UNIVERSITA' DEGLI STUDI DI NAPOLI FEDERICO II

Scuola Politecnica e delle Scienze di Base
Corso di Laurea Magistrale in Ingegneria dell'Automazione e Robotica

Field and Service Robotics

HOMEWORK 1

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Exercise n°1

Let us consider the *ATLAS Robot* from Boston Dynamics in two different configurations, as shown in the two figures below.

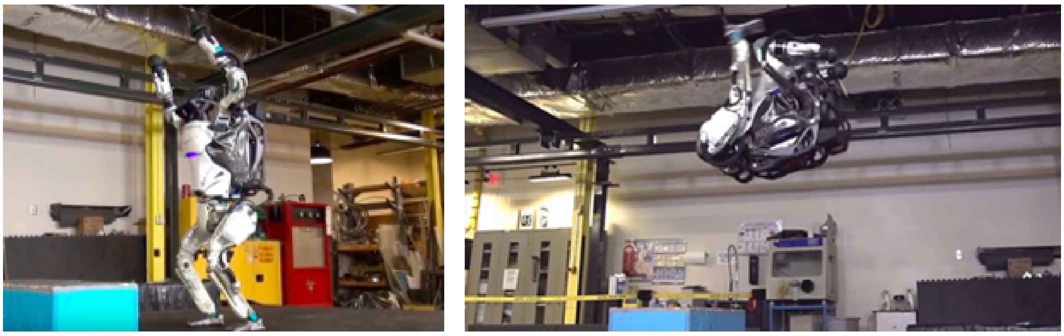

Figure 1: ATLAS Robot

ATLAS Robot is a *Humanoid Robot*, designed to perform advanced mobility and manipulation tasks in complex environments. Equipped with high agility, it can walk on rough terrain, jump, perform flips, and even execute acrobatic movements thanks to a sophisticated *Control System*. From Figure 1 it is possible to notice that, on the left, ATLAS is *standing* and on the right, ATLAS is performing a *backflip*. Considering that ATLAS actuators can produce *unbounded torques*, it is possible to say that:

a. *"While standing, ATLAS is Fully Actuated"* [1] is **TRUE**. This because, thanks to the assumption made previously, this Robot can perform an instantaneous acceleration in any arbitrary direction. In fact, in general, a Humanoid Robot has always a *'foot'* in contact with the ground (for this reason, some Humanoid Robots are very slow, in such a way to make the system *Fully Actuated* in any moment). This is possible thanks to the large number of actuators that make up the robot. Notice that we are talking about *Instantaneous Accelerations* because

[1]Without going into theoretical details, the question of the Fully Actuation is: 'It is possible to realize any instantaneous acceleration I want to my system?'.
If YES the system is *Fully Actuated*, if NO the system is *Underactuated*

we consider a given *instant* t and a given *state* x and, depending on the *joint positions* q and the *joint velocities* $\dot{q}$, there exists a *control input* τ such that the resulting acceleration $\ddot{q} = f(q, \dot{q}, \tau, t)$ is the desired one. If f is *surjective*, as in the case under examination, then the system is *Fully Actuated*.

b. *"While doing backflip, ATLAS is Fully Actuated"* is **FALSE**. This because, during a backflip, a Humanoid Robot cannot perform any arbitrary *instantaneous acceleration*. For example, the Robot cannot have a *translational acceleration*, since it is in mid-air and it is not in contact with the ground. In fact, when the Robot starts to jump, the trajectory of the motion is already *predefined* and, during it, there are no combination of *inputs* τ in such a way to move, for example, the Robot left or right.

Exercise n°2

Let us consider the *Spatial Mechanism* in Figure 2. It is a *Surgical Manipulator* developed at the National University of Singapore and it is composed by *three identical parts*, each with a *Prismatic Joint* P, and two *Universal Joints* U.

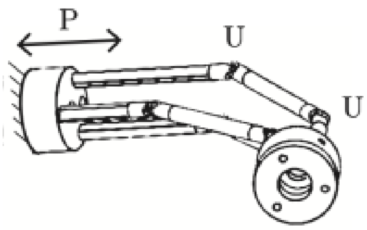

Figure 2: Surgical Manipulator

Using the *Grübler's Formula*[2] $DoFs = m(N-1-J) + \sum_{i=1}^{J} f_i$ to compute the Degrees of Freedoms of this manipulator, it is possible to obtain that:

[2]With N = number of links, J = number of joints, m = number of DoFs of a rigid body according to the specific case, f_i = number of freedoms provided by the i-th joint

$$DoFs = 6(7 + 1 - 1 - 9) + (6 * 2 + 3 * 1) = 3$$

But, not all the constraints are independent, and therefore this formula provides only a *Lower Bound* of the number of DoFs. By comparing this with my intuition about the possible motions of this mechanism, it is possible to notice that each part of the manipulator can move along the relative axis of the Prismatic Joint P with 3 DoFs. Then, there are also 2 DoFs for each of the two triplets of Universal Joints U. So, at the end, the mechanism in Figure 2 has **7 DoFs**. If instead, we consider that the 3 Prismatic Joints P can move only simultaneously, the mechanism has **5 DoFs**.

The *Configuration Space Topology*, for this robot, in the first case, is:

- I^7 considering the Joint Limits for both Prismatic and Universal Joints
- $I^3 \times S^2 \times S^2$ considering the Joint Limits only for the Prismatic Joints

Let us consider now the *Spatial Mechanism* in Figure 3. It is made by *6 identical bars* with all *Spherical Joints*. As done before, with the *Grübler's Formula* the result is:

$$DoFs = 6(7 + 1 - 1 - 12) + 12 * 3 = 6$$

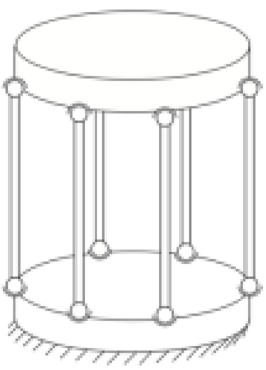

Figure 3: 6 Bars Mechanism

But, for the same reason as before, also here this is only a *Lower Bound*. In fact, according to my intuition, the mechanism in Figure 3 can perform the rotations of the 6 bars around their axis, the translation along an horizontal plane, the rotation along a vertical axis of the top platform and the vertical translation of it along the same axis. So, according to this, the 6 Bars Mechanism in Figure 3 has **9 DoFs**.

The *Configuration Space Topology*, in this case, is:

- $T^6 \times S^2 \times S^1$ without considering the limit of the Joints related to the two platform
- $T^6 \times I^3$ considering the limit of the Joints

Exercise n°3

Now let us state whether the following sentences regarding *Underactuation* or *Fully Actuation* are True or False.

a. *"A car with inputs the steering angle and the throttle is underactuated"* is **TRUE**. The system is *Underactuated* because it has only two control inputs and at least three DoFs provided by the planar rigid body. In fact, the 'problem' for the car is that it is not possible do parking so easily but, sometimes, it is necessary to perform maneuvers because it cannot generate any instantaneous acceleration, in particular lateral acceleration.

b. *"The KUKA youBot system on the slides is Fully Actuated"* is **TRUE**. This because it can generate any instantaneous acceleration in any direction thanks to the omnidirectional base and to the Mecanum-Wheels, each driven by its own motor, plus the 5-DoFs Robotic Arm. So, for these reasons, the KUKA youBot can move freely in any direction without doing maneuvers like in the car example.

c. *"The hexarotor system with co-planar propellers is fully actuated"* is **FALSE**. This because the condition $dim[\tau] < dim[q]$, which implies *Underactuation*, is only a sufficient condition. There are cases, like this, where, even having a number of actuators greater than the number of *DoFs*, this does not imply the *Fully Actuation* of the system. This situation typically occurs when the actuators do not independently control all the DoFs due to specific constraints, such as kinematic or dynamic limitations, or to the specific configuration of the actuators. Here, in fact, also if $dim[\tau] > dim[q]$ the system is *Underactuated.* The propellers are aligned to the same plane and, basically, there are *8 forces* always aligned and perpendicular to the plane itself. Only by unbalancing the forces it is possible to achieve a rotation of the body but there is no way of combining the *control inputs* such that it is possible to achieve any acceleration in the space (for example the task that let the drone to move left and right with the drone in balance). So I can never have only the *translational acceleration* along x or y. Even if the robot is made up of a greater number of propellers, this does not change the final result, and the system will always remain *Underactuated.*

d. *"The KUKA iiwa 7-DOF robot is redundant and it cannot be underactuated because we know that all redundant systems are not"* is **FALSE**. This because *Redundancy* and *Underactuation* are not related to each other. In fact, *Redundancy* is something related to the *Kinematic of the task* I want to achieve and it is related to the possibility of achieving it with different *configurations* and *input combinations.* The *Underactuation*, instead, is related to the *Dynamic Model* and to the possibility to have any acceleration in any direction. In fact, also in the previous example of the Humanoid Robot, it can be both *Fully*

Actuated and *Underactuated* depending on what the robot is doing. So, a redundant manipulator could also be Underactuated.

Exercise n°4

Let us state whether each of the following *distributions* are *involutive* or not and let us find the *Annihilator* for each of them.

a. $\Delta_1 = \left\{ \begin{bmatrix} -3x_2 \\ 1 \\ -1 \end{bmatrix} \right\}, \quad U \in \mathbb{R}^3$

It is possible to notice that, the distribution Δ_1 is a *1-Dimensional Distribution.* So, by definition, since the *Lie Bracket* $[f, f] = 0$, and the *Zero Vector* belongs to any distribution (it is the zero element of a *Vector Space*), it is **involutive**[3][4]. In this case $d = dim(\Delta_1(x)) = 1$, while the space dimension is $n = 3$, because $U \in \mathbb{R}^3$.

Since $dim(\Delta_1(x)) = 1$, there will exist an annihilator[5] $\Delta^{\perp}(x)$ of dimension equal to 2. In fact $dim(\Delta(x)) + dim(\Delta^{\perp}(x)) = n = 3$.

The annihilator of a distribution is identified by the set of co-vectors such that $\omega^* F(x) = 0$ [6]. Here $F(x) = \begin{bmatrix} -3x_2 \\ 1 \\ -1 \end{bmatrix}$

In this case it is possible to write:

$$\begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix} \cdot \begin{bmatrix} -3x_2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[3] A distribution Δ is said to be involutive if, for any pair of vector fields $f_1(x), f_2(x) \in \Delta$, the Lie Bracket of these Vector Fields belongs to the distribution $\Delta \Rightarrow [f_1(x), f_2(x)] \in \Delta$

[4] The Lie Bracket of two Vector Fields $f_1(x)$ and $f_2(x)$ is defined as $[f_1(x), f_2(x)] = \frac{\partial f_2}{\partial x} f_1(x) - \frac{\partial f_1}{\partial x} f_2(x)$

[5] The Annihilator of a distribution $\Delta(x)$ is a co-distribution $\Delta^{\perp}(x)$ such that $\Delta^{\perp}(x) = \{\omega^* \in \mathbb{R}^n : \langle \omega^*, v \rangle = 0 \quad \forall v \in \Delta(x)\}$ with $dim(\Delta(x)) + dim(\Delta^{\perp}) = n$

[6] $F(x)$ is the matrix composed by the Vector Fields

And so

$$-3x_2\omega_1 + \omega_2 - \omega_3 = 0$$

$$\omega_3 = \omega_2 - 3x_2\omega_1$$

In conclusion

$$\omega^* = [\omega_1 \quad \omega_2 \quad (\omega_2 - 3x_2\omega_1)]$$

Let us now compute the two co-vectors by assigning different values to ω_1 and ω_2. Obviously, the trivial case in which both are 0 should not be considered. Using $\omega_1 = 1$ and $\omega_2 = 0$ the co-vector is:

$$\omega_1^* = [1 \quad 0 \quad -3x_2]$$

Instead, using $\omega_1 = 0$ and $\omega_2 = 1$ the co-vector is:

$$\omega_2^* = [0 \quad 1 \quad 1]$$

At the end, the annihilator is:

$$\Delta^{\perp}(x) = \text{span}\{\omega_1^*, \omega_2^*\} = \text{span}\{[1 \quad 0 \quad -3x_2], [0 \quad 1 \quad 1]\}$$

b. $\Delta_2 = \left\{ \begin{bmatrix} -1 \\ 0 \\ x_3 \end{bmatrix}, \begin{bmatrix} x_2 \\ -9 \\ x_1 \end{bmatrix} \right\}, \quad U \in \mathbb{R}^3$

Now, the same steps as in the previous point should be followed, but in this case the *Involutiveness Property* must be verified.

The matrix associated with the *Vector Fields* of the distribution Δ_2 is $F(x) = \begin{bmatrix} -1 & x_2 \\ 0 & -9 \\ x_3 & x_1 \end{bmatrix}$ with $rank = 2$ because the determinant of the minor made by the first two rows is 9. For this reason $dim(\Delta_2) = d = 2$.

Now let us compute the Lie Bracket between the two Vector Fields

$$[f_1(x), f_2(x)] = \frac{\partial f_2}{\partial x} f_1(x) - \frac{\partial f_1}{\partial x} f_2(x) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ x_3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ -9 \\ x_1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -(1 + x_1) \end{bmatrix}$$

Let us add this new Vector Field in the matrix $F(x)$. If the rank of this new matrix remains the same, so the computed Lie Bracket is linearly dependent on $f_1(x)$ and $f_2(x)$, the Distribution Δ_2 is involutive.

$$F(x) = \begin{bmatrix} -1 & x_2 & 0 \\ 0 & -9 & 0 \\ x_3 & x_1 & -(1 + x_1) \end{bmatrix}$$

Computing the determinant of the new $F(x)$ it is possible to notice that

$$det(F(x)) = -9(1 + x_1)$$

When $x_1 \neq -1$ the matrix $F(x)$ is *Full-Rank* and so the rank increased and the distribution Δ_2 is **Not Involutive**. It is **Involutive** only if $x_1 = -1$, and so only considering this plane.

Let us compute the annihilator. In this case $dim(\Delta^{\perp}(x)) = n - dim(\Delta(x)) = 1$.

$$\begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix} \cdot \begin{bmatrix} -1 & x_2 \\ 0 & -9 \\ x_3 & x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -\omega_1 + \omega_3 x_3 = 0 \\ \omega_1 x_2 - 9\omega_2 + \omega_3 x_1 = 0 \end{cases} \Rightarrow \begin{cases} \omega_1 = \omega_3 x_3 \\ \omega_2 = \frac{1}{9}\omega_1 x_2 + \frac{1}{9}\omega_3 x_1 = \frac{1}{9}\omega_3 x_3 x_2 + \frac{1}{9}\omega_3 x_1 \end{cases}$$

$$\Rightarrow \begin{cases} \omega_1 = \omega_3 x_3 \\ \omega_2 = \dfrac{1}{9}\omega_3(x_3x_2 + x_1) \end{cases}$$

So

$$\omega^* = \begin{bmatrix} \omega_3 x_3 & \dfrac{1}{9}\omega_3(x_3x_2+x_1) & \omega_3 \end{bmatrix}$$

Avoiding the trivial choice of $\omega_3 = 0$, it is possible to choose $\omega_3 = 9$ in such a way to obtain the annihilator

$$\Delta^{\perp}(x) = \text{span}\{[9x_3 \quad x_3x_2 + x_1 \quad 9]\}$$

c. $\Delta_3 = \left\{ \begin{bmatrix} 2x_3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2x_2 \\ x_1 \\ -1 \end{bmatrix} \right\}, \quad U \in \mathbb{R}^3$

Let us do the same steps made in point **b.**.

The matrix associated with the Vector Fields of the distribution Δ_3 is $F(x) = \begin{bmatrix} 2x_3 & -2x_2 \\ 1 & x_1 \\ 0 & -1 \end{bmatrix}$ with $rank = 2$ because the determinant of the minor made by the last two rows is -1. For this reason $dim(\Delta_2) = d = 2$.

Now let us compute the Lie Bracket between the two Vector Fields

$$[f_1(x), f_2(x)] = \frac{\partial f_2}{\partial x} f_1(x) - \frac{\partial f_1}{\partial x} f_2(x) = \begin{bmatrix} 0 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2x_3 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2x_2 \\ x_1 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} -2 \\ 2x_3 \\ 0 \end{bmatrix} - \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2x_3 \\ 0 \end{bmatrix}$$

Let us add this new Vector Field in the matrix $F(x)$.

$$F(x) = \begin{bmatrix} 2x_3 & -2x_2 & 0 \\ 1 & x_1 & 2x_3 \\ 0 & -1 & 0 \end{bmatrix}$$

Computing the determinant of the new $F(x)$ it is possible to notice that

$$det(F(x)) = 4x_3^2$$

When $x_3 \neq 0$ the matrix $F(x)$ is *Full-Rank* and so the Rank increased and the distribution Δ_3 is **Not Involutive**. It is **Involutive** only if $x_3 = 0$, and so only considering this plane.

Let us compute the annihilator. In this case $dim(\Delta^{\perp}(x)) = n - dim(\Delta(x)) = 1$.

$$\begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix} \cdot \begin{bmatrix} 2x_3 & -2x_2 \\ 1 & x_1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 2\omega_1 x_3 + \omega_2 = 0 \\ -2\omega_1 x_2 + \omega_2 x_1 - \omega_3 = 0 \end{cases} \Rightarrow \begin{cases} \omega_2 = -2\omega_1 x_3 \\ \omega_3 = \omega_2 x_1 - 2\omega_1 x_2 = -2\omega_1 x_1 x_3 - 2\omega_1 x_2 \end{cases}$$

$$\Rightarrow \begin{cases} \omega_2 = -2\omega_1 x_3 \\ \omega_3 = -2\omega_1(x_1 x_3 + x_2) \end{cases}$$

So

$$\omega^* = [\omega_1 \quad -2\omega_1 x_3 \quad -2\omega_1(x_1 x_3 + x_2)]$$

Avoiding the trivial choice of $\omega_1 = 0$, it is possible to choose $\omega_1 = 1$ in such a way to obtain the annihilator

$$\Delta^{\perp}(x) = \text{span}\{[1 \quad -2x_3 \quad -2(x_1 x_3 + x_2)]\}$$

.

Exercise n°5

Let us show, using the *Accessibility Rank Condition*, that a *set of Pfaffian constraints*, that does not depend on the *generalized coordinates*, $A\dot{q} = 0$, with A constant, is always *integrable* (*completely holonomic system*). Remember that:

- *Holonomic Constraint*[7] means that some parts of the system's configuration are *not reachable* and the instantaneous motions are prevented[8]

- *Nonholonomic Constraint* means that instantaneous motions are prevented but it is possible to *reach* that configuration

So, it is important to understand the *controllability* of the system and, if it is controllable, then the set of Pfaffian constraints is nonholonomic, otherwise it is holonomic and so integrable. *Kinematic constraints*[9] are generally expressed in the Pfaffian and compact form

$$A(q)\dot{q} = 0$$

Here, the matrix $A(q)$ is *constant*, and so it does not depend on q. The directions in which I can move constitute the *Kinematic Model*, and so it is necessary to find the dual space (and so the annihilator) of the relative constraints.
Let us construct now a distribution from the *Pfaffain matrix A*. The distribution is spanned by the vectors $g_j(q) \quad j = 1, ..., m \quad$ such that $Ag_j(q) = 0$. This distribution is the *Kinematic Distribution*

$$\Delta = \text{span}\{g_j(q) \in \mathbb{R}^{\ltimes} : Ag_j(q) = 0, \quad j = 1, ..., m\}$$

So, a generic vector belonging to Δ can be expressed as *linear combination* of

[7] $h_i(q) = 0, \quad i = 1, ..., k < n$ with $h_i : \mathbb{R}^n \Rightarrow \mathbb{R}$ and $h_i \in C^\infty$ and it is Smooth

[8] There is a reduction of the space of the *admissible model configurations* to a subset of $\mathbb{R}^n$ of dimension $n - k$

[9] $a_i(q, \dot{q}) = 0 \quad i = 1, ..., k < n$

the Vector Fields spanning Δ

$$\dot{q} = \sum_{j=1}^{m} g_j(q)u_j = G(q)u \in \Delta$$

where the *control inputs* u_j are the weights of the linear combination, in such a way to obtain a *Control-Affine system*. Because A, as we know, is constant, this imply that also the Vector Fields g_j don't depend on the generalized coordinates q. So, for this reason, all the Lie Brackets that can be generated by all the Vector Fields g_j are 0. So, the *Accessibility Distribution* Δ_A [10] is composed only by the Vector Fields g_j and so it coincides with the *Kinematic Distribution* Δ. But, $rank(A) = k$, where k is the number of constraints, and so it is possible to say that:

$$dim(\Delta_A) = dim(\Delta) = m = n - k < n$$

Because the *Accessibility Rank Condition* is satisfied only if $dim(\Delta_A) = n$, the System is **NOT CONTROLLABLE**.

Thus, in conclusion, it has been demonstrated that a set of Pfaffian constraints that does not depend on the generalized coordinates, $A\dot{q} = 0$, with A constant, is always **INTEGRABLE** and so the system is **COMPLETELY HOLONOMIC**.

Exercise n°6

Let us consider the *Raibert's Hooper Robot* in Figure 4. It has the following kinematic constraint in the Pfaffian form $(I + m(l+d)^2)\dot{\theta} + m(l+d)^2\dot{\psi} = 0$, with $q = [\theta \quad \psi \quad I]^T$, with I the moment of inertia of the body and m the leg mass concentrated at the foot. It is possible to compute a *Kinematic Model* of such a

[10] The Accessibility Distribution is the distribution generated by all the Vector Fields and all the Lie Bracket generated by them

robot and show whether this system is holonomic or not. To do this, **MATLAB Symbolic Toolbox** is been used.

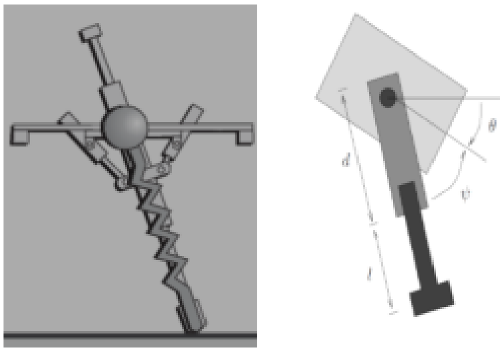

Figure 4: Raibert's Hooper Robot

To view the developed code, see the file *HW1_FSR_ES6.m* attached to this project. Thanks to this, the following results have been achieved very quickly.

$$A = [I + m(d+l)^2 \quad m(d+l)^2 \quad 0]$$

$$G = [g_1(q) \quad g_2(q)] = \begin{bmatrix} -\dfrac{m(d+l)^2}{md^2+2mdl+ml^2+I} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[g_1(q), g_2(q)] = \frac{\partial g_2}{\partial q} g_1(q) - \frac{\partial g_1}{\partial q} g_2(x) = \begin{bmatrix} -\dfrac{m(d+l)^2}{(md^2+2mdl+ml^2+I)^2} \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -\dfrac{m(d+l)^2}{md^2+2mdl+ml^2+I} & 0 & -\dfrac{m(d+l)^2}{(md^2+2mdl+ml^2+I)^2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$rank(F) = 3$$

So, it is possible to say that the system is **CONTROLLABLE** thanks to the *Accessibility Rank Condition* because $n = 3$. Therefore, the system is **COMPLETELY NONHOLONOMIC** and the Kinematic constraint introduced before is **NOT INTEGRABLE**.

```
Vector Fields of the Kinematic Model:

G =

[-(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I), 0]
[                                           1, 0]
[                                           0, 1]

Lie Bracket [g1,g2]:

Lie_Bracket =

-(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I)^2
                                             0
                                             0

F Matrix related to the Accessibility Distribution Δ_A :

F =

[-(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I), 0, -(m*(d + l)^2)/(m*d^2 + 2*m*d*l + m*l^2 + I)^2]
[                                           1, 0,                                               0]
[                                           0, 1,                                               0]

rank(F):
     3

The System is Controllable and Completely Nonholonomic
```

Figure 5: MATLAB Overview