Bayesian Latent Class Approach for Causal Estimation with Multiple Cognitive Outcomes

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Background

- Estimating causal effects with multiple outcomes in the presence of unobserved latent subgroups is challenging; treatment effects may differ by latent class, but class membership is unobserved[1,2].
- **Advantages** of Bayesian joint modeling include:
  - ✓ general framework with straightforward uncertainty quantification and sensitivity analysis;
  - ✓ an interpretable way to handle multiple outcomes in causal inference;
  - ✓ extends easily to handle missing data.
- Most latent-class causal methods are **two-step frequentist**: classes first, effects second, complicating sensitivity analysis for unmeasured confounding and class-number selection[1].

Objectives

Extending causal latent-class approaches

- Propose a Bayesian joint latent-class model **(Bayesian Joint)** beyond a two-step frequentist causal latent-class model **(Two-step Freq)**[1,3].
- Compare model performance under different simulation scenarios.

Causal Latent Class Framework

1. NOTATIONS

- n subjects indexed by i, $i = 1, \dots, n$, with a single time-point outcome measurement.
- X_i and A_i are random variables representing the baseline confounder vector and exposure for subject i. Y_{ip} denotes the p cognitive outcomes for subject i observed at the end of the study.
- C_i is introduced as the latent class membership for subject i, capturing heterogeneity in the cognitive population with K latent classes.

2. DAG

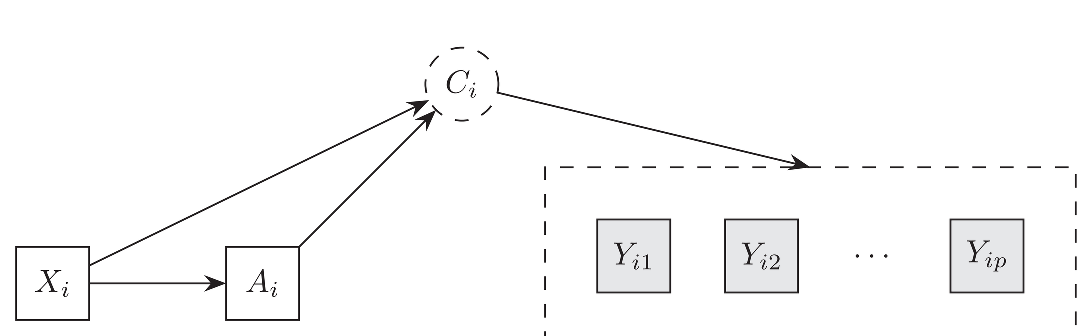

Figure 1. Causal latent-class framework among baseline confounders, treatment, and multiple outcomes.

3. ASSUMPTIONS

- **SUTVA and Positivity**.
- **Latent unconfoundedness:** $A_i \perp\!\!\!\perp C_i^a \mid X_i$.
- **Conditional independence:** Given C_i, outcomes are independent, $P(Y_{i1}, \dots, Y_{ip} \mid C_i) = \prod_{m=1}^{p} P(Y_{im} \mid C_i)$.

4. CAUSAL PARAMETER

- For each class $c = 1, \dots, K$, define the potential indicator $Z_i^a(c) = \mathbf{1}\{C_i^a = c\}$ with average potential class probability $E[Z_i^a(c)]$.
- **Causal estimand:** Average treatment effect on class probabilities, $\text{ATE}_c = E[Z_i^1(c)] - E[Z_i^0(c)]$.

5. PARAMETERIZATION

- Outcome model, For $m = 1, \dots, p$, $P(Y_{im} \mid C_i = c, \theta)$.
- Latent class membership model, $P(C_i = c \mid A_i, X_i, \alpha)$.
- Baseline exposure assignment model, $P(A_i \mid X_i, \gamma)$.
- Covariates model, $P(X_i \mid \beta)$.
- $\Lambda = \{\theta, \alpha, \beta, \gamma\}$.

Simulation Study

1. SIMULATION RESULTS

Conf.	Sep.	Imb.	Bayesian Joint: Coverage (MSE) C1	C2	C3	C4	Two-step Freq: Coverage (MSE) C1	C2	C3	C4
weak	low	balanced	0.94 (0.001812)	0.98 (0.009183)	0.96 (0.001673)	0.94 (0.001364)	0.95 (0.001969)	0.96 (0.001697)	0.98 (0.002119)	0.98 (0.001187)
		one_rare	0.94 (0.002300)	0.95 (0.001797)	0.94 (0.001653)	0.97 (0.000716)	0.96 (0.001836)	0.98 (0.001710)	0.97 (0.002019)	0.98 (0.000664)
		two_rare	0.92 (0.002188)	0.91 (0.002427)	0.94 (0.000990)	0.96 (0.000766)	0.95 (0.001695)	0.97 (0.002760)	1.00 (0.002022)	0.95 (0.000756)
		severe_one	0.98 (0.001483)	0.98 (0.001694)	0.95 (0.001579)	0.98 (0.000308)	0.99 (0.002228)	0.98 (0.002400)	0.97 (0.002415)	0.99 (0.001670)
	high	balanced	0.93 (0.002672)	0.99 (0.001450)	1.00 (0.001456)	0.93 (0.003063)	0.95 (0.001128)	0.94 (0.001083)	0.99 (0.000806)	0.94 (0.000926)
		one_rare	1.00 (0.002768)	0.97 (0.001638)	0.96 (0.001546)	1.00 (0.001657)	0.95 (0.001056)	0.93 (0.001022)	0.92 (0.001492)	0.90 (0.000550)
		two_rare	0.94 (0.002867)	0.87 (0.002454)	0.85 (0.000652)	0.94 (0.001282)	0.91 (0.001339)	0.89 (0.001864)	0.89 (0.000632)	0.89 (0.000603)
		severe_one	0.95 (0.002489)	0.94 (0.002539)	0.96 (0.001399)	0.99 (0.000585)	0.88 (0.001782)	0.92 (0.001260)	0.92 (0.001433)	0.91 (0.000373)
strong	low	balanced	0.95 (0.001709)	0.98 (0.001460)	0.97 (0.001495)	0.94 (0.001942)	0.95 (0.001946)	0.98 (0.002390)	1.00 (0.002384)	0.95 (0.001803)
		one_rare	0.92 (0.002315)	0.98 (0.001464)	0.97 (0.001924)	0.92 (0.001342)	0.98 (0.002334)	0.97 (0.002261)	0.99 (0.003310)	0.95 (0.001412)
		two_rare	0.94 (0.002384)	0.99 (0.001697)	0.96 (0.000897)	0.90 (0.001276)	0.97 (0.002406)	0.98 (0.004481)	0.99 (0.004780)	0.96 (0.001458)
		severe_one	0.98 (0.002024)	0.97 (0.002413)	0.97 (0.001823)	0.97 (0.000513)	0.97 (0.003661)	0.99 (0.002999)	0.98 (0.002789)	0.97 (0.002514)
	high	balanced	0.94 (0.002391)	1.00 (0.001190)	0.99 (0.012450)	0.94 (0.002659)	0.94 (0.001424)	0.93 (0.001470)	0.89 (0.001484)	0.96 (0.001169)
		one_rare	0.96 (0.003185)	0.98 (0.001328)	0.99 (0.001103)	0.99 (0.001673)	0.97 (0.001379)	0.97 (0.001141)	0.93 (0.001523)	0.98 (0.000626)
		two_rare	0.97 (0.003054)	0.93 (0.002021)	0.94 (0.000541)	0.98 (0.001104)	0.95 (0.001656)	0.93 (0.002065)	0.94 (0.000694)	0.93 (0.000706)
		severe_one	0.96 (0.002953)	0.95 (0.002844)	0.98 (0.001020)	1.00 (0.000683)	0.95 (0.002230)	0.92 (0.001609)	0.95 (0.001223)	0.88 (0.000323)

2. SIMULATED DATASET

- For each simulation scenario, 100 datasets were generated with $n = 800$ and $K = 4$. Coverage probability and MSE were computed to assess model performance.
- Uncertainty for the Two-step Freq model was estimated using the bootstrap.
- Data-generating parameters were optimized to achieve the desired simulation scenarios.
- $A \mid X \sim \text{Bernoulli}(\pi), \quad \text{logit}(\pi_i) = \gamma_0 + \gamma^\top X_i$.
- $C_i \mid A_i, X_i \sim \text{Cat}(\pi_{i1}, \dots, \pi_{iK})$, $\log(\frac{\pi_{ik}}{\pi_{i1}}) = \alpha_{0k} + \alpha_{A,k} A_i + \alpha_{X,k}^\top X_i$.
- $Y_{im} \mid C_i = c \sim \mathcal{N}(\theta_{cm}, \sigma^2)$.
- Imb.: Class imbalance; one/two rare – one or two small clusters; severe-one – one cluster <5% of subjects.
- C1–C4: Latent cognitive subgroups.
- Conf.: Confounding strength.
- Sep.: Class separation.

Methodology (Extensions)

1. Bayesian Causal Latent Class Model (Bayesian Joint)

- Joint likelihood:

$$\prod_{i=1}^{n} P(o_i \mid \Lambda) = \prod_{i=1}^{n} \sum_{c=1}^{K} P(Y_{i1}, \dots, Y_{ip}, A_i, X_i, C_i{=}c \mid \Lambda)$$

$$\prod_{i=1}^{n} \sum_{c=1}^{K} P(C_i \mid A_i, X_i, \alpha) P(Y_i \mid C_i, \theta)\, P(A_i \mid X_i, \gamma)\, P(X_i \mid \beta)$$

- Specify prior $P_0(\Lambda)$ for model parameters; estimate average potential outcomes using posterior predictive inference.

$$E[Z_i^a(c) \mid o_{1:n}] = \int_\Lambda E[Z_i^a(c) \mid \Lambda] P(\Lambda \mid o_{1:n}) d\Lambda$$
$$= \int_\Lambda \int_X P(C_i \mid A_i, X^*, \alpha) P(X^* \mid \beta) P(\Lambda \mid o_{1:n})\, dx\, d\Lambda,$$

where X^* refers to the synthetic covariates generated under the posterior draw of β.

☆ **Compute the distribution of average potential outcomes using Monte Carlo simulation,**

Step 1. **Given $\Lambda^{(s)}$, draw synthetic covariates $x^{(s,r)} \sim P(\cdot \mid \beta^{(s)})$, $r = 1, \dots, M$, and calculate $Z_r^{a,s}(c) = \Pr(C = c \mid A = a, x^{(s,r)}, \alpha^{(s)})$.**

Step 2. **Compute the average potential outcome by averaging over synthetic covariates and posterior draws $\frac{1}{S}\sum_{s=1}^{S} \frac{1}{M}\sum_{r=1}^{M} \Pr(C = c \mid A = a, x^{(s,r)}, \alpha^{(s)})$.**

2. Frequentist Causal Latent-Class Model (Two-step Freq)

- **Adapted from *Causal Inference in Latent Class Analysis*[1], serves as the baseline:**

Step 1. Perform multiple imputation for missing data.

Step 2. Estimate propensity scores and compute weights.

Step 3. Fit latent class analysis using weighted outcomes, with treatment as a predictor.

Step 4. Estimate causal effects based on logistic regression coefficients.

Conclusion & Future Works

- **The Bayesian Joint model showed slightly better performance than the Two-step Frequentist model in the simulation study in unbalanced class scenarios.**
- The Bayesian Joint model offers a more flexible structure, which is advantageous for real-world data applications.
- **Future work will extend the framework to longitudinal outcomes and nonparametric covariate models.**

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