Introduction to Survival Analysis

Understanding Time-to-Event Data

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What is Survival Analysis?

- Study of **time until an event occurs**
- Crucial in **medicine, engineering, business**, **insurance,** and beyond
- Events include:
  - **Death**
  - **System failure**
  - **Relapse**
  - **Customer churn**
  - **Morbidity**
  - **Retirement**

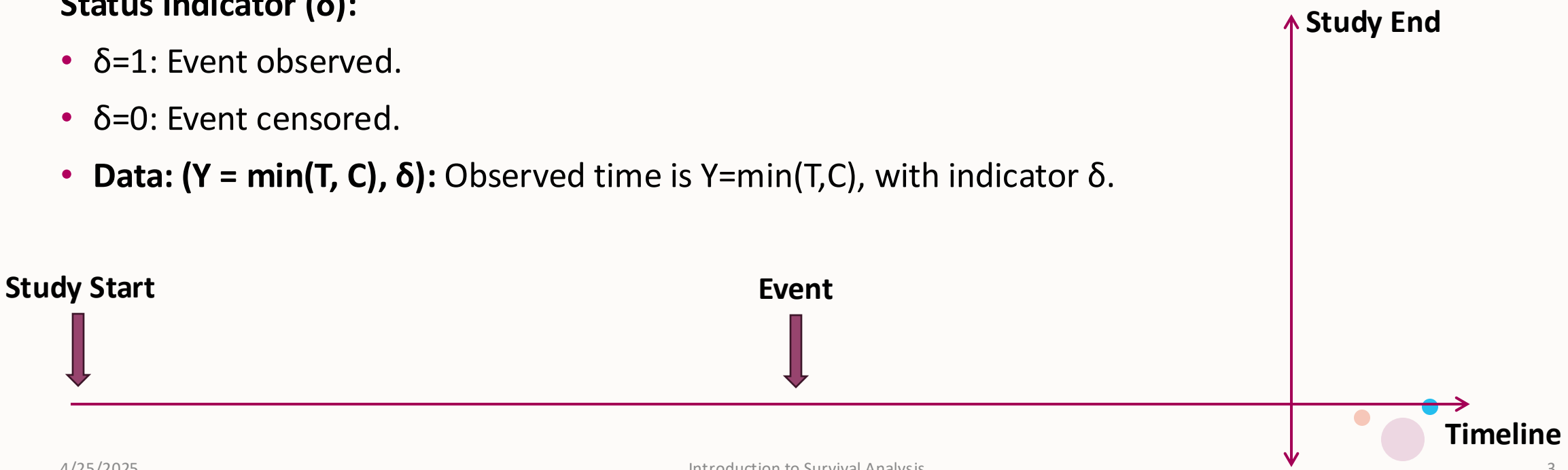
Key Variables

Event Time (T):
Time until the event occurs.

Censoring (C):
Occurs when the event is not observed by the end of the observation period.

Status Indicator (δ):

- δ=1: Event observed.
- δ=0: Event censored.
- **Data: (Y = min(T, C), δ):** Observed time is Y=min(T,C), with indicator δ.

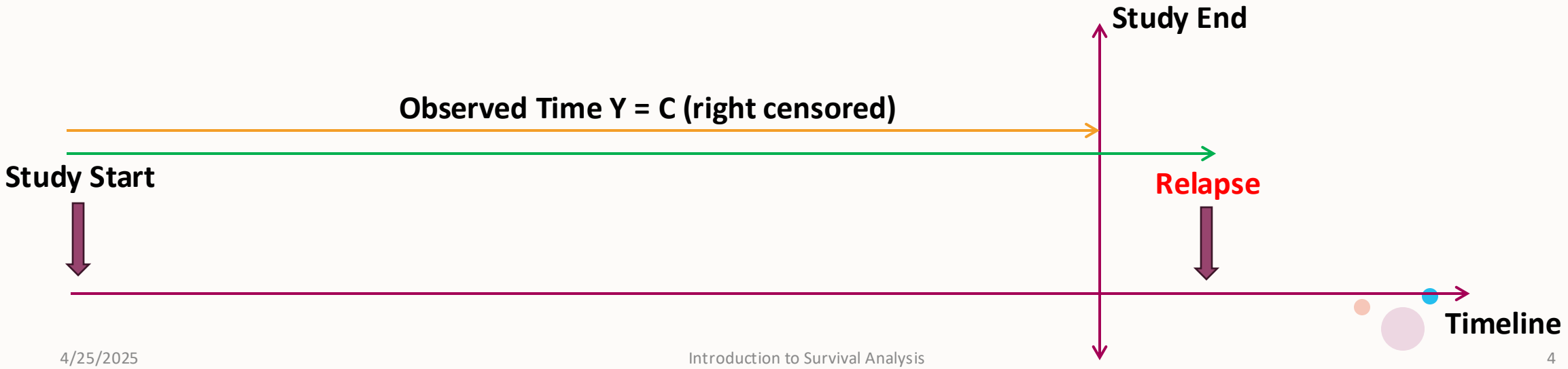


Types of Censoring

- **Right censoring** (most common): event has not occurred *yet*

Example:

- Patient A enrolls in the study and is followed for the full 5 years without **relapsing**.
- Their relapse time is right-censored at 5 years.
- Why? Because we only know that their relapse time is at least 5 years, but we don't know the exact time of relapse (if any).

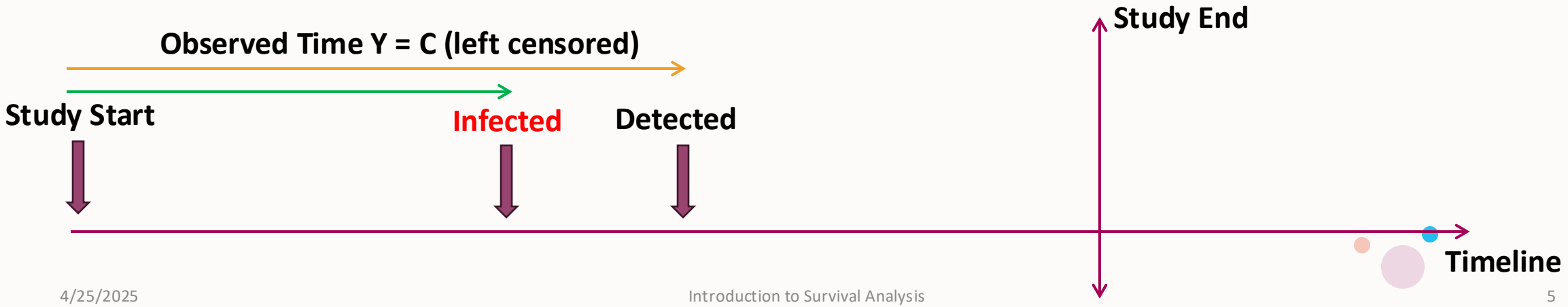


Types of Censoring

- **Left censoring**: event occurred before observation

Example:

- A patient comes in for their first test, and the results show they are already **infected**.
- However, there is no record of when the infection actually occurred.
- The infection happened before the first observation, but the exact time of infection is unknown—only that it occurred before detection.

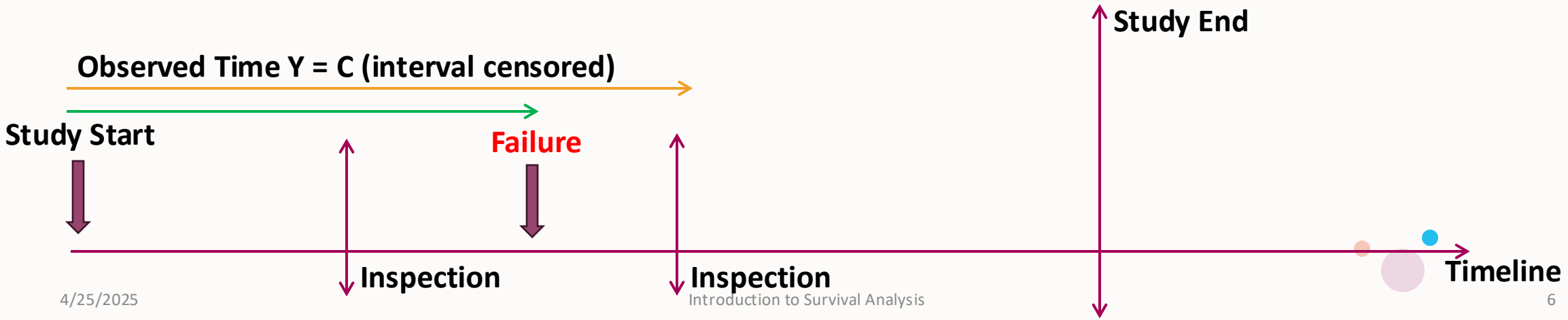


Types of Censoring

- **Interval censoring**: event occurred in a time interval

Example:

- A company monitors machines for component failure with routine inspections every 100 hours.
- A machine's component is functioning at 500 hours,
- But at the next inspection at 600 hours, the component is found to have already failed.
- The failure occurred between 500 and 600 hours, but the exact time is unknown.
- That's interval censoring — the failure time is known only to lie within a time interval.



Survival & Hazard Functions

- Definition:

$$S(t) = P(T > t) = 1 - F(t)$$

- $F(t)\ is\ the\ CDF$.
- The probability an individual survives past time t.
- Starts at 1, decreases over time
- $If\ f(t)\ is\ the\ PDF$:

$$S(t) = \int_t^{\infty} f(u)\ du = 1 - \mathrm{F(t)}$$

- Or:

$$f(t) = -\frac{dS(t)}{dt}$$

Survival & Hazard Functions

- Hazard Function:

$$\lambda(t) = \frac{f(t)}{S(t)}$$

- Think of it as: *"If you're alive at time t, what's the risk you die instantly?"*
- *Cumulative Hazard function:*

$$\Lambda(t) = \int_0^t \lambda(u)\, du$$

- Relation to survival:

$$S(t) = \exp(-\Lambda(t))$$

- It is the total accumulated risk of experiencing the event up to time t.
- Think it as the "exposure to danger" that builds up over time.
- The longer you "survive," the more risk you've accumulated — but not necessarily experienced yet.

Thank you!