Discrete Math : Logical Equivalence

Consider compound statement: "Grass is green or snow is white"
Same truth value as: "Snow is white or grass is green"
Logical Form of Statement #1 : $p \vee q$ → They both have same truth value
Logical Form of Statement #2 : $q \vee p$

Statement #1

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Statement #2

q	p	q∨p
T	T	T
T	F	T
F	T	T
F	F	F

Combined

p	q	p∨q	q∨p
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

* When 2 statement forms have the same truth-values for each possible combination for the constitute statement variables the statement forms are said to be LOGICAL EQUIV.

Logically Equivalent

"p and q" if they are Logically Equivalent "$p \equiv q$" is written.

Example Show that $\neg(\neg p) \equiv p$

p	¬p	¬(¬p)
T	F	T
F	T	F

Truth table shows $\neg(\neg p) \equiv p$
"$\neg(\neg p)$" is logically Equivalent to "p"

Example 2. Show that $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$\neg(p \vee q) \equiv \neg p \wedge \neg q$ "Logically Equivalent"
DeMorgans Laws. negation symbol switches "v" to "$\wedge$" and is distributed

Example 3. Show that $p \to q \equiv \neg p \vee q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

$p \to q \equiv \neg p \vee q$ "Logically Equivalent"

Example 4. Show that $\neg(p \to q) \not\equiv \neg p \to \neg q$

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p	q	$\neg p$	$\neg q$	$\neg p \to \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

$\neg(p \to q) \not\equiv \neg p \to \neg q$ "Not Logically Equivalent"