

Due: Monday 4/22/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

Self assessment due: Monday 4/29/2019 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope – be sure to delete any extra title pages that Gradescope attaches). For each subpart where your original answer was correct, write “correct.” Otherwise, write and explain the correct answer. **Do not leave any boxes empty.**

If you did not submit the homework (or skipped some questions) but wish to receive credit for the self-assessment, we ask that you first complete the homework without looking at the solutions, and then perform the self assessment afterwards.

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

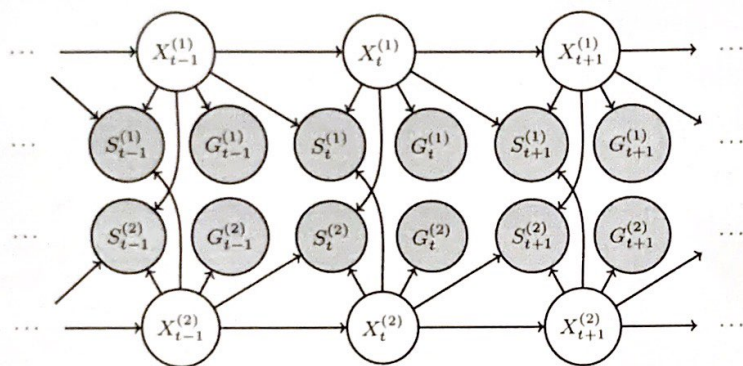
Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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Collaborators	

Q1. Particle Filtering: Where are the Two Cars?

As before, we are trying to estimate the location of cars in a city, but now, we model two cars jointly, i.e. car i for $i \in \{1, 2\}$. The modified HMM model is as follows:

- $X^{(i)}$ – the location of car i
- $S^{(i)}$ – the noisy location of the car i from the signal strength at a nearby cell phone tower
- $G^{(i)}$ – the noisy location of car i from GPS



d	$D(d)$	$E_L(d)$	$E_N(d)$	$E_G(d)$
-4	0.05	0	0.02	0
-3	0.10	0	0.04	0.03
-2	0.25	0.05	0.09	0.07
-1	0.10	0.10	0.20	0.15
0	0	0.70	0.30	0.50
1	0.10	0.10	0.20	0.15
2	0.25	0.05	0.09	0.07
3	0.10	0	0.04	0.03
4	0.05	0	0.02	0

The signal strength from one car gets noisier if the other car is at the same location. Thus, the observation $S_t^{(i)}$ also depends on the current state of the other car $X_t^{(j)}$, $j \neq i$.

The transition is modeled using a drift model D , the GPS observation $G_t^{(i)}$ using the error model E_G , and the observation $S_t^{(i)}$ using one of the error models E_L or E_N , depending on the car's speed and the relative location of both cars. These drift and error models are in the table above. **The transition and observation models are:**

$$\begin{aligned}
 P(X_t^{(i)} | X_{t-1}^{(i)}) &= D(X_t^{(i)} - X_{t-1}^{(i)}) \\
 P(S_t^{(i)} | X_{t-1}^{(i)}, X_t^{(i)}, X_t^{(j)}) &= \begin{cases} E_N(X_t^{(i)} - S_t^{(i)}), & \text{if } |X_t^{(i)} - X_{t-1}^{(i)}| \geq 2 \text{ or } X_t^{(i)} = X_t^{(j)} \\ E_L(X_t^{(i)} - S_t^{(i)}), & \text{otherwise} \end{cases} \\
 P(G_t^{(i)} | X_t^{(i)}) &= E_G(X_t^{(i)} - G_t^{(i)}).
 \end{aligned}$$

Throughout this problem you may give answers either as unevaluated numeric expressions (e.g. $0.1 \cdot 0.5$) or as numeric values (e.g. 0.05). The questions are decoupled.

(a) Assume that at $t = 3$, we have the single particle $(X_3^{(1)} = -1, X_3^{(2)} = 2)$.

(i) What is the probability that this particle becomes $(X_4^{(1)} = -3, X_4^{(2)} = 3)$ after passing it through the dynamics model?

$$\begin{aligned}
 &D(-2) \cdot D(1) \\
 &= 0.25 \times 0.10
 \end{aligned}$$

Answer: 0.025

(ii) Assume that there are no sensor readings at $t = 4$. What is the joint probability that the *original* single particle (from $t = 3$) becomes $(X_4^{(1)} = -3, X_4^{(2)} = 3)$ and then becomes $(X_5^{(1)} = -4, X_5^{(2)} = 4)$?

$$0.025 \times 0.1 \times 0.1$$

Answer: 0.00025

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

Correct

For the remaining of this problem, we will be using 2 particles at each time step.

(b) At $t = 6$, we have particles $[(X_6^{(1)} = 3, X_6^{(2)} = 0), (X_6^{(1)} = 3, X_6^{(2)} = 5)]$. Suppose that after weighting, resampling, and transitioning from $t = 6$ to $t = 7$, the particles become $[(X_7^{(1)} = 2, X_7^{(2)} = 2), (X_7^{(1)} = 4, X_7^{(2)} = 1)]$.

(i) At $t = 7$, you get the observations $S_7^{(1)} = 2, G_7^{(1)} = 2, S_7^{(2)} = 2, G_7^{(2)} = 2$. What is the weight of each particle?

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	0.0225
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	0.00105

$P(S_7^1 = 2 | X_6^1 = 0, X_7^1 = 2, X_7^1 = 2) \cdot$
 $P(G_7^1 = 2 | X_7^1 = 2)$
 $P(G_7^1 = 2 | X_6^1 = 3, X_7^1 = 4, X_7^1 = 1) \cdot$
 $P(G_7^1 = 2 | X_7^1 = 4)$

(ii) Suppose both cars' cell phones died so you only get the observations $G_7^{(1)} = 2, G_7^{(2)} = 2$. What is the weight of each particle?

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	0.25
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	0.0105

$P(G_7^1 = 2 | X_7^1 = 4) \cdot P(G_7^2 = 2 | X_7^2 = 1)$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

Correct

(c) To decouple this question, assume that you got the following weights for the two particles.

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	0.09
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	0.01

What is the belief for the location of car 1 and car 2 at $t = 7$?

Location	$P(X_7^{(1)})$	$P(X_7^{(2)})$
$X_7^{(i)} = 1$	0	0.1
$X_7^{(i)} = 2$	0.9	0.9
$X_7^{(i)} = 4$	0.1	0

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Correct

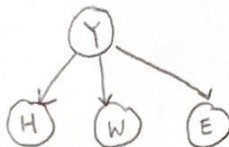
Q2. Naive Bayes

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

Your friend claims that he can write an effective Naive Bayes spam detector with only three features: the hour of the day that the email was received ($H \in \{1, 2, \dots, 24\}$), whether it contains the word 'viagra' ($W \in \{\text{yes}, \text{no}\}$), and whether the email address of the sender is Known in his address book, Seen before in his inbox, or Unseen before ($E \in \{K, S, U\}$).

(a) Flesh out the following information about this Bayes net:

Graph structure:



Parameters: H, W, E

Size of the set of parameters: $24 \times 2 \times 3 = 144$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

For Parameters, needs to include a variable for defining ham/spam

Size of the set of parameters: $1 + 23 \times 2 + 2 + 2 \times 2$

Suppose now that you labeled three of the emails in your mailbox to test this idea:

$$P(X=1) = \frac{1+1}{\text{total } 1+1}$$

spam or ham?	H	W	E
spam	3	yes	S
ham	14	no	K
ham	15	no	K

(b) Use the three instances to estimate the maximum likelihood parameters.

$$P_{ML}(W=\text{yes}) = \frac{1}{3} \quad P_{ML}(E=S) = \frac{1}{3} \quad P_{ML}(E=U) = 0$$

$$P_{ML}(W=\text{no}) = \frac{2}{3} \quad P_{ML}(E=K) = \frac{2}{3}$$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

spam = 1/3, H3,spam = 1, H14,ham = 1/2, H15, ham = 1/2, W,spam=1, E,S,spam = 1, E,K,ham = 1

(c) Using the maximum likelihood parameters, find the predicted class of a new datapoint with $H = 3$, $W = \text{no}$, $E = U$.

$$\frac{1}{3} \times \frac{2}{3} \times 0 = 0$$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

Correct - since $E=U$ is never observed, it has zero likelihood
No prediction could be made

- (d) Now use the three to estimate the parameters using Laplace smoothing and $k = 2$. Do not forget to smooth both the class prior parameters and the feature values parameters.

$$P_{ML}(W=Yes) = \frac{3}{7} \quad P_{ML}(E=S) = \frac{3}{7}$$

$$P_{ML}(W=No) = \frac{4}{7} \quad P_{ML}(E=L) = \frac{4}{7}$$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

The Laplace smoothed estimate for a categorical variable X with parameters $\theta_1, \dots, \theta_d$ for the $\{1, \dots, d\}$ values of X is $\theta_i = x_i + k / N + kd$ where x_i is the number of times value i is observed, N is the total number of observations, and d is the number of values of X .

- (e) Using the parameters obtained with Laplace smoothing, find the predicted class of a new datapoint with $H = 3$, $W = no$, $E = U$.

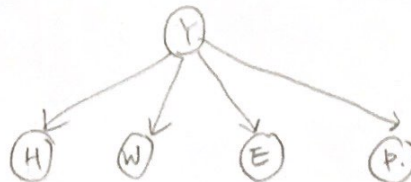
$$\frac{2}{7} \times \frac{4}{7} \times \frac{1}{3}$$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

Ham. The probability under the model for each class is computed as the product of the class prior and the feature conditionals.

- (f) You observe that you tend to receive spam emails in batches. In particular, if you receive one spam message, the next message is more likely to be a spam message as well. Specify a new graphical that most naturally captures this phenomenon.

Graph structure:



Parameters: H, W, E, P (Prev spam?)

Size of the set of parameters: $2 \times 2 \times 3 \times 2 = 24$

Self assessment If correct, write "correct" in the box. Otherwise, write and explain the correct answer.

Graph structure needs each hidden state node with three observation child nodes.

Size of the set of parameters = $1 + 2 \times 3 + 2 + 2 \times 2 + 2$