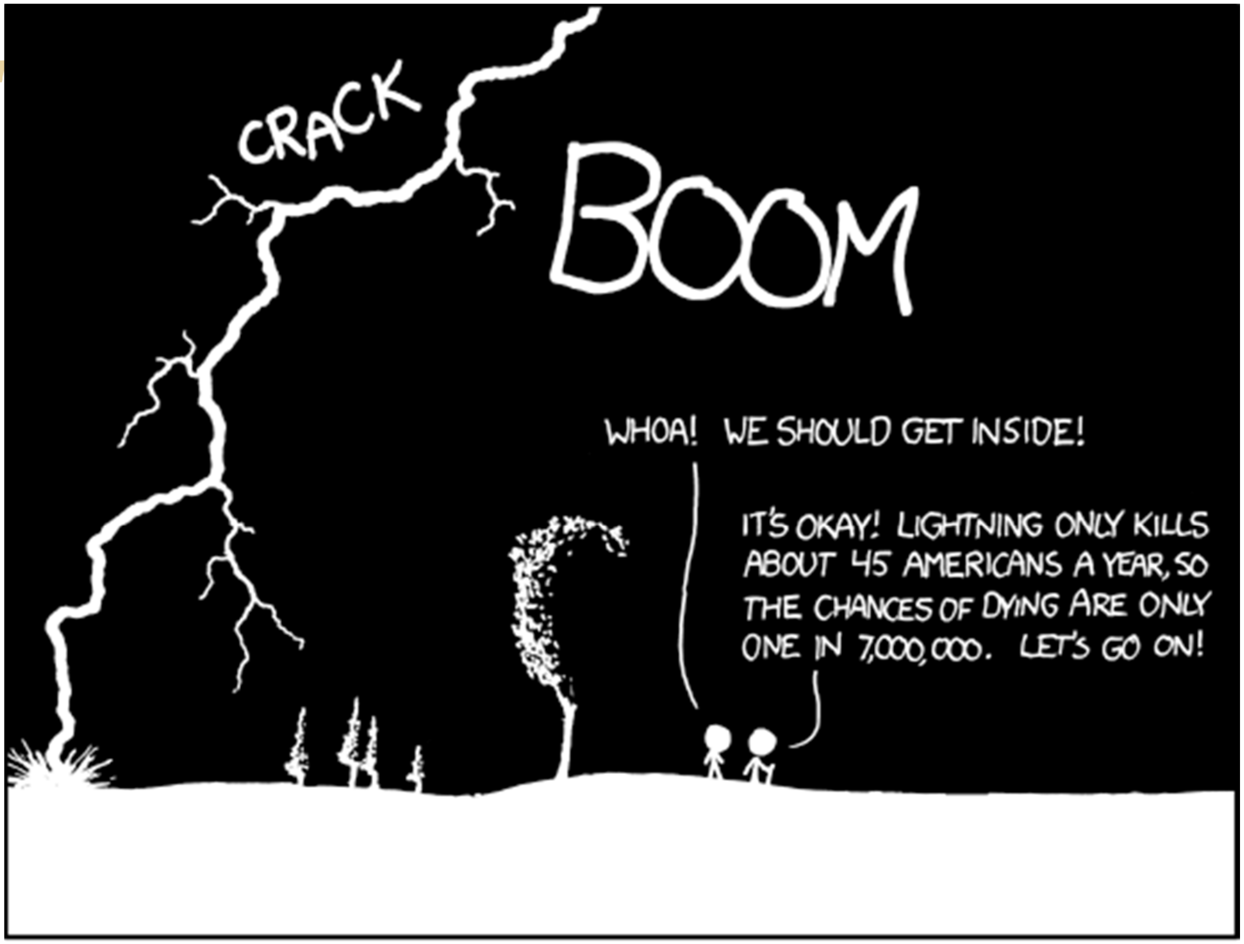
UNIVERSITY of WASHINGTON

Data Science UW
Methods for Data Analysis

Probability and More on Distributions
Lecture 2
Stephen Elston



THE ANNUAL DEATH RATE AMONG PEOPLE WHO KNOW THAT STATISTIC IS ONE IN SIX.

Topics

- Review
- Counting
- Axioms of Probability
- Probability Examples
- Conditional Probability
- More on Distributions

Review

- > Distributions
  - – Discrete: Bernoulli, Binomial, Poisson
  - – Continuous: Uniform, Normal, Student’s T
- > Numerical and Visual Exploration of Data
- > Transformations
- > Simpson’s Paradox

Review

Summary statistics

> Sample mean = μ = sum(x_i)/n

> sample var = σ = sum((μ – x_i)^2) / (n – 1)

> Sample std = sqrt(σ)

> Standard error of the sample mean = se = std/ sqrt(n)

Counting

- Combinatorics of the biggest areas of mathematics.
- Example:
  - Subway has 4 bread choices, 5 meat choices, 4 toppings. How many sandwich combinations?
  - How many different 4-beer tasters can I have in a bar with 10 beers on tap?
- Solve these using the 'Multiplication Principle'.
  - Subway Problem:

$$\underset{\text{(\# of breads)}}{\underline{\quad 4 \quad}} * \underset{\text{(\# of meats)}}{\underline{\quad 5 \quad}} * \underset{\text{(\# of toppings)}}{\underline{\quad 4 \quad}} = 80$$

  - Beer Problem:

$$\underset{\text{(\# for 1}^{\text{st}}\text{ beer)}}{\underline{\quad 10 \quad}} * \underset{\text{(\# for 2}^{\text{nd}}\text{ beer)}}{\underline{\quad 9 \quad}} * \underset{\text{(\# for 3}^{\text{rd}}\text{ beer)}}{\underline{\quad 8 \quad}} * \underset{\text{(\# for 4}^{\text{th}}\text{ beer)}}{\underline{\quad 7 \quad}} = 5{,}040$$

Multiplication Principle

> If there are A ways of doing task a, and B ways of doing task b, then there are A*B ways of completing both tasks.

> Example:

– If I have 5 books, how many ways can I *order* them on the bookshelf?

5 choices	*	4 choices	*	3 choices	*	2 choices	*	1 choice
1st book		2nd book		3rd book		4th book		5th book

= 5 factorial = 5! = 120

Factorials

- Factorials
  - Count # ways to order N things = N!
- Factorials get VERY LARGE quickly.
  - 21! Is larger than the biggest long-int in 64 bit.
    - 21! = 5.1E19
    - Biggest long int (64 bit) = 9.2E18
  - Fun fact, every 52 card shuffle is highly likely to be the only time that shuffle has ever occurred.

Counting Subgroups

- Revisit: 10 beers on tap, need a sample of 4 different beers.
- Let's assume order matters, i.e., Amber-Stout-Porter-Red is different from Red-Porter-Stout-Amber.
- Use 'Permutations' (pick):

$$10 * 9 * 8 * 7 = \frac{10!}{6!} = \frac{10!}{(10-4)!} = 10P4 = P(10,4)$$

Counting Subgroups

- Now, Let's assume order doesn't matter.
- Use 'Combinations' (choose):

$$\frac{10*9*8*7=\frac{10!}{6!}=\frac{10!}{(10-4)!}=10P4=P(10,4)}{(\#\ of\ orderings\ of\ 4\ beers)=4!}$$

$$=\frac{10!}{4!\,(10-4)!}=10C4=C(10,4)=\binom{10}{4}$$

More on Combinations

- > Combinations appear on the Pascal's Triangle!
- > C(N,x) appears on the Nth row, xth number (starting at 0)

$\binom{10}{4}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1

Counting Examples

> There are 10 Light beers on tap, and 10 Dark beers on tap, how many ways can I get a 4-beer sampler that contains exactly 1 light beer? (ordering doesn't matter)

$$\frac{(\#\ of\ ways\ for\ light\ beer) \cdot (\#\ of\ ways\ for\ dark\ beer)}{(\#\ of\ ways\ to\ order\ 1L\ and\ 3D)}$$

$$\frac{(10) \cdot \binom{10}{3}}{4} = \frac{10 * 120}{4} = 300$$

Counting Examples

> 6:5 Blackjack is dealt with a 6 shoe deck (52*6=312 cards). How many ways can someone get dealt two rank 10 cards?

$$\binom{6decks * 4ranks * 4suits}{2} = \binom{96}{2} = \frac{96!}{2!\,(94!)} = \frac{96 * 95}{2} = 4560$$

Counting Examples

> How many ways can two dice be rolled to get a sum of 10?

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Counting in R

- expand.grid() – function that creates a data frame from all combinations of vectors supplied.
- R-demo

Probability

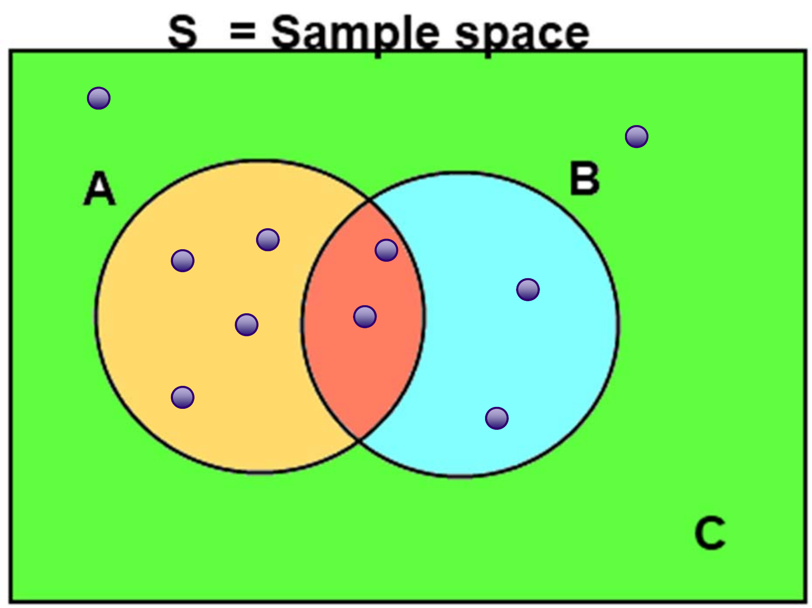
> The Probability of an event, A, is the number of ways A can occur, divided by the number of total possible outcomes in our Sample Space, S.

$$P(A) = \frac{N(A)}{N(S)}$$

> If ● is an event, then

$$P(A) = \frac{6}{10} = \frac{3}{5}$$

$$P(B) = \frac{4}{10} = \frac{2}{5}$$

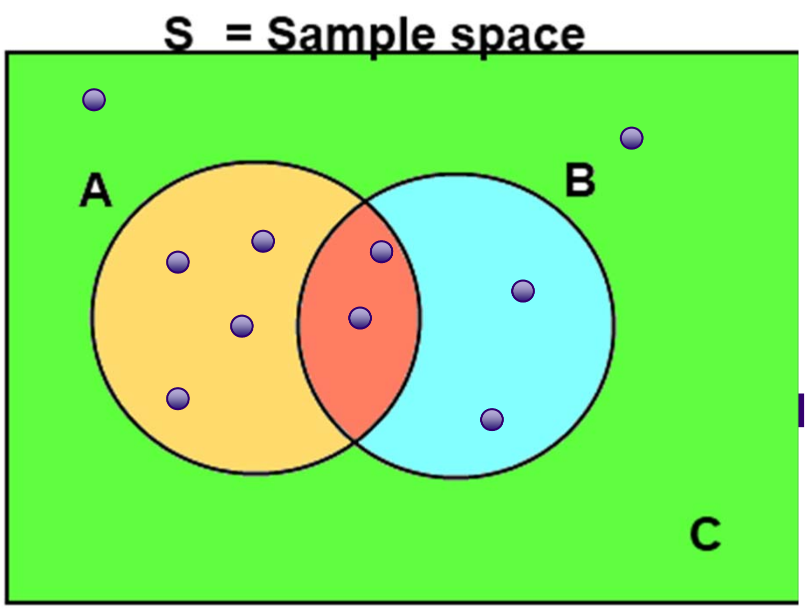


Probability

- If ● is an event, then
  - Intersection: $P(A \cap B) = \dfrac{2}{10} = \dfrac{1}{5}$
  - Union: $P(A \cup B) = \dfrac{8}{10} = \dfrac{4}{5}$
  - Negation: $P(A') = \dfrac{6}{10} = \dfrac{3}{5}$

$$P((A \cup B)') = P(C) = \frac{2}{10} = \frac{1}{5}$$

$$P(A' \cap B') = P(C) = \frac{2}{10} = \frac{1}{5}$$



Axioms of Probability

> Probability is bounded between 0 and 1.

$$0 \leq P(A) \leq 1$$

Note: “Percent” literally means per one hundred

> Probability of the Sample Space = 1.

$$P(S) = 1$$

> The probability of finite *mutually exclusive* unions is the sum of their probabilities.

$$P(A \cup B) = P(A) + P(B) \quad \text{If A and B are M.E.}$$

Probability Examples

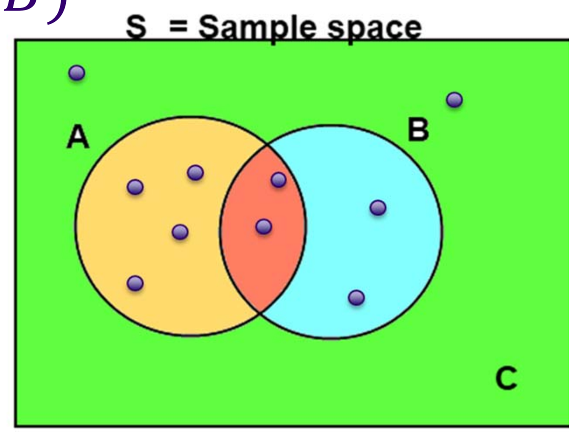
> Probability of rolling a sum of 10?

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

Mutually Exclusive Events

> In all cases, the probability of the union of A and B takes the form:

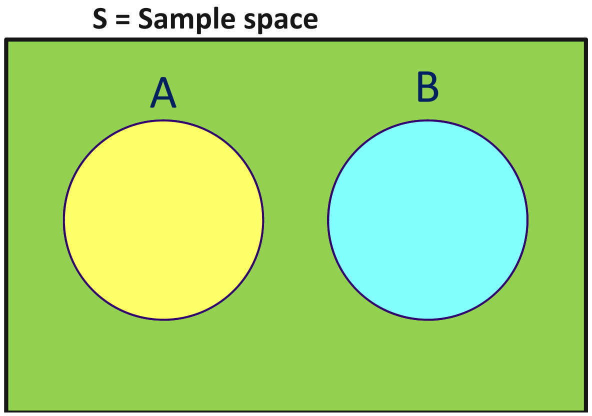
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



> If A and B are mutually exclusive that means that

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$



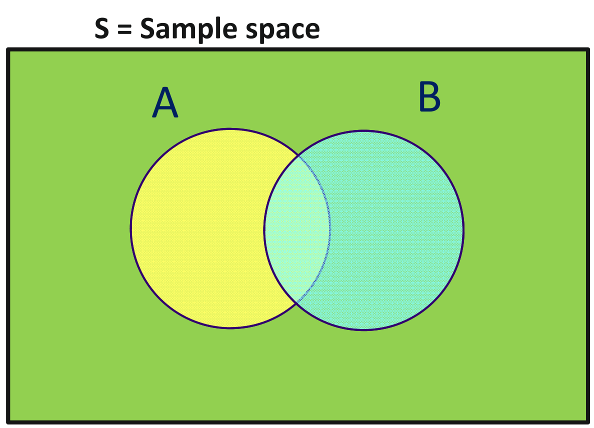
Conditional Probability

> The probability of A *given* B is written:

$$P(A|B)$$

> And is equal to:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{, compare to:} \quad P(E) = \frac{P(E)}{P(S)}$$



Independent Events

- Events A is independent of B if and only if:

$$P(A|B) = P(A)$$

- A being independent of B does NOT imply B is independent of A.

$$P(A|B) = P(A) \quad \not\Rightarrow \quad P(B|A) = P(B)$$

$$P(A|B) = P(A) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow \quad P(B)P(A) = P(A \cap B)$$

E.g. The event that my boss takes vacation has an impact on when I take vacation, but when I take vacation has no impact on when my boss takes vacation. (i.e., his vacation is independent of mine, but not vice versa)

Independence vs. Mutually Exclusive

- These are not related AT ALL and in fact, are nearly opposite ideas.
- If A is M.E. of B then: $P(A|B) = 0$

B occurring has a HUGE impact on P(A)

- If A is independent of B then: $P(A|B) = P(A)$

Example: The probability the sidewalk is wet given it is raining is very high,
But the probability that it is raining given the sidewalk is wet is lower (if I run my sprinklers often).

Odds

> Odds are expressed as (Count in event favor):(Count not in event favor)

- Make sure you reduce the fraction first

$$P(A) = \frac{n}{m} = \frac{n}{n + (m - n)}$$

Count in favor of A

Count not in favor of A

- Implies the odds are:

$$n : (m - n)$$

Examples:

If P(A)=5/6, then the odds are 5:1. 'Five to one'.

If the odds are 3:20, then P(A)=3/23

A straight up sports bet in Vegas has odds 1:1 (50%), but pays 0.95Xbet.

R Demo

Monty Hall Problem

- Famous conditional probability problem that divided statisticians when it came out.
  - Start with 3 doors. One prize behind unknown door. Pick a door. Host reveals a separate door with no prize. Then contestant can switch. Should they?

Monty Hall Problem

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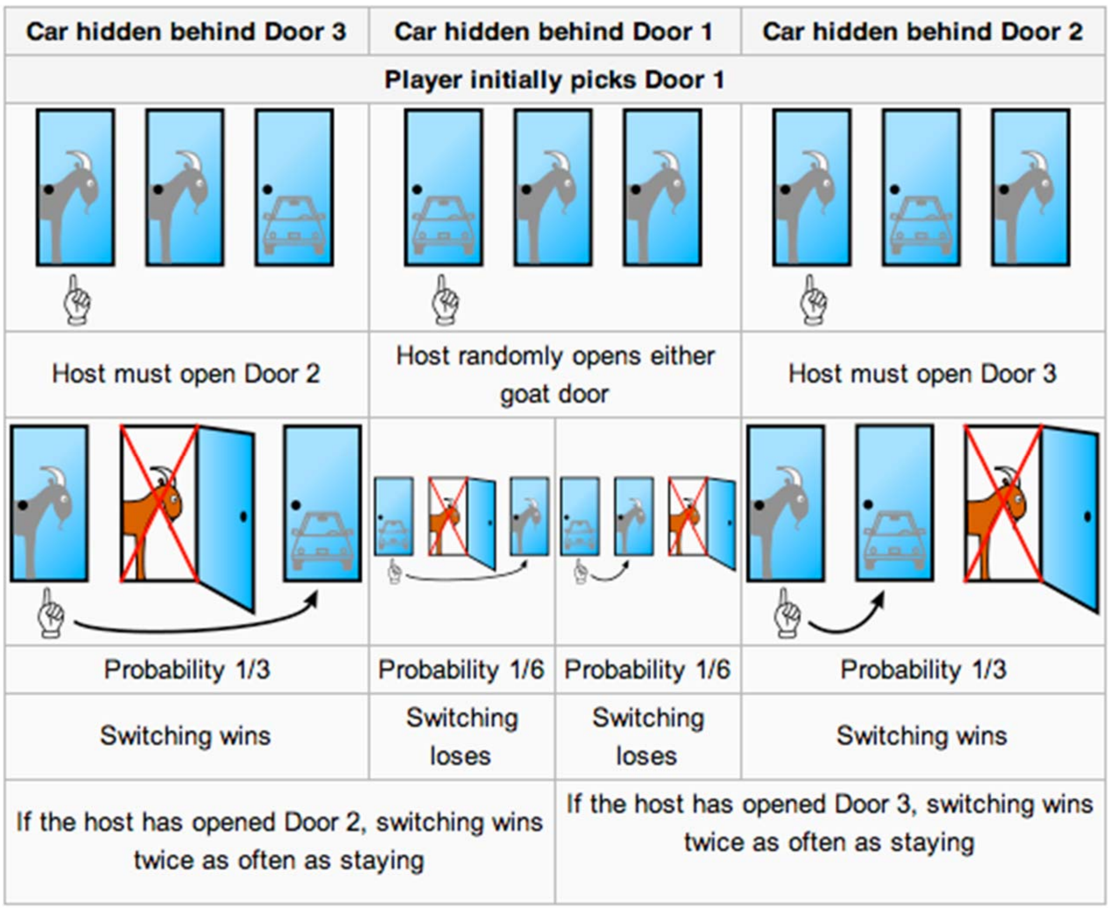
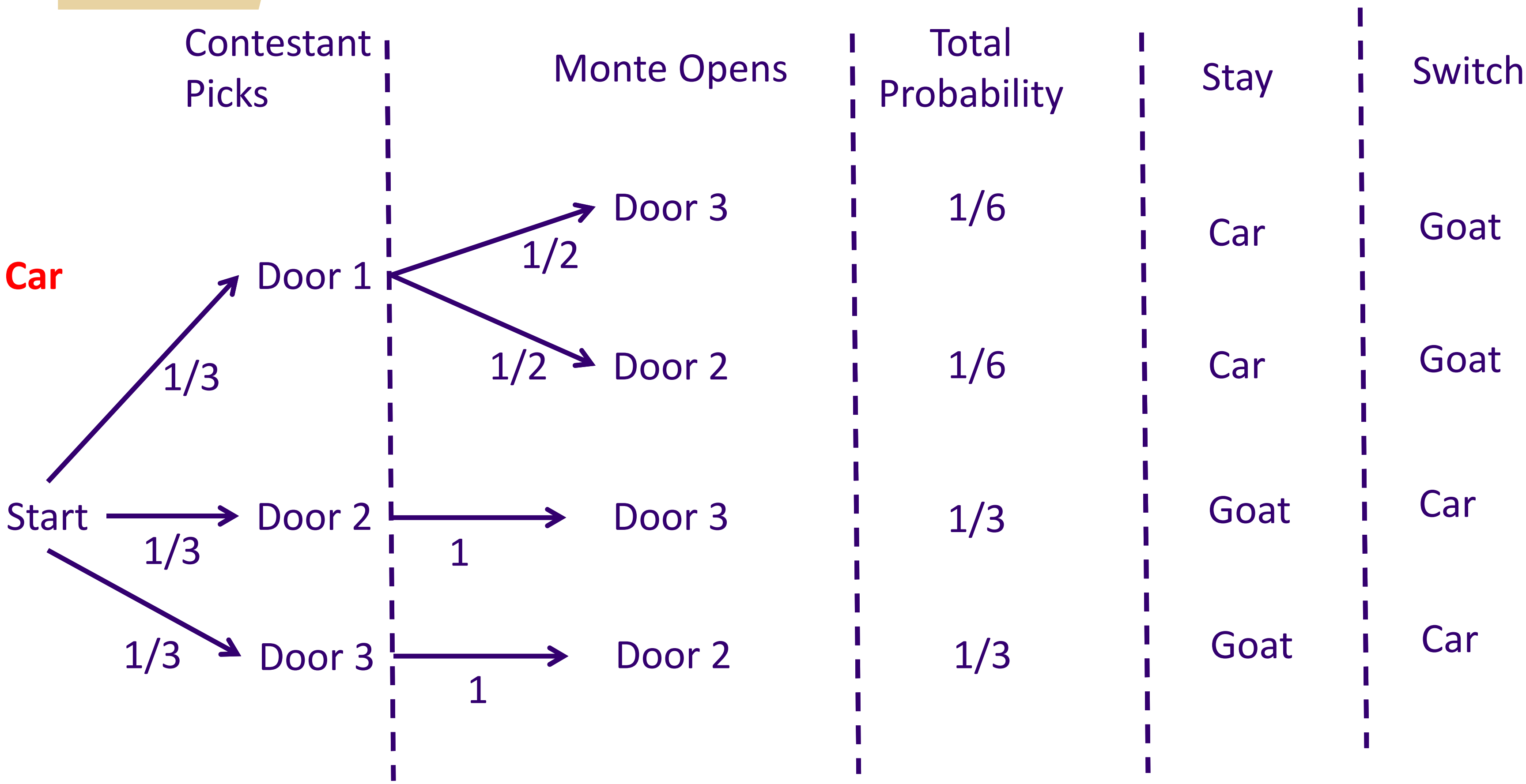


Monty Hall Problem: Conditional Probabilities



P(Car | Switch) = 2/3
P(Goat | Switch) = 1/3

Monty Hall Problem

- http://www.stayorswitch.com/
- https://en.wikipedia.org/wiki/Monty_Hall_problem

Simulations

> Used for complex distributions
> Can test distributional assumptions
> Simulate a conditional probability hierarchy
> Large number of realizations
> Use system.time() from base or microbenchmark() from microbenchmark package.
> R Demo

Testing Statistical Software

- Usual test processes apply: Need to build test cases
- Test cases must be repeatable (e.g. set.seed())
- Build test cases as you go: Test driven development

Dealing with Missing Data

- Reasons for missing data
  - Recording failure (mechanical/software failures)
  - Reporting failure (human decisions)
  - Translation failure (data transferring/parsing errors)
- Many shapes and types
  - Shapes: block, regular, random, sparse
  - Types:
    - Missing At Random (MAR): a particular variable has randomly omitted data.
    - Missing Completely At Random (MCAR) : every piece of data has equal chance of being omitted.
    - Missing Not At Random (MNAR): The value of data is related to chance of being omitted.
- Outliers may also be treated as missing data.

Dealing with Missing Data

Type	Benefits	Disadvantages	Notes
Drop Missing	-Speed	-Data Loss	
Substitution	- Speed	- Bias	
Mean/Median/Mode Fill	-No Data Loss	-Variance Reduction	
X~F(independents)	-More Accurate -No Data Loss	-Slower	-Needs most columns to be filled out -Hard on ind. data
knn	-More Accurate -No Data Loss	-Slower -Dependent on distance function - Bias	Forward fill Backward fill
X~F(y,independents)	-Very accurate -No Data Loss	-Slower -Need y	-Only on training set!

Dealing with Missing Data: Using Outside or New Data Sources

- Don’t forget to explore outside or new data sources to help fill-in missing data.
- With the advent of free public data and bigger data sources, this is gaining popularity as a tool for imputation.
- Unstructured text is a major source of data.
- Ex:
  - Caesar’s uses public reviews on websites to mine for customer sentiment about hotel rooms.
  - Zillow uses text descriptions of properties to fill in missing data about # bedrooms, # bathrooms, sq. footage, and various amenities.
  - Subject to human stupidity.

Yelp Rating for Circus-Circus: 2/5
Text Description: “My son and I stayed here. The service was great, the room was great, but it turns out my son is deathly afraid of clowns.”

Dealing with Missing Data: Variance and Multiple Imputation

- Imputation tries and maintain the intrinsic variance in the data set.
- Multiple different predictions are made for each missing data point. (Using previous methods)
- Hypothesis testing and predictions are made on all imputed sets to gauge the variance in the outcomes.
- R package ‘Amelia’ does this and creates a nested list of data frames.
- Lots of details at: https://cran.r-project.org/web/packages/Amelia/vignettes/amelia.pdf
- R demo

Getting Data

- Files
  - Csv: read.csv
  - Txt: read.table
- Web/HTML
  - readLines
  - XML, xpath
  - http://gastonsanchez.com/work/webdata/getting_web_data_r4_parsing_xml_html.pdf
- Databases
  - Sqlite: sqldf, RSQLite packages
    - Sqlite example
  - MongoDB: rmongodb package
  - Postgresql: RPostgreSQL package

Storing Data

- .csv – write.csv()
- .txt – write.txt()
- .Rdata – save()
  - Workspaces are very compressed compared to csv
- Databases
  - Sqlite: sqldf, RSQLite packages
    - Sqlite example
  - MongoDB: rmongodb package
  - Postgresql: RPostgreSQL package

SQL and R

- Handle datasets larger than memory
- Support several common databases.
- e.g. SQLite: http://www.r-bloggers.com/r-and-sqlite-part-1/
- Or with dplyr: https://cran.r-project.org/web/packages/dplyr/vignettes/databases.html
- And, many other packages and references: search around.
- R Demo

Assignment

- Homework 2:
  - Write an R-script to compute the Monty Hall Probabilities with simulations (get probabilities AND variances for switching and not switching).
  - You should submit:
    - **ONE R-script** that outputs the probabilities and variances.
    - Submission should include a proof of correct answer: chart and table.
  - Read Intro to Data Science Chapter 7 and 10.
  - Read Statistical Thinking for Programmers Ch. 4.