Homework 3

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Question 1

Refer to C++ File

Question 2

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Question 3

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Question 4

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Question 5

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Question 6

Refer to C++ File

Question 7

7.1 Exercise 3.1.1

a) True.
b) False.
c) True.
d) False.
e) True.
f) False.
g) False.

7.2 Exercise 3.1.2

a) False.
b) True.
c) True.
d) True.
e) False.

7.3 Exercise 3.1.5

b) $\{3, 6, 9, 12, ...\} = \{x \in \mathbb{N} : x \geq 3$ and x is an integer multiple of 3$\}$

The cardinality is infinite.

d) $\{0, 10, 20, 30, \ldots, 1000\} = \{x \in \mathbb{N} : 0 \leq x \leq 1000$ and x is an integer multiple of 10$\}$

The cardinality is 101.

7.4 Exercise 3.2.1

a) True.
b) True.
c) False.
d) False.
e) True.
f) True.
g) True.
h) False.
i) False.
j) False.
k) False.

Question 8

8.1 Exercise 3.2.4

b) $\{X \in P(A) : 2 \in X\} = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

Question 9

9.1 Exercise 3.3.1

c) $A \cap C = \{-3, 1, 17\}$
d) $A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$
e) $A \cap B \cap C = \{1\}$

9.2 Exercise 3.3.3

a) $\bigcap_i^5 = 2 = A\{i\} = \{1\}$
b) $\bigcup_i^5 = 2 = A\{i\} = \{1, 2, 3, 4, 9, 16, 25\}$
e) $\bigcap_i^5 = 1 = C\{i\} = \{X \in \mathbb{R} : -1/100 \leq X \leq 1/100\}$
f) $\bigcup_i^5 = 1 = C\{i\} = \{X \in \mathbb{R} : -1 \leq X \leq 1\}$

9.3 Exercise 3.3.4

b) $P(A \cap B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$
d) $P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10

10.1 Exercise 3.5.1

b) $BxAxC$ = { foam, tall, whole }
c) { (foam,non-fat), (foam,whole), (no-foam, non-fat), (no-foam, whole) },

10.2 Exercise 3.5.3

b)True. $\mathbb{Z}^2 \subseteq R^2$
c)False $\mathbb{Z}^2 \cap Z^3 \neq \emptyset$
e)True. For any three sets, A, B, and C, if $A \subseteq B$, then $AxC \subseteq BxC$

10.3 Exercise 3.5.6

d) xy where $\in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2$
Answer: xy = $\{01, 011, 001, 0011\}$

e) xy where $\in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2$
Answer: xy = $\{aaa, aaaa, aab, aaab\}$

10.4 Exercise 3.5.7

f) $P(AxB) = \{\emptyset, \{ab\}.\{ac\}, \{ab, ac\}\}$

g) $P(A)xP(B) = \{(\{\emptyset\}, \{\emptyset\}), (\{\emptyset\}, \{b, c\}), (\{\emptyset\}, \{b\}), (\{\emptyset\}, \{c\}), (\{a\}, \{\emptyset\}), (\{a\}, \{b, c\}), (\{a\}, \{b\}), (\{a\}, \{c\})\}$

Question 11

11.1 Exercise 3.6.2

b) $(B \cup A) \cup (\overline{B} \cup A) = A$

Set Identity Table	
Expressions	Set Identities
$(B \cup A) \cup (\overline{B} \cup A)$	
$A \cup (B \cap \overline{B})$	Distributive Laws
$A \cup \emptyset$	Complement Laws
A	Identity Law

c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

Set Identity Table	
Expressions	Set Identities
$\overline{A \cap \overline{B}}$	
$\overline{A} \cup \overline{\overline{B}}$	DeMorgan's Laws
$\overline{A} \cup B$	Double Complement Law

11.2 Exercise 3.6.3

b) $A - (B \cap A) = A$

Answer: If $A = \{1\}$ and $B = \{1, 2\}$ by taking the difference of A away from the intersection of B and A, we are removing the A ={1}, leaving only elements found in B = {2}.

d) $(B - A) \cup A = A$

Answer: If $B = \{1, 2\}$ and $A = \{1\}$ by taking the difference of B from A, you are left with $B - A = \{2\}$. This is then unionized with A to get a set {1,2} which is not equal to A.

11.3 Exercise 3.6.4

b) $A \cap (B - A) = \emptyset$

Set Identity Table	
Expressions	Set Identities
$A \cap (B - A)$	
$A \cap (B \cap \overline{A})$	Subtraction Law
$A \cap (\overline{A}) \cap B)$	Commutative Law
$(A \cap \overline{A}) \cap B$	Associative Law
$\emptyset \cap B$	Complement Law
$\emptyset$	Domination Law

c) $A \cup (B - A) = A \cup B$

Set Identity Table	
Expressions	Set Identities
$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	Subtraction Law
$(A \cup B) \cap (A \cup \overline{A})$	Distributive Law
$(A \cup B) \cap U$	Complement Law
$A \cup B$	Identity Law