1.12.2

B) $p \to (q \wedge r)$

$\neg q$

$\therefore \neg p$

1.	$p \to (q \wedge r)$	Hypothesis
2.	$\neg p \vee (q \wedge r)$	De Morg ~~Commutative~~ Conditional 1
3.	$(\neg p \vee q) \wedge (\neg p \vee r)$	Distrib 2
4.	$(\neg p \vee q)$	Conjunction 3 Simplification
5.	$p \to q$	Commutative 4
6.	$\neg q$	Hypothesis
7.	$\neg p$	Modens Tollens 6, 5

$p \to (q \wedge r)$

$\neg p \vee (q \wedge r)$

E) $p \vee q$

$\neg p \vee r$

$\neg q$

$\therefore r$

1.	$p \vee q$	Hypothesis
2.	$\neg p \vee r$	Hypothesis
3.	$\neg q$	Hypothesis
4.	$p \to r$	Conditional 2
5.	$q \vee p$	Commutative 1
6.	p	Disjunctive Syllogism 5, 3
7.	r	Moden Poners 6, 4

1.12.3

c) $p \vee q$
$\neg p$
$\therefore q$

1. Hypoth

1. $p \vee q$ Hypothesis
2. $\neg p$ Hypothesis
3.

1. $p \vee q$ Hypothesis
2. $\neg\neg p \vee \neg\neg q$ Double Neg 1
3. $\neg p \rightarrow \neg\neg q$ ~~Conditional~~ 2
4. $\neg p$ Hypothesis
5. $\neg p \rightarrow q$ Double Neg 3
6. q Modeus Ponens 4,5

$(p \vee q)$
$\neg\neg(p \vee q)$
$\neg(\neg p \wedge \neg q)$ x
$\neg\neg p \vee \neg\neg q$ DN
$\neg p \rightarrow \neg\neg q$ CI
$\neg p$ Hypo
$\neg\neg q$ MT
q DN

x
1.12.5

c) $(C \wedge H) \rightarrow J$
$\neg J$
$\therefore \neg C$

1. $(C \wedge H) \rightarrow J$ Hypothesis
2. $\neg(C \wedge H) \vee J$ Conditional 1
3. $(\neg C \vee \neg H) \vee J$ DeMorg 2
4. $\neg C \vee (\neg H \vee J)$ Associative 3
5. $\neg C \vee (J \vee \neg H)$ Commutative 4
6. $(\neg C \vee J) \vee \neg H$ Associative 5
7. $C \rightarrow J$ Conditional 6

1.12.5.

D) $(C \wedge H) \to J$

$\neg J$

H

$\therefore \neg C$

1 $(C \wedge H) \to J$ ~~Condit~~ Hypothesis

2 $\neg(C \wedge H) \vee J$ Conditional 1

3 $(\neg C \vee \neg H) \vee J$ DeMorgan 2

4 $\neg C \vee (\neg H \vee J)$ Commutative 3

5 $\neg C \vee (J \vee \neg H)$ Associative 4

6 $(\neg C \vee J) \vee \neg H$ Commutative 5

$\neg J$ ~~Add~~ Hypothesis

H Hypothesis

$\neg C \vee J \vee \neg H \vee \neg J$ Addition 6, 5

$\neg C \vee J \vee \neg H \vee \neg J \vee H$ Addition 7, 8

$\neg C \vee J \vee \neg J \vee \neg H \vee H$ Associative 9

$\neg C \vee T \vee \neg H \vee H$ Complement 10

$\neg C \vee T \vee T$ Complement 11

$\neg C \vee T$ Domination

T Domination

1.12.5 C)

$(C(x) \land H(x)) \rightarrow J(x)$

$\neg J(x)$

$\therefore \neg C(x)$

Invalid

$C(x)$	$H(x)$	$J(x)$	$C(x) \land H(x) \rightarrow J(x)$	$\neg J(x)$	$\neg C(x)$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
Counter EXP → T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

1.12.5 D)

$(C(x) \land H(x)) \rightarrow J(x)$

$\neg J(x)$

$H(x)$

$\therefore \neg C(x)$

$C(x)$	$H(x)$	$J(x)$	$((C(x) \land H(x)) \rightarrow J(x))$	$\neg J(x)$	$\neg C(x)$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

1 $(C(x) \land H(x)) \rightarrow J(x)$ Hypo

2 $(\neg(C(x) \land H(x))) \lor J(x)$ Conditional 1

3 $(\neg C(x) \lor \neg H(x)) \lor J(x)$ Demorgan 2

4 $J(x) \lor (\neg C(x) \lor \neg H(x))$ Commutative 3

5 $\neg J(x)$ Hypothesis

6 $(\neg C(x) \lor \neg H(x))$ Disjunctive Syllogism 4,5

7 $H(x)$ Hypothesis

8 $\neg H(x) \lor \neg C(x)$ Commutative 6

9 $H(x) \rightarrow \neg C(x)$ Conditional Identity 8

10 $\neg C(x)$ Modus Ponens 7, 9

$\exists x\,(P(x) \vee Q(x))$

$\therefore \exists x\, P(x)$

1. $\exists x\,(P(x) \vee Q(x))$ Hypothesis
2. c is a particular element
3.

$(P(a) \vee Q(a)) = 1$

$\neg(Q(a)) = 1$

$P(a) = 1$

When $P(a) \vee Q(a)$ is true, $P(a)$ is false and $Q(a)$ is true, $\neg Q(a)$ is false. However the conclusion $P(a)$ is false, invalidating the argument

$P(a) \vee Q(a)$

1.13.5

D) $M(x)$: Missed class
$D(x)$: Detention

$$\begin{array}{l} \forall x\; S(x) \to D(x) \\ \neg M(\text{Penelope}) \\ \hline \therefore \neg D(\text{Penelope}) \end{array}$$

1. $\forall x\, S(x) \to D(x)$ Hypothesis
2. $\forall x\; \neg S(x) \vee D(x)$ Conditional 1
3.

1.13.5

E) $\forall x\ (M(x) \vee D(x)) \Rightarrow \neg A(x)$

Penelope (P) is Student

A(Penelope)

$\therefore\ \neg D(\text{Penelope})$

1. $\forall x\ (M(x) \vee D(x)) \Rightarrow \neg A(x)$ Hyp
2. Penelope (P) is Student Hyp
3. $(M(P) \vee D(P)) \Rightarrow \neg A(P)$ Uni Inst. 1,2
4. $(\neg(M(P) \vee D(P))) \vee \neg A(P)$ Conditional 3
5. $(\neg M(P) \wedge \neg D(P)) \vee \neg A(P)$ DeMorg 4
6. $\neg A(P) \vee (\neg M(P) \wedge \neg D(P))$ Communative 5
7. $A(P) \Rightarrow (\neg M(P) \wedge \neg D(P))$ Conditional 6
8. $A(P)$ Hyp.
9. $(\neg M(P) \wedge \neg D(P))$ Modus Poneas 8, 7
10. $\neg D(P) \wedge \neg M(P)$ Communative 9
11. $\neg D(P)$ Simplifiantion 10

2.2.1 c) If x is a real number and $x \leq 3$ then
$12 - 7x + x^2 \geq 0$

$12 - 7x + x^2 \geq 0$ $\qquad$ $12 - 7(3) + 3^2 \geq 0$
$x^2 - 7x + 12 \geq 0$ $\qquad$ $12 - 21 + 9 \geq 0$
$(x-4)(x-3)$ $\qquad$ $0 \geq 0$

X Direct Proof: ~~Assume~~ - If x is a real number and $x \leq 3$, assume $x = 3$. Since $x = 3$, ~~the~~ we can plug 3 into each x in the expression $12 - 7x + x^2 \geq 0$.

$$12 - 7(3) + (3)^2 \geq 0$$

When evaluated we get
$0 \geq 0$

Direct Proof: Assume that x is a real number and $x \leq 3$.
Since $12 - 7x + x^2 \geq 0$, we can factor this to get

$$(x-4)(x-3) \geq 0$$

Since, ~~evaluating~~ x is a real number and $x \leq 3$ by definition we can evaluate the factorization as
$x = 3$ or $x = 4$

If $x = 3$ we can plug x into the above expression to get
$(3-4)(3-3) \geq 0$

~~A~~, ✓ which evaluates to $0 \geq 0$

X However $x = 4$ we plugged into the expression yields

2.2.1 D) The product of two odd intergers is an odd interger

$(n)(j) = nj$

• $(2k+1)(2k+1) = 4k^2 + 2k + 2k + 1 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

$(2k+1)(2j+1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$

Direct Proof

* Assume the product of two odd intergers "n" and "j"

If the product of two odd interger is an odd interger let us express two non-consecutive odd intergers as "n" and "j". If by definition n is an odd interger and j is an odd interger let us express them as $(2k+1)$ and $(2j+1)$ where "k" and "j" are integers.

If we evaluate the product of these integers we using the expressions $(2k+1)$ and $(2j+1)$, we get $4kj + 2k + 2j + 1$. We can factor this expression to $2(2kj + k + j) + 1$. We know that k and j are any interger, and the expression above shows we multiply this expression by "2" to get an even interger. We also know that an even number + an odd number is an odd number, and "1" is an odd number, thus producing and odd interger.

2.3.1 D) For every integer n, if n^2-2n+7 is even, then n is odd.

$n = 2k$ $\qquad\qquad$ $2(k^2-2k)+7$

$(2k)^2 - 2(2k)+7$ is odd

$2k^2 - 4k + 7$ $\quad$ +7 is odd $\quad$ k is integer $\quad$ $2k^2-4k$ is even (even · int = even)

Proof by Contrapositive

We assume that n is even, and have n=2k symbolizing an even integer. When "2k" is plugged into the expression n^2-2n+7, we get $2k^2-2(2k)+7$, which factored becomes $2(k^2-2k)+7$. We know that "k" is any integer and that when multiplied by 2, we get an even integer. We then add this integer by "7" an odd integer which ~~prod~~ when summed by the evaluation of "k" gives us an odd number. This odd number is contrapositive to, "n^2-2n+7 is even", validating the therom.

F) For every non-zero real number "x", if "x" is irrational, then 1/x is also irrational

$x = \frac{a}{b}$ $\quad$ $a \neq 0$, $b \neq 0$ $\quad$ $\frac{1}{a/b} = \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \cdot \frac{b}{1} = \frac{b}{a}$

if $x = a/b$ which is rational, and through ~~contrapositive~~ evaluation of

$1/x = \frac{1}{\frac{a}{b}} = b/a = x$

G) For every pair of real numbers x and y if $x^3+xy^2 \leq x^2y+y^3$ then $x \leq y$

$x(x^2+y^2) \leq y(x^2+y^2)$

2.5.1 c) If intergers x and y have the same parity, then x+y is even.

Case 1. x and y are odd

$x+y = (2k+1) + (2k+1) = 4k+2$ even $2(2k+1)$

Case 2. x and y are even

$x+y = (2k) + (2k) = 4k$ even $2(2k)$

Proof By Cases

In Case 1. we assume that x and y have a parity of odd integers. By definition, odd intergers are intergers expressed as $(2k+1)$ where k is any interger. When we substitute x and y with $(2k+1)$, we get the expression $(2k+1)+(2k+1)$, which is evaluated as $4k+2$. We can factor "2" out of $4k+2$, giving us $2(2k+1)$. "$2k+1$" is an odd interger and when multiplied by "2" an even interger, the product is always even.

In case 2. we assume that x and y have a pairity of even integers. By definition even intergers can be expressed as "2k". By substituting ~~x~~ and y with 2k, their sum x+y, is $2k+2k=4k$. We can factor 4k, to give us $2(2k)$. "2k" is an even integer, and when multiplied by another even integer "2" we know the product is also always even.

2.2.1 c)

If x is a real number and $x \leq 3$, then $12-7x+x^2 \geq 0$

$12-7x+x^2 \geq 0$

$(x-3)(x-4) \geq 0$

$x=4$ or $x=3$

$12-7x+x^2 \geq 0$

$x^2-7x+12 \geq 0$
$\quad -12 \quad -12$

$x^2-7x \geq -12$

$x(x-7) \geq -12$

$\sqrt{x^2-7x} \geq \sqrt{-12}$

$x-7x \geq \sqrt{12}$
$+7x \quad +7x$

$x \geq \sqrt{12}+7x$

Let $x=3$

$12-7x+x^2 \geq 0$

$12-7(3)+3^2 \geq 0$

$12-21+9 \geq 0$

$-21+21 \geq 0$

$0 \geq 0$

Proof

Direct Proof

Assume $x=3$

Since, $12-7x+x^2$ is a quadratic, we can factor x to either be $x=4$ or $x=3$. If we assume $x \leq 3$ and insert $x=3$ into the ~~quadrat~~ quadratic we get $0 \geq 0$ which proves true.

2.2.1 DD

The product of two odd intergers is an odd interger.

Assume arbitrary odd interger elements:

~~$a+b=c$~~ ~~$a+a=b$~~ $a+b=c$

Assume a, c are odd intergers, thus can be expressed as $(2k+1)$

$(2k+1)+(2k+1)=(2k+1)$

$4k+2=(2k+1)$

$(2k+1)(2k+1)$

$2k+1+2k+1=c$

$\frac{4k+1}{4}=\frac{c}{4}$

$k+1=\frac{c}{4}-1$
-1

$k=(c/4)-1$

$(2k+1)(2k+1)=c$

$4k^2+2k+2k+1$

$4k^2+4k+1=c$

$4k^2+4k+1=2k+1$

$\frac{(2k+1)(2k+1)}{2k+1}=\frac{2k+1}{2k+1}$

$2k+1=1 \leftarrow$ odd

1.13.5

~~E B) $\forall x\ M(x) \to D(x)$~~

E) $\forall x\ \left(\overset{T/F}{M(x)} \lor \overset{T}{D(x)}\right) \to \neg A(x)^{F}$

Penelope is student   Mx Dx Ax My

A(Penlope) T

∴ $\neg D$(Penelope) F   Valid

1. $\forall x\ (M(x) \lor D(x)) \to \neg A(x)$
2. Penlope is student
                                                    uni Inst 2
3. M(Penlope) $\lor$ D(Penlope) $\to \neg$ A(Penlope)
4. $\neg$(M(Penlope) $\lor$ D(Penlope)) $\lor \neg$ A(Penlope

$(\neg M(P) \land \neg D(P)) \lor \neg A(P)$

$(\neg A(P) \lor \neg M(P)) \land (\neg A(P) \lor \neg D(P))$

$(\neg A(P) \lor \neg D(P)) \land (\neg A(P) \lor \neg M(P))$

$(\neg A(P) \lor \neg D(P))$ simp

2.13.5

D) $\forall x\, M(x) \to D(x)$

$\neg M(Penelope)$

∴ $D(Penelope)$

1	$\forall x\, M(x) \to D(x)$	
2	Penelope is a particular student	
3	$\cancel{\forall x}\ M(Penelope) \to D(Penelope)$	Uni gen 2
4	$\neg M(Penelope)$	
5	$\neg M(Penelope) \lor D(Penelope)$	Condi 3

$\neg M(x)$	$M(x)$	$D(x)$	$M(x) \to D(x)$
F	T	T	T
F	T	F	F
T	F	T	T
T	F	F	T

2.4.1 c) The Average of 3 real numbers is geq to atleast one of the numbers.

Arg.

$(x+y+z)/3 \geq x$ or y or z

Case

I. $(x+y+z)/3 \not\geq x$ or y or z

II.

Proof.

Proof by contradiction. Assume the average of 3 real numbers is not geq to one of the numbers, if one