Critical Path Method -3rd hand in – Graphs

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Assignment description:
The figure of a project graph below is from Wikipedia "Critical path method" (See last page).
(One of the number is the diagram is not correct – find it).

Write a program with your own algorithms to find the values of the project graph. The algorithms need not to be fancy or optimal.

Make a representation of the tasks with connections and durations.
Calculate the earliest start and finish for each task.
Calculate the latest start and finish for each task.
Find the critical path.
Calculate the total floats.

"One of the number is the diagram is not correct – find it"

By looking at the graph and in my calculated project graph I have come to the conclusion that earliest finish for task C should be 35 and not 36.

"Write a program with your own algorithms to find the values of the project graph. The algorithms need not to be fancy or optimal."

I have written a program in java with my own implemented algorithm based on an example from geekforgeek. The program find all the possible project paths and calculate and prints the value for each path and task.The algorithm is not fancy or optimal.

The base for the algorithm has been taken from Print all paths from a given source to a destination the base of the code is contributed by Himanshu Shekhar. The original code represents the nodes as integer so I have modified that to use objects representing the tasks. I have also changed the code so it can make the calculations on the tasks and print the results as JSON.

From Himanshu Shekhar article about the original algorithm:
"The idea is to do Depth First Traversal of given directed graph. Start the traversal from source. Keep storing the visited vertices in an array say 'path[]'. If we reach the destination vertex, print contents of path[]. The important thing is to mark current vertices in path[] as visited also, so that the traversal doesn't go in a cycle."

"Make a representation of the tasks with connections and durations."

- The tasks are represented as objects of the Vertex class. (See code Vertex class)
- The project graph is implemented as an adjacency list. It can be traversed backwards and forwards by a given source and destination node (task represented as Vertex object) keeping the right flow direction. (See code Adjacency List and Flow Control below)
- Makes calculations for each task per path in the graph. (see code Find all paths)
- Prints the paths as JSON. (A helper class results left out in this results)

"Find the critical path."

To find the critical path I just select the path with the longest total duration.The program is not yet smart enough to take into consideration two paths with equally longest duration. *allPathsForward* contains a list of all the paths.

Result:

---------- Critical Path ----------

[{"A":{}},{"B":{}},{"C":{}},{"D":{}},{"E":{}},{"Total Duration":65}]

Logic:

```
  public void findCriticalPath(String title) {
      int max = 0;
      List<Vertex> criticalPath = null;

      for (List<Vertex> path : allPathsForward) {

          int sum = 0;
          for (Vertex i : path) {
              sum += i.duration;
          }
          if (sum > max) {
              max = sum;
              criticalPath = path;
          }
      }
      printHeadLine(title);
      PrintPathDynamic(criticalPath,new boolean[]{false,false,false,true});
      printPrintHeaderEnd();
  }
```

“Calculate the earliest start and finish for each task.

Result:

See below.

Logic:

When a path is found it traverse the nodes on that path and updates the *earlieststart/earliestfinish* of a node by the values from the previous node in the path.

The logic for when a path from source to destination is found looks like this:

```
if (u.equals(d)) {
    List<Vertex> finalPathFound = new ArrayList<Vertex>();
    Vertex previous = null;
    if(isValidPathLeftToRightFlow(localPathList)) {
        for (Vertex i : localPathList) {
            if (previous == null) {
                previous = i;
                previous.earliestStart = this.source.earliestStart;
                previous.earliestFinish = this.source.earliestFinish;
            }
            else {
                if (i.earliestStart == null || i.earliestStart <= previous.earliestFinish) {
                    i.earliestStart = previous.earliestFinish + 1;
                    i.earliestFinish = i.earliestStart + (i.duration - 1);
                }
            }
            previous = i;
            finalPathFound.add(i);
        }
        results.addPath(finalPathFound);
    }
}
```

Calculate the latest start and finish for each task. “

Logic:

When a path is found it travers the nodes on that path and updates the *lateststart/latestfinish* of a node by the values from the previous node in the path.

The logic for when a path from source to destination is found looks like this:

```
        if (u.equals(d) && isValidPathRightToLeftFlow(localPathList)) {

            List<Vertex> finalPathFound = new ArrayList<Vertex>();
            Vertex previous = null;

            for (Vertex i : localPathList) {
                if (previous == null) {
                    previous = i;
                    previous.latestStart = source.latestStart;//46;
                    previous.latestFinish = source.earliestFinish;//65;
                }

                //System.out.print(i.name+" ");
                else {
                    if (i.latestFinish == null || i.latestFinish < 0 || i.latestFinish >=
previous.latestStart) {
                        i.latestFinish  = previous.latestStart - 1;
                        i.latestStart = i.latestFinish - (i.duration - 1);
                    }
                }
                previous = i;
                finalPathFound.add(i);
            }
            results.addPathBackwards(finalPathFound);

        }
```

Calculate the total floats.

Logic:

The floats are calculated like this where *allPathsForward* contains a List of all the paths.

```
    public void updateFloat() {
        for (List<Vertex> path : allPathsForward) {
            for (Vertex i : path) {
                i.floatTime = i.latestFinish - i.latestStart + 1;
            }
        }
    }
```

Results:

---------- Complete Project Graph ----------

```
[
	[{
		"A": {
			"EarliestStart": 1,
			"EarliestFinish": 10,
			"LatestStart": 1,
			"LatestFinish": 10,
			"Float Time": 10
		}
	}, {
		"B": {
			"EarliestStart": 11,
			"EarliestFinish": 30,
			"LatestStart": 11,
			"LatestFinish": 30,
			"Float Time": 20
		}
	}, {
		"C": {
			"EarliestStart": 31,
			"EarliestFinish": 35,
			"LatestStart": 31,
			"LatestFinish": 35,
			"Float Time": 5
		}
	}, {
		"D": {
			"EarliestStart": 36,
			"EarliestFinish": 45,
			"LatestStart": 36,
			"LatestFinish": 45,
			"Float Time": 10
		}
	}, {
		"E": {
			"EarliestStart": 46,
			"EarliestFinish": 65,
			"LatestStart": 46,
			"LatestFinish": 65,
			"Float Time": 20
		}
	}, {
```

```
        "Total Duration": 65
}],
[{
        "A": {
                "EarliestStart": 1,
                "EarliestFinish": 10,
                "LatestStart": 1,
                "LatestFinish": 10,
                "Float Time": 10
        }
}, {
        "B": {
                "EarliestStart": 11,
                "EarliestFinish": 30,
                "LatestStart": 11,
                "LatestFinish": 30,
                "Float Time": 20
        }
}, {
        "C": {
                "EarliestStart": 31,
                "EarliestFinish": 35,
                "LatestStart": 31,
                "LatestFinish": 35,
                "Float Time": 5
        }
}, {
        "G": {
                "EarliestStart": 36,
                "EarliestFinish": 40,
                "LatestStart": 41,
                "LatestFinish": 45,
                "Float Time": 5
        }
}, {
        "E": {
                "EarliestStart": 46,
                "EarliestFinish": 65,
                "LatestStart": 46,
                "LatestFinish": 65,
                "Float Time": 20
        }
}, {
        "Total Duration": 60
}],
[{
```

```
        "A": {
                "EarliestStart": 1,
                "EarliestFinish": 10,
                "LatestStart": 1,
                "LatestFinish": 10,
                "Float Time": 10
        }
}, {
        "F": {
                "EarliestStart": 11,
                "EarliestFinish": 25,
                "LatestStart": 26,
                "LatestFinish": 40,
                "Float Time": 15
        }
}, {
        "G": {
                "EarliestStart": 36,
                "EarliestFinish": 40,
                "LatestStart": 41,
                "LatestFinish": 45,
                "Float Time": 5
        }
}, {
        "E": {
                "EarliestStart": 46,
                "EarliestFinish": 65,
                "LatestStart": 46,
                "LatestFinish": 65,
                "Float Time": 20
        }
}, {
        "Total Duration": 50
}],
[{
        "A": {
                "EarliestStart": 1,
                "EarliestFinish": 10,
                "LatestStart": 1,
                "LatestFinish": 10,
                "Float Time": 10
        }
}, {
        "H": {
                "EarliestStart": 11,
                "EarliestFinish": 25,
```

```
                        "LatestStart": 31,
                        "LatestFinish": 45,
                        "Float Time": 15
                }
        }, {
                "E": {
                        "EarliestStart": 46,
                        "EarliestFinish": 65,
                        "LatestStart": 46,
                        "LatestFinish": 65,
                        "Float Time": 20
                }
        }, {
                "Total Duration": 45
        }]
]
```


More on the code

The project tasks are represented as **objects called Vertex** and the project graph is represented as an **adjacency list** in the Graph class. The graph is traversed in a depth first traversal to find all possible paths. When a path is found the program makes calculations such as earliest start/finish. For every calculation the Vertex object is updated and the paths are temporarily stored and printed with a helper method called Results.

- Vertex class
- Adjacency
- Find all paths recursively
- Flow control
- Main class / Driver

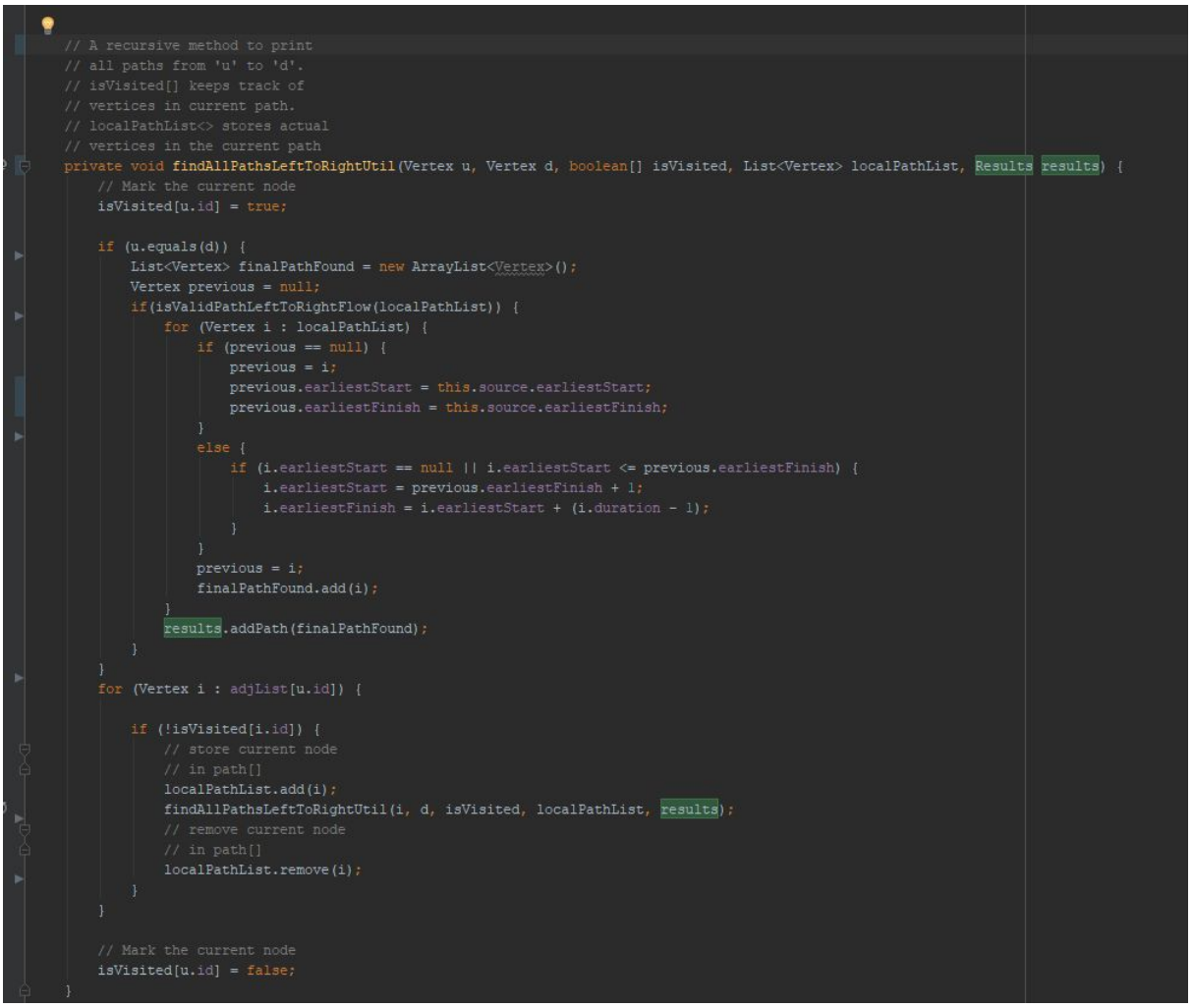
Vertex class

```java
public class Vertex {
    public int id;
    public int duration;
    public String name;
    public Integer earliestStart;
    public Integer earliestFinish;
    public Integer latestStart;
    public Integer latestFinish;
    public Integer flow;
    public Integer floatTime;

    public Vertex(int id, String name, int duration, int flow){
        this.id = id;
        this.name = name;
        this.duration = duration;
        this.flow = flow;
    }
}
```

Adjacency List

```java
// A directed graph using
// adjacency list representation
// Undirected Graph
public class Graph {

    // No. of vertices in graph
    private int numberOfVertices;
    private Results results;
    private Vertex source;
    // adjacency list
    private ArrayList<Vertex>[] adjList;
    private boolean DEBUG;

    //Constructor
    public Graph(int numberOfVertices, boolean DEBUG) {

        //initialise vertex count
        this.numberOfVertices = numberOfVertices;

        // initialise adjacency list
        initAdjList();
    }

    // utility method to initialise
    // adjacency list
    /unchecked/
    private void initAdjList() {
        adjList = new ArrayList[numberOfVertices];

        for (int i = 0; i < numberOfVertices; i++) {
            adjList[i] = new ArrayList<>();
        }
    }

    // add edge from u to v
    public void addEdge(Vertex u, Vertex v) {
        // Add v to u's list.
        adjList[u.id].add(v);
        adjList[v.id].add(u);
    }

    public void findAllVertexPaths(Vertex s, Vertex d, String flag, Results results) {
        boolean[] isVisited = new boolean[numberOfVertices];
        ArrayList<Vertex> pathList = new ArrayList<>();
        this.source = s;
```

[Find All Paths Recursively]



```
// A recursive method to print
// all paths from 'u' to 'd'.
// isVisited[] keeps track of
// vertices in current path.
// localPathList<> stores actual
// vertices in the current path
private void findAllPathsLeftToRightUtil(Vertex u, Vertex d, boolean[] isVisited, List<Vertex> localPathList, Results results) {
    // Mark the current node
    isVisited[u.id] = true;

    if (u.equals(d)) {
        List<Vertex> finalPathFound = new ArrayList<Vertex>();
        Vertex previous = null;
        if(isValidPathLeftToRightFlow(localPathList)) {
            for (Vertex i : localPathList) {
                if (previous == null) {
                    previous = i;
                    previous.earliestStart = this.source.earliestStart;
                    previous.earliestFinish = this.source.earliestFinish;
                }
                else {
                    if (i.earliestStart == null || i.earliestStart <= previous.earliestFinish) {
                        i.earliestStart = previous.earliestFinish + 1;
                        i.earliestFinish = i.earliestStart + (i.duration - 1);
                    }
                }
                previous = i;
                finalPathFound.add(i);
            }
            results.addPath(finalPathFound);
        }
    }
    for (Vertex i : adjList[u.id]) {

        if (!isVisited[i.id]) {
            // store current node
            // in path[]
            localPathList.add(i);
            findAllPathsLeftToRightUtil(i, d, isVisited, localPathList, results);
            // remove current node
            // in path[]
            localPathList.remove(i);
        }
    }

    // Mark the current node
    isVisited[u.id] = false;
}
```

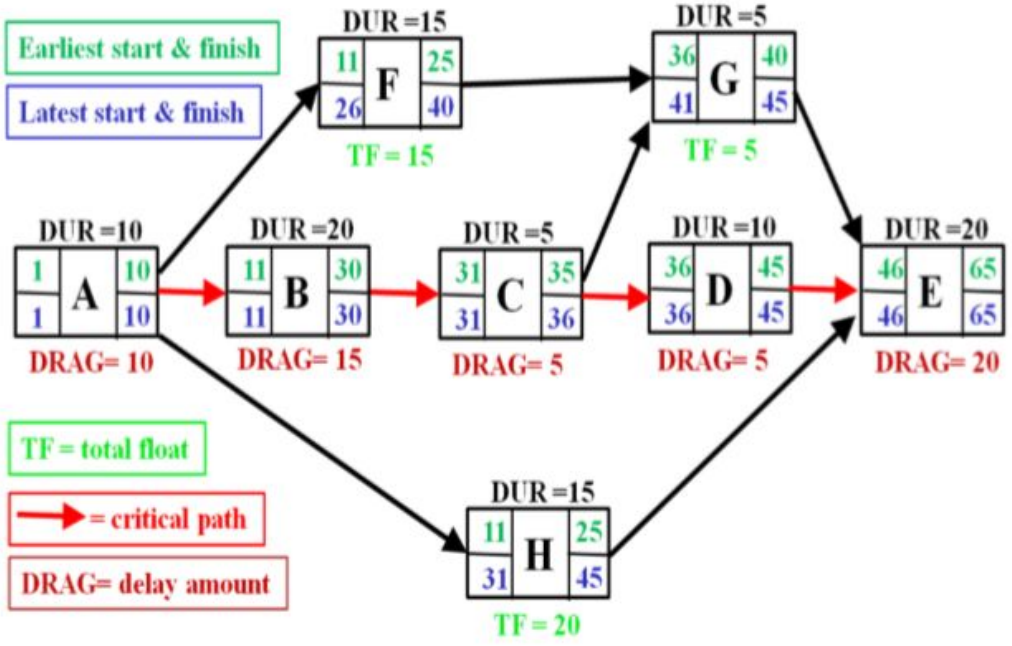
Flow Control

I have introduced a variable flow in the Vertex class. This is because I made the graph unidirectional but I still wanted control of the flow. That is Node A > F > G > C > D > E should not be possible.

```
/*Make sure we go in the right direction with help of flow value*/
boolean isValidPathLeftToRightFlow(List<Vertex> path){
    Vertex previous = null;
    for (Vertex i : path) {
        if(previous == null){
            previous = i;
        }
        else{
            if (i.flow < previous.flow){
                if(DEBUG)
                System.out.println("INVALID!!");
                return false;
            }
            previous = i;
        }
    }
    return true;
}
```

Main

```
// Driver program
public static void main(String[] args) {
    // Create a sample graph
    Results results = new Results();
    boolean DEBUG = false;
    Graph graph = new Graph( numberOfVertices: 8,DEBUG);

    Vertex a = new Vertex( id: 0,  name: "A",  duration: 10, flow: 0);
    Vertex b = new Vertex( id: 1,  name: "B",  duration: 20, flow: 10);
    Vertex c = new Vertex( id: 2,  name: "C",  duration: 5, flow: 20);
    Vertex d = new Vertex( id: 3,  name: "D",  duration: 10, flow: 30);
    Vertex e = new Vertex( id: 4,  name: "E",  duration: 20, flow: 40);
    Vertex f = new Vertex( id: 5,  name: "F",  duration: 15, flow: 15);
    Vertex g = new Vertex( id: 6,  name: "G",  duration: 5, flow: 25);
    Vertex h = new Vertex( id: 7,  name: "H",  duration: 15, flow: 12);
    graph.addEdge(a, b);
    graph.addEdge(b, c);
    graph.addEdge(c, d);
    graph.addEdge(c, g);
    graph.addEdge(d, e);
    graph.addEdge(a, f);
    graph.addEdge(f, g);
    graph.addEdge(g, e);
    graph.addEdge(a, h);
    graph.addEdge(h, e);


    System.out.println("Following are all different paths from " + a.id + " to " + e.id);
    Results res = new Results();
    a.earliestStart = 1;
    a.earliestFinish = 10;
    graph.findAllVertexPaths(a, e, flag: "earliest", res);
    res.printAllPaths( title: "Find Earliest Start/Finish",new boolean[]{true,false,false,true});
    res.findCriticalPath( title: "Critical Path");
    e.latestStart=46;
    e.latestFinish=65;
    graph.findAllVertexPaths(e, a, flag: "latest",res);
    res = new Results();
    graph.findAllVertexPaths(a, e, flag: "earliest",res);
    res.printAllPaths( title: "Find Latest Start/Finish",new boolean[]{false,true,false,true});

    res.updateFloat();
    res.printAllPaths( title: "Total Floats",new boolean[]{false,false,true,false});


    res.printAllPaths( title: "Complete Project Graph",new boolean[]{true,true,true,true});
}
```


Earliest start & finish
Latest start & finish
TF = total float
= critical path
DRAG= delay amount
DUR =10
1
1
A
10
10
DRAG= 10
DUR =15
11
26
F
25
40
TF = 15
DUR =20
11
11
B
30
30
DRAG= 15
DUR =5
31
31
C
35
36
DRAG= 5
DUR =5
36
41
G
40
45
TF = 5
DUR =10
36
36
D
45
45
DRAG= 5
DUR =20
46
46
E
65
65
DRAG= 20
DUR =15
11
31
H
25
45
TF = 20