Homework 11

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Question 1

Refer to C++ File

Question 2

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Question 3

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Question 4

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Question 5

5.1

5a) Use mathematical induction to prove that for any positive integer n, $\frac{n^3 + 2n}{3}$

Base Case: $n = 1$

$\frac{1^3 + 2(1)}{3}$

$\frac{1 + 2}{3}$

$\frac{3}{3} = 1$

Induction Step: Assume n = k is true.

$\frac{k^3 + 2k}{3}$ is true. So we can assume, $k^3 + 2k = 3j$ Where j is a positive integer.

Induction Hypothesis: If $n = k$ is true, then $n = k + 1$ will also be true.

$\frac{k^3 + 2k}{3} = \frac{(k + 1)^3 + 2(k + 1)}{3}$

$k^3 + 2k = (k^3 + 3k^2 + 3k + 1) + (2k + 2)$

$k^3 + 2k = k^3 + 3k^2 + 5k + 3$

$k^3 + 2k = k^3 + 2k + 3k + 3$

$k^3 + 2k = 3j$ via Induction Hypothesis so,

$3j = 3j + 3k^2 + 3k + 3$

$3j = 3j + 3(k^2 + k + 1)$

$3(j) = 3(j + k^2 + k + 1)$

Conclusion Step: $\frac{3(j)}{3} = \frac{3(j + k^2 + k + 1)}{3}$

If $n = k$ is divisible by 3, without a remainder for any positive integer j, and $n = k + 1$ is also an integer that is divisible by 3 without a remainder, $k = k + 1$ by means of mathematical induction.

Question 6

6.1 Exercise 7.4.1

A)$P(3)$

$$\frac{3(3+1)(2\cdot 3+1}{6}$$

$$\frac{3(4)(7)}{6}$$

$$\frac{84}{6} = 14$$

B) $P(k)$

$$\frac{k(k+1)(2\cdot k+1}{6}$$

$$\frac{k(k+1)(2k+1)}{6}$$

$$\frac{(k^2+k+1)(2k+1)}{6}$$

$$\frac{2k^3+2k^2+2k+k^2+k+k}{6}$$

$$\frac{2k^3+3k^2+4k}{6}$$

C) $P(k+1)$

$$\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(2k^2+3k+2)(2k+3)}{6}$$

$$\frac{(4k^3+6k^2+4k+6k^2+9k+6)}{6}$$

$$\frac{(4k^3+12k^2+13k+6)}{6}$$

D) The thing that must be proven in the base case is that for any number within the bounds of n, $P(n)$ is true. In the case of $\frac{n(n+1)(2\cdot n+1}{6} = j^2$, we must first prove for the lowest bound where $n = 1$, $j^2 = 1$, is true.

E) The thing that must be proven in the inductive step is the inductive hypothesis. This can be expressed as assuming $n = k$ is true within the function and, $n = k + 1$ is also true, then $k = k + 1$, the inductive hypothesis is true.

F) The inductive hypothesis would be:
$$\frac{2k^3 + 3k^2 + 4k}{6} = \frac{(4k^3 + 12k^2 + 13k + 6)}{6}$$

G) The proof would be:
$$\frac{(k)(2k^2 + 3k + 4}{6} = \frac{(4k^3 + 12k^2 + 13k + 6)}{6} \frac{(k)(k + 1)(2k + 2)}{6} = \frac{(k)(4k^2 + 12k + 13}{6}$$

6.2 Exercise 7.4.3

Prove that for $n \geq 1$, $\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{n}$

Base Case: $n = 1$
$$\frac{1}{1^2} \leq (2) - \frac{1}{1}$$
$$\frac{1}{1} \leq 1$$

For Every $n \geq 1$, if $\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{k}$, then $\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{k+1}$