7.5

Carlos Emmanuel Anguiano Pedraza

Gilberto Alexander Zing Pérez

5C

Análisis de señales

Erika Margarita Ramos Michel

I

1: $(3+2i)+(-7-i) = 3-7+2i-i$

$=(3-7)+(2i-i)$

$=-4+i$

$R=-4+i$

las factorizaciones se realizan cuando por ejemplo, tenemos divisiones y deseamos simplificar, situación que no existe en este ejercicio, sólo debían realizarse asociaciones para sumar

2: $(-7-i)-(8-6i) \rightarrow (-7-i)-2(4-3i)$

$=-2(-3i+4)=6i-8 \rightarrow -7-i+(6i-8)$

$=-7i-i-8+6i$

$=5i-15$

$=5(i-3)$

$R=5(i-3)$

$=(-7-8)+i(-1+6)$

$=-15+5i$

3: $(5+3i)+((-1+2i)+(7-5i))$

$=(5+3i)+[(-1+7)+(2i-5i)]$

$=(5+3i)+[6-3i]$

$=(5+6)+(3i-3i)$

$=11$

$R=11$

4: $(2-3i)[(-3+2i)+(5-4i)] \rightarrow (-3i+2)(-3i+5-4i)$

$=(-3i+2)(-3i+2)$

$=(-3i+2)^2$

$=4-6i-6i-9^2$

$=-5-12i$

$R=-5-12i$

5: $\frac{3-2i}{1+i} \rightarrow \frac{(-2i+3)(-i+1)}{(i+1)(-i+1)} \rightarrow \frac{(3-3i+2i-2)}{(1-i+i+1)} = \frac{1-5i}{2}$

$R=\frac{(-5i+1)}{2}$

6) $\frac{5+5i}{3-4i} + \frac{20}{4+3i} \to \frac{5(i+1)}{-4i+3} + \frac{20}{4+3i} \to \left[\frac{5(i+1)(4i+3)}{(-4i+3)(4i+3)}\right] + \frac{20}{3i+4}$

$= \frac{5(i+1)(4i+3)}{9+12i-12i+16} + \frac{20}{3i+4} = \frac{5(i+1)(4i+3)}{25} + \frac{20}{3i+4}$

$= \frac{(i+1)(4i+3)}{5} + \frac{20}{3i+4} = \frac{3+4i+3i-4}{5} + \frac{20}{3i+4} = \frac{7i-1}{5} + \frac{20}{3i+4}$

$= \frac{7i-1}{5} + \left[\frac{20(-3i+4)}{(3i+4)(-3i+4)}\right] = \frac{7i-1}{5} + \left[\frac{20(-3i+4)}{16+12i-12i+9}\right]$

$= \frac{7i-1}{5} + \left[\frac{20(-3i+4)}{25}\right] = \frac{7i-1}{5} + \frac{4(-3i+4)}{5} = \frac{7i-1}{5} + \frac{-12i+16}{5}$

$= \frac{-5i+15}{5} \to \frac{5(-i+3)}{5} \to (-i+3)$

$R = (-i+3)$

7) $\frac{3i^{30} - i^{19}}{2i-1} \to 3i^{30} = 205,891,132,094,651$

$i^{19} = 411,782,264,189,297i$

$3i^{30} \neq (3i)^{30}$

$R = \frac{205,891\,132\,094\,651}{5} + \frac{411\,782\,264\,189\,297i}{5}$

II

1) $|3z_1 - 4z_2| = |3(2+i) - 4(3-2i)|$

$= |(6+3i) + (8i-12)|$

$= |11i - 6|$

$= \sqrt{(-6)^2 + 11^2}$

$= \sqrt{36 + 121}$

$= \sqrt{157}$

$R = \sqrt{157}$

o

$157^{1/2}$

2) $z_1^3 - 3z_1^2 - 4z_1 - 8$ $(2+i)$

$\to (2+i)^3 - 3(4i+3) - 4(2+i) - 8$

$= ((i+2)(i+2)^2) - 3(4i+3) - 4(2+i) - 8$

$= (i+2)(4i+3) - 3(4i+3) - 4(2+i) - 8$

$= (6+8i+3i-4) - 3(4i+3) - 4(2+i) - 8$

$= (11i+2) - 3(4i+3) - 4(2+i) - 8$

$= (2-8) + (11i - (9+12i) - (8+4i))$

$= -5i - 23$

$R = (-5i - 23)$

Cuidar que al tomar la foto, se vea bien el texto :S

$(\bar{z}_3)^4$?

3) $(z_3)^4$ $\quad -1/2 + 3/\sqrt{2}\, i$

$= -\frac{1}{2} + \frac{3}{\sqrt{1+2i+i^2}} \to -\frac{1}{2} + \frac{3}{\sqrt{(1+i)^2}} = -\frac{1}{2} + \frac{3}{1+i}$

$= \left[\frac{3}{2} - \frac{3i}{2}\right] - \frac{1}{2} = \frac{3-3i-1}{2} = \frac{(-3i+2)}{2}$

$R = \frac{(-3i+2)}{2}$

4) $\left|\dfrac{2z_2+z_1-5-i}{2z_1-z_2+3-i}\right|^2$ $z_1=2+i$ $z_2=3-2i$

Cuidar el uso de paréntesis. Errores por no utilizarlos correctamente

$=\left|\dfrac{2(3-2i)+2+i-5-i}{2(2+i)-(3-2i)+3-i}\right|^2 = \left|\dfrac{(-3i+2)}{(3i+4)}\right|^2$

$=\left|\dfrac{(-3i+2)(-3i+4)}{(3i+4)(-3i+4)}\right| = \left|\dfrac{8-6i-12i-9}{16-12i+12i+9}\right|^2 = \left|\dfrac{(-18i-1)}{25}\right|$

$=\left|\dfrac{(-(18i+1))}{25}\right|^2 = \left|\dfrac{+1}{25}\right|^2 + \left|18i\right|^2 = \sqrt{\dfrac{1^2+18^2}{25}}$

$=\sqrt{\dfrac{325}{25}}$ $\quad R=\sqrt{\dfrac{325}{25}}$ $\quad =1$

7

III $3x + 2iy - 5x + 5y = 7 + 5i$

$= 3x + 5y = 7 \quad , \quad 2iy - ix = 5i$

$= 5x = \dfrac{7 - 5y}{3} \qquad = -ix = 5i - 2iy$

? $\qquad = x = -5i + 2iy$

$\rightarrow 3\left(\dfrac{7 - 5y}{3}\right) + 5y = 7$

$\rightarrow 2iy - (-5i + 2iy) = 5i$

$\rightarrow 7 - 5y + 5y = 7 \qquad \rightarrow 2iy + 5i - 2iy = 5i$

$= 2iy - 2iy = 5i - 5i$

$0 = 0 \qquad y = 0$

$= y = 0$

$2i(0) - ix = 5i$

$-ix = 5i$

$ix = -5i$

$3x = 7 = x = \dfrac{7}{3}$

$R = x = \dfrac{7}{3}$ y $y = 0$

Tuvieron problemas al resolver las ecuaciones

$x = 1$

$y = 2$

IV

$|z_1 z_2| = |z_1||z_2|$

① $(x_1 + y_1) + (x_2 + y_2) = x_1 + iy_1 + x_2 + iy_2$

② $(x_1 + x_2) + (y_1 + y_2) = (x_1 - 5y_1) + (x_2 - 5y_2)$

③ $(x_1 + x_2) - 5(y_1 + y_2) = (x_1 + x_2) - 5(y_1 + y_2)$

faltó trabajar la comprobación como se los indiqué en clases

Haciendo el procedimiento correspondiente, vemos que se igualan los valores

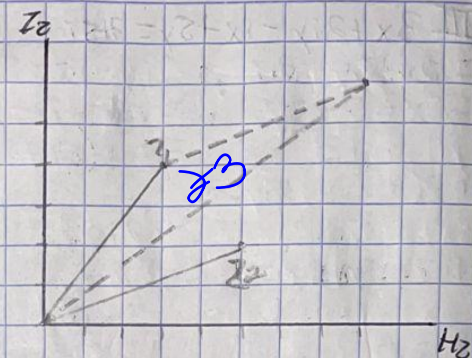
$(x + x_2) - 5(y_1 + y_2) = (x_1 + x_2) - 5(y_1 + y_2)$

V z_1 z_2

1) $(3+4i) + (5+2i) = z_3$

$= (3+5) + (2i+4i)$

$= 8+6i$ ✓



2) $(6-2i) - (2-5i)$ ✓

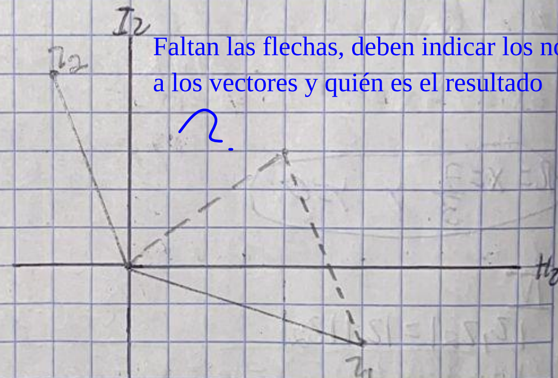
$= (6-2i) - (2-5i)$

$= \underbrace{6-2i}\ \underbrace{-2+5i}$

$= 3i+4$

Faltan las flechas, deben indicar los nombres a los vectores y quién es el resultado

?

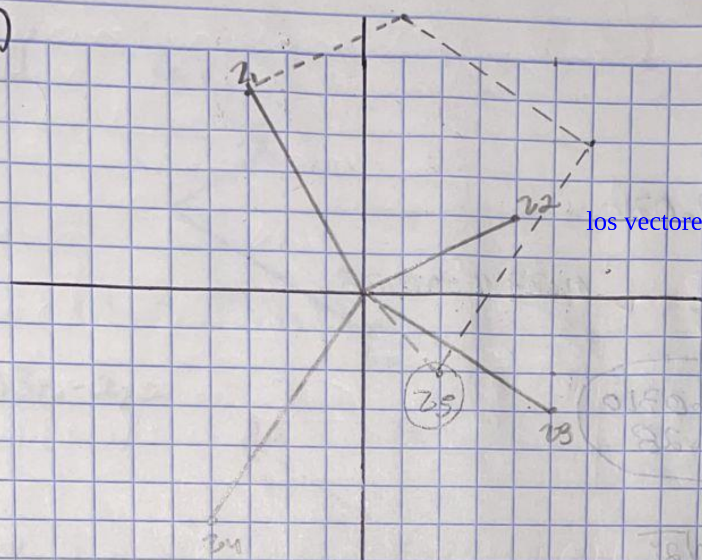


3) $(-3+5i) + (4+2i) + (5+-3i) + (-4-6i)$

$= -3+5i+4+2i+5-3i-4-6i$

$= -2i+2$ ✓

3)



los vectores no tienen sentido

VI

1) $2+2\sqrt{3i}$

$= |z| = \sqrt{2^2 + 2\sqrt{3i}^2}$

$= \sqrt{4+6}$

$= \sqrt{10}$

$= 3.1622 \rightarrow 3.17$

$4 \angle 60^\circ$

$\cancel{2}$

$\theta = \tan^{-1} \dfrac{2\sqrt{3i}}{2} = \tan^{-1}\left(\frac{4}{4} - i\sqrt{3}\right) = 0.306056$

$R = |z| = 3.17$

$\theta = 0.3060$

2) $-5+5i$

$|z| = \sqrt{-5^2+5^2}$

$= \sqrt{25+25}$

$= \sqrt{50} \rightarrow 7.0710$

$5\sqrt{2}\angle 135°$

$\theta = \tan^{-1}\frac{5}{5} \rightarrow 0.2346 \rightarrow 0.28$

R= $|z| = 7.0710$?

$\theta = 0.28$

Detalles en álgebra y cálculo de ángulos

3) $-\sqrt{6} - \sqrt{2}i$

$|z| = \sqrt{-\sqrt{6}^2 - \sqrt{2}^2}$

$= \sqrt{6-2}$

$= \sqrt{4}$

$|z| = 2$

$\theta = \tan^{-1}\frac{-\sqrt{6}}{-\sqrt{2}} = 0.8366$

R= $|z| = 2$

$\theta = 0.8366$

4) $3i$

$|z| = \sqrt{3^2}$

$|z| = \sqrt{9}$

$|z| = 3$

$\theta = \tan^{-1}\frac{\cancel{0}}{3} = 0 \rightarrow \frac{\pi}{2}$ 3/0

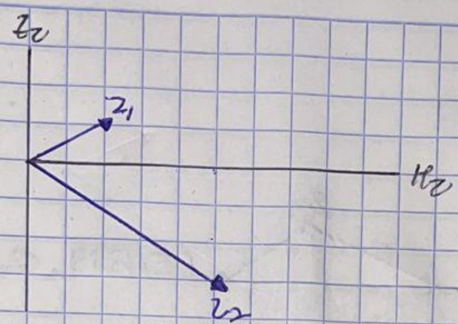
R= $|z| = 3$

$\theta = \frac{\pi}{2}$

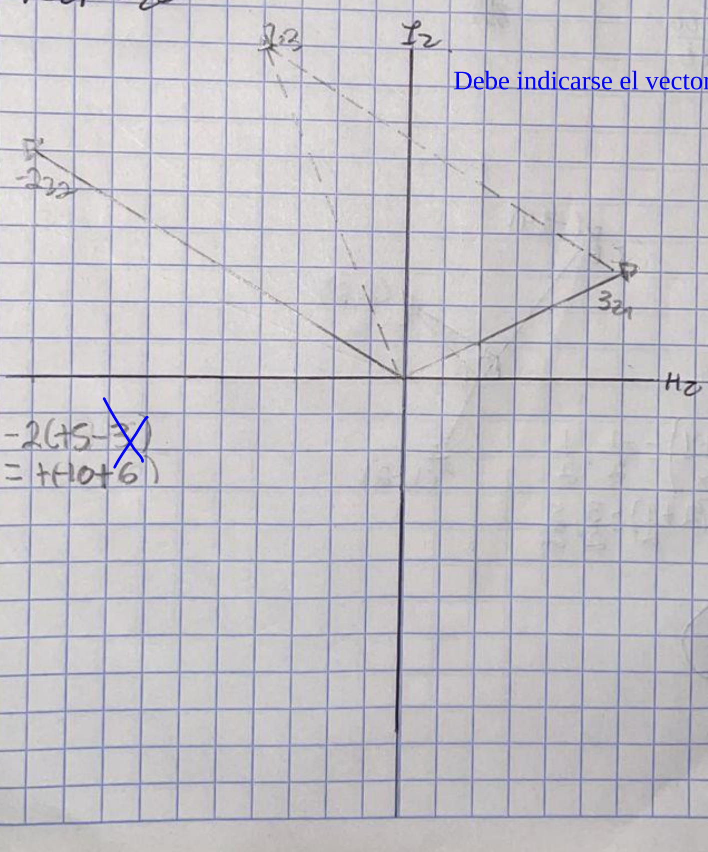
Norma

No se pidió este ejercicio

VII



1) $3z_1 - 2z_2$



Debe indicarse el vector resultante de la operación

$-2(+5-\cancel{3})$
$= +(10+6)$

2)

$\frac{1}{2}z_2+\frac{5}{3}z_1$

$0.5(5,-3)$

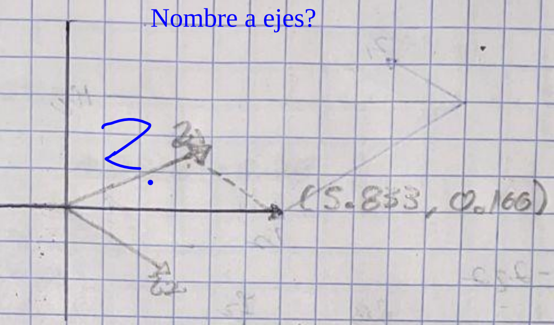
$=(2.5,-1.5)$

$5/3(2,-1)$

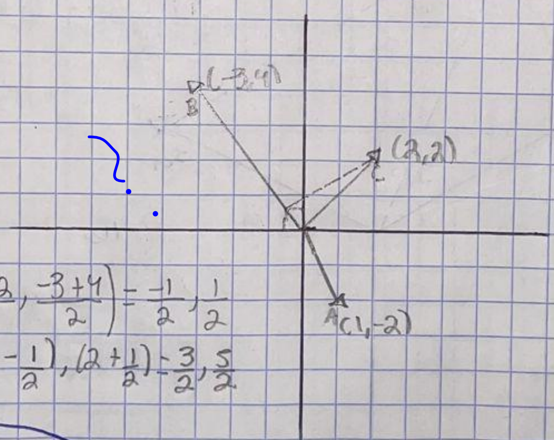
$=(3.333,-1.666)$

$$\begin{array}{r} 2.5 \\ +3.333 \\ \hline 5.833 \end{array} \qquad \begin{array}{r} -1.5 \\ +1.666 \\ \hline 0.166 \end{array}$$

Nombre a ejes?



VIII



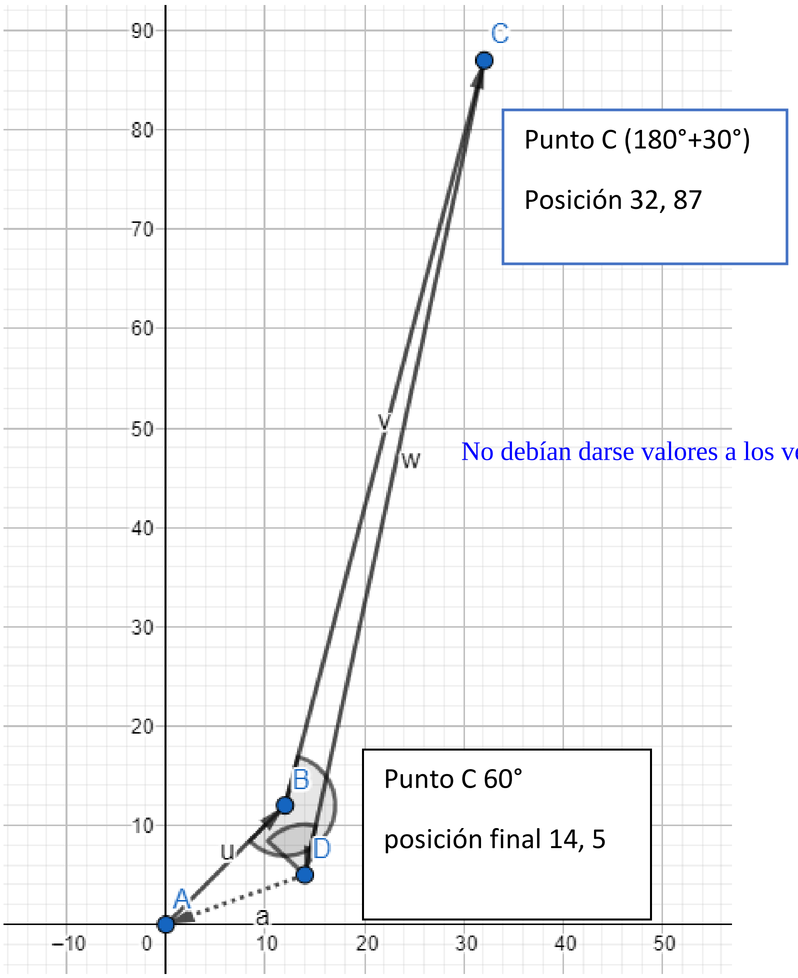
$m=\left(\frac{1-2}{2},\frac{-3+4}{2}\right)=\frac{-1}{2},\frac{1}{2}$

$m\vec{B}=\left(2-\frac{1}{2}\right),\left(2+\frac{1}{2}\right)=\frac{3}{2},\frac{5}{2}$

$R=-\frac{1}{2},\frac{1}{2}$

Norma

IX



No debían darse valores a los vectores

La razón de porque usé el 180+30 es porque en GeoGebra no me dejaba poner el ángulo de 30° así que tracé una línea recta de 180° y le agregué los 3 grados que pedía con las 20 unidades que se desplaza

X Si z es la variable compleja, encontrar:

1. z^n $\quad z = z^n \quad z^n = (\cos\theta + i\,\mathrm{sen}\,\theta)^x$

$1 = n \quad z^n = \cos(\theta + 2k\pi) + i\,\mathrm{sen}(\theta) + \mathrm{sen}\,2k\pi)$

$n = 1 \quad z = (\cos 2k\pi + i\,\mathrm{sen}\,2k\pi)^{1/n}$

$z^n = (|z| e^{j\theta})^n \quad z = e^{(i 2k\pi/n)}$

$= |z|^n e^{j\theta n}$

$z = \cos((2k\pi)/n) + i\,\mathrm{sen}(2k\pi)$

2. $z_1 \cdot z_2 \quad z_1 = 0 \quad z = r(\cos(\theta) + i\,\mathrm{sen}(\theta)) = re^{i\theta}$

$z_2 = 0$

$r = |z|, \theta$

$|z_1 z_2| = |z_1||z_2|\, e^{j(\theta_1 + \theta_2)}$

3. $\frac{z_1}{z_2} = \quad |z_1| = \sqrt{x^2+y^2}$

$|z_2| = \sqrt{p^2+q^2}$

$= \sqrt{\frac{(x^2+y^2)(p^2+q^2)}{(p^2+q^2)(p^2+q^2)}}$?

$= \frac{\sqrt{x^2+y^2}}{\sqrt{p^2+q^2}}$

$= \frac{|z_1|}{|z_2|} e^{j(\theta_1 - \theta_2)}$

XII. Demostrar que:

1. $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$= \frac{e^{\theta} + e^{\theta}}{2}$ $e^{\theta} = 1$

$= \frac{1+1}{2}$

$= \frac{2}{2}$

$\frac{e^{j\theta} + e^{-j\theta}}{2} = 1$

XIII. Demostrar que:

1. $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$

$\sin^3\theta = 0^3 = 0 = \frac{3}{4}\sin(0)$ $-\frac{1}{4}\sin 3\theta = \sin^3\theta = 0^3 = 0$

$= 0 \cdot \frac{3}{4} = 0$ $= 0 \cdot \frac{1}{4} = 0$

$= -0 + 0$

$= 0$

$\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin^3\theta$

$= \frac{3}{4} \cdot \theta - \frac{1}{4} \cdot \theta^3$

$= 0$

XV. Hallar el valor numérico de cada una de las siguientes expresiones:

1. $[3(\cos 40° + j\sin 40°)][4(\cos 80° + j\sin 80°)]$

$= \left[3\left(\cos\left(\frac{4\pi}{9}\right) + j\sin\left(\frac{2\pi}{9}\right)\right)\right]\left[4\left(\cos\left(\frac{4\pi}{9}\right) + j\sin\left(\frac{4\pi}{9}\right)\right)\right]$

$\cos(x) + j\sin(x) = e^{jx}$

$= \left[3e^{j\frac{2\pi}{9}}\right)\left(4e^{j\frac{4\pi}{9}}\right) = 3e^{j\frac{2\pi}{9}} = (-1)^{\frac{2}{9}} = 3\cdot 4(-1)^{\frac{2}{9}}e^{j\frac{4\pi}{9}}$

$= e^{j\frac{4\pi}{9}} = (-1)^{\frac{4}{9}} = 3\cdot 4(-1)^{\frac{2}{9}}(-1)^{\frac{4}{9}}$

$= 3(-1)^{2/9}\cdot 4(-1)^{4/9} = 12(-1)^{2/9}(-1)^{4/9} = 12(-1)^{\frac{2}{9}+\frac{4}{9}}$

$= 12-(1)^{2/3} = 12$ $\quad (3e^{j40°})(4e^{j80°}) = 12e^{j120°}$

2. $\frac{(2\angle 15°)^7}{(4\angle 45°)^3} = \frac{(2^7(\angle 15°)^7}{(4^3\angle 45°)^3} = \frac{(2^7(\angle 15°)^7}{(2^6\angle 45°)^3} = \frac{(2^{7-6}\angle 15°)^7}{(\angle 45°)^3}$

$= \frac{(2(\angle 15°)^7}{(\angle 45°)^3} = \frac{(2\angle 15°)^7}{\frac{\angle 180°^3}{4^3}} = \frac{2\left(\frac{\angle 480°^7}{(2)^7}\right)}{\frac{\angle 480°^3}{4^3}}$

$= \frac{\frac{\angle 180°^7}{279936}}{\angle 180°^3} = \frac{\angle 180°^7}{50388480°^3}$

$= \frac{180°^{7-3}}{279936} = \frac{180°^4}{279936} = 0.00034$

3. $\left[\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^{10}$

2

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{2\sqrt{3}i-2}{4}$

$= \frac{2\sqrt{3}i-2}{4} = \frac{-2+2\sqrt{3}i}{4} = \frac{-2}{4} + \frac{2\sqrt{3}}{4}i$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{10} = \left(\frac{-1+\sqrt{3}i}{2}\right)^{10}$

$= \frac{(-1+\sqrt{3}i)^{10}}{2^{10}} = \frac{(-1+\sqrt{3}i)^{10}}{1024}$

$= \frac{512\sqrt{3}i-512}{1024} = 512(\sqrt{3}i-1)$

$= \frac{512(\sqrt{3}i-1)}{512\cdot 2} = \frac{\sqrt{3}i-1}{2}$

$= \frac{-1+\sqrt{3}i}{2} = \frac{1}{2}+\frac{\sqrt{3}}{2}i$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

XVII Encontrar la raíz $(-1+i)^{1/3}$ y localizarla gráficamente.

$\sqrt[3]{-1+i}$

$$(-1+i)^{1/3} = i\,\mathrm{Im}(\sqrt[3]{-1+i}) + \mathrm{Re}(\sqrt[3]{-1+i})$$

$$= \sqrt[6]{2}\left(\cos\left(\frac{\pi}{4}\right) + i\,\mathrm{sen}\left(\frac{\pi}{4}\right)\right)$$

$$= 6\sqrt{2}\; e^{(i\pi)/4}$$

$$\approx 0{,}79370 + 0{,}79370i$$

2