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Recycling robot example (from Sutton, page 42) References:

- Gym documentation: https://gym.openai.com/

In [0]:

```
import numpy as np
from gym.envs.toy_text import discrete
import random
import matplotlib.pyplot as plt
```

Gym provides an environment to compare reinforcement learning algorithms. We provide the code and it provides the game and visualizations.

Consider the robot model described in Barto and Sutton Example 3.2

In [0]:

```
states = ["high", "low"]
actions = ["wait", "search", "recharge"]

P = {}

P[0] = {}
P[1] = {}

alpha = 1
beta = 1
r_wait = 0.5
r_search = 2.0

# We define a discrete environment with the corresponding transitions
def generar_ambiente(alpha=alpha, beta=beta, r_wait=r_wait, r_search=r_wait):
    P[0][0] = [(1.0, 0, r_wait, False)]
    P[0][1] = [(alpha, 0, r_search, False),
               (1-alpha, 1, r_search, False)]
    P[0][2] = [(1,0,0,False)]

    P[1][0] = [(1.0, 1, r_wait, False)]
    P[1][1] = [(beta, 1, r_search, False),
               (1-beta, 0, -3.0, False)]
    P[1][2] = [(1.0, 0, 0.0, False)]
    env = discrete.DiscreteEnv(2, 3, P, [0.0, 1.0])
    return(env)
env = generar_ambiente()
```

Implement the random strategy for 20 steps

Define a random action and see what reward it produces

In [0]:

```
def random_action(states, actions):
  return np.random.randint(0,len(actions))
```

In [0]:

```
def train(env,epoch,prefix=''):
```

```
    reward_history = [[] for _ in range(epoch)]

    for e in range(1,epoch+1):
        # At each epoch, we restart to a fresh game and get the initial state
        state = env.reset()
        # This assumes that the games will terminate
        game_over = False

        idx = 0
        while idx <20 and not game_over:
            # The agent performs an action
            action = random_action(states, actions)

            # Apply an action to the environment, get the next state, the reward
            # and if the games end
            state, reward, game_over, info = env.step(action)

            reward_history[e-1].append(reward)

            idx += 1

        print("Epoch {:03d}/{:03d} | Global_reward {:.4f}".format(e, epoch,
sum(reward_history[e-1])))

    return reward_history
```

In [5]:

```
reward_history = train(env,10)
```

```
Epoch 001/010 | Global_reward 7.5000
Epoch 002/010 | Global_reward 7.0000
Epoch 003/010 | Global_reward 6.0000
Epoch 004/010 | Global_reward 8.0000
Epoch 005/010 | Global_reward 4.5000
Epoch 006/010 | Global_reward 5.0000
Epoch 007/010 | Global_reward 4.5000
Epoch 008/010 | Global_reward 7.0000
Epoch 009/010 | Global_reward 6.0000
Epoch 010/010 | Global_reward 8.0000
```

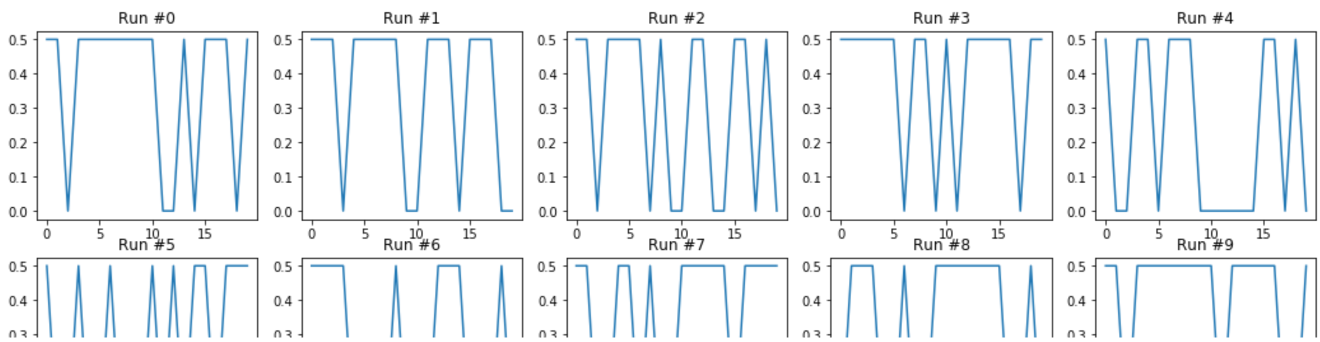
Plot the global reward

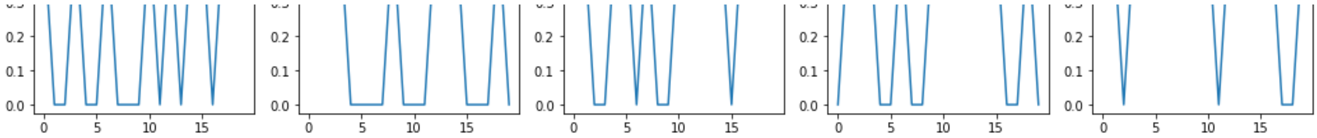
In [6]:

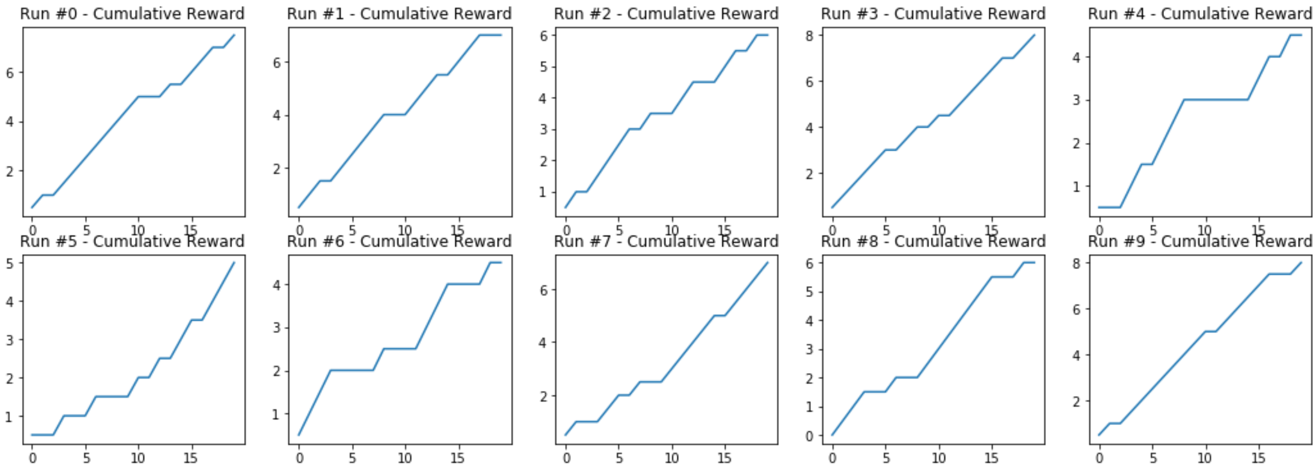
```
fg, ax = plt.subplots(2, 5, figsize=(18,6))
for idx, hist in enumerate(reward_history):
  ax[idx//5][idx%5].plot(range(len(hist)), hist)
  ax[idx//5][idx%5].set_title('Run #{}'.format(idx))
plt.show()

print('-'*125)

fg, ax = plt.subplots(2, 5, figsize=(18,6))
for idx, hist in enumerate(reward_history):
  cumulative_hist = [sum(hist[:k+1]) for k in range(len(hist))]
  ax[idx//5][idx%5].plot(range(len(hist)), cumulative_hist)
  ax[idx//5][idx%5].set_title('Run #{} - Cumulative Reward'.format(idx))
plt.show()
```







Compute theoretically the optimal value function for each state

It appears that :

- when at state 0,
  - action 0 -> stay at 0 and reward=0.5
  - action 1 -> stay at 0 and reward=0.5
  - action 2 -> stay at 0 and reward=0
- when at state 1,
  - action 0 -> stay at 1 and reward=0.5
  - action 1 -> stay at 1 and reward=0.5
  - action 2 -> move to 0 and reward=0

We have the following equation systems:

$ V^{*}(0) = \max \left\{ \begin{align*} & \text{if} \; a = 0: \quad r_{wait} + \gamma * V^{*}(0) \\ & \text{if} \; a = 1: \quad r_{search} + \gamma * (\alpha * V^{*}(0) + (1 - \alpha)*V^{*}(1)) \\ & \text{if} \; a = 2: \quad \gamma * V^{*}(0) \end{align*} \right. $

$ V^{*}(1) = \max \left\{ \begin{align*} & \text{if} \; a = 0: \quad r_{wait} + \gamma * V^{*}(1) \\ & \text{if} \; a = 1: \quad \beta * (r_{search} + \gamma * V^{*}(1)) + (1 - \beta)*(-3 + \gamma * V^{*}(0)) \\ & \text{if} \; a = 2: \quad \gamma * V^{*}(0) \end{align*} \right. $

γ being in $[0, 1)$, the $\pi(0)=2$ case must be excluded.

Solving the system yields, for each case:

$ \begin{align*} \text{ Case 1: } \quad \pi(0|0) = 1 ; \quad \pi(0|1) = 1 :\\ & V^{*}(0) = \frac{r_{wait}}{1 - \gamma} \\ & V^{*}(1) = \frac{r_{wait}}{1 - \gamma} \end{align*} $

$ \begin{align*} \text{Case 2:} \quad \pi(1|0) = 1 ; \quad \pi(0|1) = 1 : \\ \\ & V^{*}(0) = \frac{(1-\gamma)*r_{search} + (1-\alpha)*\gamma*r_{wait}}{(1-\gamma)*(1-\alpha*\gamma)}\\ & V^{*}(1) = \frac{r_{wait}}{1 - \gamma} \end{align*} $

$ \begin{align*} \text{Case 3:} \quad \pi(0|0) = 1 ; \quad \pi(1|1) = 1 : \\ \\ & V^{*}(0) = \frac{r_{wait}}{1 - \gamma} \\ & V^{*}(1) = \frac{(1-\gamma)*\beta*r_{search} -3*(1-\beta)*(1-\gamma) + \gamma*(1-\beta)*r_{wait}}{(1-\gamma)*(1-\beta*\gamma)} \end{align*} $

$ \begin{align*} \text{Case 4:} \quad \pi(1|0) = 1 ; \quad \pi(1|1) = 1 : \\ \\ & V^{*}(0) = \frac{r_{search}}{1-\gamma\alpha} + \frac{\gamma(1-\alpha)}{1-\gamma\alpha}\frac{\left[(1-\gamma\alpha)\beta +\gamma(1-\beta)\right]r_{search}-3(1-\beta)(1-\gamma\alpha)}{(1-\gamma\beta)(1-\gamma\alpha) - \gamma^2(1-\beta)(1-\alpha)} \\ & V^{*}(1) = \frac{\left[(1-\gamma\alpha)\beta

\gamma\alpha)}{(1-\gamma\beta)(1-\gamma\alpha) - \gamma^2(1-\beta)(1-\alpha)} \\ & V^{*}(1) = \frac{\left[(1-\gamma\alpha)\beta +\gamma(1-\beta)\right]r_{search}-3(1-\beta)(1-\gamma\alpha)}{(1-\gamma\beta)(1-\gamma\alpha) - \gamma^2(1-\beta)(1-\alpha)} \end{align*} $

$ \begin{align*} \text{Case 5:} \quad \pi(0|0) = 1 ; \quad \pi(2|1) = 1 : \\ \\ & V^{*}(0) = \frac{r_{wait}}{1 - \gamma}\\ & V^{*}(1) = \gamma \frac{r_{wait}}{1 - \gamma} \end{align*} $

$ \begin{align*} \text{Case 6:} \quad \pi(1|0) = 1 ; \quad \pi(2|1) = 1 : \\ \\ & V^{*}(0) = \frac{r_{wait}}{1 - \gamma\alpha -(1-\alpha)\gamma^2} \\ & V^{*}(1) = \gamma \frac{r_{wait}}{1 - \gamma\alpha -(1-\alpha)\gamma^2} \end{align*} $

Implement Value Iteration

Evaluate the optimal value function given a full description of the environment dynamics

In [0]:

```
def evaluate_value_func(env, theta, discount_factor, max_itr=10_000):
    """
    Args:

        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all
states.
        discount_factor: Gamma discount factor.
        max_itr: Maximum number of iterations in the fixed-point equation resolution

    Returns:
        Vector of length env.nS representing the value function.
    """
    n_states = env.nS
    n_actions = env.nA
    Vn = np.zeros((n_states,))
    Vnp1 = np.inf * np.ones((n_states,))
    itr = 0

    converged = False

    while not converged and itr < max_itr:
        for state in range(n_states):
            best = -np.inf
            for action in range(n_actions):
                transitions = env.P[state][action]
                value = 0
                for prob, next_state, reward, _ in  transitions:
                    value += prob*(reward + discount_factor * Vn[next_state])
                if value > best:
                    best = value
            Vnp1[state] = best
        converged = np.absolute(Vn - Vnp1).max() < theta
        Vn = Vnp1
        itr += 1

    return Vnp1
```

In [19]:

```
env = generar_ambiente()
value_func = evaluate_value_func(env, theta=0.001, discount_factor=0.9)
print("Found V = ", value_func)
```

```
Found V =  [0.95 0.95]
```

In [20]:

```
policy = []
for state in range(env.nS):
  possible_rewards = []
  best_reward = 0
  for action in range(env.nA):
```

```
  for action in range(env.nA):
    future = env.P[state][action]
    rewards = [future[idx][0]*future[idx][2] for idx in range(len(future))]
    future_rewards = [future[idx][0]*V[future[idx][1]] for idx in range(len(future))]
    action_reward = sum(rewards) + discount_factor*sum(future_rewards)

    if action_reward > best_reward:
      best_reward = action_reward
      best_action = action

  policy.append(best_action)
print('Best associated policy : {}'.format(policy))
```

```
Best associated policy : [0, 0]
```

Implement policy iteration

Then an policy optimisation function,

```
Evaluate a policy given an environment and a full description of the environment's
dynamics.

    Args:
        policy: [S, A] shaped matrix representing the policy.
        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all
states.
        discount_factor: Gamma discount factor.

    Returns:
        Vector of length env.nS representing the value function.
```

Despues una funcion de optimisacion de la politica:

```
 Policy Improvement Algorithm. Iteratively evaluates and improves a policy
    until an optimal policy is found.

    Args:
        env: The OpenAI envrionment.
        policy_eval_fn: Policy Evaluation function that takes 3 arguments:
            policy, env, discount_factor.
        discount_factor: gamma discount factor.

    Returns:
        A tuple (policy, V).
        policy is the optimal policy, a matrix of shape [S, A] where each state s
        contains a valid probability distribution over actions.
        V is the value function for the optimal policy.
```

In [0]:

```
def evaluate_policy(policy, env, theta, discount_factor, max_itr=10_000):
    """
    Args:
      policy: [S, A] shaped matrix representing the policy.
      env: OpenAI env. env.P represents the transition probabilities of the environment.
          env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
          env.nS is a number of states in the environment.
          env.nA is a number of actions in the environment.
      theta: We stop evaluation once our value function change is less than theta for all states.
      discount_factor: Gamma discount factor.
      max_itr: Maximum number of iterations

    Returns:
      Vector of length env.nS representing the value function.
```

```
    vector of length env.nS representing the value function.
    """
    n_states = env.nS
    n_actions = env.nA
    Vn = np.zeros((n_states,))
    Vnp1 = np.inf * np.ones((n_states,))
    itr = 0

    converged = False

    while not converged and itr < max_itr:
        for state in range(n_states):
            policy_value = 0
            for action in range(n_actions):
                transitions = env.P[state][action]
                for prob, next_state, reward, _ in  transitions:
                    policy_value += policy[state][action] * prob * (reward + discount_factor * Vn[ne
xt_state])
            Vnp1[state] = policy_value
        converged = np.absolute(Vn - Vnp1).max() < theta
        Vn = Vnp1
        itr += 1

    return Vnp1
```

In [0]:

```
from scipy.special import softmax

def policy_improvement(env, start_policy, policy_eval_fn=evaluate_policy, theta=.01, discount_facto
r=0.99,
                       hard_thresholding=False, max_itr=1000):
    """
     Policy Improvement Algorithm. Iteratively evaluates and improves a policy
    until an optimal policy is found.

    Args:
        env: The OpenAI envrionment.
        start_policy: matrix of shape [S, A] representing a valid policy
        policy_eval_fn: Policy Evaluation function that takes 3 arguments:
            policy, env, discount_factor.
        discount_factor: gamma discount factor.
        hard_thresholding: boolean, set to True to enforce deterministic policies.
        max_itr: Maximum number of iterations

    Returns:
        A tuple (policy, V).
        policy is the optimal policy, a matrix of shape [S, A] where each state s
        contains a valid probability distribution over actions.
        V is the value function for the optimal policy.
    """
    n_states = env.nS
    n_actions = env.nA
    env.reset()

    pn = start_policy
    Vn = policy_eval_fn(pn, env, theta, discount_factor)

    itr = 0
    converged = False

    while not converged and itr < max_itr:
        rewards = []
        for state in range(n_states):
            rewards.append([])
            for action in range(n_actions):
                rewards[state].append(0)
                transitions = env.P[state][action]
                for prob, next_state, reward, _ in  transitions:
                    rewards[state][action] += prob* (reward + discount_factor * Vn[next_state])

        if not hard_thresholding:
            pnp1 = softmax(np.array(rewards), axis=1)
        else:
            pnp1 = np.zeros(pn.shape)
            best_action = np.argmax(np.array(rewards), axis=1)
            for state in range(n_states):
```

```
            for state in range(n_states):
                pnp1[state, best_action[state]] = 1.0


        Vnp1 = policy_eval_fn(pnp1, env, theta, discount_factor)
        converged = np.absolute(Vn - Vnp1).max() < theta
        Vn = Vnp1
        pn = pnp1
        itr += 1

    return (pn, Vn)
```

In [23]:

```
max_iter = 100
theta = 1e-4
discount_factor = 0.99

policy = np.array([[1.,  0., 0.],
                   [1.,  0., 0.]])
env = generar_ambiente()

value_func = evaluate_policy(policy, env, theta=theta, discount_factor=discount_factor, max_itr=max
_iter)
print("Start policy:")
print("{}".format(policy))
print("Value function = ", value_func)

print()

new_policy, new_value_func = policy_improvement(env, policy, theta=theta, discount_factor=discount_
factor, max_itr=max_iter)
print("Improved policy (softmax):")
print("{}".format(new_policy))
print("Value function = ", new_value_func)

new_policy, new_value_func = policy_improvement(env, policy, theta=theta, discount_factor=discount_
factor, max_itr=max_iter,
                                                hard_thresholding = True)
print("Improved policy (hard thresholding):")
print("{}".format(new_policy))
print("Value function = ", new_value_func)
```

```
Start policy:
[[1. 0. 0.]
 [1. 0. 0.]]
Value function =  [0.995 0.995]

Improved policy (softmax):
[[0.38365173 0.38365173 0.23269654]
 [0.39191142 0.39191142 0.21617717]]
Value function =  [0.76346695 0.85942232]
Improved policy (hard thresholding):
[[1. 0. 0.]
 [1. 0. 0.]]
Value function =  [0.995 0.995]
```

Using the 3 algorithms do the following experiments

In [0]:

```
exp1 = generar_ambiente(alpha=0.9, beta=0.9, r_search=3, r_wait=2)
exp2 = generar_ambiente(alpha=0.8, beta=0.5, r_search=3, r_wait=2)
exp3 = generar_ambiente(alpha=0.5, beta=0.5, r_search=3, r_wait=2)
exp4 = generar_ambiente(alpha=0.9, beta=0.6, r_search=1, r_wait=0.9)
exp5 = generar_ambiente(alpha=0.9, beta=0.6, r_search=1, r_wait=0.5)
```

Compare the different strategies with the random one

In [25]:

```
max_iter = 100
theta = 1e-6
discount_factor = 0.99

random_policy = np.array([[1/3, 1/3, 1/3],
                          [1/3, 1/3, 1/3]])

for i, exp in enumerate([exp1, exp2, exp3, exp4, exp5]):
    print("-"*20)
    print("[Experience # {}]".format(i+1))

    exp.reset()

    optimal_value_func = evaluate_value_func(exp, theta=theta, discount_factor=discount_factor, max
_itr=max_iter)
    random_value_func = evaluate_policy(random_policy, exp, theta=theta, discount_factor=discount_f
actor, max_itr=max_iter)

    print("- Optimal value func:")
    for n_s, name in enumerate(states):
        print("    start = {}: \t{}".format(name.upper(), optimal_value_func[n_s]))
    print()

    print("- Value func for random policy:")
    for n_s, name in enumerate(states):
        print("    start = {}: \t{}".format(name.upper(), random_value_func[n_s]))
    print()

    improved_policy, improved_value_func = policy_improvement(exp, random_policy,
                                                              theta=theta,
discount_factor=discount_factor,
                                                              max_itr=max_iter)
    print("- Improved policy (softmax):")
    print(improved_policy)
    print()

    print("- Value func for this policy:")
    for n_s, name in enumerate(states):
        print("    start = {}: \t{}".format(name.upper(), improved_value_func[n_s]))
    print()

    improved_policy, improved_value_func = policy_improvement(exp, random_policy,
                                                              theta=theta,
discount_factor=discount_factor,
                                                              max_itr=max_iter,
                                                             hard_thresholding=True)
    print("- Improved policy (hard thresholding):")
    print(improved_policy)
    print()

    print("- Value func for this policy:")
    for n_s, name in enumerate(states):
        print("    start = {}: \t{}".format(name.upper(), improved_value_func[n_s]))
    print()
```

```
--------------------
[Experience # 1]
- Optimal value func:
    start = HIGH:  1.9405
    start = LOW:  1.9210949999999998

- Value func for random policy:
    start = HIGH:  0.9774
    start = LOW:  0.40062546666666665

- Improved policy (softmax):
[[0.31492907 0.4940568  0.19101413]
 [0.41613765 0.16898396 0.41487839]]

- Value func for this policy:
    start = HIGH:  1.269878303121338
    start = LOW:  0.7678893223865249

- Improved policy (hard thresholding):
[[0. 1. 0.]
 [1. 0. 0.]]
```

```
 [1. 0. 0.]]

- Value func for this policy:
    start = HIGH:  1.9405
    start = LOW:  0.995

--------------------
[Experience # 2]
- Optimal value func:
    start = HIGH:  1.9405
    start = LOW:  1.9210949999999998

- Value func for random policy:
    start = HIGH:  0.9774
    start = LOW:  0.40062546666666665

- Improved policy (softmax):
[[0.31492907 0.4940568  0.19101413]
 [0.41613765 0.16898396 0.41487839]]

- Value func for this policy:
    start = HIGH:  1.269878303121338
    start = LOW:  0.7678893223865249

- Improved policy (hard thresholding):
[[0. 1. 0.]
 [1. 0. 0.]]

- Value func for this policy:
    start = HIGH:  1.9405
    start = LOW:  0.995

--------------------
[Experience # 3]
- Optimal value func:
    start = HIGH:  1.9405
    start = LOW:  1.9210949999999998

- Value func for random policy:
    start = HIGH:  0.9774
    start = LOW:  0.40062546666666665

- Improved policy (softmax):
[[0.31492907 0.4940568  0.19101413]
 [0.41613765 0.16898396 0.41487839]]

- Value func for this policy:
    start = HIGH:  1.269878303121338
    start = LOW:  0.7678893223865249

- Improved policy (hard thresholding):
[[0. 1. 0.]
 [1. 0. 0.]]

- Value func for this policy:
    start = HIGH:  1.9405
    start = LOW:  0.995

--------------------
[Experience # 4]
- Optimal value func:
    start = HIGH:  1.9405
    start = LOW:  1.9210949999999998

- Value func for random policy:
    start = HIGH:  0.9774
    start = LOW:  0.40062546666666665

- Improved policy (softmax):
[[0.31492907 0.4940568  0.19101413]
 [0.41613765 0.16898396 0.41487839]]

- Value func for this policy:
    start = HIGH:  1.269878303121338
    start = LOW:  0.7678893223865249

- Improved policy (hard thresholding):
[[0. 1. 0.]
```

```
[[0. 1. 0.]
 [1. 0. 0.]]

- Value func for this policy:
    start = HIGH:  1.9405
    start = LOW:  0.995

--------------------
[Experience # 5]
- Optimal value func:
    start = HIGH:  1.9405
    start = LOW:  1.9210949999999998

- Value func for random policy:
    start = HIGH:  0.9774
    start = LOW:  0.40062546666666665

- Improved policy (softmax):
[[0.31492907 0.4940568  0.19101413]
 [0.41613765 0.16898396 0.41487839]]

- Value func for this policy:
    start = HIGH:  1.269878303121338
    start = LOW:  0.7678893223865249

- Improved policy (hard thresholding):
[[0. 1. 0.]
 [1. 0. 0.]]

- Value func for this policy:
    start = HIGH:  1.9405
    start = LOW:  0.995
```

In [0]:

```

```