Homework 2

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Question 1

Refer to C++ Files

Question 2

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Question 3

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Question 4

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Question 5

5.1 Exercise 1.12.2

B)

$p \implies (q \wedge r)$
$\neg q$
$\therefore notp$

Rules of Inference Chart		
Lines	Expressions	Rules of Inference
1.	$p \implies (q \wedge r)$	Hypothesis
2.	$\neg p \vee (q \wedge r)$	Conditional Identity 1
3.	$(\neg p \vee q) \wedge (\neg p \vee r)$	Distribution Law 2
4.	$(\neg p \vee q)$	Simplification 3
5.	$p \implies q$	Conditional Identity 5
6.	$\neg q$	Hypothesis
7.	$\neg p$	Modus Tollens 6,5

E)

$p \wedge q$
$\neg p \vee r$
$\neg notq$
$\therefore r$

Rules of Inference Chart		
Lines	Expressions	Rules of Inference
1.	$p \vee q$	Hypothesis
2.	$\neg p \vee q$	Hypothesis
3.	$\neg q$	Hypothesis
4.	$p \implies q$	Conditional Identity 2
5.	$q \vee p$	Commutative Law, 1
6.	p	Disjunctive Syllogism 5,3
7.	r	Modus Ponens 6,4

5.2 Exercise 1.12.3

C)

$p \vee q$
$\neg p$
$\therefore q$

Rules of Inference Chart		
Lines	Expressions	Rules of Inference
1.	$p \vee q$	Hypothesis
2.	$\neg\neg p \vee \neg\neg q$	Double Negative 1
3.	$\neg p \implies \neg\neg q$	Conditional Identity 2
4.	$\neg p$	Hypothesis
5.	$\neg p \implies q$	Double Negative 3
6.	q	Modus Ponens 4,5

5.3 Exercise 1.12.5

C)

$(C(x) \wedge H(x)) \implies J(x)$
$\neg J(x)$
$\therefore \neg C(x)$

Truth Table 1.12.5 C					
$C(x)$	$H(x)$	$J(x)$	$(C(x) \wedge H(x)) \implies J(x)$	$\neg J(x)$	$\neg C(x)$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

Invalid Argument: Counter-Example on Line 4 of Table.
When $(C(x) \wedge H(x)) \implies J(x)$ is true, and $\neg J(x)$ is also true, $\neg C(x)$ is false.

D)

$(C(x) \wedge H(x)) \implies J(x)$
$\neg J(x)$
$H(x)$
$\therefore \neg C(x)$

Rules of Inference Chart		
Lines	Expressions	Rules of Inference
1.	$(C(x) \land H(x)) \implies J(x)$	Hypothesis
2.	$(\neg(C(x) \land H(x))) \lor J(x)$	Conditional Identity 1
3.	$(\neg C(x) \lor \neg H(x)) \lor J(x)$	DeMorgan's Law 2
4.	$J(x) \lor (\neg C(x) \lor \neg H(x))$	Commutative Law 3
5.	$\neg J(x)$	Hypothesis
6.	$\neg C(x) \lor \neg H(x)$	Disjunctive Syllogism 4,5
7.	$H(x)$	Hypothesis
8.	$\neg H(x) \lor \neg C(x)$	Commutative Law 6
9.	$H(x) \implies \neg C(x)$	Conditional Identity 8
10.	$\neg C(x)$	Modus Ponens 7,9

5.4 Exercise 1.13.3

B)

$\exists x\ (P(x) \lor Q(x))$
$\exists x\ \neg Q(x)$
$\therefore\ \exists x\ P(x)$

$P(x)$ and $Q(x)$ Combinations		
	P	Q
a	F	T
b	F	F

Explanation: When $P(a) \lor Q(a)$ is true, $P(a)$ is false, and $Q(a)$ is true. $\neg Q(a)$ is false. However the conclusion $P(a)$ is false, invalidating the argument.

5.5 Exercise 1.13.5

D)

$\forall x\ M(x) \implies D(x)$
$\neg M(Penelope)$
$\therefore \neg D(Penelope)$

Truth Table 1.13.5 D				
$M(x)$	$D(x)$	$\neg M(x)$	$M(x) \implies D(x)$	$\neg D(x)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Argument Invalid: Counter-Example on Line 3.
When $\forall x M(x) \implies D(x)$ is true, and $\neg M(x)$ is true, $\neg D(x)$, the conclusion is false. We can substitute x for the case of Penelope based on the hypothesis, of Penelope being a particular student in the class and the argument will still be invalid.

E)

$\forall x\ (M(x) \vee D(x)) \implies \neg A$
Penelope is a student (p)
$A(p)$
$\therefore\ \neg D(Penelope)$

Rules of Inference Chart		
Lines	Expressions	Rules of Inference
1.	$\forall x\ (M(x) \vee D(x)) \implies \neg A$	Hypothesis
2.	Penelope (p) is a student	Hypothesis
3.	$(M(p) \vee D(p)) \implies \neg A(p)$	Universal Instantiation 1,2
4.	$(\neg((M(p) \vee D(p))) \vee \neg A(p)$	Conditional Identity 3
5.	$(\neg M(p) \wedge \neg D(p)) \vee \neg A(p)$	DeMorgan's Law 4
6.	$\neg A(p) \vee (\neg M(p) \wedge \neg D(p))$	Commutative Law 5
7.	$A(p) \implies (\neg M(p) \wedge \neg D(p))$	Conditional Identity 6
8.	$A(p)$	Hypothesis
9.	$\neg M(p) \wedge \neg D(p)$	Modus Ponens 8,7
10.	$\neg D(p) \wedge \neg M(p)$	Commutative Law 9
11.	$\neg D(p)$	Simplification 10

Question 6

6.1 Exercise 2.2.1

C) If x is a real number and $x \leq 3$ then $12 - 7x + x^2 \geq 0$.

$$\begin{gathered} \text{I. } 12 - 7x + x^2 \geq 0 \\ \text{II. } x^2 - 7x + 12 \geq 0 \\ \text{III. } (x-4)(x-3) \geq 0 \\ \text{IV. } (3-4)(3-3) \geq 0 \\ \text{V. } 0 \geq 0 \end{gathered}$$

Proof.
Direct Proof. Assume that $x = 3$. We will show that $12 - 7x + x^2$ is also equal to 0. We first re-arrange I. into a quadratic formula form. We then factor out II. into III. Once there, we can substitute x with 3 to have the equation greater than or equal to 0. Since when $x = 3$ the expression $12 - 7x + x^2$ is also equal to 0, we can conclude for values where $x \leq 3$, $12 - 7x + x^2$ will be also greater than or equal to 0. □

D) The product of two odd integers is an odd integer.

$$\begin{gathered} \text{I. } (n)(j) = q \\ \text{II. } (2k+1)(2k+1) = 4k^2 + 4k + 1 \\ \text{III. } 4k^2 + 4k + 1 = 2k(2k^2 + 2) + 1 \end{gathered}$$

Proof.

Direct Proof. Assume that two odd integers are n and j. We will show that $n \cdot j$ is equal to an odd integer q. We first express n and j as odd integers, and by definition an odd integer takes the form of $2k + 1$. We can then multiply these products to produce II. Because the multiplication of these expressions take the form of a quadratic equation we can factor the expression out to III. We know that the definition of an even number is $2k$, but in our final expression we add 1 an odd integer. An even integer plus and odd integer is always odd, so we can conclude the product of two odd integers is an odd integer. □

Question 7

7.1 Exercise 2.3.1

D) For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

$$\text{I. } n = 2k$$
$$\text{II. } (2k)^2 - 2(2k) + 7$$
$$\text{III. } 4k^2 - 4k + 7$$
$$\text{IV. } 2k(2k - 2) + 7$$

Proof.

Proof by contrapositive. Assume that n is an even integer. We will show that $n^2 - 2n + 7$ is odd for the case of n. We know that by definition an even integer can be expressed as $2k$. We can substitute n with $2k$ as seen in II, giving us a quadratic form III. We can factor out $2k$ from the expression, which by definition is an even integer. We are also left with 7 an odd integer, so when we add an even integer by an odd integer we get an odd integer. Concluding that for every integer n, if $n^2 - 2n + 7$ is even, then n is odd. □

F) For every non-zero real number x, if x is irrational then 1/x is also irrational.

$$\text{I. } x = a/b$$
$$\text{II. } 1/x = 1/(a/b)$$
$$\text{III. } 1/(a/b) = b/a$$

Proof.

Proof by contrapositive. Assume that x is a rational real number. We can express x as a/b where $a \neq 0$ and $b \neq 0$, by definition of rational numbers. We can substitute x in $1/x$ as $1/(a/b)$ as seen in II. When evaluated, we get the reciprocal b/a, which is itself a rational number. Concluding that for every non-zero real number x, if x is irrational then 1/x is also irrational. □

G) For every pair of real numbers x and y if $x^3 + xy^2 \leq x^2y + y^3$ then $x \geq y$.

$$\text{I.} x^3 + xy^2 > x^2y + y^3$$
$$\text{II. } x(x^2 + y^2) > y(y^2 + x^2)$$

Proof.

Proof by contrapositive. Assume that $x^3 + xy^2 > x^2y + y^3$ I. We can factor out x from $x^3 + xy^2$ and y from $x^2y + y^3$ leaving us with II. Since the factor of x is greater than the factor of y, we can conclude that for every pair of real numbers x and y if $x^3 + xy^2 \leq x^2y + y^3$ then $x \geq y$. □

L) For every pair of real numbers x and y if $x + y > 20$, then $x > 10$ or $y > 10$.

$$\text{I. } x \leq 10$$
$$\text{II. } y \leq 10$$
$$\text{III. } x + y \leq 20$$
$$\text{IV. } 10 + 10 \leq 20$$
$$\text{V. } 20 \leq 20$$

Proof.

Proof by contrapositive. Assume that x and y are both less than or equal to 10 (I and II). If x and y are both 10, we can substitute x and y as 10. We can also assume that x and y are both less than or equal to 20. We can then substitute x and y in to the expression, and when evaluated we get $20 \leq 20$ V. We can then conclude that for every pair of real numbers x and y if $x + y > 20$, then $x > 10$ or $y > 10$. $\square$

Question 8

8.1 Exercise 2.4.1

C) The average of three real numbers is greater than or equal to at least one of the numbers.

$$\text{I. } (x + y + z)/3 \geq x$$
$$\text{II. } (x + y + z)/3 \not\geq x$$
$$\text{III. } (x + y + z)/3 < x$$

Proof.

Proof by contradiction. Assume that the average number of the real numbers are not greater than or equal to one of the numbers. We can then further assume that the average number of three real numbers are not at least one of the numbers. Next we can assume that the average of three numbers is none of the numbers. And finally we can further deduce that the average of three numbers is less than all of the numbers. The negation of this statement leads to the conclusion, the average of three real numbers is greater than or equal to at least one of the numbers. □

E) There is no smallest integer.

$$\text{I. } x \not> y$$
$$\text{II. } x < y$$
$$\text{III. } x \geq y$$

Proof.

Proof by contradiction. Assume that their is a smallest integer. If there is a number that is the smallest integer, then their will be a integer that is greater than another integer. We can then further assume that there is a integer that is also greater than or equal to another integer. By negation of this statement, we can conclude that there is no smallest integer. □

Question 9

9.1 Exercise 2.5.1

C) If integers x and y have the same parity, then $x + y$ is even.

Case 1. x and y are odd

$$\text{I. } x + y$$
$$\text{II. } (2k + 1) + (2k + 1) = 4k + 2$$
$$\text{III } 4k + 2 = 2(2k + 1)$$

Case 2. x and y are even

$$\text{VI. } x + y$$
$$\text{V. } (2k) + (2k) = 4k$$
$$\text{VI. } 4k = 2(2k)$$

Proof.

Proof by cases. We consider two cases, where x and y are odd, and x and y are even. In Case 1. we assume x and y are odd integers. We can express both x and y as $2k + 1$ by definition of odd integers II. When added, their sum produces $4k + 2$ and when factored becomes $2(2k + 1)$ III. We know that when an odd integer, expressed as $(2k + 1)$ is multiplied by an even integer 2, their product is also even.

In Case 2. we assume x and y are even. We can express both x and y as $2k$ by definition of even integers and when added, their sum produces $4k$. We can factor $4k$ as $2(2k)$ VI. We know that when an even integer, expressed as $(2k)$ is multiplied by an even integer 2, their product is also even.

We can then conclude that if integers x and y have the same parity, then $x + y$ is even. □