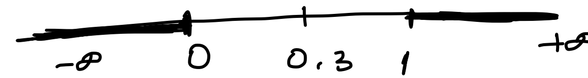
Variable Aleatoria

Ejercicio 6

Variable Aleatoria



Dada la v.a. continua X cuya función de densidad viene definida por:

$$f(x) = \begin{cases} k(1-x^2) & ,0 < x < 1 \\ 0 & ,en\ el\ resto \end{cases}$$

$k = 3/2$

a) Obtener el valor de k.
b) Calcular $P(X < 0.3)$.
c) Obtener la media y varianza de X.
d) Obtener la media y varianza de $Y = 3X - 1$.
e) Obtener la media y varianza de $H = 3X^2$.

a) $$\int_{-\infty}^{+\infty} f(x)\,dx = 1.$$

$$1 = \int_0^1 k\,(1-x^2)\,dx = k\int_0^1 (1-x^2)\,dx = k\left[\int_0^1 1\cdot dx - \int_0^1 x^2\,dx\right]$$

$$1 = k\left[\, x\Big]_0^1 - \frac{x^3}{3}\Big]_0^1 \right] = k\left[(1-0) - \left(\frac{1^3}{3} - \frac{0^3}{3}\right)\right]$$

$$1 = k\,(1 - 1/3) = k \cdot 2/3 \Rightarrow k = 3/2.$$

b) $P(X < 0.3) = \int_{-\infty}^{0.3} f(x)\,dx = \int_{0}^{0.3} 3/2\,(1-x^2)\,dx.$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)\Bigg]_0^{0.3} = \frac{3}{2}\left[\left(0.3 - \frac{(0.3)^3}{3}\right) - \left(0 - \frac{0^3}{3}\right)\right]$$

$$= \frac{3}{2}\left(0.3 - \frac{(0.3)^3}{3}\right) = 0.405$$

c) $E(X) = \int_{-\infty}^{+\infty} x \cdot f(x)\, dx. = \int_0^1 x \cdot \frac{3}{2}(1 - x^2)\, dx.$

$$= \frac{3}{2} \int_0^1 (x - x^3)\, dx = \frac{3}{2} \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{3}{2} \left[\left(\frac{1^2}{2} - \frac{1^4}{4} \right) - \left(\frac{0^2}{2} - \frac{0^4}{4} \right) \right]$$

$$= \frac{3}{2} \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{3}{2} \left(\frac{1}{4} \right) = 3/8$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$Var(X) = 1/5 - (3/8)^2$$

$$Var(X) = 0.059$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 \cdot f(x)\, dx = \int_0^1 x^2 \cdot \frac{3}{2}(1-x^2)\, dx$$

$$= \frac{3}{2} \int_0^1 (x^2 - x^4)\, dx = \frac{3}{2} \left[\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\right]_0^1$$

$$= \frac{3}{2}\left[\left(\frac{1^3}{3} - \frac{1^5}{5} - \left(\frac{0^3}{3} - \frac{0^5}{5}\right)\right)\right] = \frac{3}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{15}_{5}} = \frac{1}{5}$$

e) $Y = 3X - 1$

$$E(Y) = E(3X-1) = 3E(X) - 1 = 3 \cdot \frac{3}{8} - 1 = \frac{1}{8}$$

$$Var(Y) = Var(3X-1) = 3^2 \cdot Var(X) = 9 \times 0.059 = 0.534$$

f) $H = 3X^2$ $\qquad E(H) = E(3X^2) = 3E(X^2) = \frac{3}{5}$

$$Var(H) = Var(3X^2) = 3^2\, Var(X^2) = 9 \cdot Var(\underbrace{X^2}_{Z})$$

$$= 9 \cdot Var(Z) = 9 \cdot \left(E(Z^2) - (E(Z))^2\right) = 9\left[E(X^4) - (E(X^2))^2\right]$$

$$= 9\left[3/35 - (1/5)^2\right] = 72/175$$

$$\underline{E(X^4)} = \int_{-\infty}^{+\infty} x^4 \cdot f(x)\, dx = \int_0^1 x^4 \cdot \frac{3}{2}(1-x^2)\, dx.$$

$$= \frac{3}{2}\int_0^1 (x^4 - x^6)\, dx = \frac{3}{2}\left[\frac{x^5}{5} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{3}{2}\left(\frac{1}{5} - \frac{1}{7}\right) = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{35} = \frac{3}{35}$$