even Odd > 1	
	1 Even Odd > 1	
	1 Even Odd> 1	



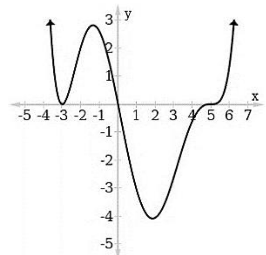
6) Determine which graph(s) of the polynomial functions below have **<u>ALL</u>** of the following properties:

- A zero at $x = -3$ with multiplicity 2, **AND**
- A zero at $x = 0$ with multiplicity 1, **AND**
- A zero at $x = 5$ with multiplicity 3.

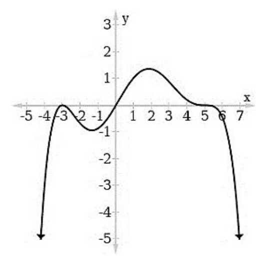
A.



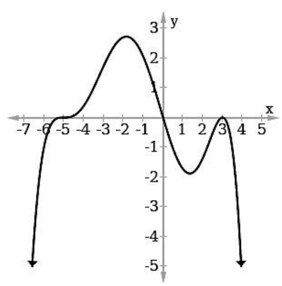
B.



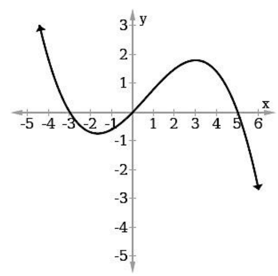
C.



D.



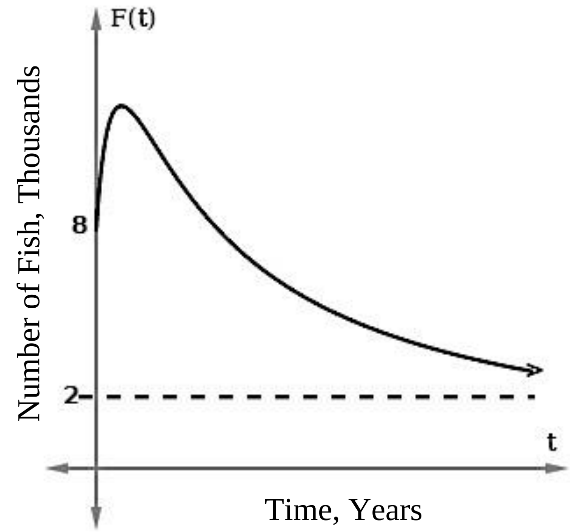
E.



2.4: Warm Up

Student Name:

1) A population of fish was introduced into Beaver Lake. The number of fish (in thousands) t years after introduction can be modelled by the function $F(t)$, graphed here.

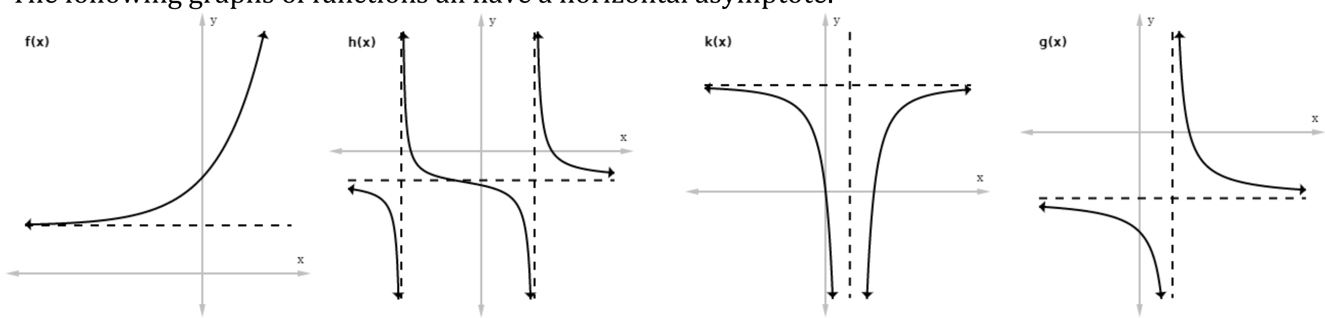


a) What happens to this population of fish in the long run?

b) How did you use the graph to determine your answer for part a)?

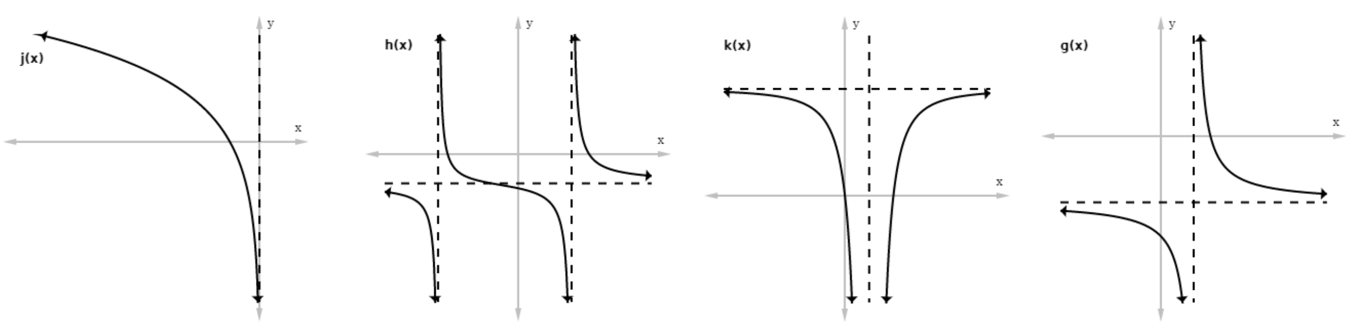
Recall, in Chapter 1, we explored asymptotic behavior of the parent functions.

2) The following graphs of functions all have a horizontal asymptote.



Write what you think it means for a function to have a horizontal asymptote.

3) The following graphs of functions all have vertical asymptotes.



Write what you think it means for a function to have a vertical asymptote.

2.4: Vertical Asymptotes & Horizontal Asymptotes

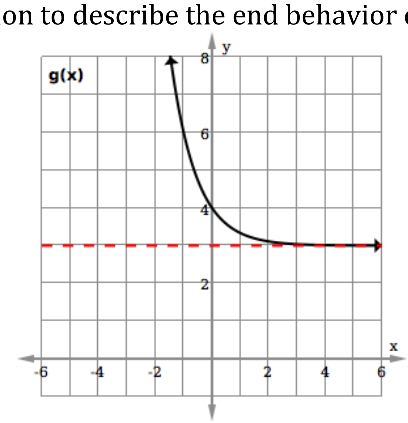
Learning Objectives
Together with your team, given the graph of a function: • Give the equation of all vertical and horizontal asymptotes. • Use the graph to determine the end behavior of a function.

Summary: Horizontal Asymptotes and End Behavior

Horizontal Asymptotes	Vertical Asymptotes

1) Write the equations of all the asymptotes (if any) for the following functions, and then complete the arrow notation to describe the end behavior of the graph.

a)

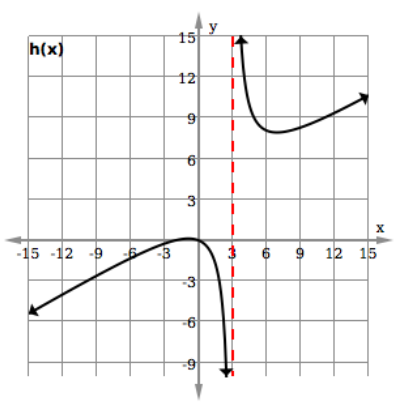


As $x \to -\infty$, $g(x) \to$______ and, as $x \to \infty$, $g(x) \to$______

Vertical Asymptote: __________

Horizontal Asymptote: __________

b)

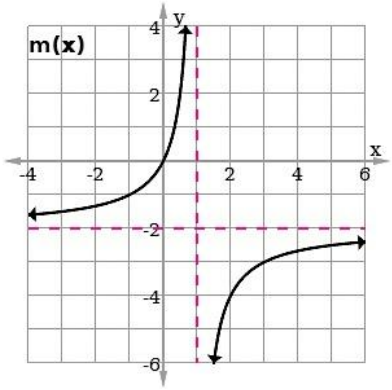


As $x \to -\infty$, $h(x) \to$______ and, as $x \to \infty$, $h(x) \to$______

Vertical Asymptote: __________

Horizontal Asymptote: __________

c)

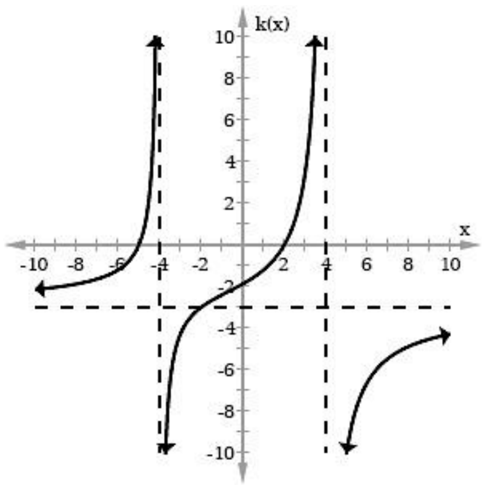


As $x \to -\infty, m(x) \to$_______ and as $x \to \infty, m(x) \to$______

Vertical Asymptote: _______________

Horizontal Asymptote: _______________

d)



As $x \to -\infty, k(x) \to$_______ and, as $x \to \infty, k(x) \to$_____

Vertical Asymptotes: _______________

Horizontal Asymptote: _______________

2) Sketch a graph of a function f with the following end behavior.

As $x \to -\infty, f(x) \to 0$

As $x \to \infty, f(x) \to \infty$

Chapter Learning Objectives

1. Recognize, symbolically and graphically, vertical and horizontal translations and reflections, vertical scaling, and combinations of those transformations.
2. Given a symbolic representation of a function (i.e. an equation), apply transformations to sketch a "good enough" graph of the function.
3. Analyze how the graphical and symbolic representations of a function and its transformations are related.
4. Use transformations to analyze the symmetry of a function from either a graphical or a symbolic representation.
5. Use the graphical and/or symbolic representations of a transformed function to model real-world situations.
6. Use the equation of a transformed parent function to determine features of its graph, including: its domain and range, its end behavior, the minimum or maximum value on the graph, the location of any asymptotes—and interpret these features in the context of a given situation.
7. Determine whether a function has even or odd symmetry.

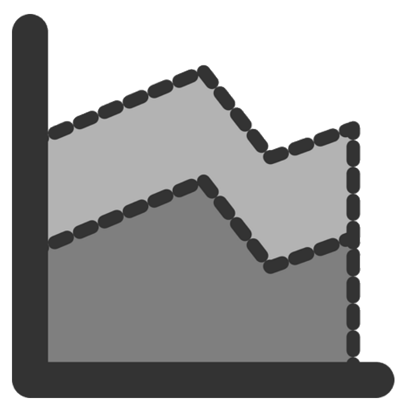
Chapter 3
How Do We Transform a Given Function?

Chapter Overview

By applying transformations, we can create new (but similar) functions from old ones. In this chapter, we will study the following six transformations:

- vertical and horizontal translations (shifts)
- reflections (flips) about the x- and y-axes
- vertical scaling (stretches and shrinks)

We will use these transformations to connect the symbolic forms with the graphical forms for each of our function families.



Chapter 3 Contents

3.1: Warm Up

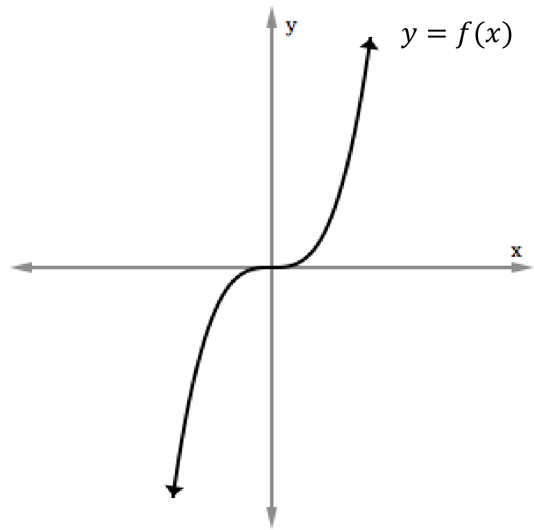
Student Name:

1) Let $y = f(x)$ be a function, and c and d be positive real numbers.

a) Fill in the blanks (Hint: to check your thinking, try using Desmos to look at a few example graphs.)

- The function $j(x) = f(x) + c$ shifts the graph of $f(x)$ ____________________ c units.
- The function $h(x) = f(x) - d$ shifts the graph of $f(x)$ ____________________ d units.

b) Sketch two example graphs here. Also, include their equations.

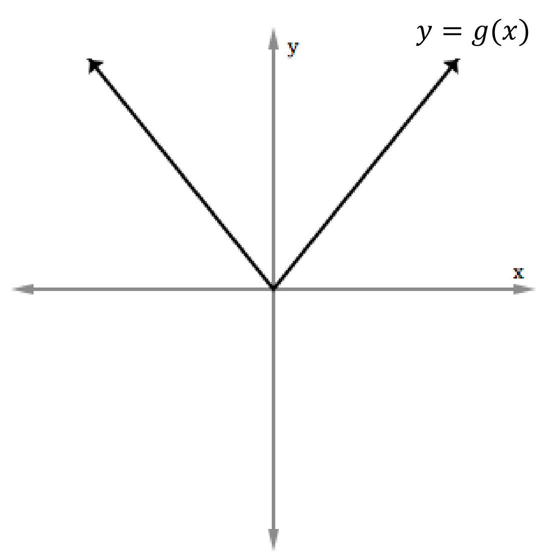


2) Let $y = g(x)$ be a function, and c and d be positive real numbers.

a) Fill in the blanks:

- The function $k(x) = g(x - c)$ shifts the graph of $g(x)$ ____________________ c units.
- The function $m(x) = g(x + d)$ shifts the graph of $g(x)$ ____________________ d units.

b) Sketch two example graphs here. Also, include their equations.



3.1: Introduction to Transformations

Learning Objectives
Together with your team: • Recognize, symbolically and graphically, vertical and horizontal translations and reflections, vertical scaling, and combinations of those transformations. • Given a symbolic representation of a function (i.e. an equation), apply transformations to sketch a "good enough" graph of the function. • Analyze how the graphical and symbolic representations of a function and its transformation are related.

This lesson involves completing a Desmos Activity that will help you explore some different types of function transformations.

Go to https://student.desmos.com/ and enter the Class Code: ______________________________

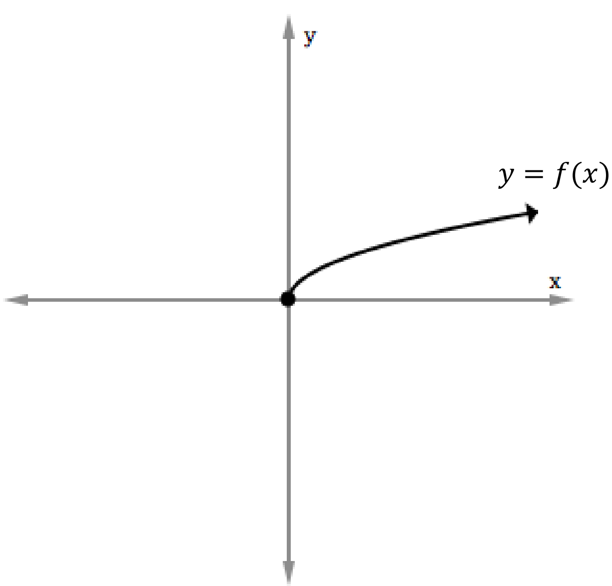
As you complete the Desmos Activity, take notes of your observations. For each observation, record an example (NOTE: an example should include both graphs and equations of functions that illustrate each type of transformation.)

1) Desmos Activity Screens 3 and 4: Let $y = f(x)$ be a function.

- The function $g(x) = -f(x)$ reflects the graph of $f(x)$ over the ____________________.

- The function $h(x) = f(-x)$ reflects the graph of $f(x)$ over the ____________________.

Sketch two example graphs here. Also, include their equations:

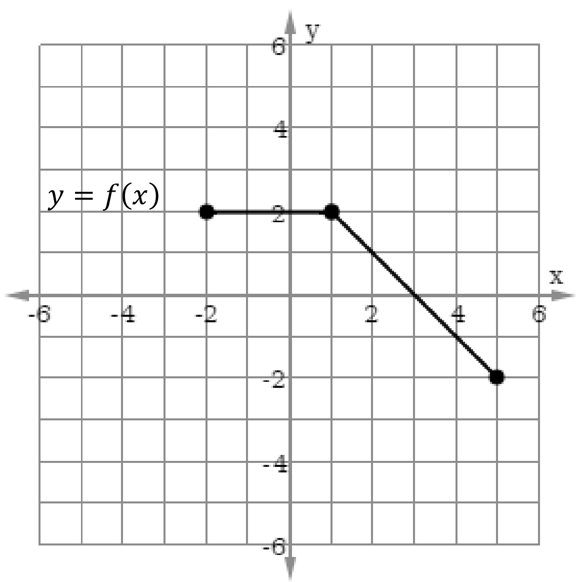


2) Desmos Activity Screens 6 and 7: Let $y = f(x)$ be a function.

- If $a > 1$, the function $g(x) = af(x)$ is a ________________________________, by a factor of a, of the graph of $f(x)$.

- If $0 < b < 1$, the function $h(x) = bf(x)$ is a ________________________________, by a factor of b, of the graph of $f(x)$.

- Choose specific values for a and b and record the values in the table below using your chosen values.



x	$f(x)$	$af(x)$ $a =$ _____	$bf(x)$ $b =$ _____
-2			
0			
1			
3			
5			

- Sketch graphs of $y = af(x)$ and $y = bf(x)$ on the same axes as $y = f(x)$ and <u>label each with its equation.</u>

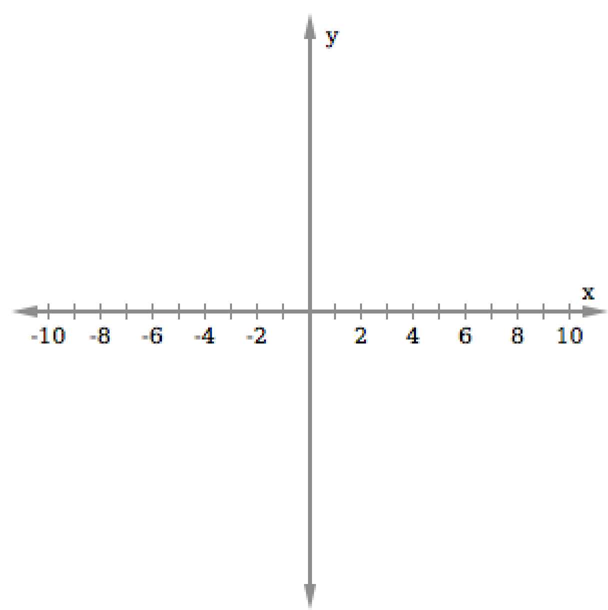
Apply what you learned in the Desmos Activity to complete the remaining graphing problems in this lesson.

3) Let $g(x) = \log_2(x + 5)$.

a) Begin by sketching a graph of the parent function $f(x) = \log_2(x)$.

b) Label the coordinates of one point on the graph of $f(x) = \log_2(x)$ and the equation of the asymptote.

c) On the same set of axes, sketch a graph of g. Label the coordinates of one point on the graph of g and the equation of the asymptote.

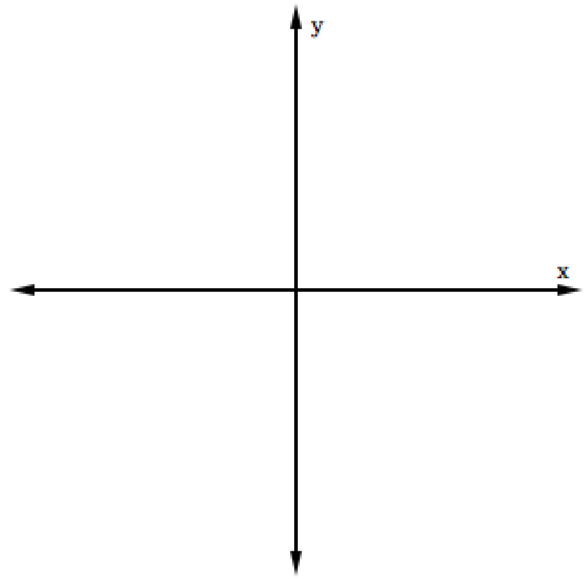


4) Let $f(x) = \sqrt{x}$.

a) Fill in the blank with the correct function, $h(x) = f(x+1) + 2 =$________________________.

b) List the transformations you need to apply to $f(x)$ to obtain the graph of $h(x)$.

c) Sketch a "good enough" graph of $h(x)$:

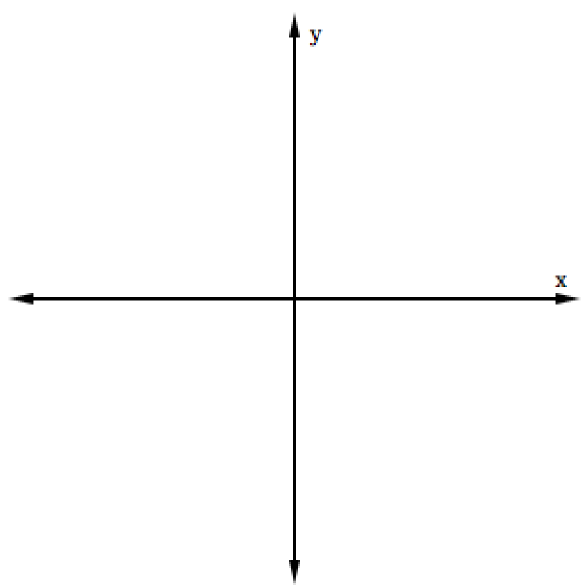


5) Let $k(x) = 2^x$.

a) Fill in the blank with the correct function, $g(x) = k(-x) - 3 =$________________________.

b) List the transformations you need to apply to $k(x)$ to obtain the graph of $g(x)$.

c) Sketch a "good enough" graph of $g(x)$.

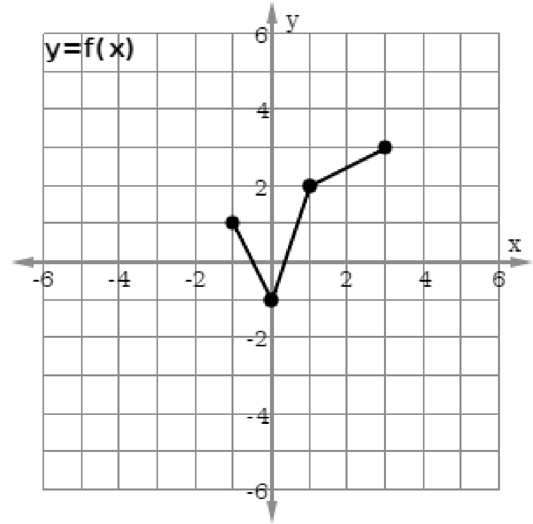


6) The function $y = f(x)$ is shown here. Let $k(x) = f(-x)$.

a) Sketch the graph of $k(x) = f(-x)$ on the same set of axes.

b) Fill in the table with points on the graph of k.

f	k
$(-1, 1)$	
$(0, -1)$	
$(1, 2)$	
$(3, 3)$	

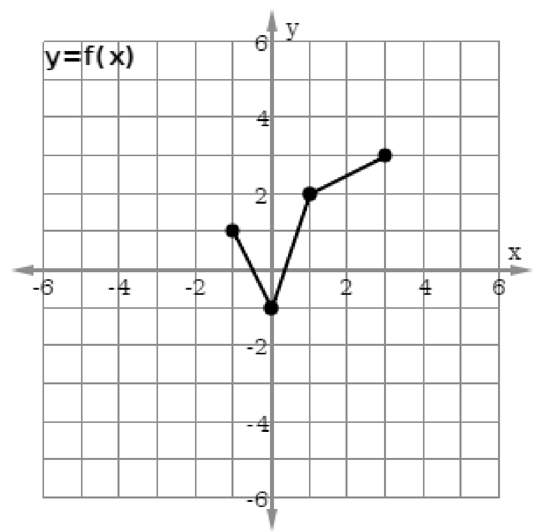


7) The function $y = f(x)$ is shown here. Let $j(x) = f(x - 3) - 4$

a) Sketch the graph of $j(x) = f(x - 3) - 4$ on the same set of axes.

b) Fill in the table with points on the graph of j.

f	j
$(-1, 1)$	
$(0, -1)$	
$(1, 2)$	
$(3, 3)$	

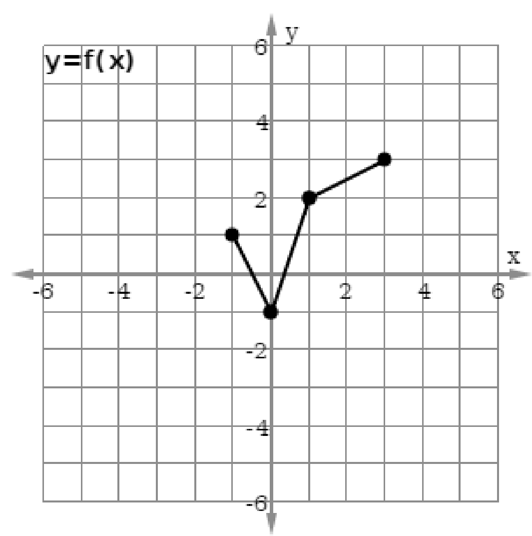


8) The function $y = f(x)$ is shown here. Let $m(x) = -2f(x)$

a) Sketch the graph of $m(x) = -2f(x)$ on the same set of axes.

b) Fill in the table with points on the graph of m.

f	m
$(-1, 1)$	
$(0, -1)$	
$(1, 2)$	
$(3, 3)$	



9) Use what you know about transformations to answer the following questions.

a) Which of the following transformations *preserve* the x-intercept(s) of the graph of the original function f (i.e. do not move them)?

Select all that apply.

A. Reflection over the x-axis

B. Horizontal Shift

C. Vertical Scaling

D. Vertical Shift

E. Reflection over the y-axis

b) Explain your reasoning for the transformations you selected in a).

10) Use what you know about transformations to answer the following questions.

a) Which of the following transformations *preserve* the y-intercept of the graph of the original function f (i.e. do not move it)? Select all that apply.

A. $y = f(x - c)$

B. $y = f(x) + d$

C. $y = f(-x)$

D. $y = -f(x)$

E. $y = af(x)$

Explain your reasoning for the transformations you selected in a).

Use what you know about transformations to answer questions **11) – 14)**. You may want to sketch "good enough" graphs.

11) Which of the following functions have the same end behavior as $m(x) = x^2$? Select all that apply.

A. $f(x) = -\frac{1}{2}x^2$

B. $g(x) = 9x^6$

C. $h(x) = -3x^4$

D. $i(x) = 2x^5$

E. $j(x) = \frac{5}{3}x^8$

F. $k(x) = -4x^7$

12) Which of the following functions have the same end behavior as $n(x) = -x^2$? Select all that apply.

A. $f(x) = -\frac{1}{2}x^2$

B. $g(x) = 9x^6$

C. $h(x) = -3x^4$

D. $i(x) = 2x^5$

E. $j(x) = \frac{5}{3}x^8$

F. $k(x) = -4x^7$

13) Which of the following functions have the same end behavior as $p(x) = x^3$? Select all that apply.

A. $f(x) = -\frac{1}{2}x^2$

B. $g(x) = 9x^6$

C. $h(x) = -3x^4$

D. $i(x) = 2x^5$

E. $j(x) = \frac{5}{3}x^8$

F. $k(x) = -4x^7$

14) Which of the following functions have the same end behavior as $q(x) = -x^3$? Select all that apply.

A. $f(x) = -\frac{1}{2}x^2$

B. $g(x) = 9x^6$

C. $h(x) = -3x^4$

D. $i(x) = 2x^5$

E. $j(x) = \frac{5}{3}x^8$

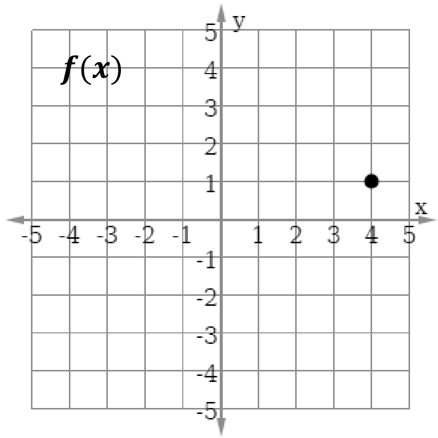
F. $k(x) = -4x^7$

Summary: End Behavior of $f(x) = ax^n$, where $n = 0, 1, 2, 3, 4, 5, 6, \ldots$			
		Degree, n	
		n Even	n Odd
Coefficient, a	a Positive		
	a Negative		

3.2: Warm Up

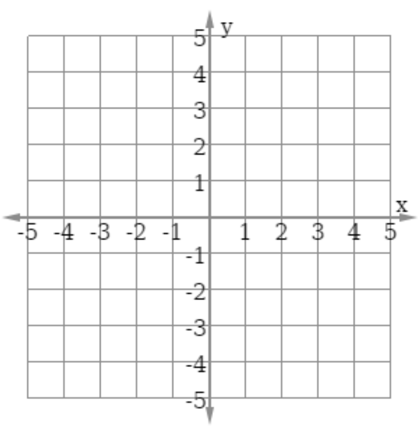
Student Name:

1) Let f be the function defined by the coordinate point $(4,1)$.



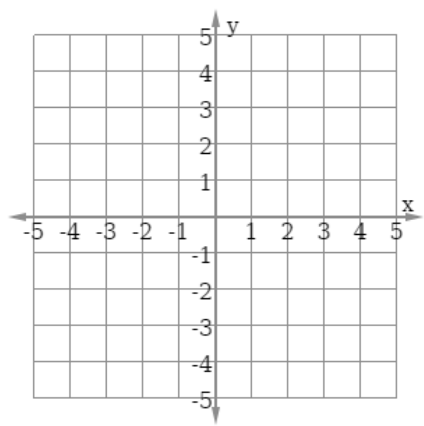
a) Apply the following transformations to f and sketch the resulting new function:

- First, shift up three units.
- Then, reflect over the x-axis.



b) Apply the following transformations to f and sketch the resulting new function:

- First, reflect over the x-axis.
- Then, shift up three units.



c) What do you notice about the two graphs you drew?

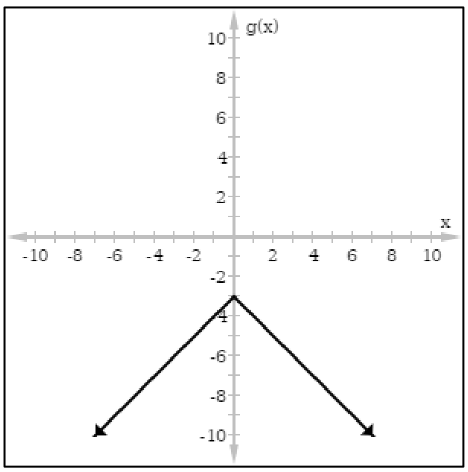
3.2: Order of Transformations & Symmetry

Learning Objectives
Together with your team: • Analyze how the graphical and symbolic representations of a function and its transformation are related. • Use transformations to analyze the symmetry of a function from either a graphical or a symbolic representation.

1) Let $f(x) = |x|$.

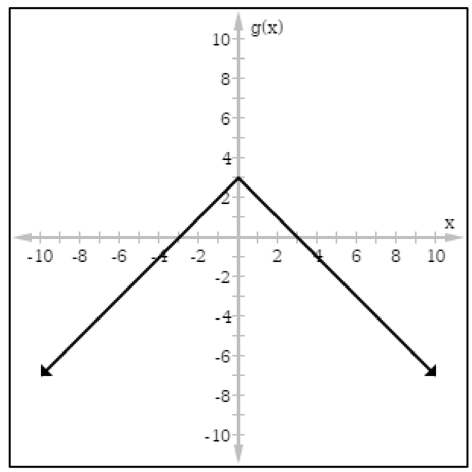
Suppose you are asked on an exam to sketch a graph of $g(x) = -f(x) + 3 = -|x| + 3$, but you have forgotten the correct order to perform the two transformations. Explain how you will decide which graph below is correct.

Shift the graph up three units, then reflect the result over the x-axis?



or…

Reflect the graph over the x-axis, then shift the result up three units?



Summary: Order of Transformations

2) Let $h(x) = 5(3)^{-x} + 1$.

a) List all the transformations applied to $f(x) = 3^x$ to obtain the graph of $h(x)$.

b) Does the order the transformations are applied to obtain $h(x)$ of matter? Explain.

c) What is a *possible* correct order in which to graph the transformations?

3) Let f and g be functions where $g(x) = -2f(x-6) - 6$.

a) The function $g(x)$ involves four transformations. In the table below:
- List the four transformations (in any correct order)
- State whether the transformation affects the input or the output value
- Describe how each point on the graph is changed by the transformation

Transformation	Does it affect the input or output?	Describe the numerical effect on the coordinates of each point.

b) Use the information you recorded in a) to determine the coordinates of the point on the graph of g given that the point $(-1, 7)$ lies on the graph of a function f.

4) Given that the point $(3, -10)$ lies on the graph of a function k, what point must be on the graph of h where $h(x) = k(x-4) + 2$?

5) Circle all the parent functions that have even symmetry (are symmetric about the y-axis) and underline all the parent functions that have odd symmetry (are symmetric about the origin).

A. $f(x) = x$

B. $f(x) = x^2$

C. $f(x) = x^3$

D. $f(x) = \sqrt{x}$

E. $f(x) = \sqrt[3]{x}$

F. $f(x) = \frac{1}{x}$

G. $f(x) = \frac{1}{x^2}$

H. $f(x) = 2^x$

I. $f(x) = \log_{0.5}(x)$

J. $f(x) = |x|$

Summary: Graphical Symmetry

Even Symmetry (symmetric about the y-axis)	Odd Symmetry (symmetric about the origin)

Reference Guide: Transformations

Vertical Transformations affect the OUTPUT

Transformation	Visual Effect	Formula	Numerical effect
Vertical translations (shifts)	Translation up by k	$f(x) + k$	Outputs increased by k
	Translation down by k	$f(x) - k$	Outputs decreased by k
Vertical scaling (stretches and shrinks)	Vertical stretch for $k > 1$	$k \cdot f(x)$	Outputs multiplied by k
	Vertical shrink for $0 < k < 1$	$k \cdot f(x)$	Outputs multiplied by k
Vertical reflection	Flip over the x-axis	$-1 \cdot f(x)$	Outputs multiplied by -1

Horizontal Transformations affect the INPUT

Transformation	Visual Effect	Formula	Numerical effect
Horizontal translations (shifts)	Translation right by k	$f(x - k)$	Inputs *increase* by k
	Translation left by k	$f(x + k)$	Inputs *decrease* by k
Horizontal scaling (stretches and shrinks)	We will not cover this type of transformation in this class!		
Horizontal reflection	Flip over the y-axis	$f(-x)$	Inputs multiplied by -1

Combining Transformations

Combination of vertical transformations of the form $af(x) + k$	Combinations of horizontal and vertical transformations are **independent**.
• First, vertically stretch by a. • Then, vertically shift by k.	• It does not matter whether horizontal or vertical transformations are performed first.

3.3: Warm Up

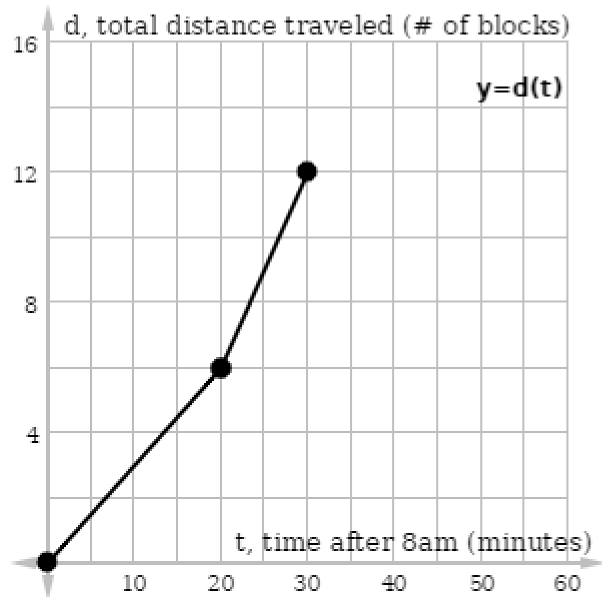
Student Name:

1) Shala walks to school along the same route every morning and the total distance she usually travels is shown in the graph. She usually leaves her house at 8am, however yesterday she left 30 minutes later than normal.

a) Sketch a graph of Shala's total distance traveled to school yesterday.

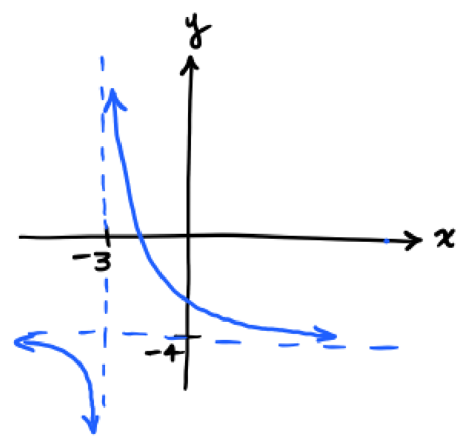
b) Write the symbolic form of the graph you drew as a transformation of d.

Shala's Total Distance



2) Omar was asked to find the domain and range of $f(x) = \frac{2}{x+3} - 4$ so, he sketched a "good enough" graph.

a) Do you think Omar's graph is "good enough" for his task? Why or why not?



b) Help Omar determine the domain and range of $f(x)$.

Domain: ______________________

Range: ______________________

3.3: Applying Transformations

Learning Objectives

Together with your team:
- Use the graphical and/or symbolic representations of a transformed function to model real-world situations.
- Use the equation of a transformed parent function to determine features of its graph, including: its domain and range, its end behavior, the minimum or maximum value on the graph, the location of any asymptotes—and interpret these features in the context of a given situation.

1) Let $k(x) = 3(x-4)^2 + 6$.

a) Determine the domain and range of the function k.

Domain: ______________________________ Range: ______________________________

b) Determine the end behavior of $k(x)$ and finish the arrow notation below.

As $x \to -\infty$, $k(x) \to$________ and as $x \to \infty$, $k(x) \to$________.

c) What is the minimum value of the function $k(x)$? Where does the minimum value occur?

Min. value of $k(x) =$________ Min. value occurs at $x =$________

d) Did you sketch a "good enough" graph to answer the question in a) – c) above? If so, explain how you used it to answer the questions. If not, explain how a graph could be useful for checking your answers to the problems in a) – c).

2) Determine the domain and range of each of the functions given in a) and b) below. (Hint: think about the graph of the given function compared to the graph of its parent function.)

a) $j(x) = \sqrt{x-5}$

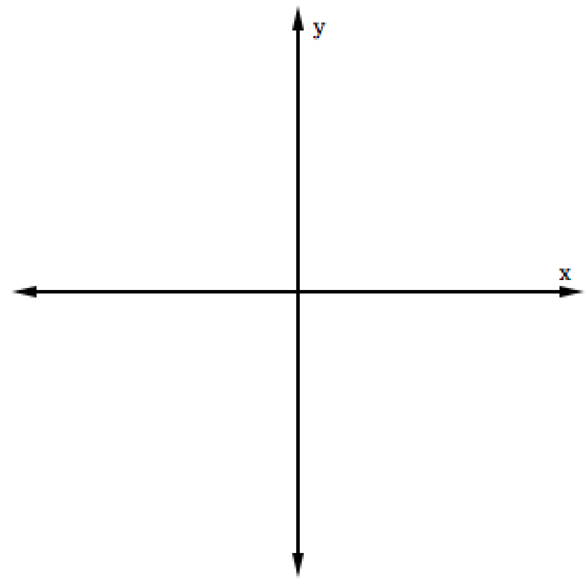
Domain: ________________________ Range: ________________________

b) $k(x) = \sqrt[3]{x-5}$.

Domain: ________________________ Range: ________________________

3) Let $f(x) = 2\ln(x+5) + 1$.

Determine the domain and range of f.



Domain: ______________________________

Range: ________________________________

4) Consider the function $h(x) = \frac{1}{x+3} + 5$.

a) Determine the end behavior of h and finish the arrow notation below.

As $x \to -\infty,\ h(x) \to$________ and as $x \to \infty,\ h(x) \to$_______.

b) Determine the domain and range of the function h.

Domain: ______________________________ Range: ________________________________

5) Fiona was walking at a constant rate. The function $D(t) = -3|t-2| + 6$ gives her distance, in miles, from home after t hours.

a) Sketch a graph of $D(t)$.

b) What is the farthest distance that Fiona gets from home?

c) When is Fiona farthest from home?

d) Determine the time interval(s) when Fiona is heading away from home.

e) Interpret the maximum value of the absolute value graph in the context of this situation. Write your answer in a complete sentence.

Chapter 4
What Can We Learn from an Equation?

Chapter Learning Objectives

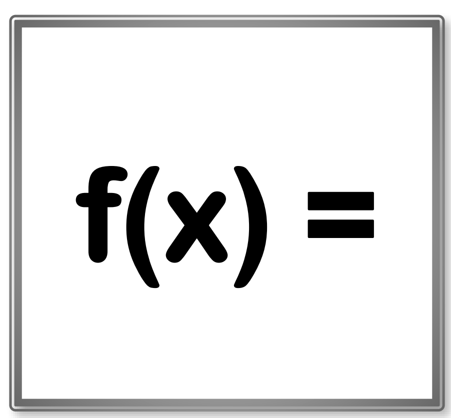
1. Use a given equation of a function to determine its:
   - domain,
   - x-intercepts (zeros), and
   - y-intercept.
2. Given an equation of a function, use it to calculate the average rate of change of the function over a specified interval.
3. Interpret the average rate of change of a function in context.
4. Determine the end behavior of a function (numerically for any function, and for polynomial functions, using the leading term).
5. Use the graph or equation of the best-fit trend line to model a real-world data set and use it to predict values not in the data set.
6. Interpret function values in the context of a given situation.

Chapter Overview

In Chapter 2, we saw how to analyze a function by interpreting various features of its graph. Here in Chapter 4, we will turn our attention to the symbolic representation (equation) of a function, using equations to determine:

- x-and y-intercepts
- domain and range
- intervals of the domain where the function values are positive and negative
- end behavior (numerically)
- its average rate of change over an interval

For polynomial functions, we will also look for connections between the standard and factored form of polynomial equations and their zeros and multiplicities, and their end behavior.



Chapter 4 Contents

4.1: Warm Up

Student Name:

1) Consider the following definition.

> The **zeros of a function** are the inputs that have an output of 0.

What are some other terms you have heard that have the same or similar meaning as "zero?"

2) Let $h(x) = \frac{-3x+5}{8}$.

a) Write an equation that would help you solve $h(x) = 0$.

b) What are the x-intercept(s) of $h(x)$?

c) If you want to find the value of $h(0)$, how is that process different from the one you did in part b) above?

d) Determine the value of the function h when $x = 0$. Is the function positive or negative?

e) How can you find the y-intercept of any function, given its equation?

4.1: Features of a Function from an Equation

Learning Objectives
Together with your team, use a given equation of a function to: • Determine its domain, x-intercepts (zeros), y-intercept, and whether it is positive or negative at a given value. • State the domain and range of a radical function given an equation. • Identify inputs and outputs and interpret these in the context of a situation.

1) Use the equation of each function given below to determine its y-intercept, if it exists. If the function has no y-intercept, explain how you used the equation to determine this.

a) $j(x) = 3\left(x - \frac{1}{2}\right)(x + 5)$

b) $w(x) = \sqrt{3x + 4}$

c) $k(x) = \log_5(x - 5)$

2) For the functions $j(x)$ and $w(x)$ given in the problem above, state whether the function is positive or negative at $x = 0$. Circle your answers.

$j(x)$ is positive/negative at $x = 0$.

$w(x)$ is positive/negative at $x = 0$.

3) Use the equation of each function given below to determine its x-intercept(s), if possible. If not possible, explain why.

a) $j(x) = 3\left(x - \frac{1}{2}\right)(x + 5)$

b) $w(x) = \sqrt{3x + 4}$

c) $f(x) = -4x^2 + 6x - 1$

d) $k(x) = \frac{3}{x} + 5$

Summary: x-intercepts and y-intercepts from the equation of a function	
x-intercepts	y-intercepts
Zero Product Property:	

4) A group of students is working together on the following problem.

$$\text{Solve } f(x) = 0 \text{ where } f(x) = 4x(x-2)(x+3)^2$$

Which of the students' ideas below are correct? Explain.

A. Plug in $x = 0$ to get $4(0)(0-2)(0+3)^2 = 0(-2)(3)^2 = 0$.

B. Solve $0 = 4x(x-2)(x+3)^2$, so we can set each factor equal to zero, like this: $4x = 0$, $x - 2 = 0$, and $x + 3 = 0$.

C. $4x(x-2)(x+3)^2$ will equal zero if we plug in $x = 0, 2$, or -3.

5) A College Algebra instructor assigned a different function to each of the following students and asked them to determine its domain, using only the given equation (i.e. not a graph).

Use the idea presented by each student to find the domain of each function. Write your answer in interval notation.

a) $k(x) = \frac{8}{(x-1)(x+2)}$

"I know I can't divide by 0, so the bottom of the fraction can't be 0."

b) $w(x) = \sqrt{-9 - x}$

"The stuff under the radical sign has to be 0 or bigger, so I need to figure out what values of x make $-9 - x$ bigger than or equal to 0."

c) $g(x) = \log(3x^2 + 5)$

"The input of a log has to be positive, so I just need to figure out what makes $3x^2 + 5$ greater than zero."

d) $j(x) = 3\left(x - \frac{1}{2}\right)(x + 5)$

"If I multiply this out, I get a quadratic function, and I already know the domain of a quadratic function."

Summary: Domain of a Function from an Equation

Rational Functions	Logarithmic Functions	Square Root Functions

6) Let $f(x) = 1$ and $g(x) = \frac{x}{x}$.

a) Are f and g the *same* function? If so, explain why. If not, what is the difference?

b) What does it mean for two functions to be the same?

7) Together with your team, determine the domain of the following functions.

a) $f(x) = \sqrt[3]{3x + 3}$

b) $g(x) = \sqrt[4]{3x + 3}$

c) $h(x) = \sqrt[5]{3x + 3}$

Summary: Domains of Higher Root Functions

8) Find the domains of the functions f and g below.

$f(x) = \sqrt[4]{9-x}$

$g(x) = \sqrt[3]{9-x}$

9) For each piecewise-defined function given below, graph its domain on a number line and write the domain in interval notation. Two of these are done for you.

Piecewise Function	Domain on a Number Line	Domain in interval notation
a) $f(x) = \begin{cases} 3x - 2 & \text{if} \quad x \leq -3 \\ 7 & \text{if} \quad 5 < x < 9 \end{cases}$		
b) $g(x) = \begin{cases} \frac{x}{2} & \text{if} \quad x = -2 \\ 6 & \text{if} \quad x \neq -2 \end{cases}$		
c) $h(x) = \begin{cases} -2 + x & \text{if} \quad -6 \leq x < -2 \\ 3 + x^2 & \text{if} \quad x \geq -2 \end{cases}$		$[-6, \infty)$
d) $k(x) = \begin{cases} -2x + 4 & \text{if} \quad x < -2 \\ x^2 + 1 & \text{if} \quad -2 \leq x \leq 6 \\ \sqrt{x + 3} & \text{if} \quad 7 < x \leq 22 \end{cases}$		

4.2: Warm Up

Student Name:

1) Let $f(x)$ be a function with the coordinate points $(a, f(a))$ and $(b, f(b))$. What is the ARC of the function between $x = a$ and $x = b$?

2) What is the average rate of change of $h(x) = -2x^3 + 3x^2$ from $x = -1$ to $x = 0$?

4.2: Average Rate of Change from an Equation

Learning Objectives

Together with your team, use a given equation of a function to:

- Calculate the average rate of change of the function over a specified interval.
- Interpret the ARC of a function in context.

1) Barry Allen is about to start college. When he was born, his grandparents opened a college savings account (no more deposits were made after the initial investment). Barry will use money from this account to pay college expenses. The amount of money in the account after t years is given by the function $A(t)$, for $0 \leq t \leq 18$.

$$A(t) = 5000e^{0.03t}$$

a) Use the equation to determine the ARC of the account balance over the time when Barry was 1 year old to 5 years old.

b) Write a sentence interpreting your answer to a) in the context of the situation.

c) Determine the ARC of $A(t)$ between $t = 15$ and $t = 18$.

d) What do you notice about the average rate of change when Barry is younger compared to when he's older? Given what you know about interest rates and bank accounts, do you think this make sense? Explain your thinking.

2) Let $d(t)$ be the total distance in meters that Lauren has travelled after t minutes, where:

$$d(t) = \begin{cases} t^2 & 0 \leq t \leq 10 \\ 100 & 10 < t \leq 12 \\ t + 88 & 12 < t \leq 20 \end{cases}$$

a) Calculate the ARC of the function d on the interval from $t = 5$ to $t = 17$.

b) Write a sentence interpreting your answer to a) in the context of the situation.

4.3: Modeling Real-World Data

Learning Objectives
Together with your team: • Use the best-fit model from Microsoft Excel to model a real-world data set and use it to predict values not in the data set. • Use a given equation of a function to interpret function values in the context of a given situation. • Use the graph or equation of the best-fit trend line to model a real-world data set and use it to predict values not in the data set.

With your team to consider data about the U.S. gender wage gap, provided by the Organization for Economic Co-operation and Development (OECD).

At their website, OECD provides data for the gender wage gap (see Table 1). The data in Table 1 defines a function, that takes the years from 1973 to 2016 as input and gives the gender wage gap for that year as output.

Table 1

Year	U.S. Gender Wage Gap
1973	38.11%
1974	39.1%
1975	37.63%
1976	37.76%
1977	38.22%
1978	38.97%
1979	37.64%
1980	36.55%
…	
2009	19.78%
2010	18.81%
2011	17.79%
2012	19.09%
2013	17.91%
2014	17.45%
2015	18.88%
2016	18.1%

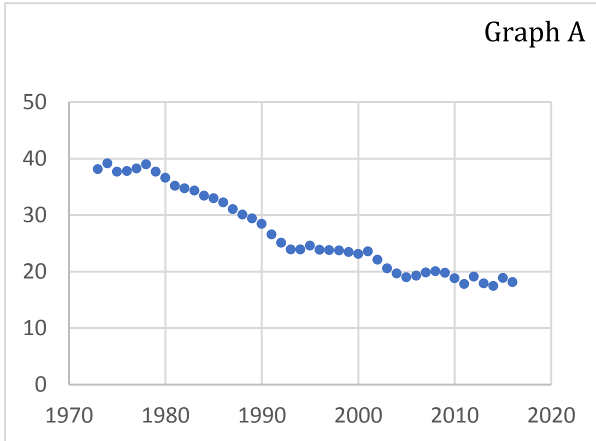
https://data.oecd.org/earnwage/gender-wage-gap.htm
(accessed 26 May 2018)

> We will use the following **definition of Gender Wage Gap:**
>
> *The difference between men's and women's median earnings as a percentage of men's median earnings.*

1) The data point (1973, 38.11%) tells us, in 1973 the gender wage gap was 38.11%; that is, the median earnings for women was 38.11% less than the median earnings for men.

a) Explain what the data point (2016, 18.1%) tells you about the situation.

b) Graph A shows the gender wage gap data from 1973 to 2016. Title the graph and give an appropriate label to each axis.



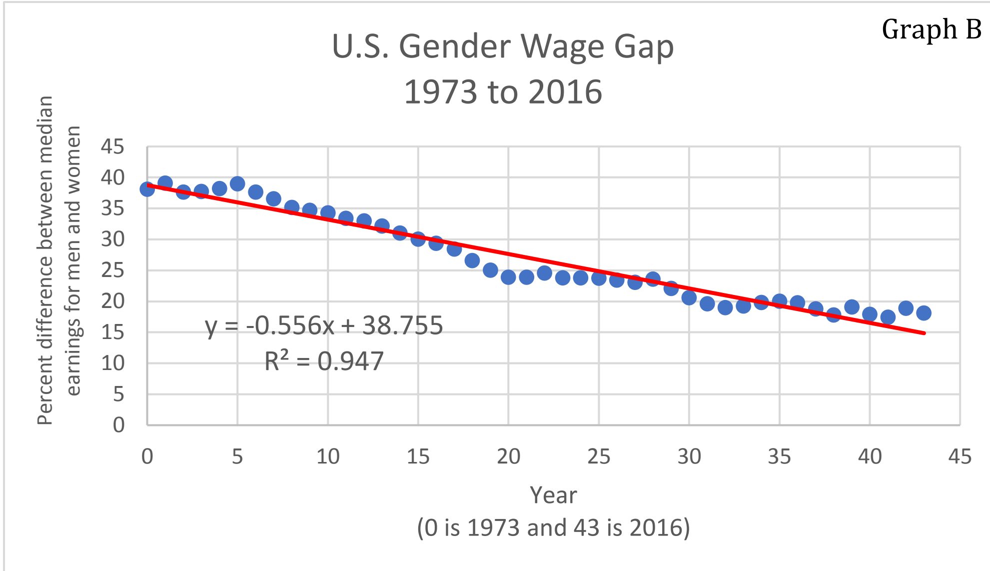
c) Based on the scatterplot, what can you conclude about the Gender Wage Gap (GWG) over time? Explain.

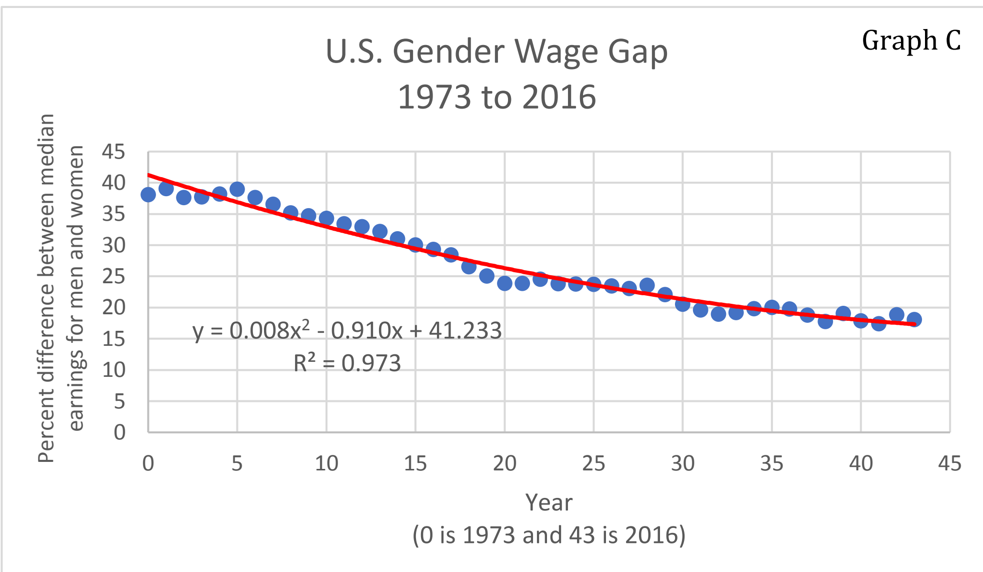
d) We could represent the GWG with an equation, as follows:

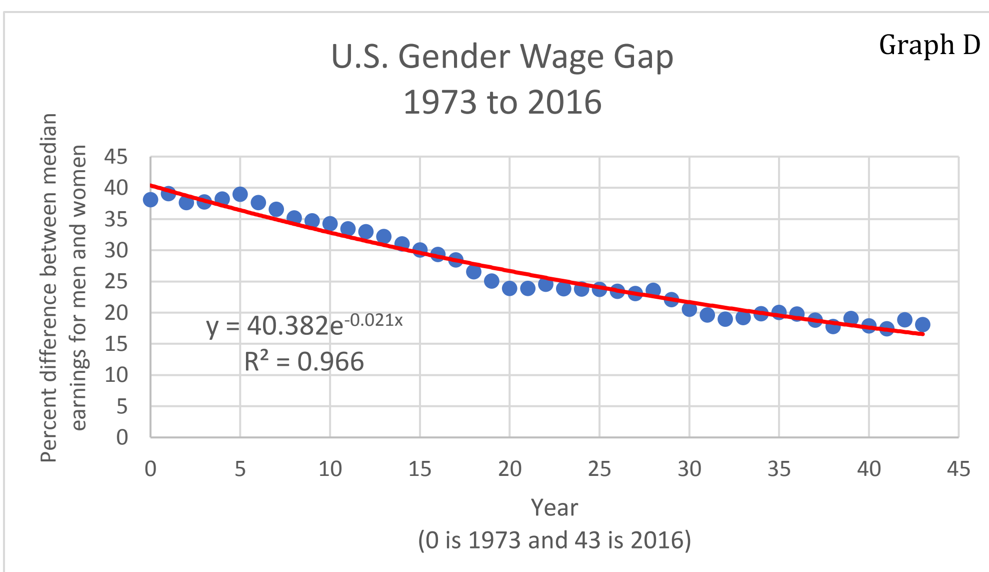
$$\text{Gender Wage Gap} = \frac{\text{Median Men's Earnings} - \text{Median Women's Earnings}}{\text{Men's Median Earnings}}$$

Write a sentence explaining what it would mean to say the Gender Wage Gap is 0%.

Using a spreadsheet program, such as Microsoft Excel or Google Sheets, we can find a 'best fit' model for a set of data. Graphs B, C, and D show three different best-fit models for the gender wage gap data, given by MS Excel. (Note: Year 0 corresponds to 1973 and Year 43 corresponds to 2016.)







2) For each model, name the *function family.*

Model B: ____________________ Model C: ____________________ Model D: ____________________

3) According to each model, what was the Gender Wage Gap in 1973? For your convenience, the models are given here.

Model	Equation	GWG in 1973
B	$f(x) = -0.556x + 38.755$	
C	$g(x) = 0.008x^2 - 0.910x + 41.233$	
D	$h(x) = 40.382e^{-0.021x}$	

4) The R^2 value is a statistical measure that indicates whether a model is a *good fit* for a dataset. The closer the R^2 value is to 1, the better the fit.

Which of the models in do you think is a good fit for the gender wage gap data? Explain.

5) Because all three models are a good fit for the recent *historical* data, we need to think about which one is likely best for predicting *future* values of the gender wage gap.

a) Discuss with your team the end behavior of each model and explain how the end behavior is related to the context of this situation. Hint: It may help to draw a sketch of each graph, that shows future years.

b) Which of the three best-fit models (linear, quadratic, or exponential) would you choose to model these data? Explain the reasons for your choice.

c) Using your chosen best fit model from b) above, what is the predicted gender wage gap in the year 2025? For convenience, the three equations are given again here.

Graph B	$f(x) = -0.556x + 38.755$
Graph C	$g(x) = 0.008x^2 - 0.910x + 41.233$
Graph D	$h(x) = 40.382e^{-0.021x}$

d) Using your chosen best fit model from b) above, in what year do you predict median earnings for women will be equal to median earnings for men? Explain.

4.4: Warm Up

Student Name:

Recall from Chapter 1, a polynomial function consists of the sum of a finite number of *terms*, each of which is a product of a real number *coefficient*, and a variable raised to a non-negative integer power; in other words, a polynomial function is a sum of power functions of the form ax^n where $n = 0, 1, 2, 3, \ldots$ and a is any real number.

1) Let g be the function given by the equation $g(x) = -4(3x+5)(4x-2)$.

a) For the polynomial function, $g(x) = -4(3x+5)(4x-2)$ what is the *leading term*?

b) For the polynomial function, $g(x) = -4(3x+5)(4x-2)$ what is the *leading coefficient*?

c) For the polynomial function, $g(x) = -4(3x+5)(4x-2)$ what is the *degree*?

2) Give an example of a polynomial function with degree 4 and leading coefficient $-\frac{1}{2}$.

Use what you know about polynomial functions and transformations (see Lesson 3.1) to answer the following question.

3) Which of the following has the same end behavior as $f(x) = 9x^8$? Select all that apply.

A. $m(x) = 6x^5$

B. $h(x) = 7x$

C. $j(x) = 4x^2$

D. $k(x) = 91x^{14}$

4.4: End Behavior of a Polynomial from an Equation

Learning Objectives
Together with your team, use a given equation of a function to: • Determine its end behavior (numerically for any function, and for polynomial functions, using the leading term).

1) Consider this table for the polynomial function $p(x) = 6x^5 + 7x + 3x^2 + 9x^8$.

x	$p(x) = 6x^5 + 7x + 3x^2 + 9x^8$
1	25
10	900,600,370
100	90,000,060,000,030,700
1000	9,000,000,006,000,000,003,007,000

a) How does the value of $p(x)$ change as x becomes large and positive?

b) Which term in the polynomial contributes most to the value of $p(x)$, for these large values of x?

c) What is the end behavior of $p(x) = 6x^5 + 7x + 3x^2 + 9x^8$ (make sure to also consider how the value of $p(x)$ changes as x become large and negative)? Complete the arrow notation.

As $x \to -\infty$, $p(x) \to$ _______ and as $x \to \infty$, $p(x) \to$ ________

2) Use the following Desmos link to explore how the leading term determines the end behavior of a polynomial:

https://www.desmos.com/calculator/jzb4eycolq

a) Which of the following functions has the same end behavior as $p(x) = 2x^3 - 6x^2 + 2x + 3$? Select all that apply.

A. $a(x) = 13x^5 - 2x^2 + x - 11$

B. $b(x) = 2x^3 + x - 11 - 6x^4$

C. $c(x) = -17x^2 + 3x$

D. $d(x) = -\frac{1}{4}x^5 - \frac{3}{4}x^3 + \frac{1}{4}$

E. $e(x) = 15x^9$

F. $f(x) = 2x^2 + x - 1$

b) Explain the selections you made in a).

Summary: End Behavior of a Polynomial Function with Leading term ax^n

		Degree, n	
		n Even	n Odd
Leading Coefficient, a	a Positive		
	a Negative		

3) Determine the degree, leading coefficient, and end behavior of the following polynomial functions.

a) $f(x) = -2x^2(x-1)(x+2) = -2x^4 + 2x^3 - 4x^2$

End Behavior: As $x \to -\infty$, $f(x) \to$ ______ and as $x \to \infty$, $f(x) \to$ ______

b) $g(x) = 4(x+2)^2(x-1)^3 = 4x^5 + 4x^4 - 20x^3 - 4x^2 + 32x - 16$

End Behavior: As $x \to -\infty$, $g(x) \to$ ______ and as $x \to \infty$, $g(x) \to$ ______

c) $h(x) = -(x+2)^2(x-1)^5$ Hint: It is not necessary to multiply this out.

Degree: ________ Leading coefficient: ________

End Behavior: As $x \to -\infty$, $h(x) \to$ ______ and as $x \to \infty$, $h(x) \to$ ______

d) $k(x) = (-4x^3 + 1)^2$

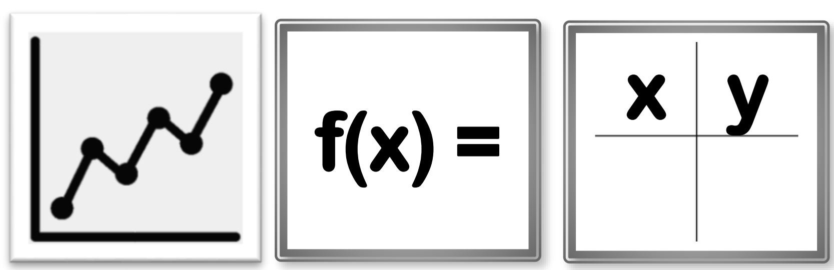
Degree: ________ Leading coefficient: ________

End Behavior: As $x \to -\infty$, $k(x) \to$ ______ and as $x \to \infty$, $k(x) \to$ ______

Chapter 5
How are Different Representations of Functions Connected?

Chapter Overview

In Chapter 5, we will explore the connections between the various ways of representing functions: equations, tables, and graphs of the same function. For example, we will use a graph or a table to write the equation of a function. Additionally, we will study the different forms that a polynomial equation can take (standard form, factored form, etc.) and consider how each form is useful for answering different types of questions.



Chapter Learning Objectives

1. Relate the attributes of the *equation* of a quadratic function to corresponding features of the *graph* of the function and interpret in context, using either:
   - standard form $(f(x) = ax^2 + bx + c)$
   - factored form,
   - vertex form
2. Convert between all three forms of the equation for a quadratic function.
3. Relate the attributes of the *equation* of a polynomial function, given in either standard form or factored form, to the corresponding attributes of the *graph* of the function.
4. Use either a graph or the equation of a polynomial function to identify the zeros of the function and their multiplicities.
5. Determine the degree of a polynomial function from a symbolic representation and the minimum possible degree from a graphical representation of a polynomial function, and use both representations to determine the end behavior of the function.
6. Sketch "good enough" graph of a polynomial function, given its equation.
7. Use a given equation of a rational function to determine features of its graph and then use this information to sketch a "good enough" graph, including:
   - any intercepts,
   - its end behavior,
   - the equation of any asymptotes.
8. From the graph of a quadratic, polynomial, or rational function, write an equation of the function.
9. Given an equation of an exponential function, determine the initial amount and growth or decay rate of the function.
10. Given a table for an exponential function, write an equation for the function
11. Relate the attributes of the *equation* of a piecewise-defined function to corresponding attributes of the *graph* of the function.
12. Given a graph for a piecewise-defined function, write an equation for the function.

Chapter 5 Contents

5.1: Warm Up

Student Name:

1) A classmate claims all three of the following equations represent the same function.

$$h(t) = -2t^2 + 12t + 14 \quad \text{Standard Form}$$
$$h(t) = -2(t - 7)(t + 1) \quad \text{Factored Form}$$
$$h(t) = -2(t - 3)^2 + 32 \quad \text{Vertex Form}$$

a) What does it mean to say that these equations represent the same function?

b) How could you determine whether these equations represent the same function?

c) Is your classmate correct? In other words, check whether the three equations above represent the same function. Show your work.

5.1: Forms of Quadratic Functions

Learning Objectives

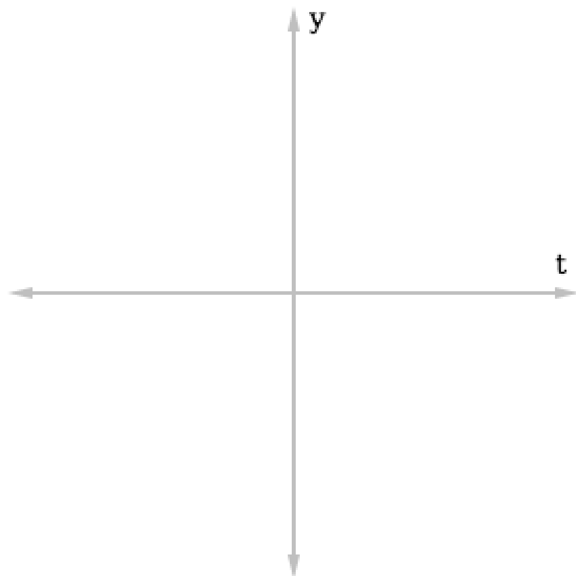
Together with your team:
• Relate the attributes of the *equation* of a quadratic function to corresponding features of the *graph* of the function and interpret in context, using either:
    ○ standard form: $f(x) = ax^2 + bx + c$
    ○ factored form: $f(x) = a(x - r_1)(x - r_2)$
    ○ vertex form: $f(x) = a(x - h)^2 + k$
• Convert between all three forms of the equation for a quadratic function.

1) A ball is thrown straight up in the air over the edge of a hotel balcony. The height in feet of the ball after t seconds is given by the function h as follows:

$h(t) = -2t^2 + 12t + 14$ Standard Form

$h(t) = -2(t - 7)(t + 1)$ Factored Form

$h(t) = -2(t - 3)^2 + 32$ Vertex Form



a) Sketch a "good enough" graph of $h(t)$. Include the coordinates of all the intercepts and the vertex of the graph on your sketch.

b) What is the maximum height of the ball?

Which form of the equation gives you that information?

c) When did the ball hit the ground?

Which form of the equation gives you that information?

d) What was the initial height of the ball; that is, how high was the ball at $t = 0$ seconds?

Which equation gives you that information?

e) What is a reasonable domain for this situation?

f) What is a reasonable range for this situation?

Summary: Forms of Quadratic Functions

- Standard Form:______________________________ y-intercept: ___________________
- Vertex Form: ______________________________ vertex: ______________________
- Factored Form: ____________________________ x-intercept(s): _________________

a is called the ____ of the quadratic.

If $a > 0$, then the graph of the quadratic will ____

And if $a < 0$, then the graph of the quadratic will ____

2) Suppose you are given a quadratic function in standard form:

$$f(x) = ax^2 + bx + c.$$

If you want to find the maximum (or minimum) point of the function, that is, the *vertex* (h, k), you can use the fact that the x-coordinate of the vertex, h, is $h = -\frac{b}{2a}$.

a) Use the Desmos link below to explore where this formula for the x-coordinate of the vertex come from.

https://www.desmos.com/calculator/naevfljlym

b) Once you find the x-coordinate of the vertex, h, how do you find the y-coordinate of the vertex, k?

3) A student was finding the vertex form of $g(x) = x^2 - 10x + 1$. They found $h = -\frac{b}{2a} = \frac{10}{2(1)} = 5$. What is the y-value of the vertex of $g(x)$?

Summary: Converting a Quadratic from Standard Form to Vertex Form

ALEKS Note:
The vertex form of a quadratic given in standard form can also be found using a process called, "completing the square," which you will see in the ALEKS explanations. You are welcome to use completing the square, but it will not be required in this class.

4) Let $f(x) = 3x^2 - 12x + 4$.

a) Find the vertex, (h, k), of $f(x)$.

b) Use the vertex you found in part a) and what you know about transformations to write the equation of $f(x) = 3x^2 - 12x + 4$ in vertex form.

5) Write $h(x) = 2x^2 + 16x + 5$ in vertex form.

5.2: Warm Up

Student Name:

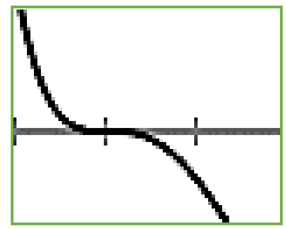
1) Match the graphical representation of each type of zero with its multiplicity.

a) Multiplicity 1: ______

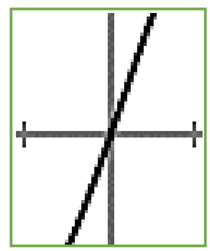
b) Even multiplicity: ______

c) Odd multiplicity greater than 1: ______

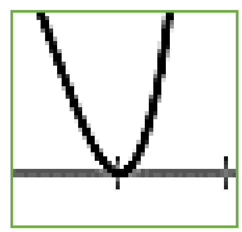
A.



B.



C.

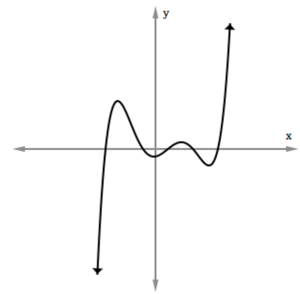


2) Match the following graphs and equations to the arrow notation that describes its end behavior.

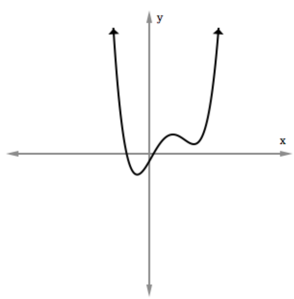
A.	B.	C.	D.
As $x \to -\infty, f(x) \to -\infty$ As $x \to \infty, f(x) \to \infty$	As $x \to -\infty, f(x) \to \infty$ As $x \to \infty, f(x) \to \infty$	As $x \to -\infty, f(x) \to \infty$ As $x \to \infty, f(x) \to -\infty$	As $x \to -\infty, f(x) \to -\infty$ As $x \to \infty, f(x) \to -\infty$

a) ________ $f(x) = x^4 - 5x^7$

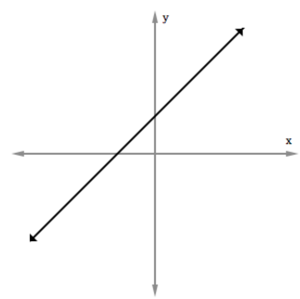
b) ________



c) ________

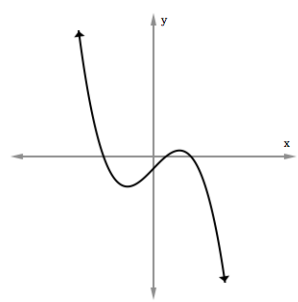


d) ________

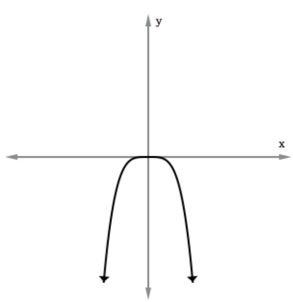


e) ________ $f(x) = 4(x-3)^2(x+5)(x-1)$

f) ________



g) ________



h) ________ $f(x) = -x^4 + x^3$

5.2: Polynomial Functions

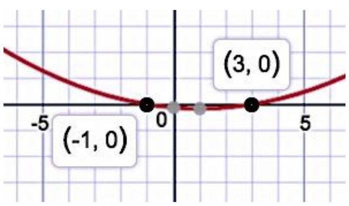
Learning Objectives
Together with your team: • Relate the attributes of the *equation* of a polynomial function, given in either standard form or factored form, to corresponding attributes of the *graph* of the function. • Use either a graph or the equation of a polynomial function to identify the zeros of the function and their multiplicities. • Determine the degree of a polynomial function from a symbolic representation and the minimum possible degree from a graphical representation of a polynomial function and use both representations to determine the end behavior of the function. • Sketch a "good enough" graph of a polynomial function, given its equation.

1) Use the following link to open a graph in Desmos. Explore the zeros and multiplicities of polynomial functions by moving the sliders.

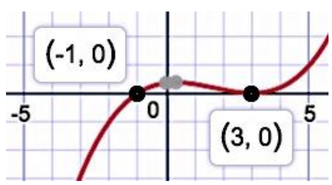
https://www.desmos.com/calculator/9ytpnfyg9f

Use the slider to help you match each function with the corresponding zoomed in portion of the polynomial graphs below. Write the letter in each blank.

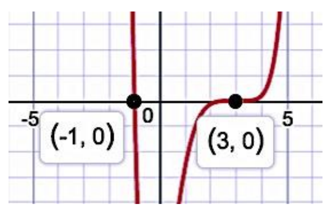
A.



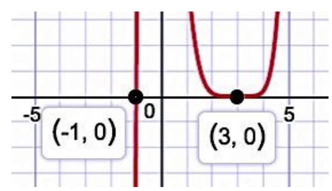
B.



C.



D.



a) $f(x) = \frac{1}{25}(x-3)(x+1)$ ________

b) $f(x) = \frac{1}{25}(x-3)^2(x+1)$ ________

c) $f(x) = \frac{1}{25}(x-3)^3(x+1)$ ________

d) $f(x) = \frac{1}{25}(x-3)^4(x+1)$ ________

e) $f(x) = \frac{1}{25}(x-3)^5(x+1)$ ________

f) $f(x) = \frac{1}{25}(x-3)^6(x+1)$ ________

g) $f(x) = \frac{1}{25}(x-3)^7(x+1)$ ________

h) $f(x) = \frac{1}{25}(x-3)^8(x+1)$ ________

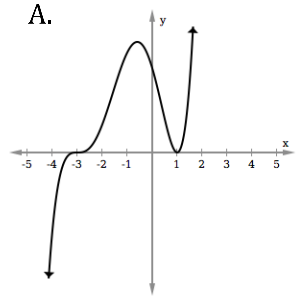
i) $f(x) = \frac{1}{25}(x-3)^9(x+1)$ ________

j) $f(x) = \frac{1}{25}(x-3)^{10}(x+1)$ ________

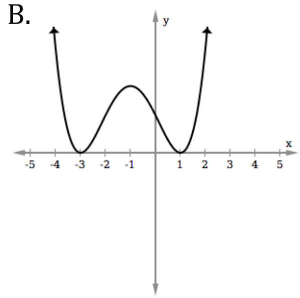
For **2) – 5),** match each polynomial function in factored form with its graph.

2)	______	$f(x) = (x-1)^2(x+3)^2$
3)	______	$g(x) = (x-1)^3(x+3)$
4)	______	$h(x) = (x-1)(x+3)^2$
5)	______	$k(x) = (x-1)^2(x+3)^3$

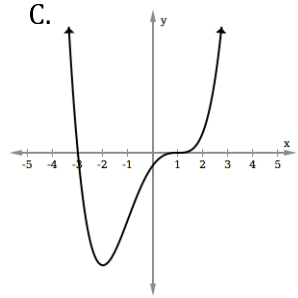
A.



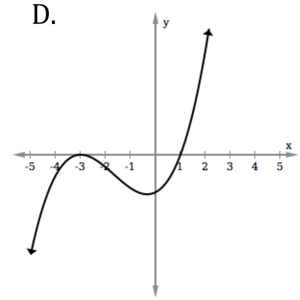
B.



C.



D.



6) Which of the following statements must be true, given that $f(x)$ is a polynomial function with a factor of $(x-r)^n$? Select all that apply.

A. $x = r$ is a zero of $f(x)$.

B. The multiplicity of the zero $x = r$ is n.

C. The multiplicity of the zero $x = n$ is r.

D. The graph of $f(x)$ has an x-intercept at $(r, 0)$.

E. The graph of $f(x)$ has an x-intercept at $(0, r)$.

F. The graph of $f(x)$ will either touch the x-axis at $x = r$ or cross the x-axis at $x = r$ depending on whether n is even or odd.

Summary: Multiplicities of Real Zeros of Polynomial Functions

	Zero with Multiplicity 1	Zero with Even Multiplicity	Zero with Odd Multiplicity > 1
Least possible multiplicity:			
Zero:			
Factor:			
x-intercept:			

7) The polynomial function f is given by the equation $f(x) = 5x(x-4)^2(x+1)^3$.

a) What is the degree and leading coefficient of the polynomial?

Degree: ____________ Leading coefficient: ____________

b) Determine the end behavior of f.

As $x \to -\infty$, $f(x) \to$_________ and as $x \to \infty$, $f(x) \to$_________

c) Solve $f(x) = 0$.

d) List the zeros of f and their multiplicities.

Zeros	Multiplicity

e) Sketch a "good enough" graph of f.

f) Solve $f(x) \leq 0$.

8) Let $k(x) = x^4 - 2x^3 - 8x^2$.

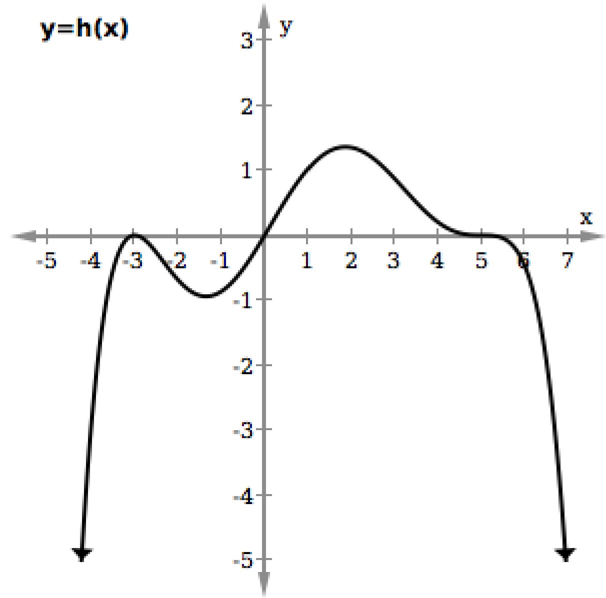
a) Solve $k(x) = 0$.

b) Sketch a "good enough" graph of k.

c) Solve $k(x) < 0$.

Summary: Solving Polynomial Inequalities by Graphing

9) The polynomial function h is graphed here.



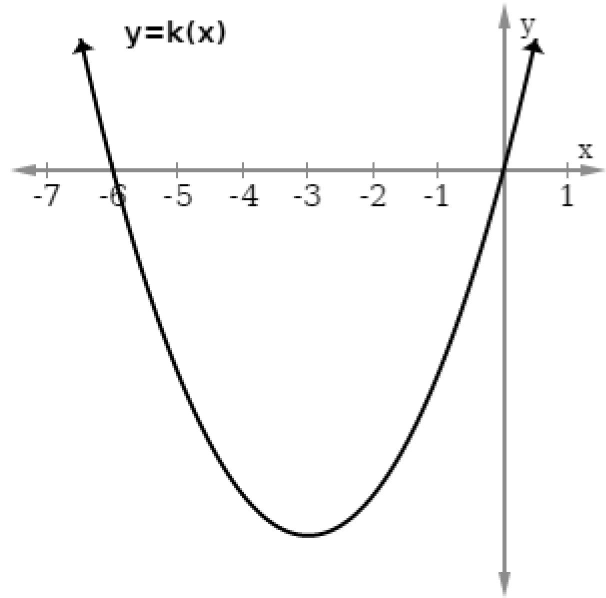
a) List four things you know about the function $h(x)$ from its graph.

-
-
-
-

b) Write a possible equation for h.

c) To determine the leading coefficient, use the fact that the graph goes through the point $(3, 1)$.

10) The quadratic function k is graphed here.



a) List at least three things you know about the function $k(x)$.

-
-
-

b) Write a possible equation for $k(x)$.

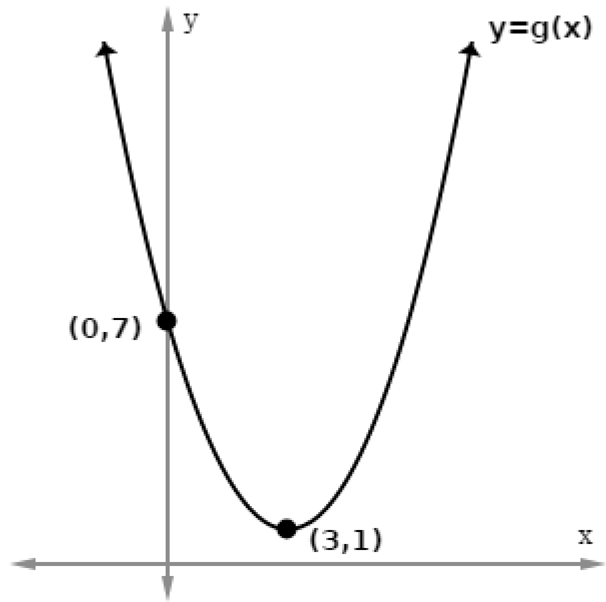
c) To determine the leading coefficient, use the fact that the graph goes through the point $(-4, -10)$.

d) Write the equation of $k(x)$ in factored form.

11) The quadratic function g is graphed here.

a) List at least three things you know about the function $g(x)$.

-
-
-



b) Use what you know about transformations to write a possible equation for $g(x)$.

c) To determine the leading coefficient, use the fact that the graph goes through the point $(0, 7)$.

d) Write the equation of $g(x)$ in vertex form.

5.3: Warm Up

Student Name:

1) Which of the following functions have the same end behavior as $g(x) = 2x^3 - 7x^2 + 1$?

A. $y = 2x^3$

B. $y = -5x^3$

C. $y = x^2$

D. $y = x$

2) Explain your answer to **1)**.

5.3: Rational Functions

Learning Objectives
Together with your team: • Use a given equation of a rational function to determine features of its graph, including: its intercepts, end behavior, the equation of any asymptotes, and then sketch a "good enough" graph of the function. • From the graph of a rational function, write an equation of the function.

1) Desmos Class Code:______________________________

Record the following information for your notes:

a) **Screen 8:** What are the zeros of $f(t) = \frac{2(t-1)}{(t-4)(t+2)}$?

b) **Screen 9:** What are the equations of the vertical asymptotes of $f(t) = \frac{2(t-1)}{(t-4)(t+2)}$? Write your answer as an equation of a vertical line, $t = d$.

c) **Screen 10**: What is the domain of $f(t) = \frac{2(t-1)}{(t-4)(t+2)}$?

d) **Screen 11:** Sketch a graph of $f(t) = \frac{2(t-1)}{(t-4)(t+2)}$. Include all asymptotes and intercepts.

2) Match the feature of the graph of a rational function with the part of its equation that it comes from:

Denominator	Zeros
	Vertical asymptotes
Numerator	x-intercepts
	x-values excluded from the domain

<u>Summary</u>: Rational Functions: x-intercepts and vertical asymptotes	
x-intercepts:	
Vertical asymptotes:	
Domain:	

3) Some College Algebra students were reviewing rational functions for their exam. They were analyzing the function $k(x)$ given below. Which students are on the right track? Select all that apply.

$$k(x) = \frac{(x-3)(x+5)}{(x+2)(x+4)}$$

A. "$x = -2$ and $x = -4$ are not in the domain of $k(x)$ because they make the denominator equal to zero."

B. "The domain of $k(x)$ is $(-\infty, -4) \cup (-4, -2) \cup (-2, \infty)$."

C. "The equations of the vertical asymptotes of $k(x)$ are $x = -2$ and $x = -4$."

D. "$x = -2$ and $x = -4$ must be in the domain of $k(x)$ because that's where the vertical asymptotes are."

4) The function f is given by the following equation.

$$f(x) = \frac{x^2 - x - 12}{x^2 + 4x - 12} = \frac{(x-4)(x+3)}{(x-2)(x+6)}$$

a) Determine the x-value(s) where $f(x)$ is undefined.

b) Give the domain of $f(x)$.

c) Give the equations of the vertical asymptotes of $f(x)$.

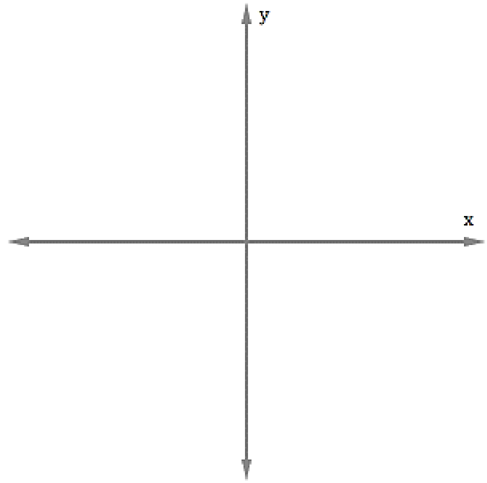
5) With your team, explain how the vertical asymptotes and the domain of a rational function are related. Be ready to share with the class.

Let's use this Desmos link to explore the end behavior of Rational Functions:
https://www.desmos.com/calculator/m3nb3jh4l2

6) Let $h(x) = \frac{5x^4+6x^3-2x^2+4x-1}{8x^4-7}$.

a) Sketch a "good enough" graph of both $h(x)$ and $y = \frac{5x^4}{8x^4} = \frac{5}{8}$ on the axes to the right.

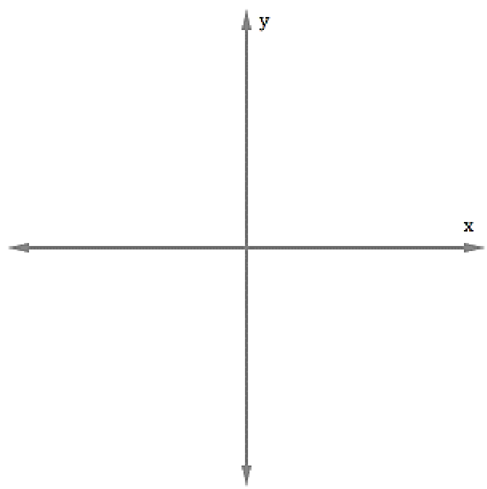
b) What do you notice about the end behavior of these two functions?



7) Let $g(x) = \frac{8x^4-x^3+2x^2-5x+1}{x^6-3x^4-x^3+9x^2+2}$.

a) Sketch a "good enough" graph of both $g(x)$ and $y = \frac{8x^4}{x^6} = \frac{8}{x^2}$ on the axes to the right.

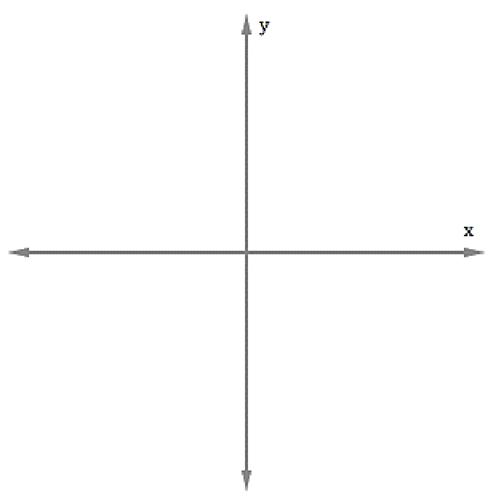
b) What do you notice about the end behavior of these two functions?



8) Let $f(x) = \frac{x^5-2x-3}{x^4-7x}$.

a) Sketch a "good enough" graph of both $f(x)$ and $y = \frac{x^5}{x^4} = x$ on the axes to the right.

b) What do you notice about the end behavior of these two functions?



9) Some College Algebra students are trying to find the horizontal asymptote of $r(x) = \frac{5x-3}{x^9-7x+4}$. Who do you think is on the right track?

A. "$r(x)$ has the same end behavior $g(x) = \frac{5}{x^8}$."

B. "We can find the horizontal asymptote of $g(x) = \frac{5}{x^8}$ by looking at end behavior."

C. "I can find the end behavior of $g(x) = \frac{5}{x^8}$. As $x \to \infty$ $g(x) = \frac{5}{x^8} \to 0$ because when x is big $g(x) = \frac{5}{x^8}$ gets small."

D. "Oh, I see, since $g(x) = \frac{5}{x^8}$ looks kind of like our parent function $f(x) = \frac{1}{x^2}$, the horizontal asymptote must be $y = 0$."

10) The function $p(x) = \frac{3x^2-5x-7}{x+1}$ has the same end behavior as $y = 3x$.

a) Explain why this is true about the end behavior of $p(x)$.

b) Write the end behavior of $p(x)$ using arrow notation.

c) Determine the equation of the horizontal asymptote of $p(x)$.

<u>Summary</u>: End Behavior and Horizontal Asymptotes of Rational Functions

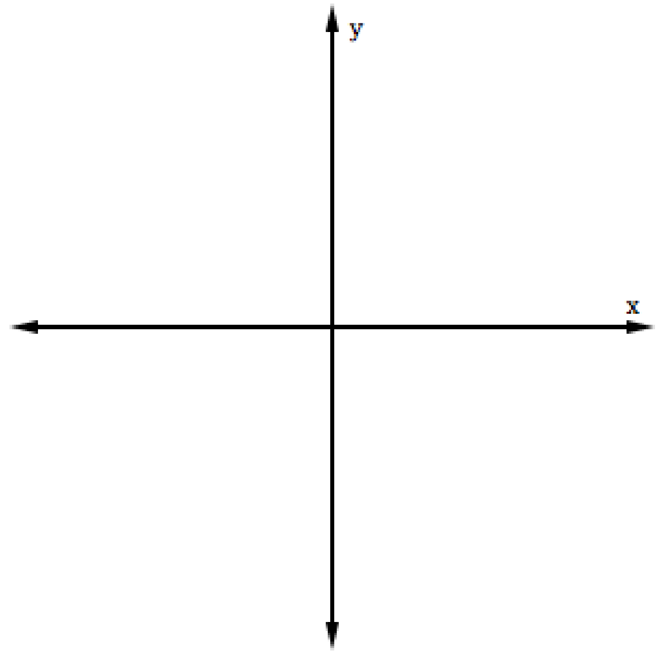
ALEKS Note:
The ALEKS explanations related to end behavior and asymptotes of rational functions discuss a different approach than the one above. These approaches are equivalent, but the one above may require less memorization.

For the following three questions, a) use the leading terms to find a simpler function with the same end behavior and b) determine the equation of the horizontal asymptote of each function, if it exists.

11) $j(x) = \frac{15x^4 - x^2 - 1}{3x^4 + 4x - 12}$ a)	b)
12) $f(x) = \frac{x+1}{3x^2 + 5x - 2}$ a)	b)
13) $l(x) = \frac{3x^4 - x - 9}{x+1}$ a)	b)

14) Consider the rational function $f(x) = \frac{x-2}{x+7}$.

a) What are the zero(s) of $f(x)$?



b) Write the equations of any vertical asymptote(s) of $f(x)$. Draw this as a dotted line on the graph.

c) Use a simpler function with the same end behavior to determine the equation for the horizontal asymptote of $f(x)$. Draw this as a dotted line on the graph.

d) Determine the y-intercept of $f(x)$, if it exists. Label the (x, y) coordinates of the y-intercept on the graph.

e) Complete your "good enough" graph of the function $f(x)$, and label the graph you sketched with all of its features you found in a) – d) above.

f) Use your graph to solve $f(x) < 0$.

15) Find the following features of $p(x)$.

$$p(x) = \frac{2x^3 - 10x^2 + 14x - 6}{x^2 + 9x + 14} = \frac{2(x-1)^2(x-3)}{(x+2)(x+7)}$$

a) What are the zeros of $p(x)$?

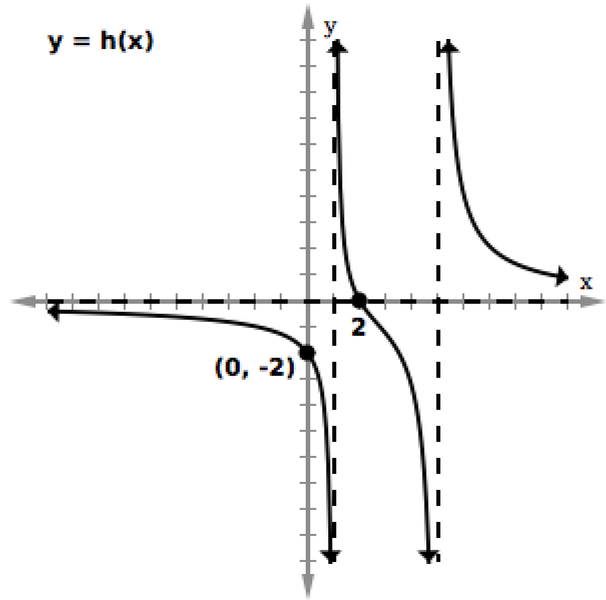
b) What is a function, in simplest form, with the same end behavior as $p(x)$?

c) Write the equations of any vertical and horizontal asymptote(s) of $p(x)$.

d) Determine the y-intercept of $p(x)$, if it exists.

e) Check your answers to a) – d) by graphing $p(x)$ in Desmos.

16) The graph of the rational function h has vertical asymptotes at $x = 1$ and $x = 5$, and horizontal asymptote at $y = 0$. Its graph also has an x-intercept at $(2,0)$. The equation for $h(x)$ has one of the five forms shown below.



a) Choose the appropriate form for $h(x)$.

A. $h(x) = \frac{a}{x-b}$

B. $h(x) = \frac{a(x-b)}{x-c}$

C. $h(x) = \frac{a}{(x-b)(x-c)}$

D. $h(x) = \frac{a(x-b)}{(x-c)(x-d)}$

E. $h(x) = \frac{a(x-b)(x-c)}{(x-d)(x-e)}$

b) Determine the equation of the function h from a), given that its graph passes through the point $(0, -2)$.

5.4: Warm Up

Student Name:

1) Let $m(x) = 7(4)^x$.

a) Fill out the table of values for the function $m(x)$.

x	$m(x) = 7(4)^x$
-2	
-1	
0	
1	
2	
3	
4	

b) What is the y-intercept of $m(x)$?

c) Describe how the output values, $m(x)$, change as x increases by one unit.

d) Explain why $(0, a)$ is the y-intercept of $f(x) = a(b)^x$.

e) How does the output value of $f(x) = a(b)^x$ change as x increases by 1 unit?

2) Use what you learned about the y-intercept and the base of an exponential function in the form $y = a(b)^x$ to answer the following questions.

x	$g(x)$
-1	$\frac{1}{8}$
0	$\frac{1}{2}$
1	2
2	8
3	32
4	128

a) Determine the y-intercept of $g(x)$.

b) Describe how the output values of $g(x)$ change, as x increases by one unit.

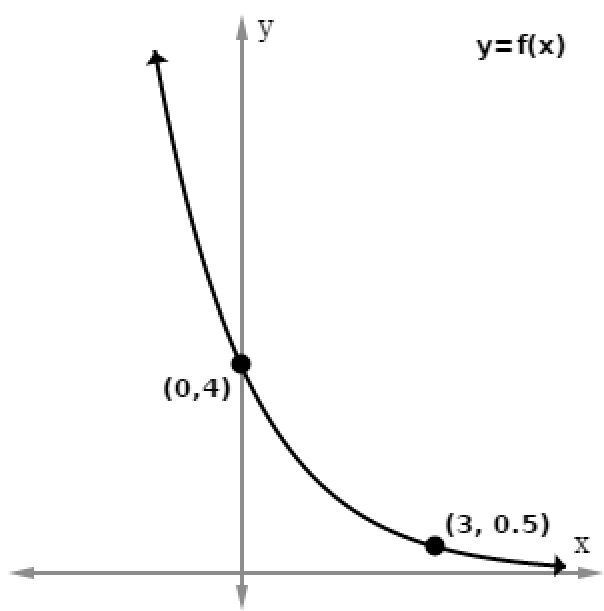
c) Write the equation of the exponential function, $g(x)$, in the form $y = a(b)^x$.

5.4: Exponential Functions

Learning Objectives
Together with your team: • Relate the attributes of the *equation* of an exponential function to corresponding attributes of the *graph* of the function. • Given an equation of an exponential function, determine the initial amount and growth or decay rate of the function. • Given a table for an exponential function, write an equation for the function.

Summary: General Form of an Exponential Function, $y = a(b)^x$

1) Determine the equation of the exponential function, $f(x)$, in the form $y = a(b)^x$.



2) Determine the equation of an exponential function that contains the points $(3, 162)$ and $(4, 486)$ of the form $f(x) = a(b)^x$.

Summary: Determining the Equation of an Exponential Model, $f(x) = a \cdot b^x$, Given Two Points	
When the y-intercept is given:	
When the y-intercept is not given:	

3) Maike decided to make a risky investment into her favorite cousin's start-up company (she loves her cousin, but is not sure about the company, which gives "history of mathematics" bicycle tours). Maike's financial adviser ran some data through her software and found the prediction formula $A(t) = 350(0.91)^t$, where $A(t)$ is the predicted value of Maike's investment, in dollars, after t years.

a) How much money does Maike initially invest? Include units.

b) Does this function represent exponential growth or decay? Explain your reasoning.

c) The amount of money in the account increases/decreases (circle) by __________ % each year.

d) Evaluate: $A(7) =$ ________________

e) Interpret your answer to part d) in the context of this situation.

4) A group of students is trying to determine if $g(x) = 3(0.95^{-0.9})^x$ represents exponential growth or exponential decay. Which of the following students are showing correct reasoning?

A. "g represents exponential growth because $0.95^{-0.9}$ is approximately 1.05 or 105% which is bigger than 100%."

B. "The graph of g is increasing so it must be growth."

C. "g represents exponential decay because there is a negative exponent."

D. "I created a table and saw that as x gets bigger y gets bigger so this is exponential growth."

E. "This function is decay because $0.95 < 1$."

5) For the following exponential functions of the form $f(x) = a \cdot b^x$, identify the y-intercept, the value of the base, and whether the function is exponential growth or decay.

Function	y-intercept	Base	Growth or Decay?
$h(t) = 56(0.998)^t$			
$g(x) = 2.3(e^{-0.23})^x$			
$n(x) = 12e^{0.65x}$			

5.5: Warm Up

Student Name:

1) Evaluate: $\log_2(16) =$_________

2) $\log_9(81) = 2$ because $9^2 = 81$. Both statements tell us that 2 is the exponent you raise 9 to in order to get 81. Fill in the following table.

Log Form	Exponential Form
$\log_9(81) = 2$	$9^2 = 81$
$\log_3(x) = 7$	
$\log_5(157) = x + 2$	
$\log_6(2x - 1) = 4$	
$\log_4(2x - 1) = 3$	
$\ln(x) = 10$	

Sometimes writing log equations in the equivalent exponential form or exponentials in the equivalent logarithmic form makes it possible to solve for a variable in an exponent or inside a logarithm. We will do solving of this type in Chapter 7.

5.5: Properties of Logarithms

Learning Objectives
Together with your team: • Use properties of logs and exponentials to rewrite logarithmic or exponential expressions.

Reference Guide: Properties of Logarithms

Property Name	Log Property	Related Exponential Property
Product rule for logarithms	$\log_a(MN) = \log_a(M) + \log_a(N)$	$a^x \cdot a^y = a^{x+y}$
Quotient rule for logarithms	$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$	$\frac{a^x}{a^y} = a^{x-y}$
Power rule for logarithms	$\log_a(M^p) = p\log_a(M)$	$(a^x)^p = a^{xp}$
One-to-one property	If $\log_a(M) = \log_a(N)$, then $M = N$	If $a^x = a^y$ then $x = y$
First power rule	$\log_a(a) = 1$	$a^1 = a$
Zero exponent rule	$\log_a(1) = 0$	$a^0 = 1$

1) Fill in the missing values to make the equations true.

a) $\log_8(8^2) = \square \log_8(8)$

b) $\ln(11) - \ln(3) = \ln \square$

c) $\log_3 \square + \log_3(10) = \log_3(70)$

2) Martin was asked to use properties of logarithms to expand the logarithmic expression:

$$\ln\left(\frac{x(x+5)}{yz}\right)$$

Where did Martin go wrong?

$\ln\left(\frac{x(x+5)}{yz}\right)$

$= \ln\big(x(x+5)\big) - \ln(yz)$

$= \ln(x) + \ln(x+5) - \ln(y) + \ln(z)$

$= \ln(x) + \ln(x) + \ln(5) - \ln(y) + \ln(z)$

3) Use properties of logarithms to expand $\log\left(\frac{\sqrt[3]{x}}{y^2(z-1)}\right)$.

4) Josie is trying to solve this equation for x: $\log_5(x+4) = 1 - \log_5(x+8)$. Her first step is shown. Apply the Product Rule for logarithms to write the next step for Josie. **DO NOT solve this equation. We will learn how to solve this in Chapter 7.**

$$\log_5(x+4) = 1 - \log_5(x+8)$$

$$\log_5(x+8) + \log_5(x+4) = 1$$

$$\underline{\hspace{6cm}} = 1$$

5) Write as a single log:

a) $3\ln(x-2) - 2\ln(x) - \frac{1}{2}\ln(x+5)$

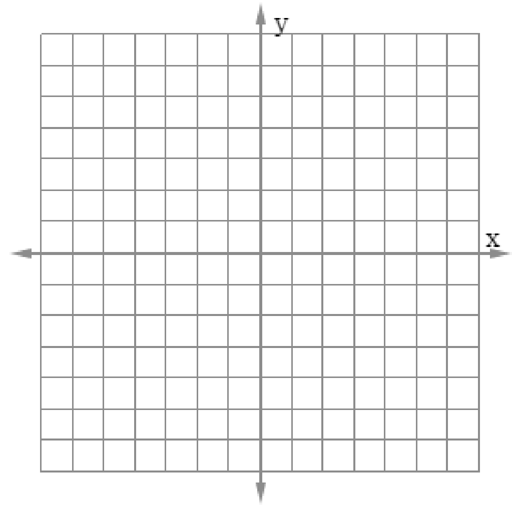
b) $\frac{1}{2}\log_3(x) + 2\log_3(y) - 4\log_3(z) + 3\log_3(w)$

5.6: Warm Up

Student Name:

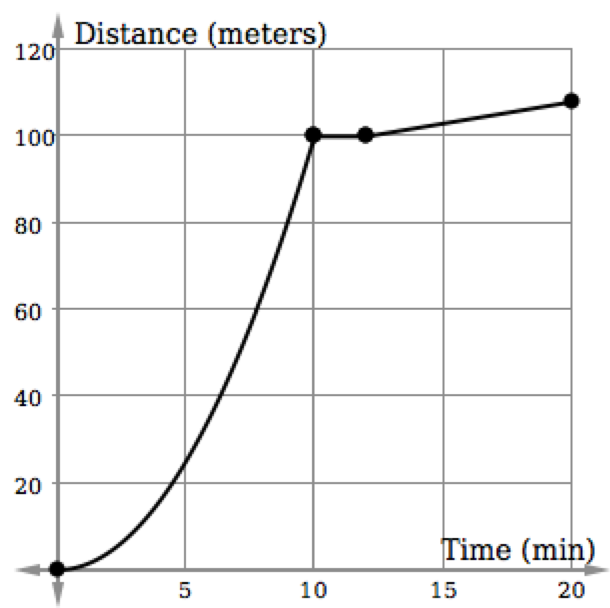
1) Graph the piecewise function f given by the equation.

$$f(x) = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$



2) The piecewise function models the total distance in meters, d, Lauren has travelled after t minutes. Using the graph of $d(t)$, write the domain for each piece.

$$d(t) = \begin{cases} t^2 & \text{if} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ 100 & \text{if} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ t + 88 & \text{if} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$$



5.6: Graphs and Equations of Piecewise-defined Functions

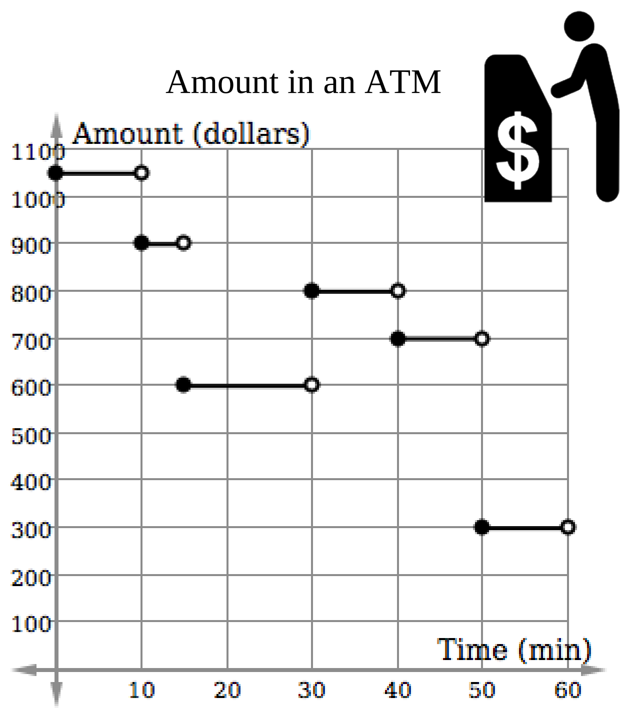
Learning Objectives
Together with your team: • Relate the attributes of the *equation* of a piecewise-defined function to corresponding attributes of the *graph* of the function. • Given a graph for a piecewise-defined function, write an equation for the function.

1) Read the following piecewise-defined function and determine which action, A, you will perform based on your birthday month, m.

$$A(m) = \begin{cases} \text{Clap your hands} & \text{if} \quad \text{Your birthday is in January} - \text{March} \\ \text{Raise your hand} & \text{if} \quad \text{Your birthday is in April} - \text{May} \\ \text{Stand up} & \text{if} \quad \text{Your birthday is in July} - \text{September} \\ \text{Put your hand on your head} & \text{if} \quad \text{Your birthday is in October} - \text{Decemeber} \end{cases}$$

2) This graph models the amount of money in a particular automatic teller machine (ATM) over a one-hour period.

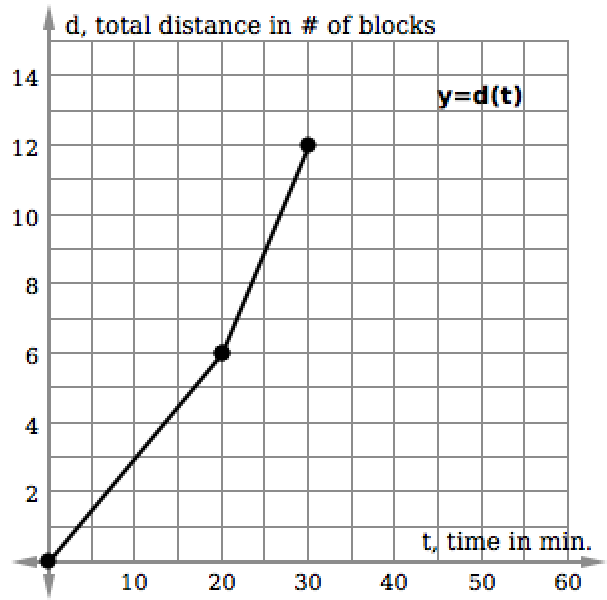
Write an equation for the function represented here.



3) Shala walks to school along the same route every morning and the total distance she usually travels is shown in the graph.

Write an equation for the function represented here.

Shala's Total Distance



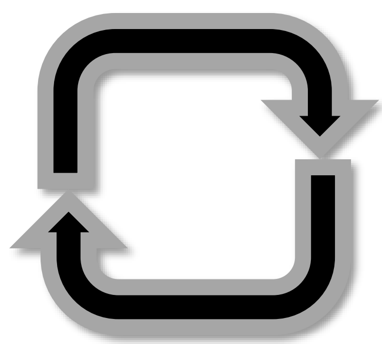
Chapter Learning Objectives

1. Given two functions symbolically, graphically, or numerically, combine the functions using:
   - Addition,
   - Subtraction,
   - Multiplication,
   - Division, or
   - Composition
2. Identify the domain and range of combined functions.
3. Evaluate the sum, difference, product, quotient, or composition of two functions for a given value, and interpret in a given context.
4. Express a function given symbolically as a composition of two functions.
5. Represent the composition of two functions using arrow diagrams.
6. Describe the relationship between a function and its inverse and interpret in a given context.

Chapter 6
How Do We Combine Functions?

Chapter Overview

In Chapter 3, you learned how we could create a new function by starting with a function and applying transformations to it. In Chapter 6, we will again create new functions, this time by combining parent functions either through (1) arithmetic operations (sum, difference, product, and quotient), or (2) function composition. We will also identify the inverse function that should be applied to both sides of an equation in order to help us solve. We will solve equations in Chapter 7.



Chapter 6 Contents

6.1: Function Algebra

Learning Objectives
Together with your team: • Use arithmetic operations to combine two functions (given symbolically, graphically, or numerically) to create a new function. • Identify the domain and range of the new function.

1) A small publishing company is releasing a new book. Their total cost C, in dollars, can be modeled by the function $C(N) = 750 + 17.95N$, where N is the number of books the company produces. The total revenue earned, in dollars, from selling N books can be modeled by the function $R(N) = 32.80N$.

Let $P(N)$ be the company's profit, in dollars, for selling N books.

Recall: **Cost** is the amount of money spent.

Revenue is the amount of money collected.

Profit is the difference in the amount of money collected and money spent.

a) Complete the table, using the given values of N.

N	50	100	500
$C(N)$			
$R(N)$			
$P(N)$			

b) Write a function $P(N)$ that models the profit, P, for selling N books.

2) Let $u(x) = \frac{x}{x+1}$ and $v(x) = \frac{x-3}{x+6}$. Let the quotient of u and v be the function $w(x) = \left(\frac{u}{v}\right)(x) = \frac{u(x)}{v(x)}$.

a) Which of the following x-values are in the domain of the function $w(x) = \frac{u(x)}{v(x)}$? Select all that apply.

x	0	1	-1	6	-6	3	2
$u(x)$	0	1/2	undefined	6/7	$-6/5$	3/4	2/3
$v(x)$	$-1/2$	$-2/7$	$-4/5$	1/4	undefined	0	$-1/8$

A. 0

B. 1

C. -1

D. 6

E. -6

F. 3

G. 2

b) The equation for the *quotient function*, w, is ________________________________

c) What is the domain of the function w?

Summary: Domain of a Combined Function

3) Let $f(x) = x$, $j(x) = \frac{1}{x}$, and the *product function* $k(x) = (fj)(x) = f(x) \cdot j(x)$.

a) Fill in the missing values in the following table.

x	-2	-1	0	2	5
$f(x)$					
$j(x)$					
$k(x)$					

b) What is the domain of the function k?

c) Explain your reasoning for b).

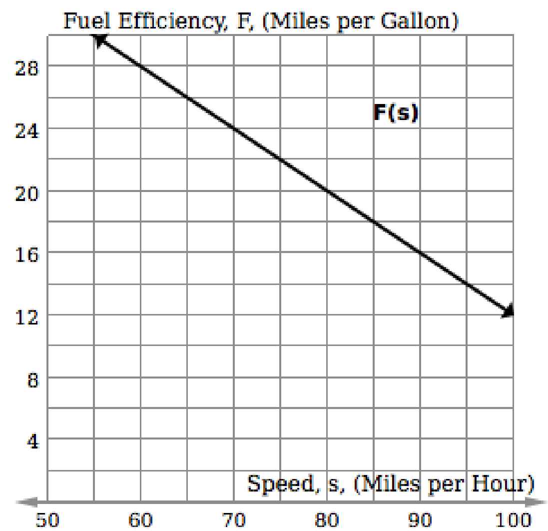
d) Sketch a quick graph of the function k and check your domain from b).

6.2: Function Composition

Learning Objectives
Together with your team: • For two functions, given symbolically, graphically, or numerically: form their composition and identify the domain and range of the new function, evaluate the composition function for a given value, and interpret in a given context. • Express a function as a composition of two functions.

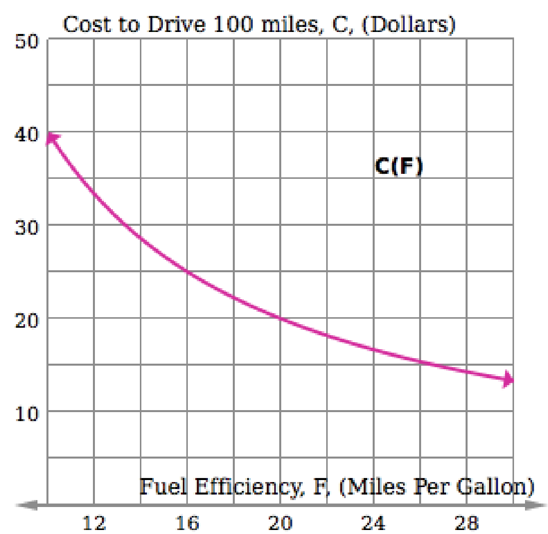
1) The function $F(s)$ gives the fuel efficiency, F, for a certain sportscar driving at a speed of s miles per hour. The function $C(F)$ gives the cost, C, to drive 100 miles when driving with a fuel efficiency of F miles per gallon, when the price of gas is \$4 per gallon.

a) Show how to use the two graphs to determine the cost to drive the sportscar 100 miles if you are driving a speed of 90 miles per hour.



b) Complete this same process for more inputs, s, and fill in the table with the cost to drive 100 miles associated with each speed.

s	$C(s)$
80	
75	
95	
60	



c) Let $F(s) = -0.4s + 52$ and $C(F) = \frac{400}{F}$.

Create a model that would allow you to find the cost to drive the BMW 100 miles driving at any given speed, s mph.

$C(s) =$ ________________________

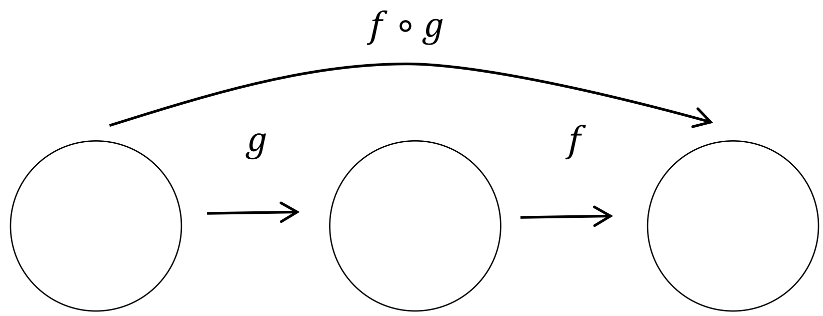
The operation of combining these functions, so that the output of the Fuel Efficiency function is used as the input for the Cost function, is known as *composition of functions*. The resulting function, $C(s)$, is called the *composite function*, and expresses Cost directly as a function of speed.

Summary: Function Composition

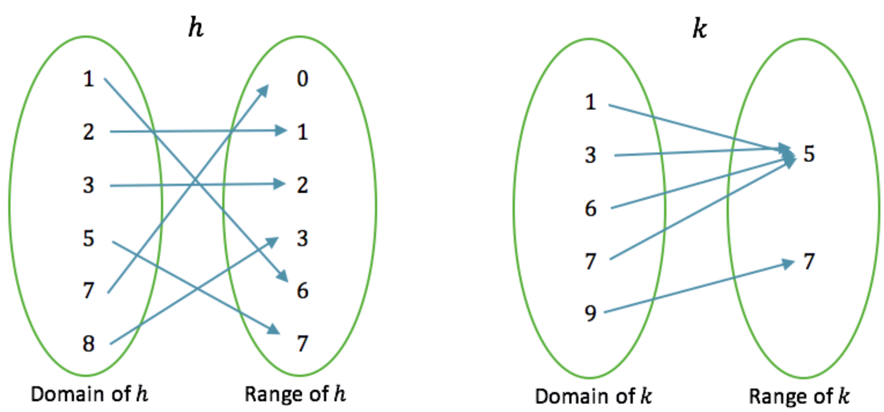
We represent function composition by the following notation: $(f \circ g)(x) = f(g(x))$

We read the left side as "f composed with g at x," and the right-hand side as "f of g of x." The two sides of the equation have the same mathematical meaning and are equal. The open circle symbol $\circ$ is called the composition operator (just as we use $+$ for function addition).

Here is an arrow diagram that illustrates the operation of function composition:

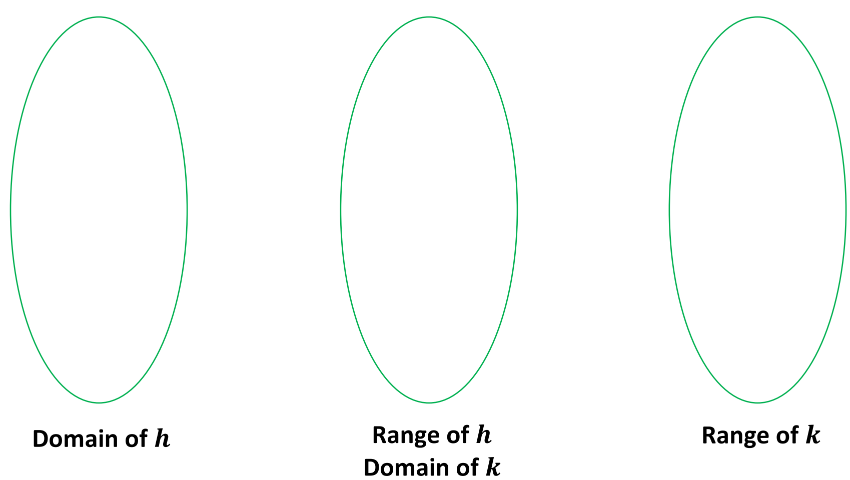


2) Two functions h and k are defined in the following two arrow diagrams.



a) Evaluate $k(h(8))$.

b) Fill in the arrow diagram below to represent the composition $(k \circ h)(x)$.



c) Explain why $k(h(7))$ is undefined.

d) What is the domain of $(k \circ h)(x)$?

e) What is the range of $(k \circ h)(x)$?

3) Let $g(x) = x^2$ and $f(x) = \sqrt{3x + 15}$.

a) Evaluate: $(f \circ g)(-5)$ = ______

b) Evaluate: $(g \circ f)(-5)$ = ______

c) Based on your answers to a) and b), what can you conclude about the order of composition? (Do not over think this.)

d) Write an equation for the composite function $g(f(x))$.

e) What is the domain of $g(f(x))$?

4) Let $h(x) = \sqrt{x-2}$ and $j(x) = \frac{1}{x^2-3}$.

a) Write an equation for the composition function $j\big(h(x)\big)$.

b) Laverna and Gentry are trying to determine the domain of $j\big(h(x)\big)$. Each of their reasoning is below. Whose solution do you think is correct? Explain your reasoning.

Laverna's solution:

I think the domain is $x \neq 5$ because division by 0 is undefined.

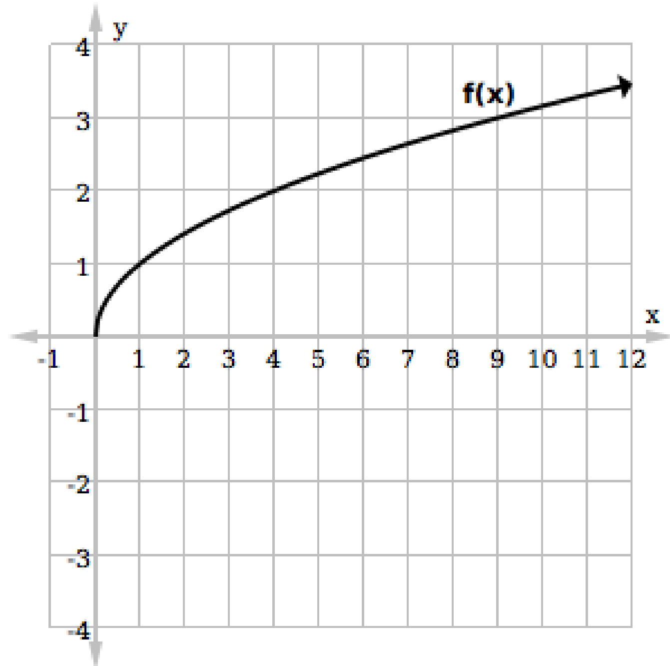
Gentry's solution:

I think the domain is $[2, 5) \cup (5, \infty)$ because you can't have a negative number under a square root, but also x cannot be 5, because you can't divide by 0.

<u>Summary</u>: Domain of a Composition

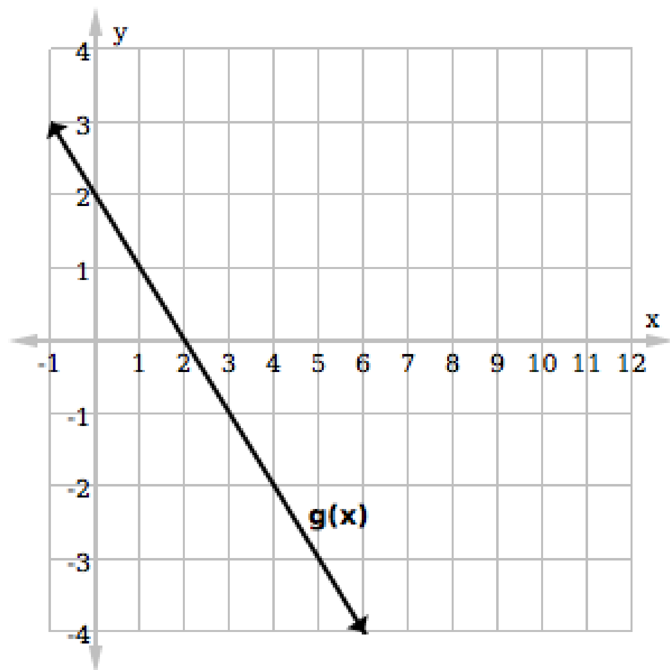
5) Estimate the following from the graphs provided.

a) $f(g(2))$



b) $g(f(9))$

c) $(g + f)(0) = g(0) + f(0)$



d) $(g \cdot f)(4) = g(4)f(4)$

e) $(f \circ g)(4) = f(g(4))$

f) What is the domain of $(f \circ g)(x) = f(g(x))$?

6) Students were asked to determine functions f and g, so that $f\left(g(x)\right) = H(x) = 2\sqrt{x^3 - 6}$. Select all of the students you think are correct. For each one, explain why they are correct or incorrect.
Note: Neither function can be the identity function.

A. $f(x) = 2\sqrt{x}$ and $g(x) = x^3 - 6$	B. $f(x) = 2x$ and $g(x) = \sqrt{x^3 - 6}$
C. $f(x) = 2\sqrt{x - 6}$ and $g(x) = x^3$	D. $f(x) = x$ and $g(x) = 2\sqrt{x^3 - 6}$

7) Let $D(x) = 3x^6 - 2$.

a) Write the equations of any functions h and k, so that $D(x) = h(k(x))$. Do not use $k(x) = x$ or $h(x) = x$.

$h(x)$=______________________________ $k(x)$=______________________________

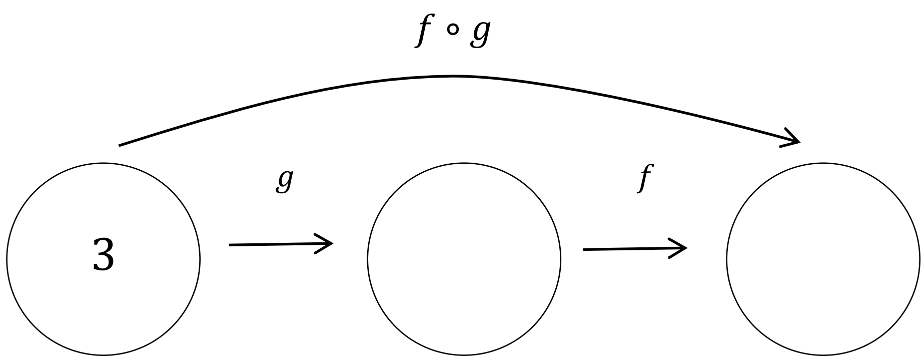
b) Use your answer from a) to determine $D(x) = h(k(x))$.

6.3: Inverse Functions

Learning Objectives
Together with your team: • Describe the relationship between a function and its inverse and interpret in a given context.

1) Consider the two functions $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$.

a) Fill in the arrow diagram for the composition of these two functions, $f(g(x))$, using the input $x = 3$.

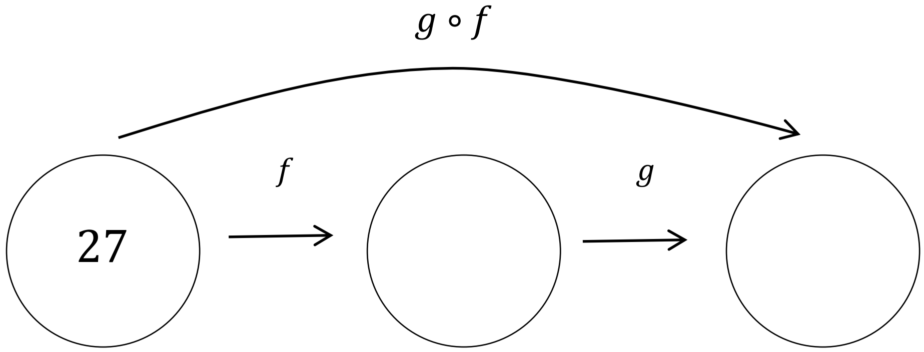


b) Fill in the table to calculate several more outputs of the composition function $f(g(x))$. Also, write an expression for the composition function in the blank.

x	$g(x) = x^3$	$f(g(x)) =$____________
1		
0		
−2		

2) Consider again the two functions $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$.

a) Fill in the arrow diagram for the *other* composition of these two functions, $g\big(f(x)\big)$, using the input $x = 27$.



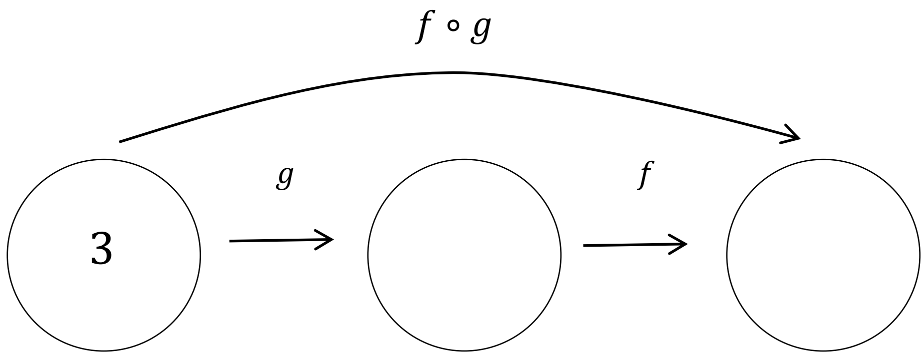
b) Fill in the table to calculate several more outputs of the composition function $g(f(x))$. Also, write an expression for the composition function in the blank.

$\boldsymbol{x}$	$\boldsymbol{f(x) = \sqrt[3]{x}}$	$\boldsymbol{g\big(f(x)\big) =}$ ____________
8		
0		
−1		

c) What relationship do you notice between the two functions $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$?

3) Consider the two functions $f(x) = \log(x)$ and $g(x) = 10^x$.

a) Fill in the arrow diagram for the composition of these two functions, $f(g(x))$, using the input $x = 3$.

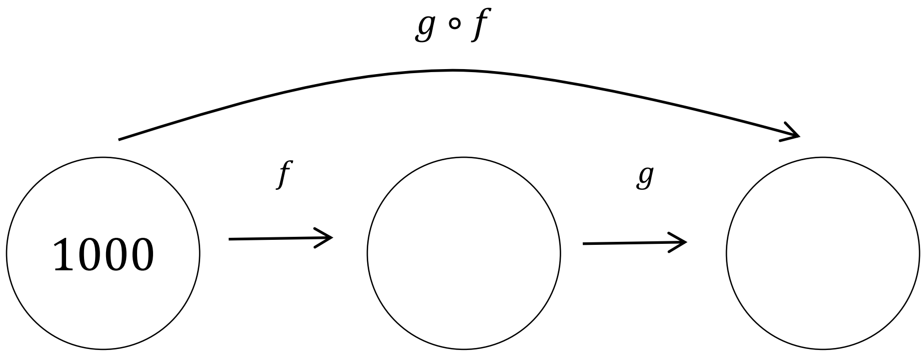


b) Fill in the table to calculate several more outputs of the composition function $f(g(x))$. Also, write an expression for the composition function in the blank.

$\boldsymbol{x}$	$\boldsymbol{g(x) = 10^x}$	$\boldsymbol{f(g(x))}$ = ____________
2	10^2	$\log(10^2) =$
1	10^1	
0	10^0	
−1	10^{-1}	
−2	10^{-2}	

4) Consider again the two functions $f(x) = \log(x)$ and $g(x) = 10^x$.

a) Fill in the arrow diagram for the *other* composition of these two functions, $g\big(f(x)\big)$, using the input $x = 1000$.



b) Fill in the table to calculate several more outputs of the composition function $g(f(x))$. Also, write an expression for the composition function in the blank.

$\boldsymbol{x}$	$\boldsymbol{f(x) = \log(x)}$	$\boldsymbol{g\big(f(x)\big)} =$____________
100	$\log(100)$	$10^{\log(100)} =$
10	$\log(10)$	
1	$\log(1)$	
$\boldsymbol{\frac{1}{10}}$	$\log\left(\frac{1}{10}\right)$	
$\boldsymbol{\frac{1}{100}}$	$\log\left(\frac{1}{100}\right)$	

c) What relationship do you notice between the two functions $f(x) = \log(x)$ and $g(x) = 10^x$?

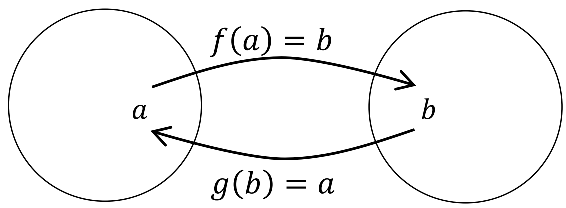
<u>Summary</u>: Relationship Between Logarithms and Exponentials

A definition of **inverse functions** we will use in this class:

Two functions f and g are inverses of each other if, when they are composed, the final output is the original input:

$$f(g(x)) = x \text{ and } g(f(x)) = x$$

We can also represent the relationship between a function f and its inverse g using an arrow diagram like this:



For Questions **5) – 12)**, match the equation with the inverse function that should be applied to both sides to solve the equation for x. Write the letter in the blank provided.

5) $e^x = 5$ ________ A. Base 10 Exponential

6) $\sqrt{x} = 7$ ________ B. Square Root

7) $\log(x) = 3.2$ ________ C. Square

8) $x^3 = 125$ ________ D. Cube

9) $\ln(x) = 2.4$ ________ E. Natural Log

10) $x^2 = 7$ ________ F. Log Base 3

11) $\sqrt[3]{x} = -6$ ________ G. Base e Exponential

12) $3^x = 11$ ________ H. Cube Root

Chapter 7
How Do We Solve Equations and Inequalities?

Chapter Learning Objectives

1. Apply inverse functions to solve basic radical, power, rational, exponential, and logarithmic equations symbolically.
2. Use "good enough" graphs to check or make sense of solutions found symbolically.
3. Determine if a found solution to an equation is extraneous.
4. Solve logarithmic, exponential polynomial, rational, radical and absolute value equations symbolically and graphically.
5. Use the test-point method and/or graphs to solve logarithmic, exponential, polynomial, rational, radical, and absolute value inequalities.
6. Relate the solutions of an equation to the graph of the associated function as well as to the context of the situation that the equation models.
7. Find the domain of a function, given its equation.

Chapter Overview

In Chapter 7, our focus will be on symbolic methods for solving equations and inequalities, starting with those that can be solved by "working backward," applying inverse functions. Not all equations can be solved using inverses alone, so we will introduce several other useful symbolic techniques.

Throughout this final chapter, keep your "good enough" graphing skills in mind! Whether you sketch by hand or use Desmos, graphs are helpful tools for checking solutions found symbolically.

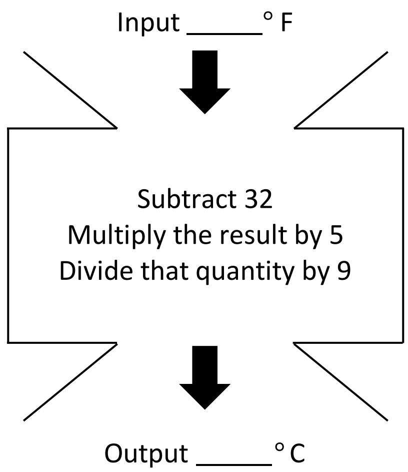
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Chapter 7 Contents

7.1: Warm Up

Student Name:

1) The diagram below is a "function machine," representing a function that takes a temperature in degrees Fahrenheit as the input, and gives the equivalent temperature in degrees Celsius as the output.



a) Room temperature is approximately 73.4° Fahrenheit.

Use the function machine to find room temperature in degrees Celsius.

73.4°F ≈__________°C

b) The boiling temperature of water is 100° Celsius.

What is the boiling temperature of water in degrees Fahrenheit?

100°C ≈__________°F

c) In general, how can you solve for a Fahrenheit temperature (input) given a certain Celsius temperature (output)? That is, make a list of directions to tell how to find the equivalent temperature in °F given a temperature in °C.

2) Rewrite the following exponential expressions in *radical* form.

a) $x^{\frac{1}{2}} =$ ________

b) $x^{\frac{3}{5}} =$__________

7.1: Working Backward

Learning Objectives
Together with your team: • Apply inverse functions to solve basic equations symbolically. • Use "good enough" graphs to check or make sense of solutions found symbolically. • Determine if a found solution to an equation is extraneous.

1) Use the inverse function to solve each of the following equations for x. (Hint: see your answers to problems **5) – 12)** in Lesson 6.3.) Write the *exact* solution, and also give a decimal rounded to the nearest tenth.

a) $e^x = 5$

b) $\sqrt{x} = 7$

c) $\log(x) = 3.2$

d) $x^3 = 125$

e) $\ln(x) = 2.4$

f) $x^2 = 7$

g) $\sqrt[3]{x} = -6$

h) $3^x = 49$

2) A team of students in a College Algebra class got stuck on solving $3^x = 49$.

Who is on the right track? Select all that apply.

A. "x must be between 3 and 4 because $3^3 = 27$ and $3^4 = 81$ and 49 is in between 27 and 81."

B. "We should graph $y = 3^x$ and $y = 49$ in Desmos and see where the graphs intersect."

C. "x must be $\frac{49}{3} \approx 16.33$."

D. "x is probably around 3.5 because $3^{3.5} \approx 46.77$ which is close to 49."

E. "x must be $\log_3(49)$ but I'm not sure how to calculate the log base 3 of a number."

Most calculators will only compute common logarithms (base 10) and natural logarithms (base e), so if you want to approximate the value of $\log_3(49)$ using your calculator, you will need one of the Change of Base Formulas, given below.

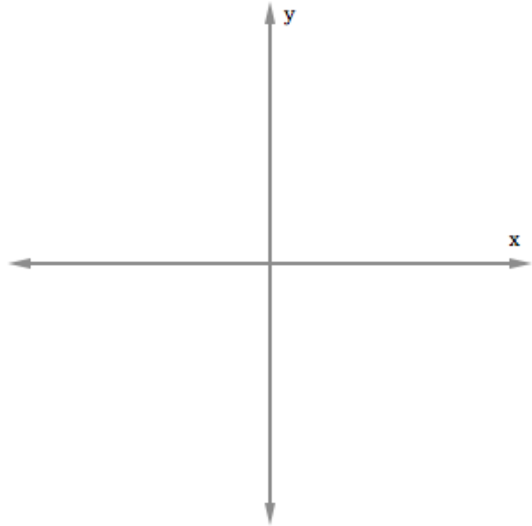
Change from base a log to common log	Change from base a log to natural log
$\log_a(b) = \dfrac{\log(b)}{\log(a)}$	$\log_a(b) = \dfrac{\ln(b)}{\ln(a)}$

3) Use a calculator and a Change of Base Formula to compute to the nearest tenth: $\log_3(49) \approx$__________

Summary: Solving $b^x = c$	
Inverse Method	Base 10 Method

4) Let $f(x) = (x+1)^2 - 20$.

a) Sketch a "good enough" graph of f.



b) How can you use your graph to determine <u>how many</u> input(s) x result in an output of $f(x) = -4$?

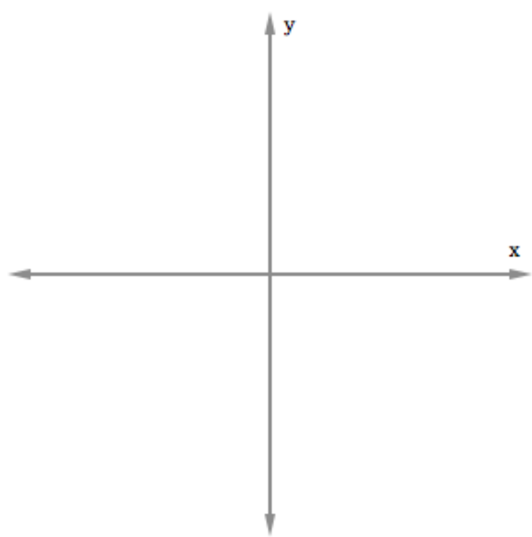
c) Next, work backward (applying inverse functions) to solve the equation for x:

$$f(x) = -4$$

d) Check your answer from c). Show your work.

5) Consider $p(t) = \sqrt{t+1}$.

a) Solve $p(t) = -4$ by working backward (applying inverse functions.)



b) Check your answer from a) by sketching a "good enough" graph of $p(t)$.

c) What does your graph reveal about the solution we find by applying inverse functions?

Summary: Extraneous Solutions

6) What would be your FIRST step to solve the following logarithmic equation for x?

$$-2\ln(x-5) = -4?$$

A. Divide both sides by -2

B. Add 5 to both sides

C. Add 4 to both sides

D. Apply the power rule to make -2 an exponent

E. Divide both sides by ln

F. There is no solution because a natural log cannot equal a negative number.

7) Let $f(x) = -2\ln(x-5)$.

a) Apply inverse functions to solve for the input(s) x that results in the output $f(x) = -4$.

b) Check your solution using your preferred method.

8) The salinity of the oceans changes with latitude and with depth. In the tropics, the salinity increases on the surface of the ocean due to rapid evaporation. In the higher latitudes, there is less evaporation and rainfall causes the salinity to be less on the surface than at lower depths.

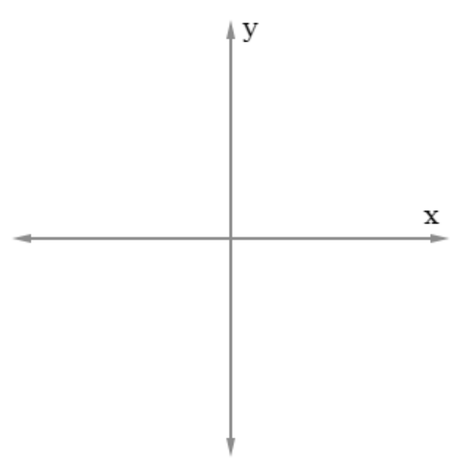
The function $S(x)$ models salinity to depths of 1000 meters at a latitude of 57.5 degrees N (we are approximately at 45 degrees N). The input x is the depth in meters and the output $S(x)$ is in grams per kilogram of seawater.

$$S(x) = 31.5 + 1.1\log(x+1)$$

a) Solve the equation $S(x) = 33$.

b) Interpret your answer to a) in the context of the given situation.

c) Use your solution to a), together with a "good enough" graph, to solve the inequality, $S(x) \geq 33$. Give your answer in interval notation.



Apply inverse functions to find all solutions for the equations in **9) – 12)**. Make sure to check your answers symbolically or graphically!

9) $-4 = -e^{3x+7} + 6$

10) $(w+3)^{\frac{3}{4}} + 2 = 66$

11) $\frac{4}{3} = \frac{1}{x} - \frac{1}{3}$

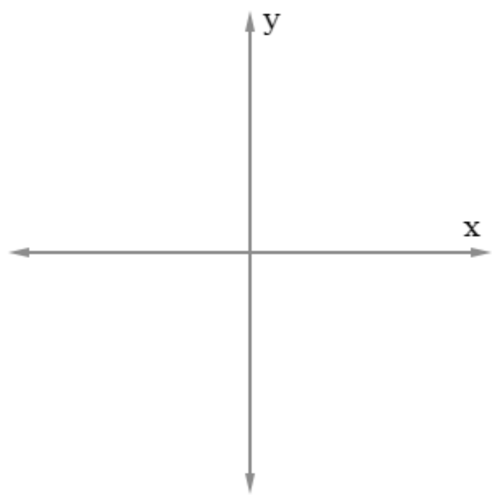
12) $0 = \sqrt{6^{3x-5}} - 10$. (Hint: you may need the Change of Base Formula.)

13) Consider the function $f(x) = \log(x - 5)$.

a) First, work backward to solve this equation for x:

$$\log(x - 5) = 0$$

b) Now, sketch a "good enough" graph of f and include your solution to a) on the graph.



c) Use your graph from b) to solve this *inequality* for x, $\log(x - 5) > 0$. Write your answer in interval notation.

7.2: Warm Up

Student Name:

1) Explain *why* the following equation cannot be solved using the working backward technique from Lesson 7.1.

$$75 = -16t^2 + 80t + 4$$

2) Use the Quadratic Formula to solve for t.

$$75 = -16t^2 + 80t + 4$$

3) Which of the following approaches do you think could help solve the inequality given below? Select all that apply.

$$(x + 2)^3(x + 5) \geq 0.$$

A. Multiply out the factors.

B. Set each factor greater than or equal to 0, $(x + 2)^3 \geq 0$ and $(x + 5) \geq 0$, and solve the individual inequalities

C. Find the zeros of $y = (x + 2)^3(x + 5)$ and test x-values to the left and right of each zero

D. Sketch a graph of $y = (x + 2)^3(x + 5)$ to determine where it is on or above the x-axis.

7.2: Solving Inequalities

Learning Objectives
Together with your team: • Use the test-point method and/or graphs to solve inequalities. • Relate the solutions of an equation or inequality to the graph of the associated functions as well as to the context of the situation that the equation models.

1) You serve as the Health and Safety Engineer during the New Year's Eve celebration in Mt. Hood, Oregon and must make sure to meet city fire safety regulations.

- Fire safety regulations require a firework shell to explode at a height greater than 75 feet above the ground.
- The fireworks will be launched from the available 4-foot platform.
- The height, W (in feet), of the Willow Firework shell t seconds after it is launched is given by the firework manufacturer to be:

$$W(t) = -16t^2 + 80t + 4$$

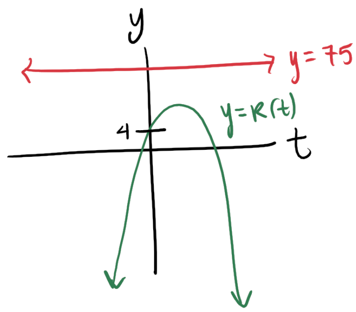
a) To determine when the Willow Firework will meet the city fire safety regulation your co-worker suggests solving the inequality $-16t^2 + 80t + 4 > 75$, but isn't quite sure how. You agree that your co-worker is on the right track and want to help. Suggest a first step.

b) Solve the inequality from a). (Look back at your work on the 7.2 Warm Up!)

c) Based on your answer to b), write up your conclusions for the City Manager related to the safety of the Willow Firework.

2) The height R (feet) of the shell of a **Ring firework**, t seconds after it is launched is modeled by the function $R(t) = -16t^2 + 60t + 4$.

Another co-work sketched this "good enough" graph of R.



a) What does this graph tell you about the situation?

b) If you were using <u>symbolic methods</u> (not using a graph) to solve the following inequality for t, how would you know *it has no solution*?

$$-16t^2 + 60t + 4 > 75$$

3) Recall, in the last part of the 7.2 Warm Up, you selected methods that could help solve the inequality:

$$(x+2)^3(x+5) \geq 0$$

a) Now, use one of your selected methods to solve the inequality.

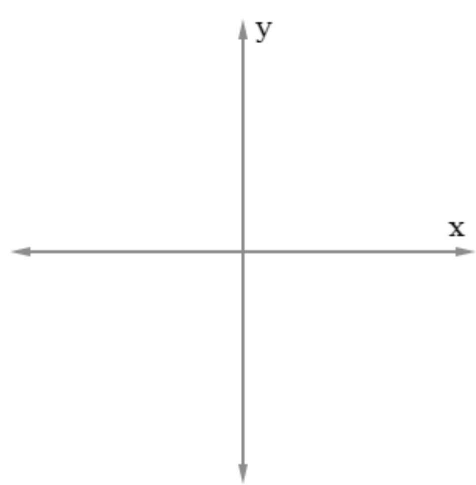
b) Use another of your selected methods to check your solution set in a).

4) Solve $-2(x+5)(x+4)^2(x-6) > 0$ using any method you prefer. Write your answer in interval notation.

5) Find the solutions to the inequality $6x^2 + x - 15 > 0$.

6) Using a "good enough" graph, solve the inequality for x.

$$\frac{x+2}{x-5} < 0$$



7) Consider $p(t) = \sqrt{t+1}$.

a) Solve $p(t) < 7$ using any method you prefer.

b) Check your answer from a) by sketching a "good enough" graph of $p(t)$.

Summary: Solving Inequalities

7.3: Warm Up

Student Name:

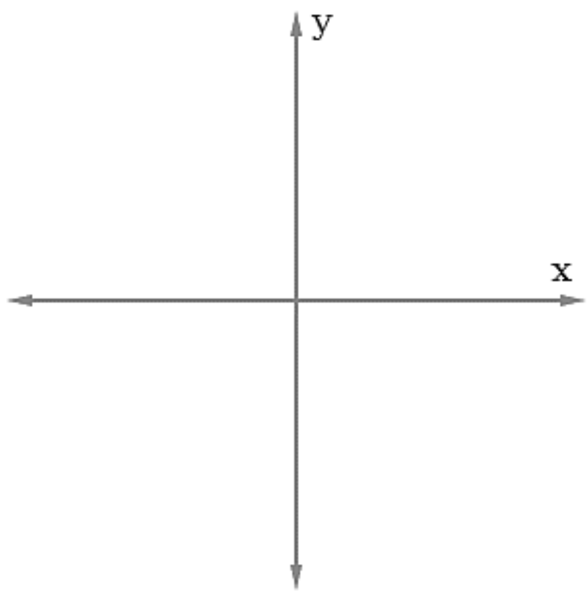
1) Fill in the blanks for each absolute value equation: $|\underline{\quad\quad}| = 5$ or $|\underline{\quad\quad}| = 5$

2) Consider $|x - 3| = 5$

a) Solve for x.

b) $|x - 3| = 5$ can be represented by the intersection between the graph of the function $j(x) = |x - 3|$ and the horizontal line $y = 5$.

Draw a graphical representation of this on the axes below and label your solutions from a).



c) Use your graph to solve $|x - 3| \geq 5$. Write your answer in interval notation.

7.3: When You Need More than Working Backward

Learning Objectives
Together with your team: • Solve equations and inequalities symbolically that cannot be solved by working backward alone. • Determine if a solution to an equation is extraneous

1) When Dasani fills their 20 oz. water bottles, the water bottle is considered defective if the volume in the bottle varies by more than 0.109 oz.

a) You are the Quality Control Manager for Dasani and the Legal Department asks you come up with a function in which they can input the volume of any water bottle and the output will tell them if the water bottle is defective or not.

b) Determine the acceptable volumes of Dasani water bottles to report to your boss. Be sure to interpret these values in the context of this situation.

c) Sketch a visual representation that could help explain this function to your boss.

2) Some students were asked to solve $-5|x-7|+6 = -9$ for x. They have different ideas on what their first step should be.

With which student do you most agree?

A. Elvis thinks they should distribute the -5.

B. Anna Maria thinks they should add 9 to both sides to set the equation equal to 0.

C. Deshawn asks, shouldn't we try to isolate the absolute value?

D. Radu says there is no solution because an absolute value cannot equal a negative number.

E. Connor thinks he remembers something about splitting the equation up into separate equations, one equal to 9 and one equal to -9.

3) Solve $-5|x-7|+6 = -9$

4) Solve $-5|x-7|+6 \geq -9$ for x. Write your answer in interval notation.

Summary: Solving Absolute Value Equations and Inequalities

5) Becka is solving the equation for x.

$$\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x-2}$$

$$x(x-2)\left(\frac{10}{x(x-2)} + \frac{4}{x}\right) = \left(\frac{5}{x-2}\right)x(x-2)$$

$$\cancel{x(x-2)}\left(\frac{10}{\cancel{x(x-2)}}\right) + \cancel{x}(x-2)\left(\frac{4}{\cancel{x}}\right) = \left(\frac{5}{\cancel{x-2}}\right)x(\cancel{x-2})$$

$$10 + 4(x-2) = 5x$$

$$10 + 4x - 8 = 5x$$

$$\boxed{2 = x}$$

Is $x = 2$ the solution to $\frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2}$? Explain.

6) Solve the equation for x.

$$\frac{2}{x^2 + 10x + 24} = \frac{4}{x+4} - \frac{5}{x+6}$$

Be sure to check your answer(s).

7) Solve $\sqrt{2z+1} - 1 = 3z$ for z. Be sure to check your answer(s).

8) In Chapter 5, Josie was trying solve this equation for x: $\log_5(x+4) = 1 - \log_5(x+8)$.
Her first step is shown here:

$$\log_5(x+4) = 1 - \log_5(x+8)$$

$$\log_5(x+8) + \log_5(x+4) = 1$$

a) Help Josie solve for x.

b) Check your answer(s) from a).

9) This graphical representation of the previous equation shows that one of the solutions was extraneous.

https://www.desmos.com/calculator/aqete33dmc

Let's explain why one of the solutions found in **8)** works in $\log_5((x+8)(x+4)) = 1$ but not in the original equation $\log_5(x+4) = 1 - \log_5(x+8)$.

10) Solve for x: $\ln(4-x) - \ln(x+1) = \ln(2)$

11) When solving equations, which types of functions can result in extraneous solutions? Select all that apply.

A. Polynomial
B. Rational
C. Radical
D. Absolute Value
E. Logarithmic
F. Exponential