The Left-Truncated Normal Distribution

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Abstract

For performance assessment of scientific software, running times can be modeled as a left-truncated normal distribution.

This document synopsises the mathematical background of the left-truncated normal distribution to better understand the associated *R* file, `ltnorm.R`.

1 Background: Standard Normal Distribution

1.1 PDF

The probability density function, $\phi(\xi)$ of the standard normal distribution with mean μ and standard deviation σ is

$$\phi(\xi) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2}\right) \tag{1}$$

... where

$$\xi = \frac{x - \mu}{\sigma} \tag{2}$$

1.2 CDF

The cumulative distribution function of the standard normal distribution is:

$$\Phi(\xi) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{\xi}{\sqrt{2}}\right)\right) \tag{3}$$

... where ξ is as in (2) and the *error function* defined as usual:

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} \exp(-t^2)\,\mathrm{d}t$$

2 Left-Truncated Normal Distribution

Let X be a normally distributed random variable with mean μ and standard deviation σ, and let its distribution be left-truncated such that $X > a$.

2.1 Expected Value

If

$$\alpha = \frac{a - \mu}{\sigma} \tag{4}$$

and

$$\lambda(\alpha) = \frac{\phi(\alpha)}{1 - \Phi(\alpha)} \tag{5}$$

... with ϕ and Φ as in (1) and (3) respectively, then the expected value is given by:

$$\mathrm{E}(X|X > a) = \mu + \sigma\lambda(\alpha)$$

2.2 Variance

If

$$\delta(\alpha) = \lambda(\alpha)(\lambda(\alpha) - \alpha) \tag{6}$$

... then the **variance** is:

$$\mathrm{Var}(X|X > a) = \sigma^2(1 - \delta(\alpha))$$

2.3 PDF

The probability density function, recalling that ξ (2) is a function of x, is:

$$f(x) = \frac{\phi(\xi)}{\sigma(1 - \Phi(\alpha))}$$

2.4 CDF

The cumulative distribution function,, recalling that ξ (2) is a function of x, is:

$$F(x) = \frac{\Phi(\xi) - \Phi(\alpha)}{1 - \Phi(\alpha)}$$

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