1.7.1

C) $\forall x\, Q(x) \vee P(3)$

True. $P(3)$ is prime in all instances. Proposition

D) $\exists x\, (Q(x) \wedge P(x))$ $Q(1) = \text{True}.$ $P(1) = 1 = 1^2 = 1$ True

True when variable x is $\underline{1}$. Proposition

E) $\forall x\, (\neg Q(x) \vee P(x))$ $Q(3) = \text{False}$ $\neg(F) = T$ $P(3) = \text{True}$

False for variable x is 4 $Q(4) = \text{True}$ $\neg(T) = F$ $P(4) = \text{False}$

Not Proposition.

1.7.2

C) $\forall x\, (T(x) \wedge E(x))$

D) $\exists x\, (E(x) \wedge \neg T(x))$

1.5.2

b) $p \wedge (\neg p \rightarrow q) \equiv p$

$p \wedge (\neg p \rightarrow q)$

$p \wedge (\neg\neg p \vee q)$ Conditional

$p \wedge (p \vee q)$ Double Neg.

$(p \wedge p) \vee (p \wedge q)$

$T \vee (p \wedge q)$

$(p \wedge q)$ x

$p \wedge (\neg p \rightarrow q)$

$p \wedge (\neg\neg p \vee q)$

$(p \wedge \neg\neg p) \vee (p \wedge q)$

$(p \wedge p) \vee (p \wedge q)$

$T \vee (p \wedge q)$

$(p \wedge q)$ x

$p \wedge (\neg p \rightarrow q)$

$p \wedge (\neg\neg p \vee q)$ Cond.

$p \wedge (p \vee q)$ Double Neg

p Absorb.

1.5.2.

C) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \wedge (p \to r)$
$(\neg p \vee q) \wedge (p \to r)$ Cond.
$(\neg p \vee q) \wedge (\neg p \vee r)$ Cond.
$\neg p \vee (q \wedge r)$ Distrib.
$p \to (q \wedge r)$ Condit.

D) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$
$\neg\neg p \vee (q \to r)$ Cond.
$\neg\neg p \vee (\neg q \vee r)$ Cond.
$p \vee (\neg q \vee r)$ Double Neg.
$\neg q \vee (p \vee r)$ Commut.
$q \to (p \vee r)$ Conditi.

E) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \to r) \vee (q \to r)$
$(\neg p \vee r) \vee (q \to r)$ Cond.
$(\neg p \vee r) \vee (\neg q \vee r)$ Cond.
$(\neg p \vee \neg q) \vee (r \vee r)$ Assoc.
$(\neg p \vee \neg q) \vee r$ Idempotent
$\neg (p \wedge q) \vee r$ DeMorgan
$(p \wedge q) \to r$ Cond.

1.5.2

G) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \equiv p \wedge \neg r$

$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$

$p \wedge ((q \wedge \neg r) \vee (\neg q \wedge \neg r))$ Distrib.

$p \wedge (\neg r \wedge (q \vee \neg q)$ Distrib.

$p \wedge (\neg r \wedge T)$ Complement

$p \wedge \neg r$ Identity

H) $p \leftrightarrow (p \wedge r) \equiv \neg p \vee r$

$p \leftrightarrow (p \wedge r)$

$(p \rightarrow (p \wedge r)) \wedge ((p \wedge r) \rightarrow p)$ Cond.

$(\neg p \vee (p \wedge r)) \wedge ((p \wedge r) \rightarrow p)$ Cond.

$(\neg p \vee (p \wedge r)) \wedge (\neg(p \wedge r) \vee p)$ Cond.

$(\neg p \vee (p \wedge r)) \wedge (p \vee \neg(p \wedge r))$ Commutative

$(\neg p \vee (p \wedge r)) \wedge (p \vee (\neg p \vee \neg r))$ DeMorg.

$((\neg p \vee p) \wedge (\neg p \vee r)) \wedge (p \vee (\neg p \vee \neg r)$ Distrib.

$((T) \wedge (\neg p \vee r)) \wedge (p \vee (\neg p \vee \neg r))$ Complement

$(T \wedge (\neg p \vee r)) \wedge ((p \vee \neg p) \vee \neg r)$ Associative

$(T \wedge (\neg p \vee r)) \wedge ((T) \vee \neg r)$ Complement

$(\neg p \vee r) \wedge (T \vee \neg r)$ Identity

$(\neg p \vee r) \wedge T$ Dommination

$(\neg p \vee r)$ Identity

1.12.2

A) $p \to q$
$q \to r$
$\neg r$

$\therefore \neg p$

1. $p \to q$ Hypothesis
2. $\neg p \vee q$ Conditional Identity, 1
3. $q \to r$ Hypothesis
4. $\neg q \vee r$ Conditional Identity, 3
5. $\neg r$ Hypothesis
6. ~~X~~

X

1. $q \to r$ Hypothesis
2. $\neg r$ Hypothesis
3. $\neg q$ Modus Tollens 1, 2
4. $p \to q$ Hypothesis
5. $\neg p$ Modus Tollens 3, 4

B) $p \to (q \wedge r)$
$\neg q$

$\therefore \neg p$

1. $p \to (q \wedge r)$ Hypothesis
2. $\neg p \vee (q \wedge r)$ Conditional 1
3. $\neg q$ Hypothesis
4. $(\neg p \vee q) \wedge (\neg p \vee r)$ Distrib 2
5. X

1. $p \to (q \wedge r)$ Hypothesis
2. $\neg p \vee (q \wedge r)$

D) $(p \vee q) \to r$
p
$\therefore r$

1. $(p \vee q) \to r$ Hypo.
2. $\neg(p \vee q) \vee r$ Cond.
3. $(\neg p \wedge \neg q) \vee r$ DeMorg
4. $(r \vee \neg p) \wedge (r \vee \neg q)$
5. p Hypo.
6.

B) $p \to (q \wedge r)$
$\neg q$
$\therefore \neg p$

✓

1. $p \to (q \wedge r)$ Hypothesis
2. $\neg p \vee (q \wedge r)$ Conditional 1
3. $(\neg p \vee q) \wedge (\neg p \vee r)$ Distributive 2
4 $(\neg p \vee q)$ Simp 3
5 $(\neg p \vee r)$ Simp 3
6. $\neg q$ Hypo.
7. $\neg p$ Dysjunctive 4 & 6.

E) $p \vee q$
$\neg p \vee r$
$\neg q$
$\therefore r$

✓

1. $p \vee q$ Hypo.
2. $\neg q$ Hypo.
3. $q \vee p$ Comm. 1
4 p Disjunct. 3, 4
5. $\neg p \vee r$ Hypo.
6. $p \to r$ Conditional 5
7 r Modus Ponens 4,6

D) $(p \vee q) \to r$
p
$\therefore r$

✓

1 $(p \vee q) \to r$ Hypo
2. $\neg(p \vee q) \vee r$ Condit. 1
3 $(\neg p \wedge \neg q) \vee r$ DeMorg. 2
4 $(r \vee \neg p) \wedge (r \vee \neg q)$ Distrib. 3
5 $(r \vee \neg p)$ Simp 4
6 $(r \vee \neg q)$ Simp 4
7 p Hypothesis
8 $\neg p \vee r$ Commutative 5
9 $p \to r$ Conditional 8
9 r Modus Ponens 7, 9

F) $p \to q$
$r \to u$
$p \wedge r$
$\therefore q \wedge u$

✓

1. $p \wedge r$ Hypothesis.
2. p Simp 1
3. r Simp 1
4. $r \to u$ Hypo.
5. u Modus Ponens 3, 4
6. $p \to q$ Hypothesis
7 q Modus Ponens 2,6
8 $q \wedge u$ Conjunction 7,5

Review

$A = \{1, 2, 3, \{1,2,3\}, \{2,3\}\}$

1.a $\{2,3\} \in A = T$

1.b $\{2,3\} \subset A = T$

1.c $\{\{1,2,3\}\} \in A = F$

1.d $\{\{1,2,3\}\} \subset A = T$

1.13.1

c) The domain U: Set of all paintings

~~$\forall x$ $B(x)$~~

~~$\exists x$~~

$$\begin{array}{l} \forall x\ (M(x) \rightarrow B(x)) \\ \exists x\ (S(x) \wedge M(x)) \\ \hline \exists x\ (S(x) \wedge B(x)) \end{array}$$

If its included in the arguement, and is a hypothesis of that argument the element is <u>Praticular</u>.

If its not included in the arguement, and is deduced by the rules of inference, the element would be either <u>Praticular</u> or <u>Arbitory</u>

We can always State an Arbitory element in an arguement

2.2.1 Direct Proofs

c) If r and s are rational numbers, then the product of r and s is a rational number.

Proof. Direct Proof
Assume r and s are rational numbers. Rational numbers by definition is $\frac{a}{b}$: $a \neq 0$ & $b \neq 0$. We can then have $r = \frac{a}{b}$ and $s = \frac{a}{b}$. $r \cdot s = \frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2} = \frac{a}{b}(\frac{a}{b})$. When we find the product of r and s using this definition, the product can be factored to have $\frac{a}{b}$ which is a rational number concluding the theory

2.2.2 Direct Proof or Counter Example

B) If $x+y$ is an even integer. x and y are both even.

False. Counter example : $1+3=4$

2.3.1 Contrapositive Proofs

L) For every pair of $\mathbb{R}$, x and y if $x+y > 20$, the $x>10$ & $y>10$

Proof. Contrapositive Proof.
Assume $x \leq 10$ and $y \leq 10$, we will prove $x+y \leq 20$. If $x \leq 10$, we can have $x=10$. If $y \leq 10$ we can have $y=10$. When we add $x+y=10+10$ we get 20, which satisfies $x+y \leq 20 = 20 \leq 20$. thus concluding the proof.

2.4.1 Proofs by Contradiction.

F) There is no largest rational negative number.

Proof. Proof by contradiction

Assume there is a largest rational negative number defined by r. We can then multiply r by 2 to get $2r$. $2r > r$ which contradicts the ~~argument~~ assumption statement, proving that there is no largest rational negative number.