UNIVERSITY of WASHINGTON

Data Science UW
Methods for Data Analysis

More on Hypothesis Testing, The Central Limit Theorem,
And an introduction to Regression
Lecture 4
Nick McClure

Excellent health statistics - smokers are less likely to die of age related illnesses.'

Topics

- Review
- Outliers
- Analysis of Variance
- Central Limit Theorem
- Introduction to resampling
- Presentation of data science results

Review

- Sampling Methods
- Law of Large Numbers
- Hypothesis Testing
  - Normal testing
  - One tailed vs Two tailed
  - P-values
  - T-test (Student’s, Welch’s)
  - Chi-Squared
  - Fisher’s Exact

Outliers

- Outlier causes:
  - Bad data
    - Sensor misread, human error, software error
  - Non-representative data
    - Real data that can be argued to be out of our interest. E.g. a sample of annual salaries that includes Warren Buffet.
  - Must provide a legitimate argument to consider as outlier.
  - Or, an **interesting aspect of the dataset** previously overlooked?

Finding Outliers – Statistical methods

> Alpha trimmed mean – aka truncated mean
  - Trim percentage (alpha) of outliers
  - Upper, lower or balanced trimming
  - Iterative method
  - Biased estimator
  - Windsor mean – replace outlier values with trim point values

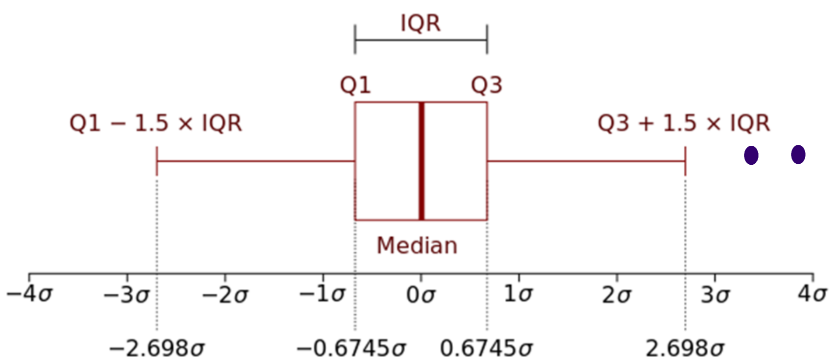
  Tukey et. al. 1947

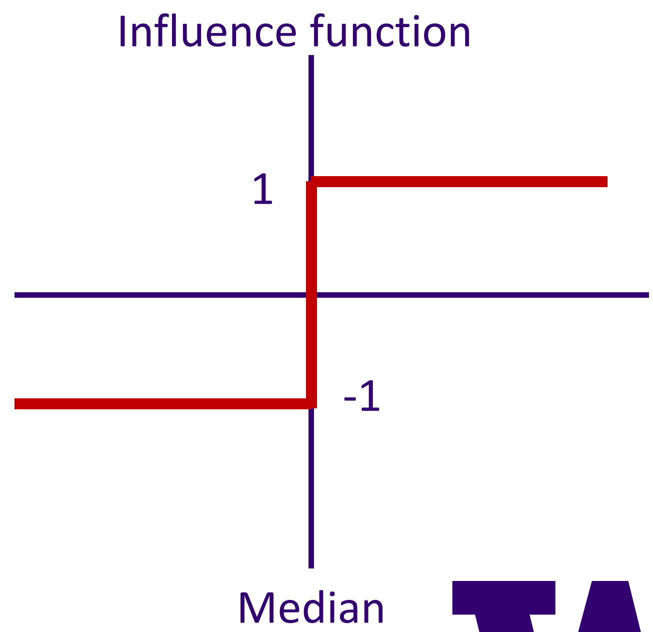
Example with two-sided alpha = 1/3

$$\frac{X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12}}{12 - 4}$$

Finding Outliers – Statistical methods

> Median
  - Median is a robust estimator
  - Use interquartile range to detect outliers
  - Biased estimator

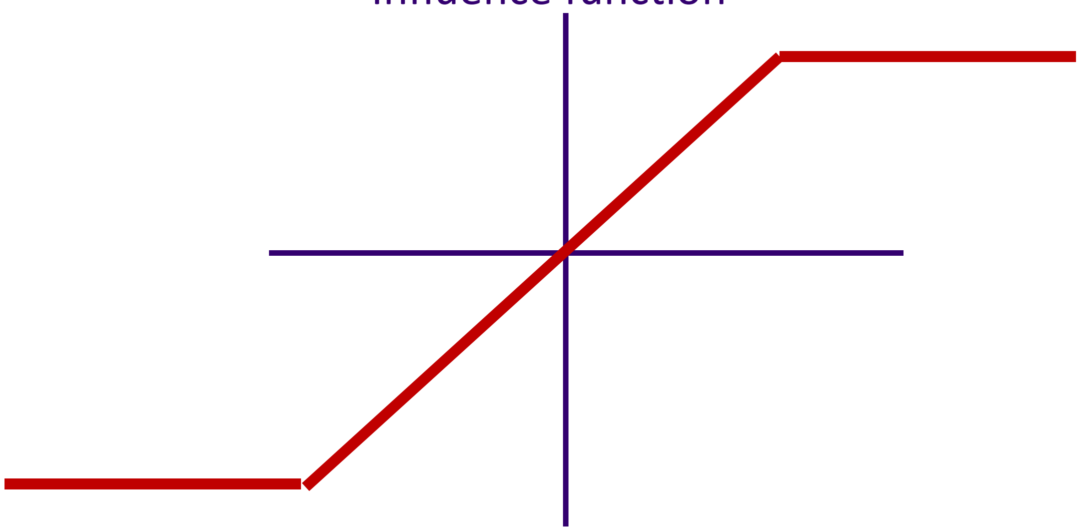




Finding Outliers – Statistical methods

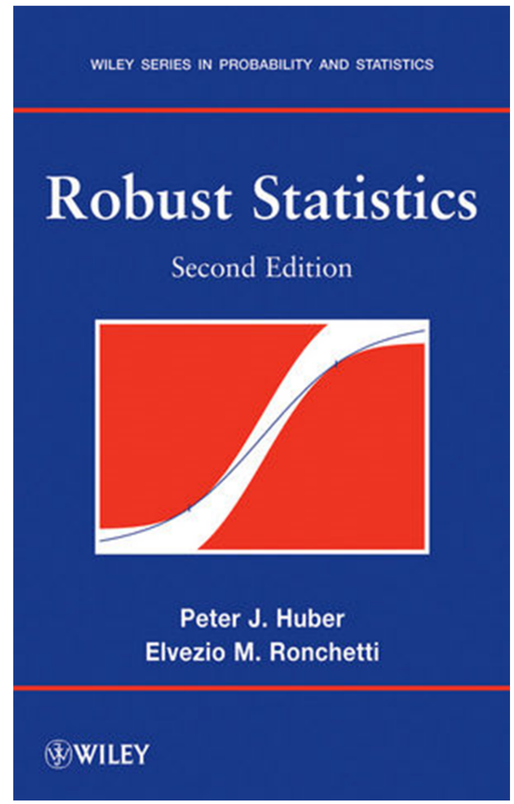
> Huber estimator
  – Attempt to reduce bias
  – Limit influence of outliers
  – Use piecewise influence function

Influence function



Huber Estimate

First Edition 1981



Finding Outliers – Statistical methods

> Resampling

- Find points with exceptional ‘influence’ on the estimate
- Special case of Jackknife method
- Computationally intensive

Validating Outliers

Is an outlier an error or a valuable case?

> Investigate multiple relationships in dataset to validate outlier
> Think what interesting or important relationship the 'outlier' might represent.

Treating Outliers

- Censor
- Interpolate new value
- Use substitute values

Hypothesis Testing Summary (so far)

- If data is normal,
  - If you know population mean and variance,
    - Use standard normal ‘z-test’.
  - If you just know population mean,
    - Use t-test (unpaired data).
    - Use Welch’s t-test (paired data).
- For categorical comparison tests,
  - If the sample/subgroup size is large enough,
    - Use Chi-squared test
  - If the sample/subgroup size is small,
    - Use Fisher’s Exact test.
- How do we know the data is normal?

Testing Between Multiple Groups

> What if we had multiple groups and we wanted to compare their means?

> Why can't we just do multiple two-sample t-tests for all pairs?

  – Results in increased probability of accepting a false hypothesis.

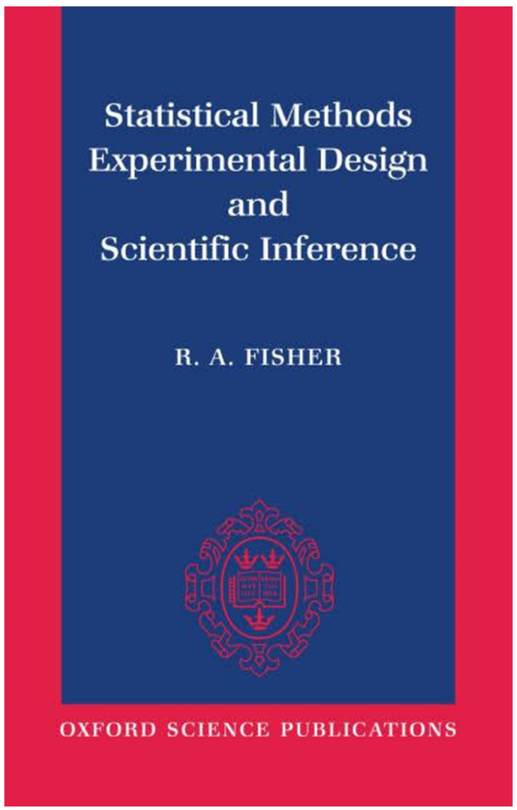
  – E.g., if we had 7 groups, there would be (7 Choose 2)=21 pairs to test. If our alpha cutoff is 5%, then we are likely to accept about 1 false hypothesis (21*0.05).

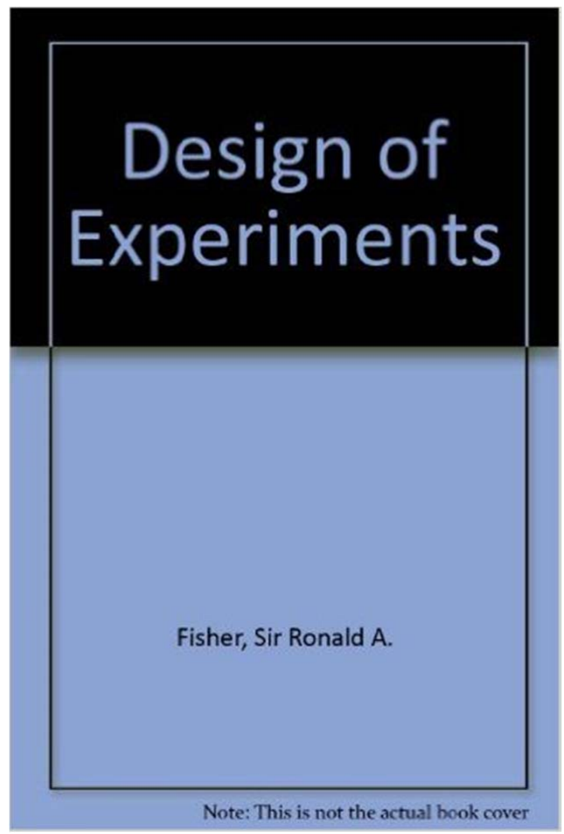
Testing Between Multiple Groups

- Null Hypothesis:
  - All groups are just samples from the same population.
- Alternative Hypothesis:
  - At least one group has a statistically different mean.
- This type of analysis is called “ANalysis Of VAriance”, or ANOVA.
  - We make data independence and normality assumptions first.

ANOVA

- > Laplace, 1827
- > Fisher, 1922, 1925, 1935 – F statistic





ANOVA Calculations

Basic ANOVA calculations:

$I = number\ of\ treatments$

n = number of data

SS = sum of squares

$SS_{Total} = SS_{Treatment} + SS_{Error}$

$DF_{Total} = DF_{Treatment} + DF_{Error} = (I - 1) + (n - I) = \mathrm{n} - 1$

$$F\ statistic\ with\ I - 1\ DF = \frac{Variance\ between\ treatments}{Variance\ within\ treatments}$$

$$= \frac{\dfrac{SST}{DF_{Treatment}}}{\dfrac{SSE}{DF_{Error}}}$$

Performing ANOVA

> ANOVA table lays out calculation
> F statistic determines significance (P value)
> Significance (P value) is key

Example;

One-way balanced ANOVA for 5 treatments, 30 samples

ANOVA

		Sum of Squares	df	Mean Square	F	Sig.
Self-Concept	Between Groups	1914.087	4	478.522	1.297	.298
	Within Groups	9223.687	25	368.947		
	Total	11137.8	29			

Performing Multiple Hypothesis Tests

- For non-ANOVA methods, remember that performing many hypothesis tests increases our risk of incorrectly rejecting a null-hypothesis.
- To compensate for this we decrease the p-value cutoff.
- The most common way of doing this is with the Bonferroni Correction.

$$p' = \frac{p}{(\#\ of\ Hypotheses)}$$

- This correction is argued to be too strong and other approximations for a new-p can be used instead.
  - Tukey's Range Test
- This is VERY important in genetics/bioinformatics.

Additional Hypothesis Testing

> Tests may lack power!
  - Need sufficient sample size
  - Size of the effect must large enough
  - 'Reasonable' significance level

$$\text{Power} = P(reject\ H_0 | H_1\ is\ true)$$

> Parametric test types:
  - Mean comparison
  - Variance comparison
  - More distribution comparisons

> R Example

Central Limit Theorem

> Sample a population many times, the distribution of means of all samples are normally distributed, regardless of the population distribution.

$\bar{X}$=sample mean.

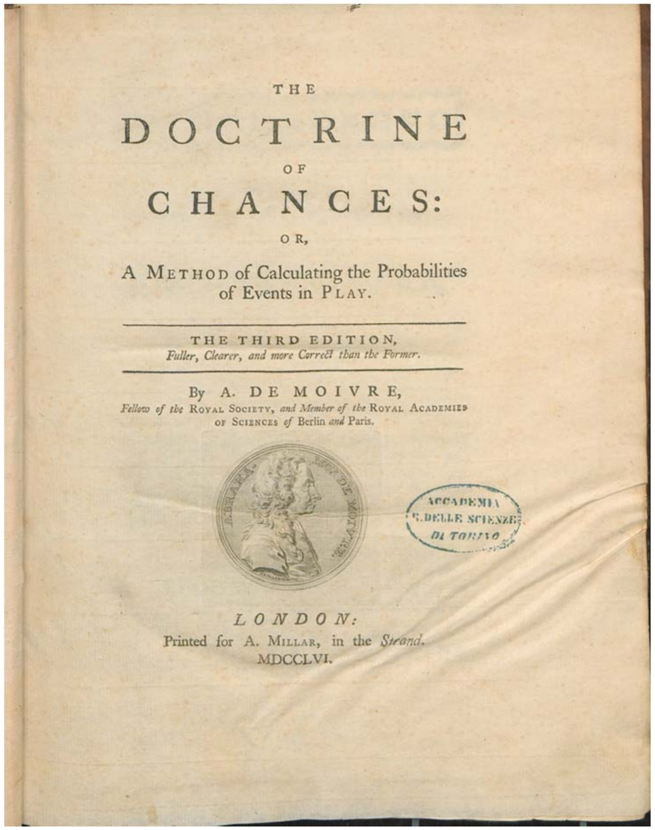
$$\bar{X} \sim N(mean, \frac{st.dev}{\sqrt{n}})$$

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

> Compare to Law of Large numbers (‘proof’ by R), shown in previous class.

Central Limit Theorem

> de Moivre, 1738 – prof of special case for Bernoulli trials
> Laplace 1776, 1785, 1820
> Chebyshev, 1887 – rigorous proof



Central Limit Theorem

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

> We can use this central limit theorem to generate confidence intervals on expressing the population mean.
> We know the sample mean, sample variance, and number of samples.
> Then we know how our estimate of the population mean is distributed (from above formula).
> We can then generate 90%, 95%, … confidence intervals around our sample mean.

Confidence Intervals

> Confidence intervals are a way to express uncertainty in *population* parameters, as estimated by the sample.

> E.g. If we create a 95% confidence interval for the population mean, say $\hat{\mu} = \bar{X} = 10 \pm 5$

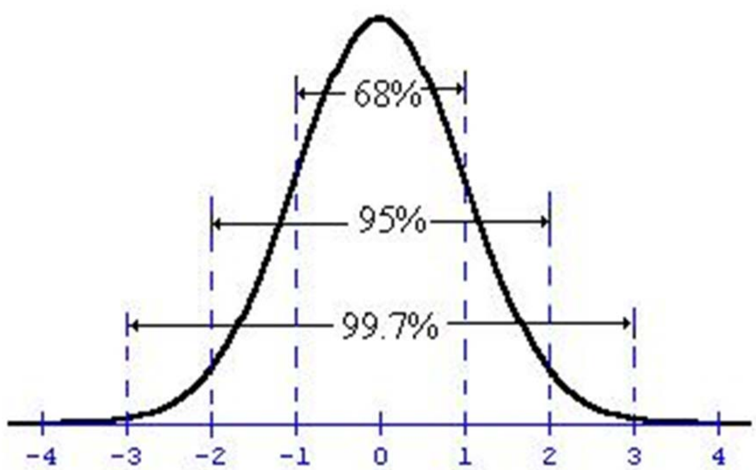
– Then we say that the true population mean, μ, has a 95% chance of being between 5 and 15.

It is **not** correct to say:

- ~~"95% of the sample values are in this range."~~
- ~~"There is a 95% chance that the mean of another sample will be in this range."~~

Confidence Intervals

> To create confidence intervals for population means, we use the central limit theorem and create confidence intervals based on the normal distribution.
  - Repeatedly sample from the population.
  - Calculate the mean for each sample.
  - Use the average of the sample means as the population estimate and create a C.I. based on the s.d. of the sample means.
  - R demo



How to work with limited sample size?

Sample size is always limited

> Point estimates are only computed once
> How reliable are point estimates?

Resampling Methods

What are resampling methods?

- Resampling methods allow computation of statistics from limited data
- Compute statistic from multiple subsamples of dataset
- Minimal distribution assumptions
- Computationally intensive

Resampling Methods

Common resampling methods

> Permutation methods
> Bootstrap: resample with equivalent size and replacement
> Jackknife: leave one out resampling
> Cross validation: resample into folds without replacement

Bootstrap Methods

- Efrom, 1979
- Re-compute statistic many times with sample with replacement
- Randomly subsample (e.g. Bernoulli sample) data with replacement
- Subsamples have the same size as original sample
- Works with any statistic … in principle

Example compute bootstrap mean

$$\text{Meanboot} = \left(\sum \text{mean}(\text{sample}_i)\right)/\text{nsample}$$

$$\text{sample}_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_1 + X_5$$

Jackkife Methods

- Quenouille, 1949, 1956; Tukey, 1958
- Re-compute statistic many times with sample with replacement
- Randomly leave one (or n) out sampling
- Only use with statistics with continuous derivatives

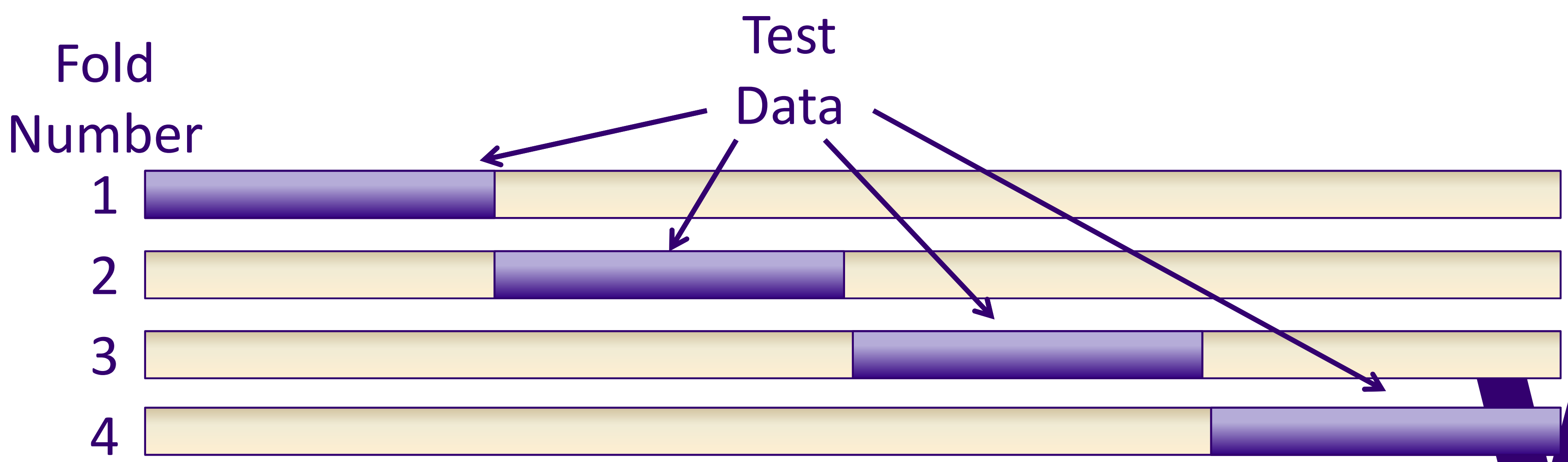
Example compute jackknife mean

$$\text{Meanjackknife} = \left(\sum \text{mean}(\text{sample}_i)\right)/\text{nsample}$$

$$\text{sample}_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_{10}$$

Cross Validation Methods

- Mosteller and Tukey, 1968
- Divide dataset into N subsamples
- N – 1 Folds train model
- One Fold evaluate model
- Nest cross validation to compare models



Resampling Pitfalls

There is no free lunch

> If sample is biased, resample statistic is biased
> Sample variance and Cis are no better than sample allows

Presentation and story telling

Important part of data science

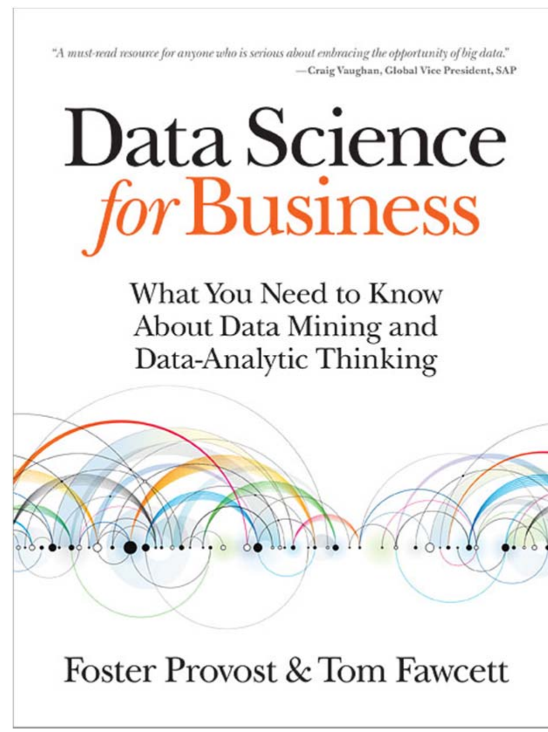
> Data science must have **impact**
> Results **only** have impact if they are understood
> Need to **‘tell the story’**
> **Draw clear conclusion**
> Evidence supports conclusion

Presenting results is hard!

Data analytic thinking

Thinking about problems using objective analysis of data

- Define problem in terms of the business impact
- Review available data sources
- Explore the data
- Try various models
- Actionable results generate value
- Support recommendations with data and analysis
- Define metrics of success



Tips for story telling

Make the story clear

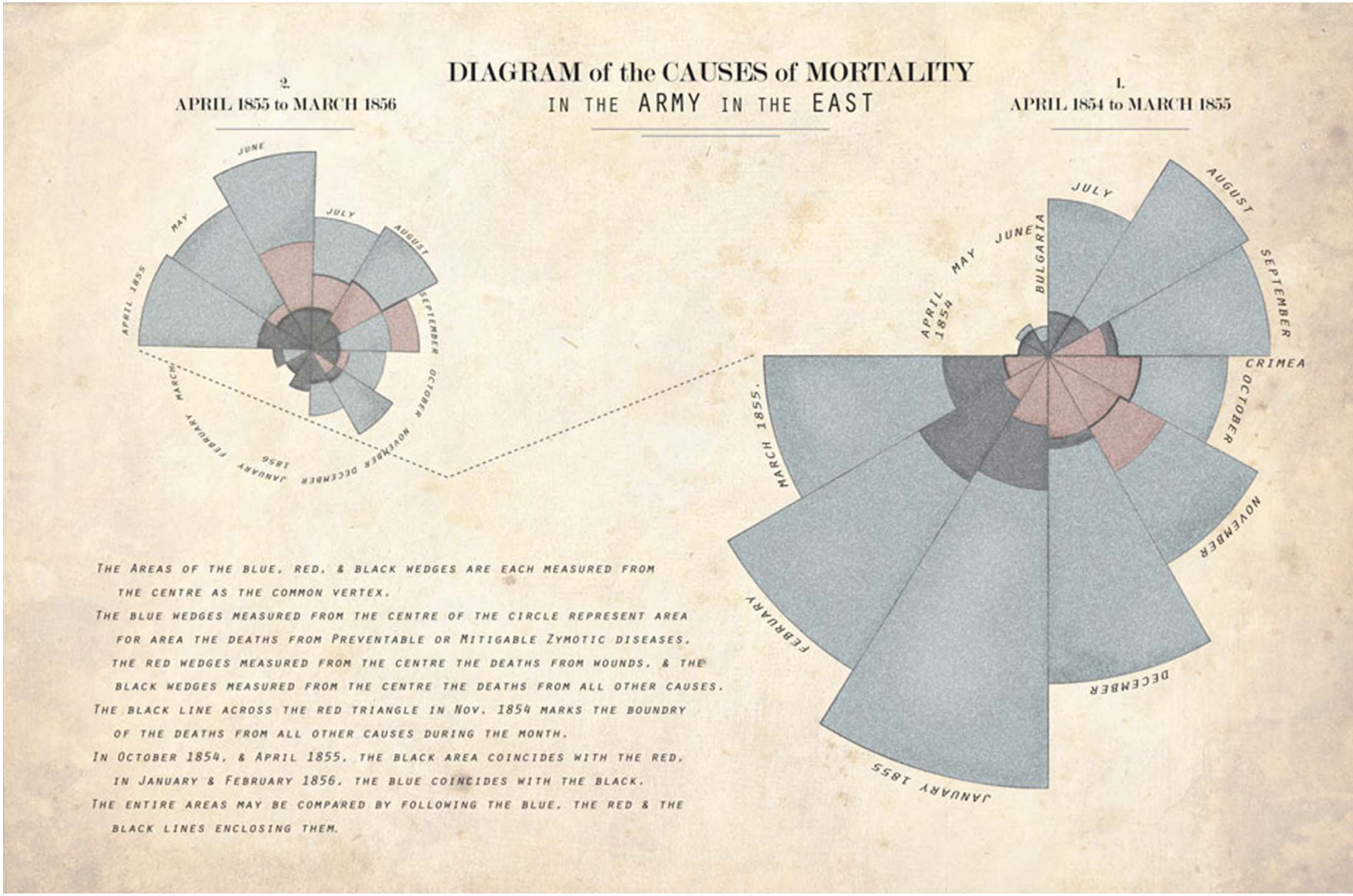
- Occam’s Razor
- You will only hold attention for a short time
- Don’t distract your audience
- Start with your conclusion
- Support your conclusion with evidence
- Few words = greater impact!

Don’t obfuscate your message!

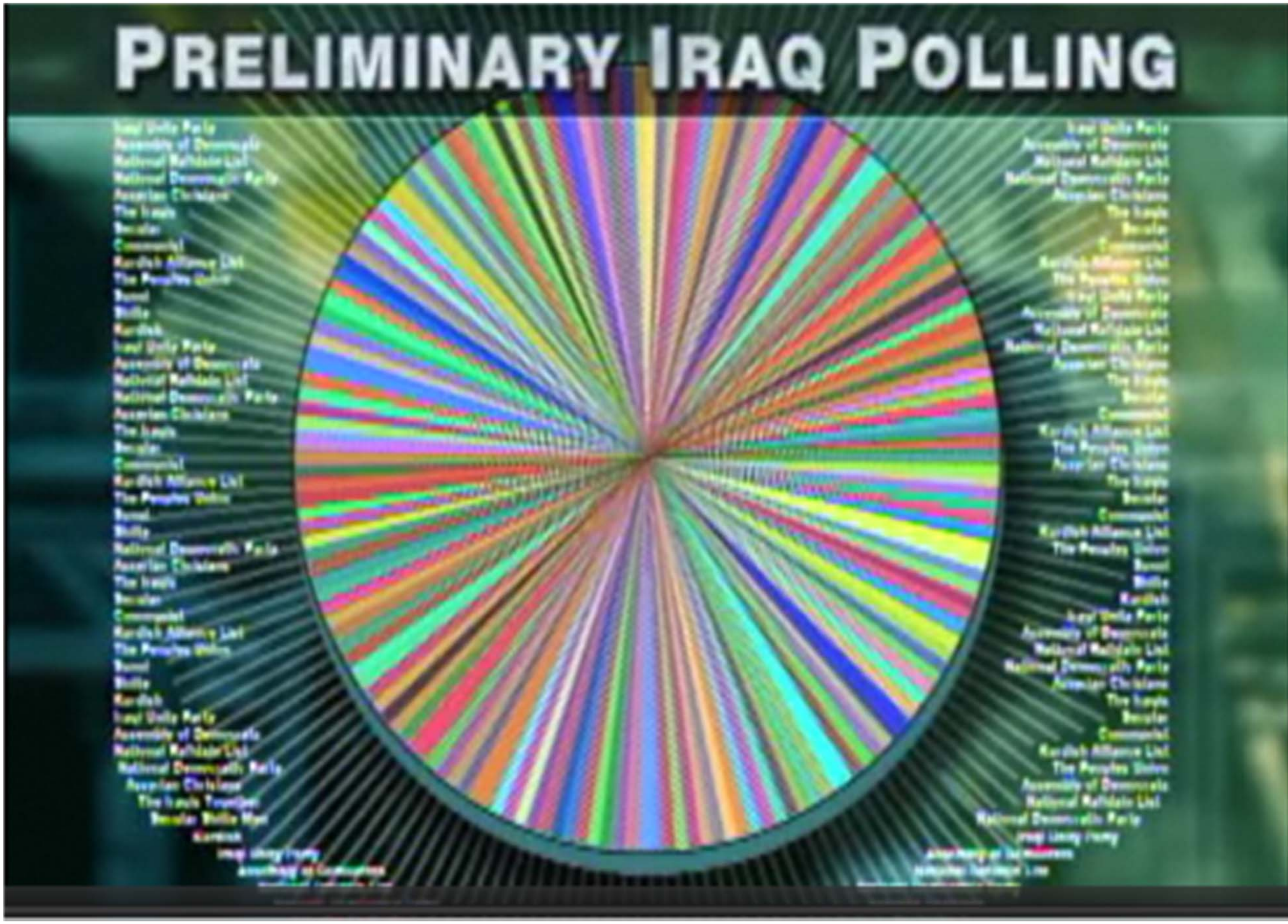
Short and simple has business impact

- Minimize discussion of methodology and technical detail
- Clear charts
  - Label axis
  - Minimize over-plotting
  - Simplify
- Short simple tables
  - Label rows and columns
  - Highlight key point
  - Minimal rows and columns

Historical example



What does this mean?



Presenting Data Science Results

Suggested reading

- > www.unomaha.edu/mahbubulmajumder/data-science/fall-2014/lectures/28-presenting-result/28-presenting-result.html#/
- > http://dupress.com/articles/telling-a-story-with-data/
- > http://www.kaushik.net/avinash/data-presentation-tips-focus-think-simplify-visualize/

Demo of testing statistical function

Ran out of time before

Homework and final project

- Grading is based on results and clear and complete presentation
  - Quality, completeness and clarity, not volume, count!
- Presentation must explain specific conclusions
  - Specific conclusions have impact
- Support conclusions with charts and tables
  - Narrative must call out the evidence
  - Presentation of evidence to maximize impact
- **Simplify you presentation!**

Assignment

- Complete Homework 4:
  - Apply ANOVA to the auto price data:
    - Compare the price (log price) of autos for several multi-valued categorical variables – number of doors, body style, drive wheels, number of cylinders, engine type
    - Graphically explore the differences – Hint, make sure you have enough data for each category.
    - Use standard ANOVA and Tukey ANOVA in R
    - Use the bootstrap distribution CIs of the (differences of) means – Hint write a function for to perform this calculation for any number of categories (levels) pairs
  - You must submit:
    - One R-script.
    - Document discussing and supporting your conclusions
- Read Statistical Thinking for Programmers Chapters 6 and 7.