CS 3823 - Theory of Computation: Homework Assignment 1

FALL 2023 **Due:** Friday, September 8, 2023

Related Reading. Chapter 1.1 and Chapter 2
Instructions. Near the top of the first page of your solutions please list clearly **all** the members of the group (please see the syllabus for the collaboration policy) who have created the solutions that you are submitting. Listing the names of the people in the group implies their full name and their 4x4 IDs. Alternatively, you can use the space below and provide the relevant information in case you submit the solutions using this document.

Student Information for the Solutions Submitted

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Grade

Exercise	Pages	Your Score	Max
1	2		4
2	3		4
4	4-5		12
5	6-7		12
6	8		4
3	9		4
Total	2-9		40

Additional Help and Resources. Did you use help and/or resources other than the textbook? Please indicate below.

1 Set Theory [4 points]

Let A be the set $\{x, y, z\}$ and B be the set $\{a, x, y\}$.

(i) Is A a subset of B and why?

(ii) What is $A \setminus B$?

(iii) What is $A \times B$?

(iv) What is the powerset of A?

i. Given the definition of a subset, A can not be a subset of B given the set A contains z and the set B does not contain z.
ii. $A \setminus B = \{z\}$
iii. $A \times B = \{(x, a), (x, x), (x, y), (y, a), (y, x), (y, y), (z, a), (z, x), (z, y)\}$
iv. $P(A) = \{\{\ \}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

2 Induction [4 points]

Prove by induction on n that $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$.

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Let k =1,

$$(1)^2 = \frac{(1)(1+1)(2(1)+1)}{6} = \frac{(1)(2)(2+1)}{6} = \frac{(2)(3)}{6} = \frac{6}{6} = 1$$

Assume the statement is true for n = m. So, $\sum_{k=1}^{m} k^2 = \frac{m(m+1)(2m+1)}{6}$.

Show that this implies the statement is true for n = m+1. $\sum_{k=1}^{m+1} k^2 =$

$$\sum_{k=1}^{m} k^2 + (m+1)^2$$

$$\frac{m(m+1)(2m+1)+(m+1)^2}{6}$$

$$= \frac{m(m+1)(2m+1)+6(m+1)^2}{6}$$

$$= \frac{m(m+1)(2m+1)+6(m+1)(m+1)}{6}$$

$$= \frac{(m+1)(m(2m+1)+6(m+1))}{6}$$

$$= \frac{(m+1)(2m^2+m+6m+6)}{6}$$

$$= \frac{(m+1)(2m^2+7m+6)}{6}$$

$$= \frac{(m+1)(2m^2+4m+3m+6)}{6}$$

$$= \frac{(m+1)(2m(m+2)+3(m+2))}{6}$$

$$= \frac{(m+1)(m+2)(2m+3)}{6}$$

$$= \frac{(m+1)((m+1)+1)(2(m+1)+1)}{6}$$

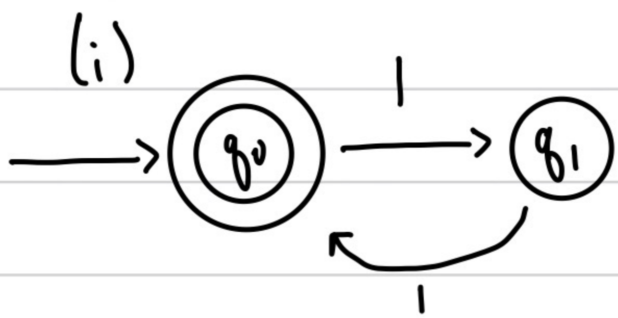
3 Drawing State Diagrams [12 points; 4 points each]

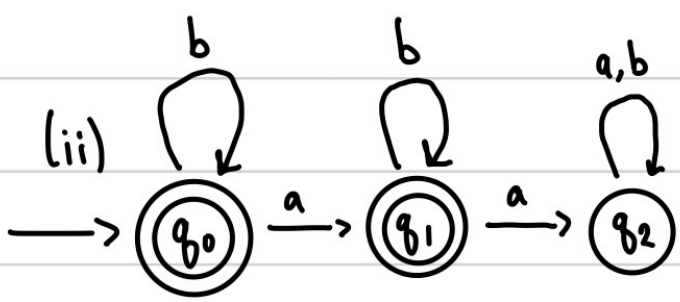
Draw state diagrams for DFAs recognizing the following languages:

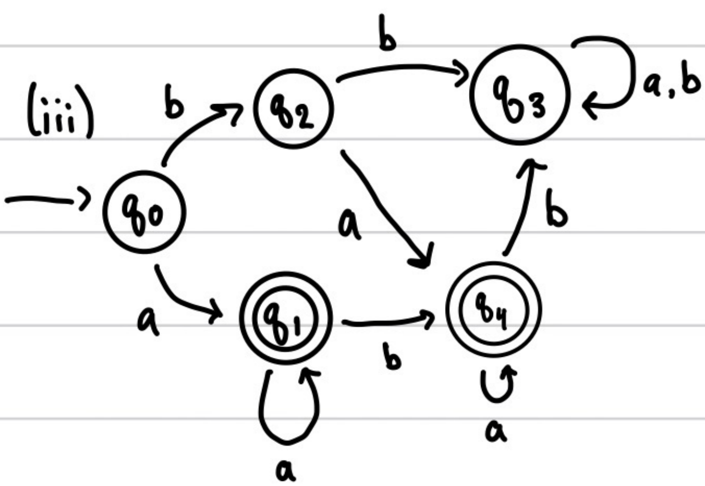
(i) $L_1 = \{w \mid \text{length of } w \text{ is even}\}, \Sigma = \{1\}$.

(ii) $L_2 = \{w \mid w \text{ has at most one occurrence of the symbol } a\}, \Sigma = \{a, b\}$.

(iii) $L_3 = \{w \mid w \text{ has at least one occurrence of the symbol } a \text{ and at most one occurrence of the symbol } b\}, \Sigma = \{a, b\}$.



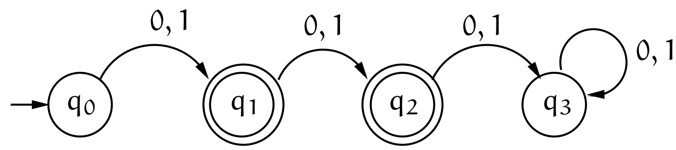




4 Reading State Machines [12 points]

Let $\Sigma = \{0, 1\}$. For each of the following DFAs explain what language they recognize.

(i) **[6 points]** Please see the DFA of machine M_1 below. For this machine M_1, also give its formal



description as a 5-tuple. You do not need to do this for the machine M_2 that follows in part (ii).

$L(M_1) = A$ where $A = \{w \mid$ if and only if the length of w is 1 or 2$\}$

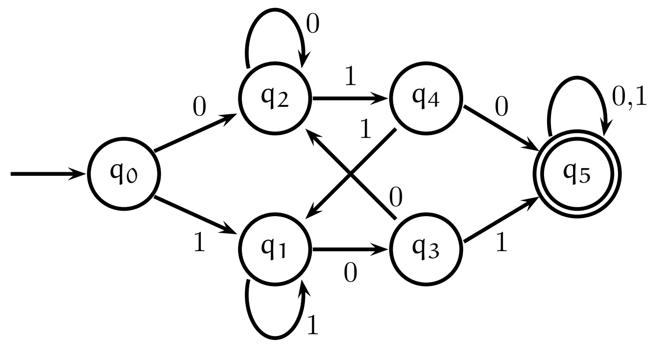
$M_1 = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_1, q_2\})$

$\delta =$

	0	1
q_0	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_3	q_3

(continuation of exercise 5)

(ii) **[6 points]** Please see the DFA of machine M_2 below.



$L(M_2) = A = \{ w \mid w$ contains the substring 101 or $010 \}$

5 Closure [4 points]

Let A and B be regular languages. Show that A△B is also regular.
Recall that A△B denotes the symmetric difference of the sets A and B. That is, $A \triangle B = (A \setminus B) \cup (B \setminus A)$.

Since A & B are regular we have:

$M_A = (Q_A, \Sigma_A, \delta_A, q_{A0}, F_A)$ $\qquad$ $M_B = (Q_B, \Sigma_B, \delta_B, q_{B0}, F_B)$

$A \Delta B$:

$M_\Delta = (Q_\Delta, \Sigma_\Delta, \delta_\Delta, q_\Delta, F_\Delta)$

$Q_\Delta = Q_A \times Q_B$

$\Sigma_\Delta = A \cup B$

$\delta_\Delta = \delta_\Delta(q, t) = \delta((r,s), t)$

$q \in Q,\ q = (r,s)$ where $\begin{cases} r \in Q_A \\ s \in Q_B \end{cases}$

$= (x, y) \mid \begin{cases} x \in Q_A \\ y \in Q_B \end{cases}$

$x = \delta_A(r, t) \qquad y = \delta_B(s, t)$

$q_\Delta = (q_{A0}, q_{B0})$

$F_\Delta = \left\{ (r,s) \;\middle|\; \begin{array}{c} r \in F_A \text{ and } s \in Q_B' F_B \\ \text{or} \\ r \in Q_A' F_A \text{ and } s \in F_B \end{array} \right.$

6 Candidate Exam Question [4 points]

For the material that is covered for this homework assignment, formulate one question that you could expect to appear in a midterm or a final exam. Then, provide a solution to the proposed question.
IMPORTANT: At least for this question you should submit a .tex file with the LaTeX code that has the proposed question and answer on Canvas. If you don't submit a .tex file answering this question, you will receive 0 (zero) points in this question.