Homework 8

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March 6, 2021

Question 1

Please refer to C++ File.

Question 2

Please refer to C++ File.

Question 3

Please refer to C++ File.

Question 4

Please refer to C++ File.

Question 5

Please refer to C++ File.

Question 6

Please refer to C++ File.

Question 7

7.1 Exercise 6.1.5

B) $\frac{\binom{13}{1}\binom{4}{2}\binom{49}{2}}{\binom{52}{5}}$

C) $\frac{\binom{52}{1}\binom{4}{1}\binom{13}{4}}{\binom{52}{5}}$

D) $\frac{\binom{52}{1}\binom{4}{2}\binom{50}{3}}{\binom{52}{5}}$

7.2 Exercise 6.2.4

A) $1 - \frac{\binom{39}{5}}{\binom{52}{5}}$

B) $1 - \frac{\binom{13}{5}\binom{4}{1}^5}{\binom{52}{5}}$

C) $\frac{2 \cdot \binom{13}{1}\binom{39}{4} - 13^2\binom{26}{3}}{\binom{52}{5}}$

D) $1 - \frac{\binom{26}{5}}{\binom{52}{5}}$

Question 8

8.1 Exercise 6.3.2

A) $P(A) = 6!/7! = 1/7$
$P(B) = 7!/(2 \cdot 7!) = 1/2$
$P(C) = 5!/7! = 1/42$

B) $P(A|C)$ is determined if the letter "b" falls in the middle, and the letters "def" occur together. So we can determine that this is the intersection of both $P(A)$ and $P(C)$. If "b" falls in the middle, then there are $2 \cdot 3!$ ways that "def" can be configured from the 7! spaces. $|P(A) \cap P(C)| = 2 \cdot 3!$ and $|P(A) \cap P(C)|/|P(C)| = (2 \cdot 3!)/5! = 1/10$

C) $P(B|C)$ is determined if the letter "c" appears to the right of b, and if the letters "def" occur together. So we can determine that this is the intersection of both $P(B)$ and $P(C)$, which would give us 5!/2.We can then find $P(B|C) = |P(B) \cap P(C)|/|P(C)| = 5!/(2 \cdot 5!) = 1/2$

D) $P(A|B)$ is determined if the letter "b" falls in the middle, and i the letter "c" appears to the right of "b". So we can determine that this is the intersection of both $P(A)$ and $P(B)$, which would give us $3 \cdot 5!$.We can then find $P(A|B) = |P(A) \cap P(B)|/|P(A)| = (3 \cdot 5!)/(7! \cdot 2) = 1/7$

E) A is independent in the event of A|B. $P(A) = 1/7 = |P(A) \cap P(B)|/|P(A)|$
B is independent in the event of B|C. $P(B) = 1/2 = |P(B) \cap P(C)|/|P(C)|$

8.2 Exercise 6.3.6

B) $(1/3)^5 \cdot (2/3)^5$

C) $(1/3) \cdot (2/3)^9$

8.3 Exercise 6.4.2

A) If there are two Dice, where one is a fair dice, and one is an unfair dice, then there is a probability of one of the two dice being picked to be rolled. This can be expressed with $P(F) = 1/2$ where F is the probability of a fair dice and $P(\bar{F}) = 1 - 1/2 = 1/2$ where $\bar{F}$ is the probability that dice is not fair. If the dice is a fair dice, then their is a probability of $P(Y|F) = (1/6)^6$ out of the 6 rolls. If the dice is unfair then out of the six rolls $P(Y|\bar{F}) = (5/20)^2 \cdot (3/20)^4$ accounting for the 2 rolls where the outcome was 6, and the remaining for rolls, being those other than 6. We can then use Bayes's Theorem to prove the possibility of whether the dice is fair, $\frac{(1/6)^6 \cdot (1/2)}{(1/6)^6 \cdot (1/2) + (5/20)^2 \cdot (3/20)^4 \cdot (1/2)}$, which gives us approximately 40.4% chance that the dice is fair, and an approximately 59.6% chance that the dice is not fair.

Question 9

9.1 Exercise 6.5.2

A) There are 4 Aces in the deck, if we choose 5 cards then there can either be $A = \{0, 1, 2, 3, 4\}$ where A is the set of aces that could be in the hand.

B) The distribution of A from the hand of 5 depends on how many aces occur in the hand. If $A = 0$ then the probability of the amount of hands that can be picked from the hand is $\binom{48}{5}/\binom{52}{5}$. We can then apply this same probability accounting for the amount aces that are in the hand. For $A = 1$ we can choose 1 out of the 4 aces. $4 \cdot \binom{48}{4}/\binom{52}{5}$. For $A = 2$, we choose 2 out of the four aces, $\binom{4}{2} \cdot \binom{48}{3}/\binom{52}{5}$. If we continue this pattern, the rest of the range is $(3, \binom{4}{3} \cdot \binom{48}{2}/\binom{52}{5}), (4, \binom{4}{4} \cdot \binom{48}{1}/\binom{52}{5}$.

9.2 Exercise 6.6.1

A) $(0 \cdot 3/45 + 21/45 + 2 \cdot 21/45) = 1.4$

9.3 Exercise 6.6.4

A) A fair die has a 1/6 chance of rolling on any number on it. If each side of the die is squared, then $E[X] = 1(1/6) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6) = 91/6$

B) If a fair coin is tossed three times, and the coin has two side, then $(1/2)^3 = 1/8$ chance for any combination of heads and tails for each flip. If the number of heads is squared, for each variation of flips, then $E[Y] = 0(1/8) + 1(3/8) + 4(3/8) + 9(1/8) = 24/8 = 3$

9.4 Exercise 6.7.4

A) The probability that any student who grabs the right jacket could be described by $E[P_1] = 1/10 + E[P_2] = 2/(10 \cdot 9) + E[P_3] = 3/(10 \cdot 9 \cdot 8)...E[P_{10}] = 10/10!$ which would approximate to 12.7%.

Question 10

10.1 Exercise 6.8.1

A) $\binom{100}{2} \cdot (.01^2) \cdot (.99^{98})$

B) $1 - \binom{100}{1} \cdot (01^1) \cdot (.99^{99}) - \binom{100}{0} \cdot (.01^0) \cdot (.99^{100})$

C) $(.01) \cdot (100)$

D) $.01 \cdot (50) \cdot (2)$
$1 - \binom{50}{0} \cdot (.01^1) \cdot (.99^{49})$
$1 - (.99^{50})$

10.2 Exercise 6.8.3

B) If the coin is biased, then the number of heads would be greater than four. If the coin is flipped ten times, then the coin needs to be heads greater than or equal to 4, out of the flips 10 flips. The probability of this happening can then be calculated as: $\binom{10}{4} \cdot (.03)^{10} + \binom{10}{5} \cdot (.03)^{10} + \binom{10}{6} \cdot (.03)^{10} + \binom{10}{7} \cdot (.03)^{10} + \binom{10}{8} \cdot (.03)^{10} + \binom{10}{9} \cdot (.03)^{10} + \binom{10}{10} \cdot (.03)^{10}$