Homework 1

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January 15, 2021

Question 1

1.1 A

Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2=$1 \times 2^0 + 1 \times 2^1 + 1 \times 2^3 + 1 \times 2^3 + 1 \times 2^4 + 1 \times 2^7$=$(155)_{10}$

2. 456_7=$6 \times 7^0 + 5 \times 7^1 + 4 \times 7^2 = (237)_{10}$

3. $38A_{16}$=$10 \times 16^0 + 8 \times 16^1 + 3 \times 16^2$=$(906)_{10}$

4. 2214_5=$4 \times 5^0 + 1 \times 5^1 + 2 \times 5^2 + 2 \times 5^3$=$(309)_{10}$

1.2 B

Convert the following numbers to their binary representation:

1. 69_{10}=$2^6 + 2^2 + 2^0$=$(1000101)_2$

2. 485_{10}=$2^8 + 2^7 + 2^6 + 2^5 + 2_2 + 2_0 = (111100101)_2$

3. $6D1A_{16}$=0110110100011010_2 (Hexadecimal to Binary Table)

1.3 C

Convert the following numbers to their hexadecimal representation:

1. 1101011_2=$(6B)_{16}$

2. 895_{10}=$(37F)_{16}$

Question 2

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = (14303)_8$

$$\begin{array}{rccccc} & 7^1 & 5^1 & 6^1 & 6 & {}_8 \\ + & 4 & 5 & 1 & 5 & {}_8 \\ \hline 1 & 4 & 3 & 0 & 3 & {}_8 \end{array}$$

2. $10110011_2 + 1101_2 = (11000000)_2$

$$\begin{array}{rcccccccc} & 1 & {}^1 0 & {}^1 1 & {}^1 1 & {}^1 0 & {}^1 0 & {}^1 1 & 1_2 \\ + & & & & & 1 & 1 & 0 & 1_2 \\ \hline & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0_2 \end{array}$$

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

$$\begin{array}{rccccc} & {}^1 7 & {}^1 A & 6 & 6 & {}_{16} \\ + & 4 & 5 & C & 5 & {}_{16} \\ \hline & C & 0 & 2 & B & {}_{16} \end{array}$$

4. $3022_5 + 2433_5 = (34)_5$

$$\begin{array}{rccccc} & {}^2\cancel{3} & 0^{\cancel{1}9} & \cancel{1}^{1^1} & 6 & {}_5 \\ - & 2 & 4 & 3 & 3 & {}_5 \\ \hline & 0 & 0 & 3 & 4 & {}_5 \end{array}$$

Question 3

3.1 A

Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = (01111100)_{\text{8-bit two's complement}}$

$$124_{10} \text{ is a Positive Number.}$$

$$124_{10} < 2^7$$

$$2^6 + 2^5 + 2_4 + 2^3 + 2^1 = 124$$

$$\text{0 is the 8th digit, which signifies a positive number.}$$

$$01111100_2 = 124_{10}$$

2. $-124_{10} = (10000100)_{\text{8-bit two's complement}}$

$$-124_{10} \text{ is a Negative Number. So we need the positive binary representation of } 124_{10}$$

$$124_{10} < 2^7$$

$$2^6 + 2^5 + 2_4 + 2^3 + 2^1 = 124$$

$$\text{0 is the 8th digit, which signifies a positive number.}$$

$$01111100_2 = 124_{10} = (+x)$$

$$\begin{array}{rccccccccl} & {}^{1}0 & {}^{1}1 & {}^{1}1 & {}^{1}1 & {}^{1}1 & {}^{1}1 & 0 & 0_2 & (+x) \\ + & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0_2 & (-x) \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0_2 & \end{array}$$

$$(-x) = (10000100)_{\text{8-bit two's complement}}$$

3. $109_{10} = (01101101)_{\text{8-bit two's complement}}$

$$109_{10} \text{ is a Positive Number.}$$

$$109_{10} < 2^7$$

$$2^6 + 2^5 + 2^3 + 2^1 + 2^0 = 109$$

$$\text{0 is the 8th digit, which signifies a positive number.}$$

$$01101101_2 = 109_{10}$$

4. $-79_{10} = (10110001)_{\text{8-bit two's complement}}$

$$-79_{10} \text{ is a Negative Number. So we need the positive binary representation of } 79_{10}$$

$$79_{10} < 2^7$$

$$2^6 + 2^3 + 2_2 + 2^1 + 2^0 = 79$$

$$0 \text{ is the 8th digit, which signifies a positive number.}$$

$$1001111_2 = 79_{10} = (+x)$$

$$\begin{array}{rccccccccl} & {}^10 & {}^11 & {}^10 & {}^10 & {}^11 & {}^11 & {}^11 & 1_2 & (+x) \\ + & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1_2 & (-x) \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0_2 & \end{array}$$

$$(-x) = (10110001)_{\text{8-bit two's complement}}$$

3.2 B

Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{\text{8 bit 2's comp}} = (+30)_{10}$

$$0 \text{ is the 8th digit, which signifies a positive number}$$

$$2^4 + 2^3 + 2_2 + 2^1 = 30_{10}$$

2. $11100110_{\text{8 bit 2's comp}} = (-26)_{10}$

$$1 \text{ is the 8th digit, which signifies a negative number.}$$

$$\begin{array}{rccccccccl} & {}^11 & {}^11 & {}^11 & {}^10 & {}^10 & {}^11 & {}^11 & 0_2 & (+x) \\ + & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0_2 & (-x) \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0_2 & \end{array}$$

$$(+x) = (00011010)_{\text{8-bit two's complement}} = (00011010)_2$$

$$2^4 + 2^3 + 2_1 = 26_{10}$$

$$x = 26_{10} \text{ so, } -x = -26_{10}$$

3. $00101101_{\text{8 bit 2's comp}} = (+45)_{10}$

$$0 \text{ is the 8th digit, which signifies a positive number}$$

$$2^5 + 2^3 + 2_2 + 2^0 = 45_{10}$$

4. $10011110_{\text{8 bit 2's comp}} = (-98)_{10}$

$$1 \text{ is the 8th digit, which signifies a negative number.}$$

$$\begin{array}{rccccccccl} & {}^11 & {}^10 & {}^10 & {}^11 & {}^11 & {}^11 & 1 & 0_2 & (-x) \\ + & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0_2 & (+x) \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0_2 & \end{array}$$

$$(+x) = (01100010)_{\text{8-bit two's complement}} = (01100010)_2$$

$$2^6 + 2^5 + 2_1 = 98_{10}$$

$$x = 98_{10} \text{ so, } -x = -98_{10}$$

Question 4

4.1 Exercise 1.2.4

Truth Table 1.2.4 b			
p	q	p $\vee q$	$\neg(p \vee \neg q)$
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	F

Truth Table 1.2.4 c				
p	q	r	p $\wedge \neg q$	r $\vee(p \wedge \neg q)$
T	T	T	F	F
T	T	F	F	F
T	F	T	T	F
T	F	F	T	F
F	T	T	F	F
F	T	F	F	T
F	F	T	F	F
F	F	F	F	F

4.2 Exercise 1.3.4

Truth Table 1.3.4 b				
p	q	p $\implies q$	q $\implies p$	(p $\implies q) \implies (q \implies p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Truth Table 1.3.4 d				
p	q	p $\iff q$	p $\iff \neg q$	(p $\iff q) \oplus (p \iff \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

5.1 Exercise 1.2.7

B) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

Answer: $(B \wedge D) \vee (B \wedge M) \vee (M \wedge D)$

Explanation: In order for the applicant to apply for a credit card, they need to satisfy two of the three conditions. So they can have either: 1) Birth Certificate and Drivers licences OR 2)Birth Certificate and Marriages Licences OR 3)Marriage Licences and Drivers Licences

C) Applicant must present either a birth certificate or both a driver's license and a marriage license.

Answer: $B \vee (D \wedge M)$

Explanation: The applicant needs to satisfy having a Birth Certificate OR Driver's licences, along with a Marriage Licence.

5.2 Exercise 1.3.7

B) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

Answer: $(s \vee y) \implies p$

Explanation: The person can go to the parking lot if they satisfy the condition of being 1)They are a senior OR 2)At least 17 years old

C) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

Answer: $y \implies p$

Explanation: Being 17 years old implies that a person is able to park in the school parking lot.

D) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

Answer: $p \iff (s \wedge y)$

Explanation: A person can park in the school parking lot IF AND ONLY IF they satisfy the conditions of being both 1) A senior AND 2) At least 17 years of age.

E) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

Answer: p $\implies (s \lor y)$

Explanation: To park in the school parking lot, the person has to be 1) A senior OR 2) At least 17 years old.

5.3 Exercise 1.3.9

C) The applicant can enroll in the course only if the applicant has parental permission.

Answer: c $\implies p$

Explanation: The applicant needs to satisfy having parental permission in order to enroll in a course.

D) Having parental permission is a necessary condition for enrolling in the course.

Answer: p $\implies c$

Explanation: Before enrolling in the course, the person needs parental permission.

Question 6

6.1 Exercise 1.3.6

B) Maintaining a B average is necessary for Joe to be eligible for the honors program.

Answer: Joe can be eligible to enroll in the honors program if he maintains a B average.

Explanation: The statement is expressed in the logical format p $\implies q$.

So we can express this in English using q if p.

C) Rajiv can go on the roller coaster only if he is at least four feet tall.

Answer: If Rajiv can go on the roller coaster, then he is at least four feet tall.

Explanation: The statement is expressed in the logical format p $\implies q$.

So we can express this in English using if p then q.

D) Rajiv can go on the roller coaster only if he is at least four feet tall.

Answer: Rajiv going on the roller coaster implies that he is at least four feet tall.

Explanation: The statement is expressed in the logical format p $\implies q$.

So we can express this in English using p implies q.

6.2 Exercise 1.3.10

C) (p $\vee r) \iff (q \wedge r)$

Answer: False

Explanation: If r is true, hypothesis is true, conclusion is false, expression is false. If r is false, hypothesis is true and conclusion is false, expression is false.

D) (p $\wedge r) \iff (q \wedge r)$

Answer: Unknown

Explanation: If r is true, hypothesis is true, conclusion is false, expression is false. If r is false, hypothesis is false, conclusion is false, expression is true.

E) p $\iff (r \vee q)$

Answer: Unknown

Explanation: If r is true, hypothesis is true, conclusion is true, expression is true. If r is false, hypothesis is true, conclusion is false, expression is false.

F) (p $\wedge q) \iff r$

Answer: True

Explanation: If r is true, hypothesis is false, conclusion is true, expression is true. If r is false, hypothesis is false, conclusion is false, expression is true.

Question 7

7.1 1.4.5

B) If Sally did not get the job, then she was late for interview or did not update her resume. If Sally updated her resume and was not late for her interview, then she got the job.

Answer: $\neg j \implies (l \vee \neg r)$
$(r \wedge \neg l) \implies j$

Logically Equivalent.

Explanation:

Truth Table 1.4.5 b				
j	l	r	$\neg j \implies (l \vee \neg r)$	((r $\wedge\neg l) \implies j$
T	T	T	T	T
T	T	F	T	T
T	F	F	T	T
T	F	T	T	T
F	T	F	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

C) If Sally got the job then she was not late for her interview. If Sally did not get the job, then she was late ofr her interview.

Answer: J $\implies \neg l$
$\neg j \implies l$

Not Logically Equivalent.

Truth Table 1.4.5 c			
j	l	j $\implies \neg l$	$\neg j \implies l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

D) If Sally updated her resume or she was not late for her interview, then she got the job. If Sally got the job, then she updated her resume and was not late for her interview.

Answer: (r $\vee\neg l) \implies j$
$j \implies (r \wedge \neg j)$

Not Logically Equivalent

Explanation:

Truth Table 1.4.5 d				
j	l	r	$(r \vee \neg l) \implies j$	$j \implies (r \wedge \neg j)$
T	T	T	T	F
T	T	F	T	F
T	F	F	T	T
T	F	T	T	F
F	T	F	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8

8.1 Exercise 1.5.2

C): (p $\implies q) \wedge (\implies r) \equiv p \implies (q \wedge r)$

Logical Equivalence (p $\implies q) \wedge (\implies r) \equiv p \implies (q \wedge r)$	
(p $\implies q) \wedge (r))$	
$(\neg p \vee q) \wedge (pr)$	Conditional Law
$(\neg p \vee q) \wedge (\neg p \vee r)$	Conditional Law
$\neg p \vee (q \wedge r)$	Distributive Laws
p $\implies (q \wedge r)$	Conditional Identities

F) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Logical Equivalence ($\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$	
$\neg(p \vee (\neg p \wedge q))$	
$\neg p \wedge \neg(\neg p \wedge q)$	Demorgan's Law
$\neg p \wedge (\neg\neg p \vee \neg q)$	Demorgan's Law
$\neg p \wedge (p \vee \neg q)$	Double Negation
$(\neg p \wedge p) \vee (p \wedge \neg q)$	Distributive Law
F $\vee(\neg p \wedge \neg q)$	Complement Law
$\neg p \wedge \neg q$	Identity Law

I) (p $\wedge q) \implies r \equiv (p \wedge \neg r) \implies q$

Logical Equivalence (p $\wedge q) \implies r \equiv (p \wedge \neg r) \implies q$	
(p $\wedge q) \implies r$	
$\neg(p \wedge q) \vee r$	Conditional Law
$(\neg p \vee \neg q) \vee r$	Demorgan's Law
$(\neg p \vee r) \vee \neg q$	Associative Law
$(\neg p \vee \neg\neg r) \vee \neg q$	Double Negative
$\neg(p \wedge \neg r) \vee \neg q$	Demorgan's Law
(p $\wedge\neg r) \implies q$	Conditional Law

8.2 Exercise 1.5.3

C) $\neg r \vee (\neg r \implies p) \equiv T$

Logical Equivalence $\neg r \vee (\neg r \implies p) \equiv T$	
$\neg r \vee (\neg r \implies p)$	
$\neg r \vee (\neg\neg r \vee p)$	Conditional Law
$\neg r \vee (r \vee p)$	Double Negation
$(\neg r \vee r) \vee p$	Associative Law
T $\vee p$	Complement Law
T	Domination Law

D) $\neg(p \implies q) \implies \neg q \equiv T$

Logical Equivalence $\neg(p \implies q) \implies \neg q \equiv T$	
$\neg(p \implies q) \implies \neg q$	
$\neg(\neg p \vee q) \implies \neg q$	Conditional Law
$(\neg\neg p \wedge \neg q) \implies \neg q$	Demorgan's Law
(p $\wedge \neg q) \implies \neg q$	Double Negation
$\neg(p \wedge \neg q) \vee \neg q$	Conditional Law
$(\neg p \vee \neg\neg q) \vee \neg q$	Demorgan's Law
$(\neg p \vee q) \vee \neg q$	Double Negation
$\neg p \vee (q \vee \neg q)$	Associative Law
$\neg p \vee T$	Complement Law
T	Domination Law

Question 9

9.1 Exercise 1.6.3

C) There is a number that is equal to its square.
Answer: $\exists x\ (x = x^2)$

D) Every number is less than or equal to its square.
Answer: $\forall x\ (x \leqslant x^2)$

9.2 Exercise 1.7.4

B) Everyone was well and went to work yesterday.
Answer: $\forall x\ (\neg s(x) \implies w(x))$

C) Everyone who was sick yesterday did not go to work.
Answer: $\forall x\ (S(x) \implies \neg w(x))$

D) Yesterday someone was sick and went to work.
Answer: $\exists x\ (s(x) \land w(x))$

Question 10

10.1 Exercise 1.7.9

C) $\exists x\ ((x = c) \implies P(x))$

Answer:True.

Explanation: P(c) would evaluate true so statement evaluates to true.

D) $\exists x\ ((Q(x) \wedge R(x))$

Answer: True.

Explanation: When x = e , Q(x) is True AND R(x) is true.

E) Q(a) $\wedge P(d)$

Answer = True.

Explanation: Q(a) is true AND P(d) is True.

F) $\forall x\ ((x \neq b) \implies Q(x))$

Answer: True.

Explanation: Q(x) is False only when (x = b)

G) $\forall x\ (P(x) \vee R(x))$

Answer: False.

Explanation: When x = c, P(c) OR R(c) are both false, statement then evaluates to false.

H) $\forall x\ (P(x) \implies R(x))$

Answer: True.

Explanation: When x = a, P(a) is True; R(a) is False; so True THEN False is false.

I) $\exists(Q(x) \vee R(x))$

Answer: True.

Explanation: When x= e, Q(e) is True OR R(e) is true.

10.2 Exercise 1.9.2

B) $\exists x\ \forall y\ Q(x, y)$

Answer: True.

Explanation: For Q(2,y) all of the values evaluate to True.

C) $\exists x\ \forall y\ P(y, x)$

Answer: True.

Explanation: For P(1,x),P(2,x),P(3,x) there is an x that evaluates to True.

D) $\exists x\ \exists y\ S(x, y)$

Answer: False.

Explanation: There doesn't exists an X or Y where this true on the table.

E) $\forall x\ \exists y\ Q(x, y)$

Answer: False.

Explanation: When Q(1,y) there is no y that evaluates to True.

F) $\forall x \exists y\ P(x, y)$

Answer: True.

Explanation: For P(1,y), P(2,y), P(3,y) there is a y that evaluates to True.

G) $\forall x\ \forall y\ P(x, y)$

Answer: False.

Explanation: When x=2 and y=2, the statement evaluates to false.

H) $\exists x\ \exists y\ Q(x, y)$

Answer: True.

Explanation: When Q(2,1), Q(x,y) evaluates to True.

I) $\forall x\ \forall y\ \neg S(x, y)$

Answer: True.

Explanation: Every x and y on S(x,y) evaluates False. The negation of False is True.

Question 11

11.1 Exercise 1.10.4

C) There are two numbers whose sum is equal to their product.

Answer: $\exists x \; \exists y \; (x + y = x \cdot y)$

Explanation: There are two numbers that exists where their sum AND product are equal.

D) The ratio of every two positive numbers is also positive.

Answer: $\forall X \; \forall y \; ((x > 0) \land (y > 0)) \implies (x/y > 0)$

Explanation: If there are two positive numbers, it is IMPLIED that ratio of those two numbers are positive.

E) The reciprocal of every positive number less than one is greater than one.

Answer: $\forall x \; (x > 0) \land (x < 1) \implies (x^{-1} > 1)$

Explanation: If x is a number between 0 and 1, it is IMPLIED that the reciprocal of that x is greater than 1.

F) There is no smallest number

Answer: $\neg \exists x \; \forall y \; (x \leqslant y)$

Explanation: There does not exist a number (x), for which all numbers (y), is less than or equal to x.

G) Every number besides 0 has a multiplicative inverse

Answer: $\forall x \exists y \; (x \neq 0) \implies (x \cdot y = 1)$

Explanation: When a number (x) does not equal to 0, it is IMPLIED that there exist a number (y) which satisfies its multiplicative inverse.

11.2 1.10.7

C) There is at least one new employee who missed the deadline.

Answer: $\exists x \; (D(x) \land N(x))$.

Explanation: There exist an employee who missed the deadline.

D) Sam knows the phone number of everyone who missed the deadline.

Answer: $\exists x \; \forall y \; D(y) \implies P(Sam, y)$

Explanation: There is an employee who knows the number for everyone who missed the deadline. If such an employee exist, then that employee is Sam.

E) There is a new employee who knows everyone's phone number.

Answer: $\exists x\ N(x) \land P(x, y)$

Explanation: There exists an employee who is new and knows everyone's phone number.

F) Exactly one new employee missed the deadline

Answer: $\exists x\ [(N(x) \implies D(x) \land \forall y((x \neq y) \implies \neg D(x)]$

Explanation: There exist an employee who is new and missed the deadline.

11.3 1.10.10

C) Every student has taken at least one class besides Math 101.

Answer: $\forall x\ \exists y\ (y \neq Math101) \land T(x, y)$

Explanation: All students have taken at least a class that is not Math 101.

D) There is a student who has taken every math class besides Math 101.

Answer: $\exists x\ \forall y\ (y \neq Math101) \implies T(x, y)$

Explanation: There exists a student who took all math classes, but not Math 101.

E) Everyone besides Sam has taken at least two different math classes.

Answer: $\forall x\ y\ T(x, y) \land \exists((x \neq Sam \land (z \neq y))T(x, z))$

Explanation: Every student who is not Sam has taken more than two math classes.

F) Sam has taken exactly two math classes

Answer: $\exists x\ \exists y \forall z\ (x \neq y \land T(Sam, y) \land T(Sam, z) \land (w \neq y \land w \neq z)) \implies T(Sam, w)$

Explanation: There exists a student who is Sam, that has taken exactly two math classes that does not include a class that he may have taken twice.

Question 12

12.1 Exercise 1.8.2

B) Every patient was given the medication or the placebo or both.

Answer: There is a patient that did not get both the medication and placebo.

Explanation:

Statement Form: $\forall x\ D(x) \wedge P(x)$	
$\neg\forall x\ (D(x) \vee P(x))$	Negation
$\exists x\ \neg(D(x) \vee P(x))$	DeMorgan's Law
$\exists x\ (\neg D(x) \wedge \neg P(x))$	Demorgan's Law

C) There is a patient who took the medication and had migraines.

Answer: Every patient who did not take the drug, did not also have migraines.

Explanation:

Logical Statement Form: $\exists x\ D(x) \wedge M(x)$	
$\neg\forall x\ (D(x) \wedge P(x))$	Negation
$\exists x\ \neg(D(x) \wedge P(x))$	DeMorgan's Law
$\exists x\ (\neg D(x) \vee \neg P(x))$	Demorgan's Law

D) Every patient who took the placebo had migraines.

Answer: There is a patient who took the placebo and did not have migraines.

Explanation:

Logical Statement Form: $\forall x\ P(x)M(x)$	
$\neg\forall x\ P(x)M(x)$	Negation
$\exists x\ \neg(P(x) \implies M(x))$	Demorgan's Law
$\exists x\ \neg(\neg P(x) \vee M(x))$	Conditional Identity
$\exists x\ \neg\neg P(x) \wedge \neg M(x)$	Demorgan's Law
$\exists x\ P(x) \wedge \neg M(x)$	Double Negation

12.2 Exercise 1.9.4

C) $\exists x\ \forall y\ (P(x, y) \implies Q(x, y))$

Answer: $\exists x\ \forall y\ (P(x, y) \implies Q(x, y)) \equiv \forall x\ \exists y\ P(x, y) \wedge \neg Q(x, y)$

Logical Equivalence: $\exists x\ \forall y\ ((P(x, y) \implies Q(x, y))$	
$\neg\exists x\ \forall y\ (P(x, y)Q(x, y))$	Negation
$\forall x\ \neg\forall y\ (P(x, y)Q(x, y))$	Demorgan's Law
$\forall x\ \exists y\ \neg(P(x, y) \implies Q(x, y))$	Demorgan's Law
$\forall x\ \exists y\ \neg(\neg P(x, y) \vee Q(x, y)$	Conditional Identity
$\forall x\ \exists y\ \neg\neg P(x, y) \wedge \neg Q(x, y)$	Demorgan's Law
$\forall x\ \exists y\ P(x, y) \wedge \neg Q(x, y)$	Double Negation

D) $\exists x\ \forall y(P(x, y) \iff P(y, x))$

Answer: $\exists x\ \forall y(P(x,y) \iff P(y,x)) \equiv \forall x\ \exists y\ (\ (P(x,y) \wedge \neg P(y,x))\ \vee\ (P(y,x) \wedge \neg P(x,y))\ )$

Logical Equivalence: $\exists x\ \forall y(P(x,y) \iff P(y,x))$	
$\neg\exists x\ \forall y(P(x,y) \iff P(y,x))$	Negation
$\forall x\ \neg\forall y(P(x,y) \iff P(y,x))$	Demorgan's Law
$\forall x\ \exists y\ \neg(P(x,y) \iff P(y,x))$	Demorgan's Law
$\forall x\ \exists y\ \neg(\ (P(x,y) \implies P(y,x)) \wedge (P(y,x) \implies P(x,y)))$	Conditional Identity
$\forall x\ \exists y\ \neg\ ((\neg(P(x,y) \vee P(y,x))\ \wedge\ (P(y,x) \implies P(x,y)))$	Conditional Identity
$\forall x\ \exists y\ \neg\ (\ (\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y))$	Conditional Identity
$\forall x\ \exists y\ (\neg\ (\neg P(x,y) \vee P(y,x))\ \vee\ (\neg(\neg P(y,x) \vee P(x,y))$	Demorgan's Law
$\forall x\ \exists y\ (\ (\neg\neg P(x,y) \wedge \neg P(y,x))\ \vee\ ((\neg\neg P(y,x) \wedge \neg P(x,y))\ )$	Demorgan's Law
$\forall x\ \exists y\ (\ (P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y))\ )$	Double Negation

E) $\exists x\ \exists y\ P(x,y)\ \wedge\ \forall x\ \forall y\ Q(x,y)$

Answer: $\exists x\ \exists y\ P(x,y)\ \wedge\ \forall x\ \forall y\ Q(x,y) \equiv \forall x\ \forall y\ \neg\ (P(x,y))\ \wedge \exists x\ \exists y\ \neg Q(x,y)$

Logical Equivalence: $\exists x\ \exists y\ P(x,y)\ \wedge\ \forall x\ \forall y\ Q(x,y)$	
$\neg\exists x\ \exists y\ P(x,y)\ \wedge\ \neg\forall x\ \forall y\ Q(x,y)$	Negation
$\forall x\ \neg\exists y\ P(x,y)\ \wedge \neg\forall x\ \forall y\ Q(x,y)$	Demorgan's Law
$\forall x\ \forall y\ \neg\ P(x,y)\ \wedge \neg\forall x\ \forall y\ Q(x,y)$	Demorgan's Law
$\forall x\ \forall y\ \neg\ P(x,y)\ \wedge \exists x\ \neg\forall y\ Q(x,y)$	Demorgan's Law
$\forall x\ \forall y\ \neg\ (P(x,y))\ \wedge \exists x\ \exists y\ \neg Q(x,y)$	Demorgan's Law