Discrete Mathematics: Logical Equivalences

Let p, q, r be statement variables
Let t be a tautology
Let c be a contradiction

1. Identity Laws: $p \wedge t \equiv p$; $p \vee c \equiv p$
2. Domination laws: $p \vee t \equiv p$; $p \wedge c \equiv c$
3. Negation Laws: $p \vee \neg p \equiv t$; $p \wedge \neg p \equiv c$
4. Negations of t and c: $\neg t \equiv c$; $\neg c \equiv t$
5. Commutative Laws: $p \wedge q \equiv q \wedge p$; $p \vee q \equiv q \vee p$
6. Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$; $(p \vee q) \vee r \equiv p \vee (q \vee r)$
7. Distributive laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$;
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

8. Idempotent Laws: $p \wedge p \equiv p$; $p \vee p \equiv p$
9. Double-Negation Law: $\neg(\neg p) \equiv p$
10. De Morgan's Laws: $\neg(p \wedge q) \equiv \neg p \vee \neg q$; $\neg(p \vee q) \equiv \neg p \wedge \neg q$
11. Absorption Laws: $p \vee (p \wedge q) \equiv p$; $p \wedge (p \vee q) \equiv p$

Example. Use the table of Logical Equivalences to Show
$(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$

Start with left-hand side:

$(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv (\neg q \vee p) \wedge (\neg q \vee \neg p)$ by Commutative Laws
$\equiv (\neg q) \vee (p \wedge \neg p)$ by Distributive Laws
$\equiv (\neg q) \vee (c)$ by Negation Laws $(p \wedge \neg p \equiv c)$
$\equiv \neg q$ by Identity Law $(p \vee c \equiv p)$

Logical Equivalence with Conditionals and Bi Conditionals

Contrapositive of $p \to q$: $p \to q \equiv \neg q \to \neg p$

Example : "If a bagel cost \$2, then a bagel cost more than \$1"
Contrapositive: "If a bagel DOES NOT cost more than \$1, then a bagel does NOT cost \$2."

If the conditional statement is true, so is the contrapositive statement furthermore, if the contrapositive statement is true (T) so is the conditional statement.

$p \to q \equiv \neg q \to \neg p$

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Biconditional Symbol ($\leftrightarrow$) is made up of ($\leftarrow$) and ($\rightarrow$)

Consider: $p \leftrightarrow q \equiv p \to q$ and $q \to p$

Example: "A bagel cost \$2 if and only if a bagel costs 8 quarters."
"If a bagel cost \$2 then a bagel cost 8 quarters"
and
"If a bagel cost 8 quarters then a bagel cost \$2."

If the Biconditional statement is true, then each derived statements are true.