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5C

Análisis de señales

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I

1) $w = z + z^2$

$= (x + jy) + (x + jy)^2$

$= x + jy + [x^2 + 2xjy + (j)^2 y^2]$

$= x^2 + x + jy + 2xjy + jy^2$ (Agrupar)

$f(x+jy) = (x^2 + x + y^2 + y) + 2xjy$

$f(z) = (x^2 + x + y^2 + y) + 2xjy$

$u(x,y) = x^2 + x - y^2 + y$

$v(x,y) = 2xy^2 +$

$R = v(x,y) = 2x^2y^2$

2) $w = z - 1$

$= (x + jy) - 1$

$f(x+jy) = (x + y - 1) + jy$

$f(x+jy) = (x + y - 1) + j$

$u(x,y) = x + y - 1$

$v(x,y) = \cancel{xy - 1}\ y$

$R = u(x,y) = xy - 1$

3) $w = e^{-z}$

$= e^{-(x+jy)}$

$= e^{-x}[\cos(y) + j\,sen(y))$

$= e^{-x}\cos(y) + sen(y)$

$R = e^{-x}\cos(y)\ j\,sen(y)$

Deben cuidar sus operaciones, al tener errores no obtienen adecuadamente la parte real de la imaginaria

4) $w = e^{z^2}$

$= e^{(x+jy)^2} = e^{x^2 + 2xjy + jy^2} = e^{x^2 + 2xjy}[(\cos(y))^2 + sen(y)]^2$

$= e^{x^2 + 2xjy}\cos(y)^2 + sen(y)^2$

$R = e^{x^2 + 2xy}\cos(y)^2 + sen(y)^2$

$e^{x^2 - y^2}\cos(2xy)$

$+ sen(2xy)$

5) $\cosh(z-i) = \cosh z = \frac{e^{z}+e^{-z}}{2}$

faltó trabajar z-i

$= \cosh = \frac{e^{(x+jy)-i}+e^{-(x+jy)-i}}{2} = \frac{e^{x}e^{y}+e^{-x}e^{-y}}{2}$

$= \frac{e^{y}(\cos x + i\,\mathrm{sen}\,x)+e^{-y}(\cos x - i\,\mathrm{sen}\,x)}{2} = \frac{\cos x(e^{-y}-e^{y}) + i\,\mathrm{sen}\,x(e^{-y}+e^{y})}{2}$

$= \cos x\left(\frac{e^{-y}-e^{y}}{2}\right) + i\,\mathrm{sen}\,x\left(\frac{e^{-y}+e^{y}}{2}\right) = i\cos x(-\sinh y) + \mathrm{sen}\,x(\cosh y)$

$= \mathrm{sen}\,x\cosh y + i\cos x\,\mathrm{senh}\,y$

$U = \mathrm{sen}\,x\cosh y \quad , \quad V = \cos x\,\mathrm{senh}\,y$

para pasar a cosenos y senos debe tenerse e^{jz}

6) $2^{z^2} = 2^{(x+iy)^2} = 2(\cos y + i\,\mathrm{sen}\,y)^2$ x

$= 2(\cos y) + 2(i\,\mathrm{sen}\,y) = 2(\cos y(-\sinh y)) + 2(i\,\mathrm{sen}\,y(\cosh y))$

$= 2(\mathrm{sen}\,x\cosh y)^2 + 2i(\cos x - \mathrm{senh}\,y)^2$

$U = 2(\mathrm{sen}\,x\cosh y) \quad , \quad V = 2(\cos x\,\mathrm{senh}\,y)^2$

7) $\mathrm{senh}\,z = \frac{e^{z}-e^{-z}}{2}$

exp(x) no es igual a cos(x)+jsen(x)

$= \mathrm{senh} = \frac{e^{(x+jy)}-e^{-(x+jy)}}{2} = \frac{e^{x}e^{j}-e^{-x}e^{-j}}{2}$

$= \frac{e^{x}(\cos y + j\,\mathrm{sen}\,y)-e^{-y}(\cos x - \mathrm{sen}\,x)}{2} = \frac{\cos x(e^{-y}-e^{y}) - \mathrm{sen}\,x(e^{-y}+e^{y})}{2}$

$= \cos x\left(\frac{e^{-y}-e^{y}}{2}\right) - \mathrm{sen}\,x\left(\frac{e^{-y}+e^{y}}{2}\right) = \cos x(-\sinh y) - \mathrm{sen}\,x(\cosh y)$

$= -\mathrm{sen}\,x\cosh y + \cos x\sinh y$

$U = -\mathrm{sen}\,x\cosh y \quad , \quad V = \cos x\sinh y$

8) $w = \tan z = \dfrac{\sin z}{\cos z} = \dfrac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}$

obtenido (de otro ejercicio)

$= \dfrac{e^{(x+jy)} - e^{-(x+jy)}}{ie^{(x+jy)} + ie^{-(x+jy)}} = \dfrac{e^{x}e^{jy} - e^{-x}e^{jy}}{ie^{x}e^{jy} + ie^{-x}e^{jy}} = \dfrac{-\operatorname{sen} x \cosh y + \cos x \operatorname{senh} y}{\operatorname{sen} x \cosh y + \cos x \operatorname{senh} y}$

$u = \dfrac{-\operatorname{sen} x \cosh y}{\operatorname{sen} x \cosh y}$, $v = \dfrac{\cos x \operatorname{senh} y}{\cos x \operatorname{senh} y}$

II $\operatorname{senh} z$; $z_1 = 1 + i\frac{\pi}{2}$

$w = \operatorname{senh} z(1+i\pi/2) = \frac{e^{z} - e^{-z}}{2} = \frac{e^{1+i\pi/2} - e^{-(1-i\pi/2)}}{2}$

?

$= \frac{e^{1i\pi/2} - e^{-1+i\pi/2}}{2}$ = donde $\frac{\pi}{2} + (\operatorname{Arg}\frac{\pi}{2} + 2k\pi) = \frac{\pi}{2} + j\frac{\pi}{2}k\pi$

?

$w = \frac{e^{(\frac{\pi}{2}+j\frac{\pi}{2})+j\pi} + e^{(\frac{\pi}{2}+j\frac{\pi}{2})-j\pi}}{2} = \frac{e^{\frac{\pi}{2}-j\frac{\pi}{2}+j\pi} + e^{\frac{\pi}{2}+j\frac{\pi}{2}-j\pi}}{2}$

no le encuentro sentido a los desarrollos :(

$= \frac{e^{-\pi/2}(\cos\pi(1-1) + j\operatorname{sen}\pi(1+1)) + e^{\pi/2}(\cos\pi(1-1) + j\operatorname{sen}(1-1))}{2}$

$w = 2^{-1}\left(\frac{\quad}{2}\right)$ Poner divisor el 2

$= \frac{1}{4}(\cos\pi(1-1k) + j\operatorname{sen}\pi(1-1k)) + (\cos(1k-1) + j\operatorname{sen}(1k-1))$

$= \left(\frac{1}{4}\cos\pi(1-1k) + \cos\pi(1k-1)\right) + j\left(\frac{1}{4}\operatorname{sen}\pi(1-1k)\operatorname{sen}\pi(-1k+1)\right)$

A $(\cos\theta = \cos(-\theta)$ y $\operatorname{sen}\theta = (-\theta)$

$= \frac{5}{4}\cos\pi(1k-1) + j\frac{3}{4}\cancel{\operatorname{sen}\pi(1k-1)}$

$w = \frac{5}{4}\cos\pi(1k-1) = -1$

$w = \frac{5}{4}$ ó $w = -\frac{5}{4} = \begin{cases} |w| = 5/4 \\ \angle w = \pi(1k-1) \end{cases}$

tan(z) ?

III $\underline{\pi}$?

a) e: Si $\ln z = \ln|z| + i(\angle(z) + 2k\pi)$

$= \ln e = \ln|e| + j(\angle(e) + 2k\pi\} \ \ln|e| + i(\angle(e) + 2k\pi)$

$= \ln e + j\left(+\frac{0}{2} + 2k\pi\right) = 0 + j\left(\frac{0}{2} + 2k\pi\right)$

$\therefore \ln e = +j\pi 2k \left(+ \frac{1}{2}\right)$, donde $(k = 0, \pm 1, \pm 2, \ldots)$

b) i = Si $\ln z = \ln|z| + i(\angle(z) + 2k\pi$

$= \ln i = \ln|i| + i(\angle(i) + 2k\pi - \ln|i| + i(\angle(i) + 2k\pi)$

$= \ln 1 + i\left[+\frac{\pi}{2} + 2k\pi\right] = 0 + i\underbrace{\left(+\frac{\pi}{2} + 2k\pi\right)} \checkmark$

$\therefore \ln i = i\pi\left(2k + \frac{1}{2}\right)$, donde $(k = 0, \pm 1, \pm 2, \ldots)$

c) $= -1 - i$ ¿ Si $-1 - i = 0$, entonces $\ln(-1-i)$

$-i = +1$, $\ln(-i) = \ln(1)$, $-i = \ln(1)$,

$-i = \ln(1) + 1(-(1) + 2k\pi$, $-i = \ln 1 + 1\left(-\frac{\pi}{2} + 2k\pi\right)$,

$-i = \pi\left(-\frac{1}{2} + 2k\pi\right)$, $(-1)i = \pi\left(2k - \frac{1}{2}\right)$ donde $(k = 0, \pm 1, \pm 2)$

d) $3 - 2i$ = Si $3 - 2i = 0$, entonces

$-2i = -3$, $\ln(-2i) = \ln(3)$, $= -2i = \ln(3)$,

$-2i = \ln(-3) + 1(-(-3) + 2k\pi$, $-2i = \ln -3 + i\left(-\frac{3\pi}{2} + 2k\pi\right)$,

$-2i = \pi\left(-\frac{3}{2} + 2k\pi\right)$, $i = \frac{\pi}{-2}\left(-\frac{3}{2} + 2k\pi\right)$ donde $(k = 0, \pm 1, \pm 2, \ldots)$

ln(3-2i)=ln|3-2i|+j(arg(3-2i)+2kpi)

esto parece notación de derivada :(

IV. Resolver las siguientes operaciones:

- $i^i = \frac{d}{di}(e^{i\ln(i)}) = \frac{d}{du}(e^u) = e^u = e^u \frac{d}{di}(i(\ln(i)) = e^{i\ln(i)} \frac{d}{di}(i\ln(i))$

$= e^{i\ln(i)} \frac{d}{di}(i\ln(i)) = \frac{d}{di}(i)\ln(i) + \frac{d}{di}(\ln(i))i$

$= \frac{d}{di}(i) = 1 \; ; \; \frac{d}{di}(\ln(i)) = \frac{1}{i} \; = 1 \cdot \ln(i) + \frac{1}{i} i = \ln(i)+1$

$= e^{i\ln(i)} = i^i = i^i(\ln(i)+1)$

- $i^{1/i} = \frac{d}{di}(e^{\frac{1}{i}\ln(i)}) = \frac{d}{du}(e^u)\frac{d}{di}(\frac{1}{i}\ln(i)) = \frac{d}{du}(e^u)$

$= e^u \frac{d}{di}(\frac{1}{i}\ln(i)) = e^{\frac{1}{i}\ln(i)} \frac{d}{di}(\frac{1}{i}\ln(i))$ X

$= \frac{d}{di}(\frac{1}{i})\ln(i) + \frac{d}{di}(\ln(i))\frac{1}{i} \; ; \; \frac{d}{di}(\frac{1}{i}) = -\frac{1}{i^2} \; ; \; \frac{d}{di}(\ln(i)) = \frac{1}{i}$

$= (-\frac{1}{i^2}\ln(i) + \frac{1}{i}\cdot\frac{1}{i}) = -\frac{\ln(i)}{i^2} + \frac{1}{i^2} = \frac{-\ln(i)+1}{i^2}$

$= e^{\frac{1}{i}\ln(i)} \frac{-\ln(i)+1}{i^2} = i^{\frac{1-2i}{i}}(-\ln(i)+1)$

- $1^i = \frac{d}{di}(1^i) = \frac{d}{di}(1) \quad 1^i = 1$

$= 0 \quad \frac{d}{di}(1) = 0$

$e^{\ln i^i} = e^{i\ln(i)} = e^{i(\ln(1) + i(\frac{\pi}{2} + 2k\pi))} = e^{-(\frac{\pi}{2} + 2k\pi)}$ $k = 0, \pm 1, \pm 2, \ldots$

• $\left(\frac{\sqrt{2}}{2}+\frac{i}{\sqrt{2}}\right)^{1+i} = \cancel{\frac{d}{di}}\Big(\left(\frac{\sqrt{2}}{2}+\frac{i}{\sqrt{2}}\right)^{1+i} = \left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{1+i} = \left(\frac{1+i}{\sqrt{2}}\right)^{1+i}$

$= \frac{d}{di}\left(\frac{1+i}{\sqrt{2}}\right)^{1+i} = \frac{d}{di}\Big(e^{(1+i)\ln\left(\frac{1+i}{\sqrt{2}}\right)}\Big) = e^{(1+i)\ln\left(\frac{1+i}{2}\right)}\frac{d}{di}(1+i)\ln\left(\frac{1+i}{\sqrt{2}}\right)$

$= \frac{d}{di}\Big((1+i)\ln\left(\frac{1+i}{\sqrt{2}}\right)\Big) = \ln(1+i)-\frac{1}{2}\ln(2)+1 = 1\cdot\Big(\ln(1+i)-\frac{1}{2}\ln(2)$

$+\frac{1}{i+1}(1+i = \ln(1+i)-\frac{1}{2}\ln(2)+1 = e^{(1+i)\ln\left(\frac{1+i}{\sqrt{2}}\right)}\Big(\ln(1+i)-\frac{1}{2}(2)+1$

$= e^{(1+i)\ln\left(\frac{1+i}{\sqrt{2}}\right)} = e^{\ln\left(\frac{1+i}{\sqrt{2}}\right)(1+i)} = \left(\frac{1+i}{\sqrt{2}}\right)^{(1+i)}$

$= \left(\frac{1+i}{\sqrt{2}}\right)^{1+i}\Big(\ln(1+i)-\frac{1}{2}\ln(2)+1\Big)$

• $\left(\frac{1+i}{\sqrt{2}}\right)^{2i} = \cancel{\frac{d}{di}}\Big(e^{2i\ln\left(\frac{1+i}{\sqrt{2}}\right)}\Big) = \frac{d}{du}(e^{u})\frac{d}{di}\Big(2i\ln\left(\frac{1+i}{\sqrt{2}}\right)\Big) = e^{u}\frac{d}{di}\Big(2i\ln$

$\left(\frac{1+i}{\sqrt{2}}\right)\Big) = e^{2i\ln\left(\frac{1+i}{\sqrt{2}}\right)}\frac{d}{di}\Big(2i\ln\left(\frac{1+i}{\sqrt{2}}\right)\Big) = \frac{d}{di}\Big(2i\ln\left(\frac{1+i}{\sqrt{2}}\right)\Big) =$

$= 2\Big(\ln(1+i)+\frac{i}{1+i}\Big)-\ln(2) = e^{2i\ln\left(\frac{1+i}{\sqrt{2}}\right)}\Big(2\Big(\ln(1+i)+\frac{i}{1+i}\Big)-\ln(2)\Big)$

$= e^{2i\ln\left(\frac{1+i}{\sqrt{2}}\right)2i} = \left(\frac{1+i}{\sqrt{2}}\right)^{2i}$

$= \left(\frac{1+i}{\sqrt{2}}\right)^{2i}\Big(2\Big(\ln(1+1)+\frac{i}{1+i}\Big)-\ln(2)\Big)$

de dónde sacaron este procedimiento?

V. Encontrar el módulo y el argumento de los siguientes números complejos.

- $10^i = \cancel{\frac{d}{di}}(10^i) = \frac{d}{di}(10^i) = 10^i \ln(10)$?

- $3^{2-i} = \cancel{\frac{d}{di}}(3^{2-i}) = \frac{d}{di}(e^{(2-i)}\ln(3)) = e^{(2-i)}\ln(3)\,\frac{d}{di}((2-i)\ln(3))$

$= \ln 3\left(\frac{d}{di}(2) - \frac{d}{di}(i)\right) = \frac{d}{di}(2) = 0 \; ; \; \frac{d}{di}(i) = 1 = \ln(3)(0-1)$
$= \ln(3)$

$= e^{(2-i)\ln(3)}(-\ln(3)) = -e^{(2-i)\ln(3)}\ln(3) = 3^{(2-i)}$
$= -\ln(3)\cdot 3^{-i+2} = -\ln(3)\cdot 3^{2-i}$

VI Resolver las siguientes ecuaciones:

- $\operatorname{sen} z = 3 = z = \pi - \operatorname{sen}^{-1}(3) + 2\pi n_1 , \in \mathbb{Z}$ +
$= z = 2\pi n_2 + \operatorname{sen}^{-1}(3) , n_2 \in \mathbb{Z}$

- $e^{-z} + 1 = 0 = e^{-z} + 1 - 1 = 0 - 1$
$= e^{-z} = -1$ $\quad -z = \ln(-1) ; z = -\{\ln(1) + j2k\pi\}$

- $4\cos z + 5 = 0 = 4\cos z + 5 - 5 = 0 - 5$
$= 4\cos z = -5$
$= \frac{4\cos z}{4} = \frac{-5}{4}$ deben despejar la variable

$= \cos z = -\frac{5}{4}$

• $\operatorname{sh} iz = -i \quad \frac{e^{iz}}{2} - \frac{e^{-iz}}{2} = -iz \quad i = \emptyset$

$iz = \operatorname{sh}^{-1}(-i)$

• $\operatorname{sen} z = -\pi \quad z = 2\pi n - \operatorname{sen}^{-1}(\pi),\ n \in \mathbb{Z}$

$z = 2\pi n + \operatorname{sen}^{-1}(\pi) + \pi,\ n \in \mathbb{Z}$

• $e^{2z} + 2e^{z} - 3 = 0 = (e^{z})^2 + 2e^{z} - 30 = (u)^2 + 2u - 3 = 0$

$= u_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = u_{1,2} = \frac{-2 \pm 4}{2 \cdot 1}$

$= \frac{-2+4}{2 \cdot 1} = \frac{2}{2 \cdot 1} = \frac{2}{2} = 1 \ ; \ \frac{-2-4}{2 \cdot 1} = \frac{-6}{2 \cdot 1} = \frac{-6}{2} = -\frac{6}{2} = -3$

$= u = 1,\ u = 3 - 3 \qquad e^{z} = 1 = 0$ ✓ $\qquad e^{z} = 3$

$= \ln(e^{z}) = \ln(1) \quad z \quad z\ln(e) = \ln(1) = \ln(1)\,0$

$= 0$

despejar z

• $\operatorname{ch} z = i = \frac{e^{z} + e^{-z}}{2} = i = \frac{e^{z} + e^{-z}}{2} \cdot 2 = i \cdot 2$

$= e^{z} + e^{-z} = i \cdot 2 = u + (u)^{-1} = i \cdot 2 = u + u^{-1} = i \cdot 2 = u + \frac{1}{u} = i \cdot 2$

$uu + \frac{1}{u} u = i \cdot 2u = u^2 + 1 = i \cdot 2u = u = i\sqrt{i^2 - i},\ u = i - \sqrt{i^2 - 1}$

$u = i + \sqrt{i^2 - 1},\ u = i - \sqrt{i^2 - 1} = e^{z} = i + \sqrt{i - 1} = \ln(i + \sqrt{i^2 - 1})$

$= \ln(i + \sqrt{i^2 - 1}),\ z = \ln(i - \sqrt{i^2 - 1})$

• $\ln(z+i)=0$; $\ln(z+i)=0$

; $z+i=e^0$

; $z+i=1$

; $z+i=1 = z+i-i=1-i$

$z=1-i$

$z=1-i$ ✓

• $\ln(i-z)=1$; $\ln(i-z)=1$

; $i-z=e^1$

; $i-z=e$

; $i-z-i=e-i$

; $-z=e-i$

; $\frac{-z}{-1}=\frac{e}{-1}-\frac{i}{1}$

; $\frac{z}{1}=\frac{z}{1}=\frac{e}{-1}-\frac{i}{-1}=$; $\frac{e-i}{1}=-(e-i)$

$=-(e)-(-i)=-e-i$

$= z=e+i$