7.5

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5C

Análisis de señales

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Transformada de Laplace

1) $\mathcal{L}\{3\cos(5t)\}$

$3\mathcal{L}\{\cos(5t)u(t)\}$

$3\,\frac{s}{s^2+25} = \frac{3s}{s^2+25}$

$\cos(wt) = \frac{s}{s^2+w^2}$

$R = \frac{3s}{s^2+25}$

• $\mathcal{L}\{10\,sen(6t)\}$

$10\,\mathcal{L}\{sen(6t)u(t)\}$

$10\,\frac{6}{s^2+36} = \frac{60}{s^2+36}$

$R = \frac{60}{s^2+36}$

• $\mathcal{L}\{6\,sen(2t) - 5\cos(2t)\}$

$6\mathcal{L}\{sen(2t)u(t)\} - 5\mathcal{L}\{\cos(2t)u(t)\}$

$6\,\frac{2}{s^2+4} - 5\,\frac{s}{s^2+4} = \frac{12}{s^2+4} - \frac{5s}{s^2+4}$

$R = \frac{12}{s^2+4} - \frac{5s}{s^2+4}$

• $\mathcal{L}\{2\,sen(t) + 3\cos(2t)\}$

$2\mathcal{L}\{sen(t)u(t)\} + 3\mathcal{L}\{\cos(2t)u(t)\}$

$2\,\frac{1}{s^2+1} + 3\,\frac{s}{s^2+4} = \frac{2}{s^2+1} + \frac{3s}{s^2+4}$

$R = \frac{2}{s^2+1} + \frac{3s}{s^2+4}$

Norma

- $\mathcal{L}\{e^{-t}\cos(2t)\}$

$\mathcal{L}\{\cos(2t)\,u(t)\}_{s=s+1}$

$= \frac{s}{s^2+4}\Big|_{s=s+1} = \frac{s+1}{(s+1)^2+4}$ ✓

$R = \frac{s+1}{(s+1)^2+4}$

- $\mathcal{L}\{2e^{3t}\operatorname{sen}(4t)\}$

$2\mathcal{L}\{\operatorname{sen}(4t)\,u(t)\}_{s=s-3}$

$= 2\,\frac{4}{s^2+16}\Big|_{s=s-3} = \frac{8}{(s-3)^2+16}$ ✓

$R = \frac{8}{(s-3)^2+16}$

Incorrecta factorización. Separar en dos transformadas

- $\mathcal{L}\{5t-3\}$

$5\mathcal{L}\{t-3\}u(t)$ $\quad e^{-as}$ $\quad \frac{5}{s^2} - \frac{3}{s}$

$5\; e^{-3s} = 5e^{-3s}$

$R = 5e^{-3s}$ ✗

0.6

$\mathcal{L}\{2t^2 - e^{-t}\}$

$2\mathcal{L}\{t^2\, u(t)\}\; s = s+1$ $\qquad \frac{n!}{s^{n+1}} = \frac{2!}{s^{2+1}} \qquad \frac{1}{s} = \frac{1}{s+1}$

$2\left\{\frac{2\, u(t)}{s^3}\right\} \left[\frac{1}{s}\, u(t)\right]\Big|_{s=s+1}$ $u(t)$ $\rightarrow 2\,\frac{2}{(s+1)^3} - \frac{1}{s+1} \rightarrow \frac{4}{(s+1)^3} - \frac{1}{s+1}$

nunca puede haber t y s dentro de L o L inversa

$R = \frac{4}{(s+1)^3} - \frac{1}{s+1}$

$\mathcal{L}\{2e^{4t}\}$

$2\mathcal{L}\{1\, u(t)\}\; s = s-4 \rightarrow \cancel{2s-4}$ $\qquad \frac{2}{s-4}$

$R = 2s-4$

Laplace inversa

$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}\Big|_{t=t-3} = \text{Sen}(t)\, u(t)$

$= \text{Sen}(t-3)\, u(t-3)$ ✓

$R = \text{Sen}(t-3)\, u(t-3)$

$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}\Big|_{t=t-\pi} = \text{Sen}(t)\, u(t)$ ✓

$= \text{Sen}(t-\pi)\, u(t-\pi)$

$R = \text{Sen}(t-\pi)\, u(t-\pi)$

Norma

0.4

$\cdot \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$ $\frac{F(s)}{s} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}\Big|_{t = t-1}$

Fracc Par $= \frac{A}{s} + \frac{B}{s+1}$

$\frac{1}{s(s+1)} = \frac{A(s+1) + B(s)}{s(s+1)} = 1 = A(s+1) + B(s)$

$s = 0 \therefore A(0+1) + B(0) = A = 1;$ $\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

$s = -1 \therefore A(-1+1) + B(-1) = B = -1;$

$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = u(t) - e^{-t}[u(t)]\Big|_{t = t-1}$

$R = -\cancel{\ldots}$ $-e[u(t)]u(t-s)$

$\cdot \mathcal{L}^{-1}\left\{\frac{4}{s(s^2+4)}\right\} = \frac{F(s)}{s}$ $\mathcal{L}\left\{\int_0^t f(u)\,du\right\}$ $= \int_0^t \mathcal{L}^{-1}\left\{\frac{4}{s^2+4}\right\} dt$

$= \frac{4}{2}\int_0^t \operatorname{sen}(2t)\,dt \therefore \mathcal{L}^{-1}\left\{\frac{4}{s(s^2+4)}\right\} = \frac{4}{2}\int_0^t \operatorname{sen}(2u)\,du$

$= \frac{4}{4}[1 - \cos(2t)]$

0.4

$R = \frac{4}{4}[1 - \cos(2t)]u(t) = -\cos(2t)u(t)$

Nota: No puse el 4/4 a 1 porque no se si de el mismo resultado pero si se que se iguala a 1, o sea

$[u(t) - \cos(2t)]u(t)$, así quedaría sustituido