Variable Aleatoria

Ejercicio 5

Variable Aleatoria

Dada la v.a. discreta X cuya función de probabilidad viene definida por:

$$P(X = x) = kx \quad , x = 1,2,3,4,5$$

$\hookrightarrow k = 1/15$

a) Calcular el valor de la constante k.
b) Calcular $P(X > 2)$.
c) Calcular $E(X)$ y $Var(X)$.
d) Calcular $E(Y)$ si $Y = 2X + 5$.

a) $\sum_{i=1}^{\infty} p_i = 1$.

$p_i = P(X = x_i)$

$1 = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$1 = k \cdot 1 + k \cdot 2 + k \cdot 3 + k \cdot 4 + k \cdot 5.$

$1 = 15 \cdot k \Rightarrow k = 1/15$

b) $P(X>2)$

Vía 1

$$P(X>2) = P(X=3) + P(X=4) + P(X=5)$$
$$= \frac{1}{15}\cdot 3 + \frac{1}{15}\cdot 4 + \frac{1}{15}\cdot 5 = \frac{12}{15} = \frac{4}{5}$$

Vía 2

$$P(X>2) = 1 - P(X\leq 2)$$
$$= 1 - [P(X=1) + P(X=2)]$$
$$= 1 - \left[\frac{1}{15}\cdot 1 + \frac{1}{15}\cdot 2\right] = 1 - \frac{3}{15} = \frac{12}{15} = \frac{4}{5}$$

c) $E(X) = \sum_{i=1}^{n} x_i \cdot p_i = 1 \times \frac{1}{15} + 2 \cdot \frac{2}{15} + 3 \cdot \frac{3}{15} + 4 \cdot \frac{4}{15} + 5 \cdot \frac{5}{15}$

$p_i = P(X = x_i)$.

Valor esperado.

$E(X) = 11/3$.

$Var(X) = E(X^2) - (E(X))^2$

$= 15 - (11/3)^2$

$= \frac{14}{9}$.

$$E(x^2) = \sum x_i^2 \cdot p_i$$

$$= 1^2 \times \frac{1}{15} + 2^2 \cdot \frac{2}{15} + 3^2 \cdot \frac{3}{15} + 4^2 \cdot \frac{4}{15} + 5^2 \cdot \frac{1}{15}.$$

$$= 15.$$

Q) $Y = 2 \cdot X + 5$

$$E(Y) = E(2X+5) = 2 \cdot E(X) + 5.$$

$$= 2 \cdot \frac{11}{3} + 5 = 12.33$$

$$Var(Y) = Var(2X+5) = 2^2 \cdot Var(X) = 4 \cdot \frac{14}{9} = 6.22$$