Discrete Math

Sets:

* A set is an unsorted collection of objects

4 ways to describe sets:

(1) Roster Method:

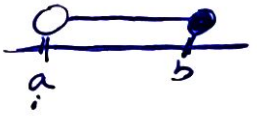
$A = \{10, 11, 12, 13, 14, 15\}$

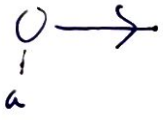
(2) Set Builder Method:

$A = \{x \mid \underbrace{(x \text{ is an integer}) \wedge (10 \leq x \leq 15)}_{P(x)}\}$

(x such that)

$(a, b] = \{x \mid a < x \leq b\}$





$(a, \infty) = \{x \mid a$

Definitions

(1) Membership: x is a member of A $x \in A$

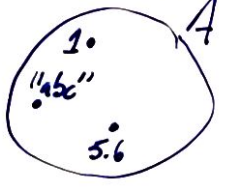
$A = \{1, \text{"abc"}, 5.6\}$

$1 \in A \Rightarrow T$ 1 has membership to A

$2 \in A \Rightarrow F$

$2 \notin A \Rightarrow$

Drawing



(2) Subset: A collection is a subcollection of a larger Set

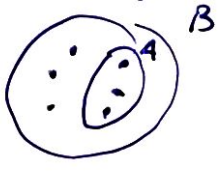
$A \subseteq B$: A subset of B

Definition:

$A \subseteq B$ if $\forall x\,(x \in A \rightarrow x \in B)$

for all x x is in A then x is in B

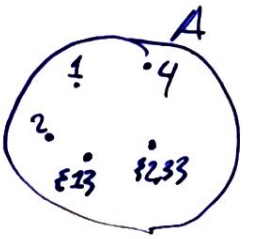
Drawing



$A \{1, 2, 4\}$

$B \{1, 2, 3, 4, 5, 6\}$ $A \subseteq B \Rightarrow T$ *More Interesting

$A = \{1, 2, \{1\}, \{2,3\}, 4\}$ →



$2 \in A \Rightarrow T$ $\quad 4 \in A \Rightarrow T$

$3 \in A \Rightarrow F$ $\quad 4 \subseteq A \Rightarrow F$ (4 isn't a set)

$\{4\} \subseteq A \Rightarrow T$

$\{4\} \in A \Rightarrow F$

$1 \in A \Rightarrow T$

$\{1\} \in A \Rightarrow T$

$\{1\} \subseteq A \Rightarrow T$

$\{3\} \subseteq A \Rightarrow F$

3 isn't an element in A so can't be a subset

$\{1, 2\} \in A \Rightarrow F$

$\{1, 2\} \subseteq A \Rightarrow T$

1 & 2 are elements/members so they are a set

$\{2, 3\} \in A \Rightarrow T$

$\{2, 3\} \subseteq A \Rightarrow F$

$\{\{2, 3\}\} \subseteq A \Rightarrow T$

$M \subseteq N$

$\forall x \; x \in M \rightarrow x \in N$

$x = 1 \quad 1 \in M \rightarrow 1 \in N \Rightarrow T$ (F → T)

$x = 2 \quad 2 \in M \rightarrow 2 \in N \Rightarrow T$ (F → T)

$x = 3 \quad 3 \in M \rightarrow 3 \in N \Rightarrow T$ (F → T)

$x = 4 \quad 4 \in M \rightarrow 4 \in N \Rightarrow T$ (T → T)

The element is in a subset of 4 in the set of N

$\{4\} \subseteq A$

$\forall x \; x \in \{4\} \rightarrow x \in A$

For

$\emptyset \in A \Rightarrow F$

$\emptyset$ is an empty set $\{\}$

$\{\} \subseteq A \Rightarrow T$

$\forall x \; x \in \{\emptyset\} \rightarrow x \in A$

F

T

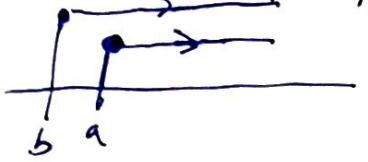
T

$\mathbb{R}$ = Set of all real numbers

$\mathbb{N}$ = Set of all natural numbers (either positive or zero and positive)
$\{0, 1, 2, 3, \dots\}$

$\mathbb{Q}$ = Set of all rational numbers

Example: Let $a, b \in \mathbb{R}$, show that $[a, \infty) \subseteq [b, \infty) \rightarrow a \geq b$



Assume $[a, \infty) \subseteq [b, \infty)$ we now show that $a \geq b$

(I) Since $[a, \infty) \subseteq [b, \infty)$ we have $\forall x \; x \in [a, \infty) \rightarrow x \in [b, \infty)$

(II) By using U.I. for $x = a$, we get $a \in [a, \infty) \rightarrow a \in [b, \infty)$

(III) Since $[a, \infty) = \{x / x \geq a\} \wedge x \in \mathbb{R}\}$ and since $a \in \mathbb{R} \wedge a \geq a$ we get $a \in [a, \infty)$

(IV) By using M.P on (II) and (III) we get that $a \in [b, \infty)$

(V) Since $[b, \infty) = \{x / x \in \mathbb{R} \wedge x \geq b\}$, and since $a \in [b, \infty)$ we get $a \in \mathbb{R} \wedge a \geq b \Rightarrow a \geq b$

Definitions

(1) $A = B$ if $(A \subseteq B) \wedge (B \subseteq A)$

equivalence $\{1,2,3\} = \{2,3,3,2,1,3,2\}$

Yes they are subsets of each other so they are equal

(2) $|A|$ = # of "DISTINCT" Elements in A

cardinality $|\{2,3,3,2,1,3,2\}| = 3$

(3) $P(A) = \{X \underbrace{/ X \subseteq A}_{P(x)}\}$; $|P(A)| = 2^{|A|}$

$A = \{1,2,3\}$

$P(A) = \{ \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{\} \}$

(4) $A \times B = \{$

Cartesian product

First component from A & Second Component is from B

$A \times B = \{a,a\}, \{a,b\}, \{a,c\}, \{b,a\}, \{b,b\}, \{b,c\}$

$\{a,b\}$ $\{a,b,c\}$ All are cartesian products for A and B

Lecture 3

Program session:

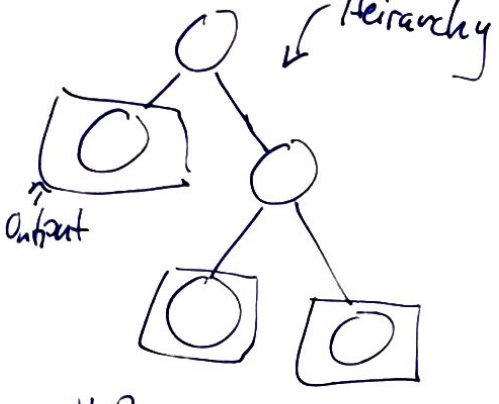
Please enter 2 grades, seperated by a space

Input → 78 , 97

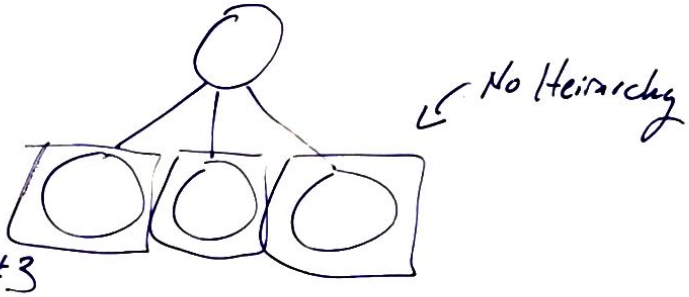
Output → Student Graduated

- If 1 grade under 60 student failed
- Student graduate
- Student graduate with honors
  ↳ Both over 90

if... else



if... else if... else...



#2 ≡ ¬(#1 ∨ #3) ≡ ¬#1 ∧ ¬#3

((grade 1 || grade 2) < 60) NO, NO OR's B/W Constants ONLY BOOL'S!!

Please order (& Call) positive intergers, seperated by a space

78 5 23 25 31

- More even numbers
- More odd numbers
- Same number of odds and evens

Memory Sketch