

# Chapter 3

# Symmetric Key Crypto

Decorative diagonal stripes in blue and grey on the left side of the slide.A horizontal blue arrow pointing to the right, positioned below the title.

Conducted by:  
Dr. Muhammad Iqbal Hossain  
Associate Professor  
Department of CSE, Brac University  
Summer 2024

# Appendix



Stream Ciphers

Block Ciphers

A5/1

DES

AES

Block Cipher Modes

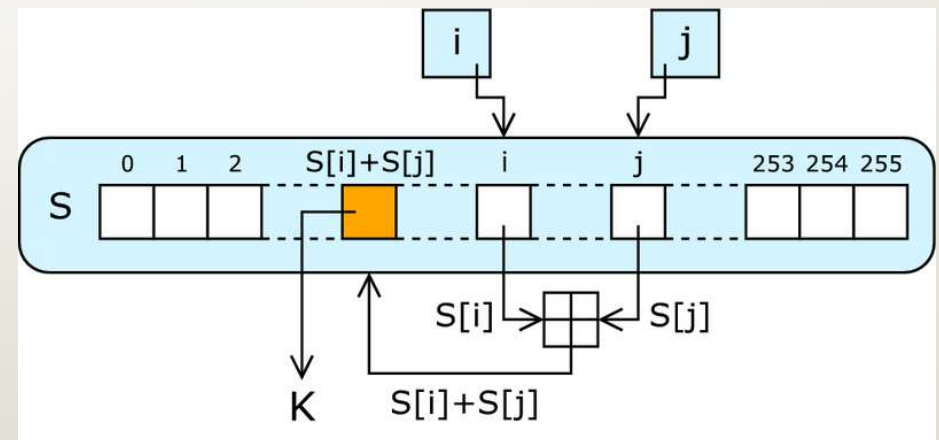
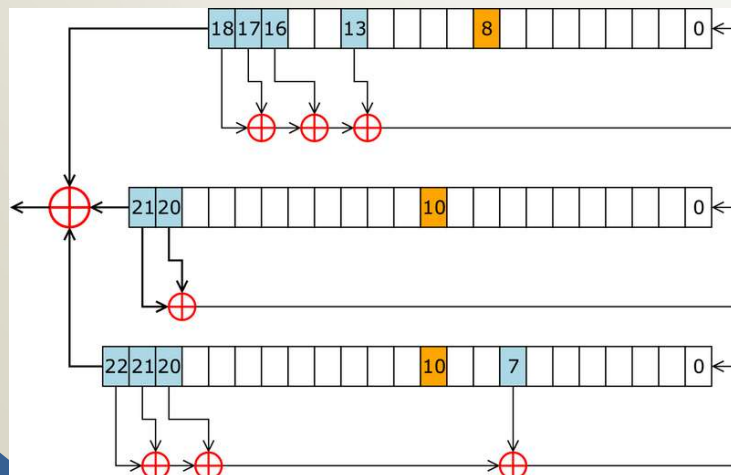
MAC Integrity

# Symmetric Key Crypto

- Stream cipher — like a one-time pad
  - Key is relatively short
  - Key is stretched into a long **keystream**
  - Keystream is then used like a one-time pad except provable security
  - Employ **confusion only**

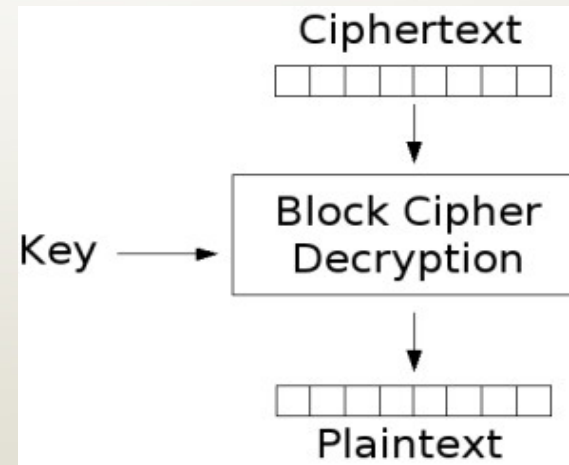
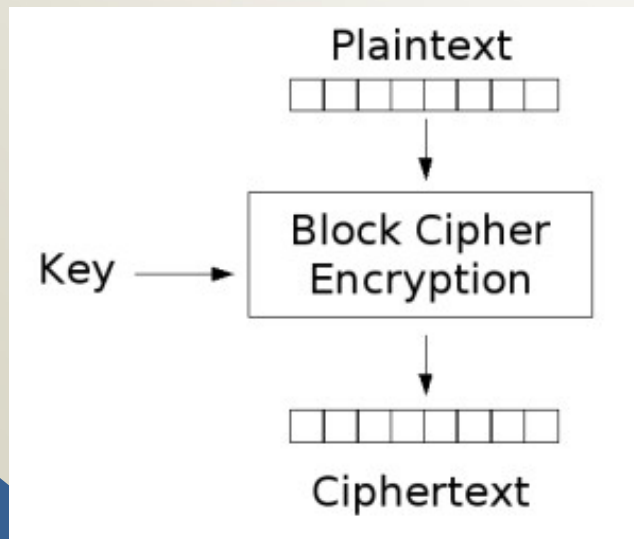
# Symmetric Key Crypto

- Examples of Stream cipher
  - A5/1: employed GSM cell phones
    - Representative stream cipher based in **H/W** (shift register)
  - RC4: used SSL protocol (lookup table)
    - Almost unique stream cipher since efficiently implemented in **S/W**



# Symmetric Key Crypto

- Block cipher—based on codebook concept
  - Block cipher key determines a “electronic” codebook
  - Each key yields a different codebook
  - Employ both confusion(relation) and diffusion(one-many)



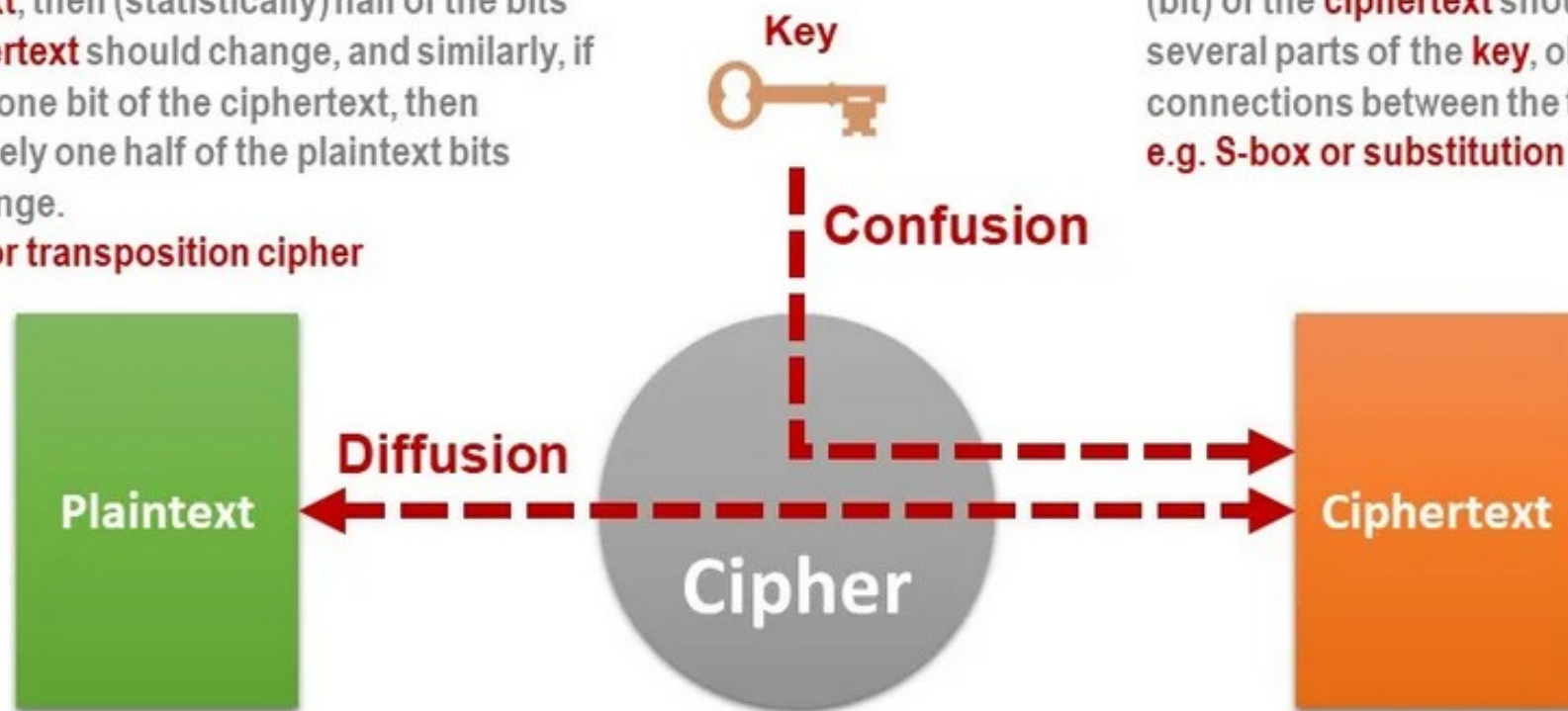
# Confusion and Diffusion

**Diffusion** means that if we change a single bit of the **plaintext**, then (statistically) half of the bits in the **ciphertext** should change, and similarly, if we change one bit of the ciphertext, then approximately one half of the plaintext bits should change.

e.g. P-box or transposition cipher

**Confusion** means that each binary digit (bit) of the **ciphertext** should depend on several parts of the **key**, obscuring the connections between the two.

e.g. S-box or substitution cipher



# Symmetric Key Crypto

## Examples of Block cipher

- **Data Encryption Standard(DES)**: relatively simple,
- **Advanced Encryption STD(AES)**
- International Data Encryption Alg.(IEDA)
- Blowfish,
- RC6
- Tiny Encryption Algorithm

# Symmetric Key Crypto

## ☐ Mode of Operation of block cipher

- Examples of block cipher mode Op
  - Electronic codebook (EOB)
  - Cipher-block chaining (CBC)
  - Cipher feedback (CFB)
  - Output feedback (OFB)
  - Counter (CTR)

## ☐ **Data integrity** of block cipher

- Message Authentication code (MAC)



# Stream Ciphers

- Not as popular today as block ciphers
- Key K of n bits stretches it into a long keystream
- Function of stream cipher
  - $\text{StreamCipher}(K) = S$  where K:key, S:keystream
  - S is used like a one-time pad
    - $c_0 = p_0 \oplus s_0, c_1 = p_1 \oplus s_1, c_2 = p_2 \oplus s_2, \dots$
    - $p_0 = c_0 \oplus s_0, p_1 = c_1 \oplus s_1, p_2 = c_2 \oplus s_2, \dots$
- Sender and receiver have same stream cipher algorithm and both know the key K

# Stream Ciphers



Inspiring Excellence

- A<sub>5</sub>/1
  - Based on [linear feedback shift registers](#)
  - Used in [GSM mobile phone system](#)
    - A<sub>5</sub>/1 is used in Europe and the United States;
    - [A<sub>5</sub>/2](#), is used in countries that are not considered trustworthy enough to have strong crypto.
- RC<sub>4</sub>
  - Based on a changing lookup table
  - Used many places – [SSL](#)

# A5/1

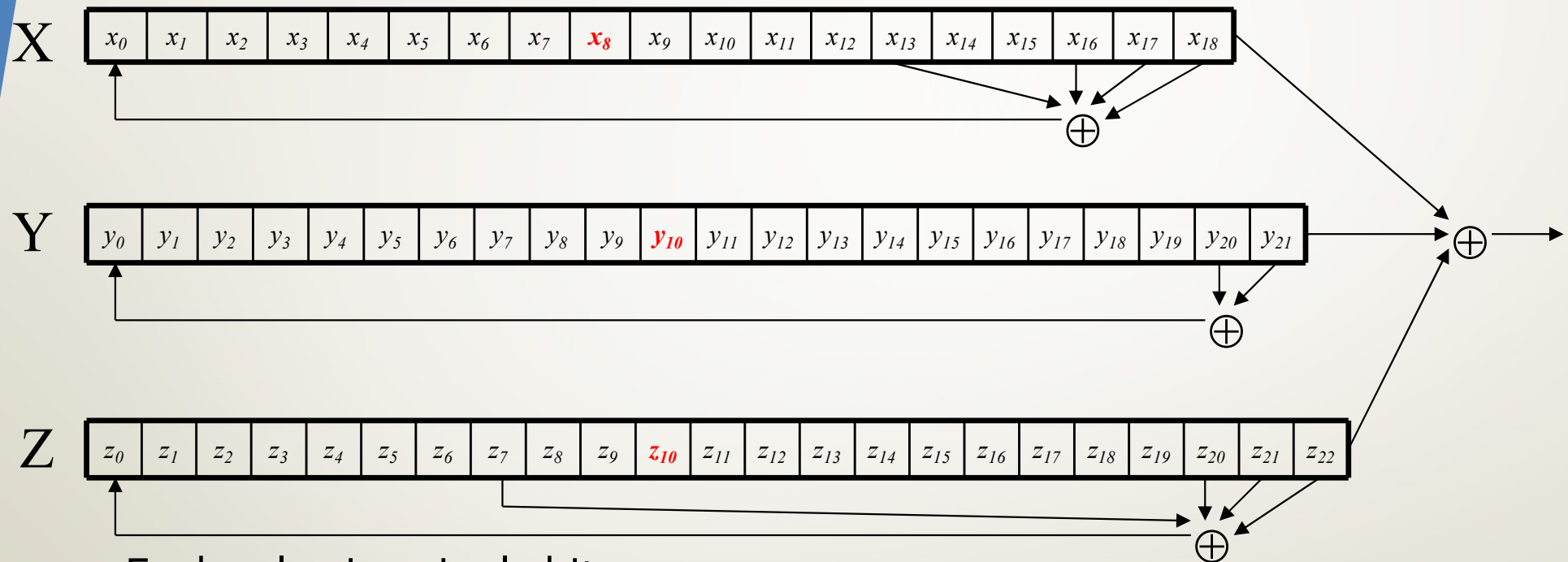
- A5/1 is Representative stream cipher based in H/W
  - Consists of 3 Linear feedback shift registers
    - X: 19 bits ( $x_0, x_1, x_2, \dots, x_{18}$ )
    - Y: 22 bits ( $y_0, y_1, y_2, \dots, y_{21}$ )
    - Z: 23 bits ( $z_0, z_1, z_2, \dots, z_{22}$ )
    - $X+Y+Z = 64$  bits

# A5/1

- At each step:  $m = \text{maj}(x_8, y_{10}, z_{10})$ 
  - Examples:  $\text{maj}(0,1,0) = 0$  and  $\text{maj}(1,1,0) = 1$
- If  $x_8 = m$  then
  - $t = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
  - $x_i = x_{i-1}$  for  $i = 18, 17, \dots, 1$  and  $x_0 = t$
- If  $y_{10} = m$  then
  - $t = y_{20} \oplus y_2$
  - $y_i = y_{i-1}$  for  $i = 21, 20, \dots, 1$  and  $y_0 = t$
- If  $z_{10} = m$  then
  - $t = z_7 \oplus z_{20} \oplus z_{21} \oplus z_{22}$
  - $z_i = z_{i-1}$  for  $i = 22, 21, \dots, 1$  and  $z_0 = t$

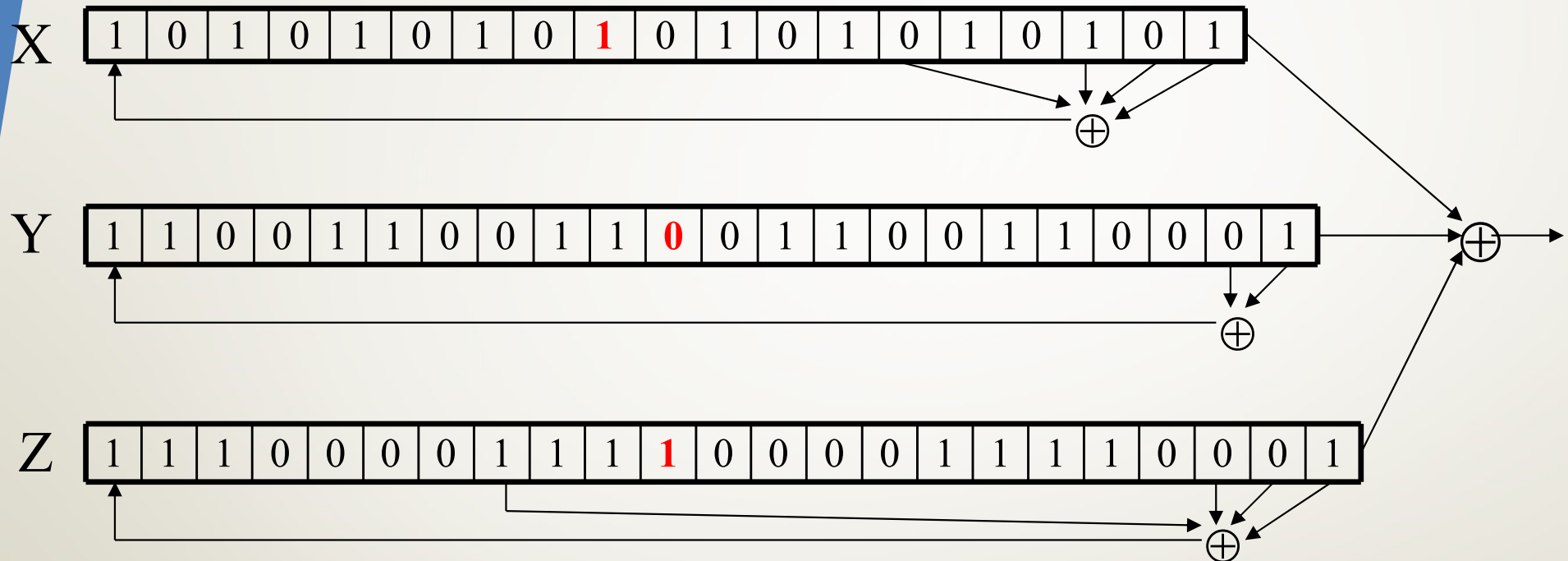
**Keystream bit is  $x_{18} \oplus y_{21} \oplus z_{22}$**

# A5/1



- Each value is a single bit
- **Key is used as initial fill of registers**
- Each register steps or not, based on  $(x_8, y_{10}, z_{10})$
- Keystream bit is XOR of right bits of registers

# A5/1



- In this example,  $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(\mathbf{1}, \mathbf{0}, \mathbf{1}) = \mathbf{1}$
- Register X steps, Y does not step, and Z steps
- Keystream bit is XOR of right bits of registers
- Here, keystream bit will be  $0 \oplus 1 \oplus 0 = 1$

# Shift Register Crypto

- Shift register crypto efficient in hardware
- Often, slow if implemented in software
- In the past, very, very popular
- Today, more is done in software due to fast processors
- Shift register crypto still used some
  - Especially in resource-constrained devices

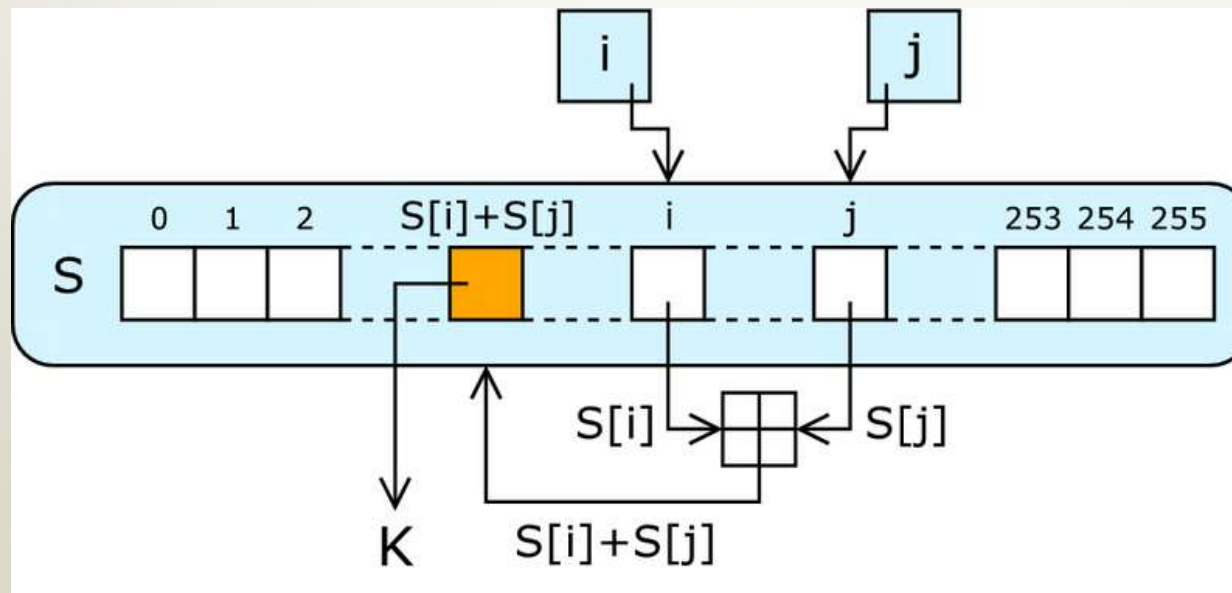
# RC<sub>4</sub>

- A self-modifying lookup table
- Table always contains a permutation of the byte values 0,1,...,255
- Initialize the permutation using key
- At each step, RC<sub>4</sub> does the following
  - Swaps elements in current lookup table
  - Selects a keystream byte from table
- Each step of RC<sub>4</sub> produces a **byte**
  - Efficient in software
- Each step of A5/1 produces only a bit
  - Efficient in hardware



# RC<sub>4</sub>

- RC<sub>4</sub> Optimized for software implementation, whereas A<sub>5</sub>/1 for hardware
- RC<sub>4</sub> produces a keystream BYTE at each step, whereas A<sub>5</sub>/1 only produce a single keystream bit



# RC<sub>4</sub>



Inspiring Excellence

- RC<sub>4</sub> is remarkably simple
  - Because it is essentially just lookup table containing permutation of the  $256(2^8)$ -byte values
  - Each time a byte of keystream is produced, the lookup table is modified in such a way that the table always contains a permutation of  $\{0,1,2,\dots,256\}$

# Stream Ciphers

- Stream ciphers were big in the past
  - Efficient in hardware
  - Speed needed to keep up with voice, etc.
  - Today, processors are fast, so software-based crypto is fast enough
- Future of stream ciphers?
  - Shamir: "the death of stream ciphers"
  - May be exaggerated...

Decorative diagonal lines in blue and black on the left side of the slide.

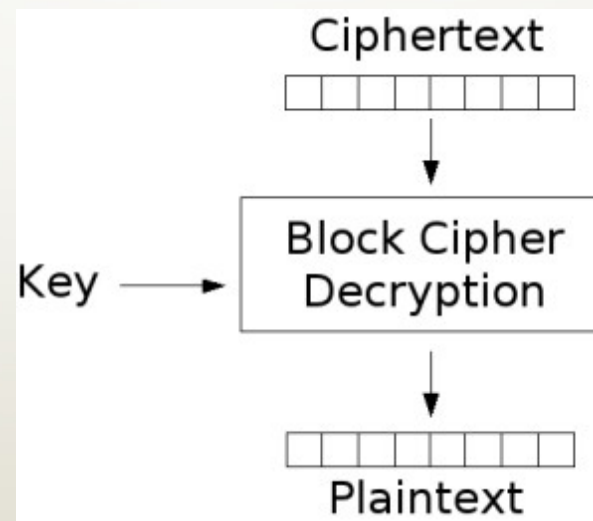
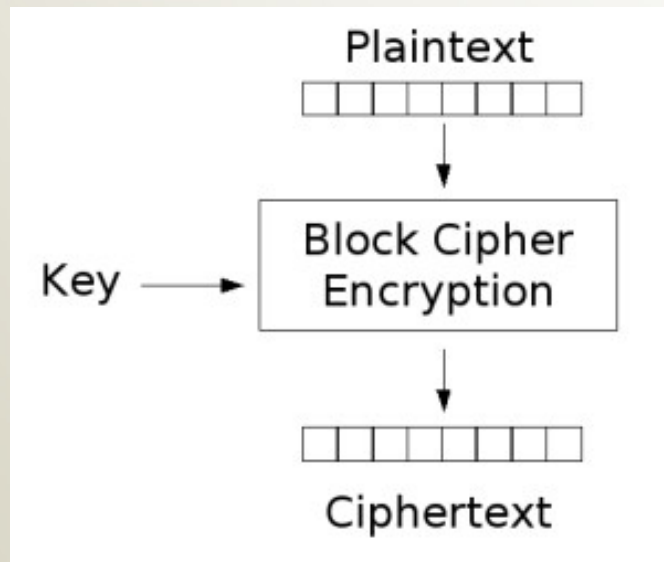
End of segment

# Block Ciphers

## BLOCK CIPHER AND DES

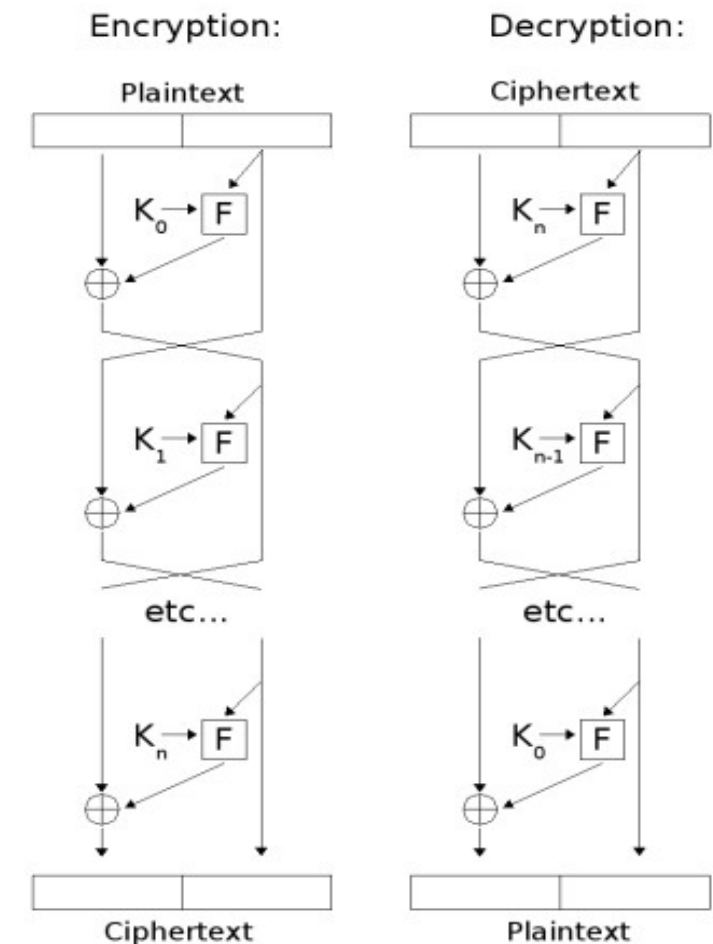
# Block Cipher

- Plaintext and ciphertext consists of **fixed sized blocks**
- **Design goal:** security and efficiency
  - It is not easy to design a block cipher that is secure and efficient



# (Iterated) Block Cipher

- Plaintext and Ciphertext consist of fixed-sized blocks
- Ciphertext obtained from plaintext by iterating a **round function**
- Input to round function consists of key and the output of previous round
- Usually implemented in software  
Typical Type is Feistel Cipher



Feistel Cipher

# Feistel Cipher

- ❑ **Feistel cipher** refers to a type of block cipher design, not a specific cipher
- ❑ Split plaintext block into left and right halves:  
Plaintext =  $(L_0, R_0)$
- ❑ For each round  $i=1,2,...,n$ , compute  

$$L_i = R_{i-1}$$

$$R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$$
 where  $F$  is **round function** and  $K_i$  is **subkey**
- ❑ Ciphertext =  $(L_n, R_n)$



# Feistel Cipher

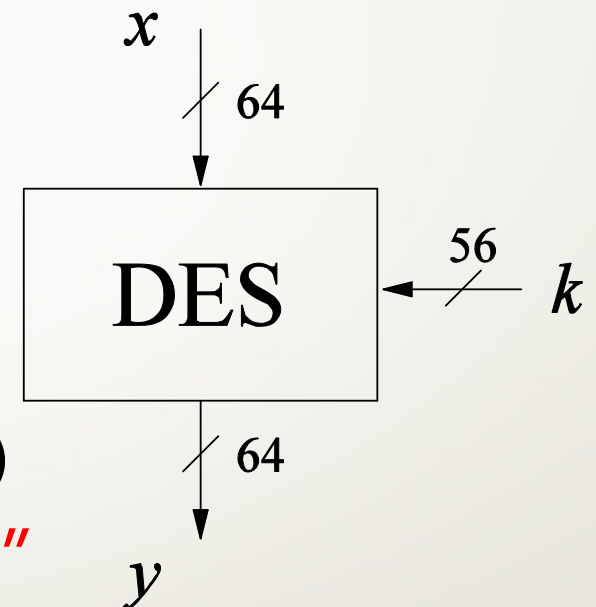
- ❑ Decryption: Ciphertext =  $(L_n, R_n)$
- ❑ For each round  $i=n, n-1, \dots, 1$ , compute
  - $R_{i-1} = L_i$
  - $L_{i-1} = R_i \oplus F(R_{i-1}, K_i)$
  - where  $F$  is round function and  $K_i$  is subkey
- ❑ Plaintext =  $(L_0, R_0)$
- ❑ Formula “works” for any function  $F$
- ❑ **But only secure for certain functions  $F$** 
  - ❑ Ex:  $F(R_{i-1}, K_i) = 0$  for all  $R_{i-1}$  and  $K_i$  -> not secure

# Data Encryption Standard (DES)

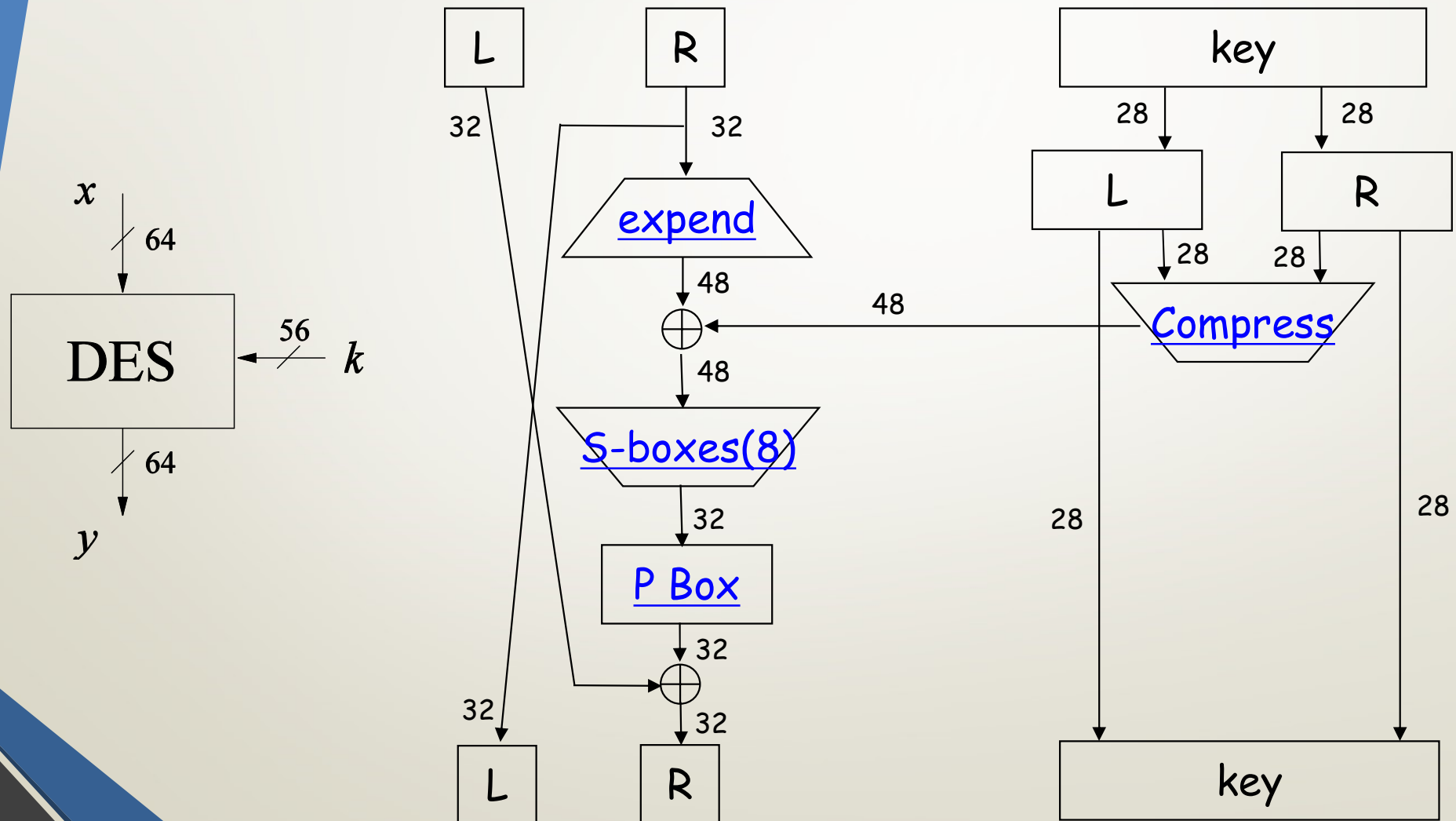
- DES developed in 1970's
- Based on IBM Lucifer cipher
- U.S. government standard
- DES development was controversial
  - NSA was secretly involved
  - Design process not open
  - Key length was reduced
  - Subtle changes to Lucifer algorithm

# DES Numerology

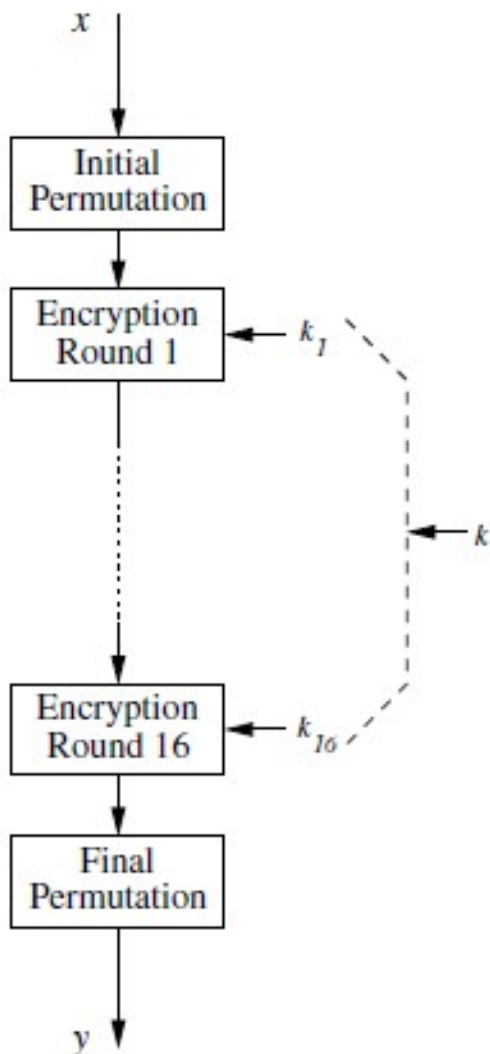
- DES is a **Feistel cipher**
  - 64 bit block length
  - 56 bit key length
  - 16 rounds
  - 48 bits of key used each round (subkey)
- Each round is simple (for a block cipher)
- Security depends primarily on **"S-boxes"**
  - Each S-boxes maps 6 bits to 4 bits
  - Total 8 S-boxes



# One Round of DES



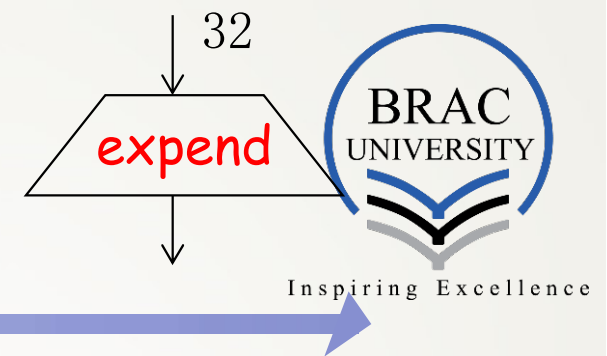
# Initial permutation



$IP$							
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

$IP^{-1}$							
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

# DES Expansion Permutation



- Input 32 bits

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	1	0	0	1	0	1	1	1	0	0	1	0	0	0	1
index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	0	1	1	0	0	1	1	1	1	0	0	1	0	0	1	1

## Output 48 bits

index	0	1	2	3	4	5	6	7	8	9	10	11
index	12	13	14	15	16	17	18	19	20	21	22	23
index	24	25	26	27	28	29	30	31	32	33	34	35
index	36	37	38	39	40	41	42	43	44	45	46	47

# DES Expansion Permutation

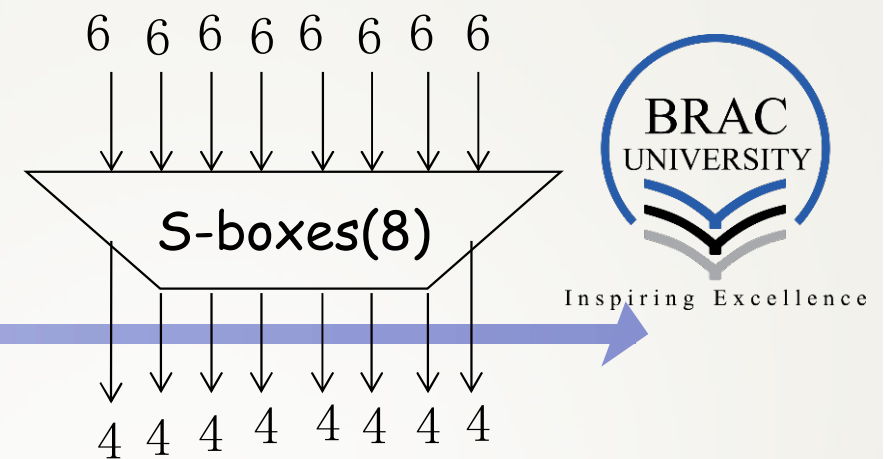
- Input 32 bits

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	1	0	0	1	0	1	1	1	0	0	1	0	0	0	1
index	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	0	1	1	0	0	1	1	1	1	0	0	1	0	0	1	1

- Output 48 bits

index	0	1	2	3	4	5	6	7	8	9	10	11
	31	0	1	2	3	4	3	4	5	6	7	8
index	12	13	14	15	16	17	18	19	20	21	22	23
	7	8	9	10	11	12	11	12	13	14	15	16
index	24	25	26	27	28	29	30	31	32	33	34	35
	15	16	17	18	19	20	19	20	21	22	23	24
index	36	37	38	39	40	41	42	43	44	45	46	47
	23	24	25	26	27	28	27	28	29	30	31	0

# DES S-box



- 8 “substitution boxes” or S-boxes
- Each S-box maps 6 bits to 4 bits
- S-box number 1

input bits (0,5)

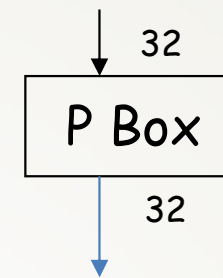


input bits (1,2,3,4)

	00	00	00	00	01	01	01	01	10	10	10	10	11	11	11	11
	00	01	10	11	00	01	10	11	00	01	10	11	00	01	10	11
0	11	01	11	00	00	11	10	10	00	10	01	11	01	10	00	01
0	10	00	01	01	10	11	11	00	11	10	10	00	01	01	00	11
0	00	11	01	01	11	00	11	00	10	01	11	10	10	01	00	10
1	00	11	11	00	10	10	01	01	10	10	00	11	01	01	11	00
1	01	11	11	10	11	01	00	10	11	11	10	01	00	10	01	00
0	00	01	10	00	01	10	10	11	11	00	01	11	11	10	01	00
1	11	11	10	00	01	10	00	01	01	10	00	11	10	00	01	11
1	11	00	00	10	00	01	01	11	11	11	11	10	10	00	10	01



# DES P-box



Inspiring Excellence

- Input 32 bits

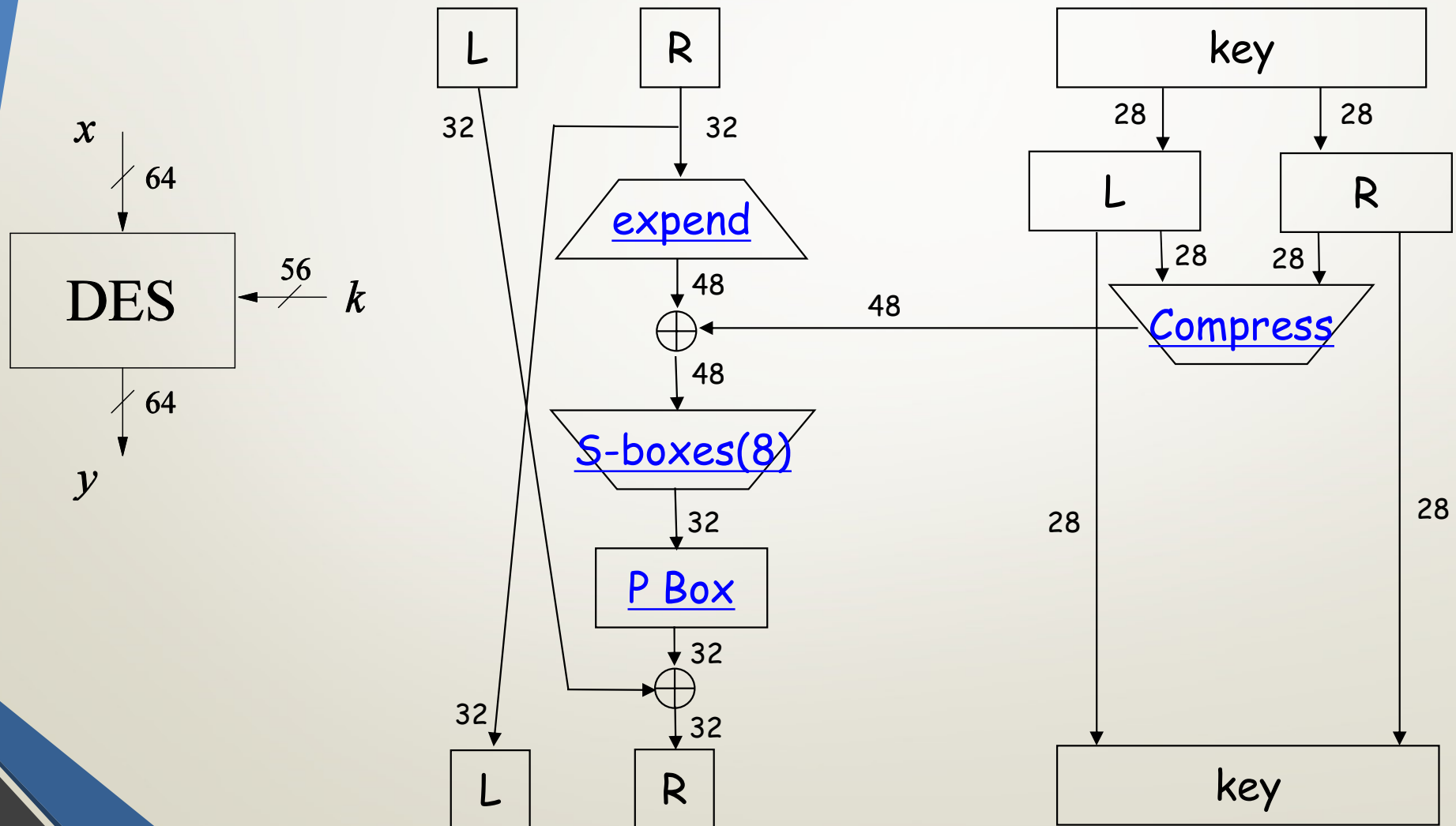
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

- Output 32 bits

15	6	19	20	28	11	27	16	0	14	22	25	4	17	30	9
1	7	23	13	31	26	2	8	18	12	29	5	21	10	3	24

[BACK](#)

# DES subkey



# DES Subkey

- 56 bit DES key, numbered 0,1,2,...,55

Left half key bits,  $LK$

49	42	35	28	21	14	7
0	50	43	36	29	22	15
8	1	51	44	37	30	23
16	9	2	52	45	38	31

Right half key bits,  $RK$

55	48	41	34	27	20	13
6	54	47	40	33	26	19
12	5	53	46	39	32	25
18	11	4	24	17	10	3

# DES Subkey

- ❑ For rounds  $i=1, 2, \dots, 16$ 
  - ❑ Let  $LK = (LK \text{ circular shift left by } r_i)$
  - ❑ Let  $RK = (RK \text{ circular shift left by } r_i)$
  - ❑ Left half of subkey  $K_i$  is of LK bits

13	16	10	23	0	4	2	27	14	5	20	9
22	18	11	3	25	7	15	6	26	19	12	1

- ❑ Right half of subkey  $K_i$  is RK bits

12	23	2	8	18	26	1	11	22	16	4	19
15	20	10	27	5	24	17	13	21	7	0	3

# DES Subkey

- For rounds 1, 2, 9 and 16 the shift  $r_i$  is 1, and in all other rounds  $r_i$  is 2
- Bits 8,17,21,24 of LK omitted each round
- Bits 6,9,14,25 of RK omitted each round
- **Compression permutation** yields 48 bit subkey  $K_i$  from 56 bits of LK and RK
- **Key schedule** generates subkey

[BACK](#)

# DES Last Word (Almost)

- An initial perm  $P$  before round 1
- Halves are swapped after last round
- A final permutation (inverse of  $P$ ) is applied to  $(R_{16}, L_{16})$  to yield ciphertext
- None of these serve any security purpose

# Security of DES

- ❑ Security of DES depends a lot on S-boxes
  - ❑ Everything else in DES is linear
- ❑ Thirty years of intense analysis has revealed **no**  
**"back door"**
- ❑ Attacks today use exhaustive key search
- ❑ Inescapable conclusions
  - ❑ Designers of DES knew what they were doing
  - ❑ Designers of DES were ahead of their time

# History of Attacks on DES

Year	Proposed/ implemented DES Attack
1977	Diffie & Hellman, (under-)estimate the costs of a key search machine
1990	Biham & Shamir propose differential cryptanalysis ( $2^{47}$ chosen ciphertexts)
1993	Mike Wiener proposes design of a very efficient key search machine: Average search requires 36h. Costs: \$1.000.000
1993	Matsui proposes linear cryptanalysis ( $2^{43}$ chosen ciphertexts)
Jun. 1997	DES Challenge I broken, 4.5 months of distributed search
Feb. 1998	DES Challenge II--1 broken, 39 days (distributed search)
Jul. 1998	DES Challenge II--2 broken, key search machine <i>Deep Crack</i> built by the Electronic Frontier Foundation (EFF): 1800 ASICs with 24 search engines each, Costs: \$250 000, 15 days average search time (required 56h for the Challenge)
Jan. 1999	DES Challenge III broken in 22h 15min (distributed search assisted by <i>Deep Crack</i> )
2006-2008	Reconfigurable key search machine <i>COPACOBANA</i> developed at the Universities in Bochum and Kiel (Germany), uses 120 FPGAs to break DES in 6.4 days (avg.) at a cost of \$10 000.



# Breaking S-BOX

Sample S-box

	00	01	10	11
0	10	01	11	00
1	00	10	01	11

$$X_1 = 110, X_2 = 010$$

$$K = 011$$

$$X_1 \oplus K = 110 \oplus 011 = 101$$

$$X_2 \oplus K = 010 \oplus 011 = 001$$

$$Sbox(X_1 \oplus K) = S(101) = 10$$

$$Sbox(X_2 \oplus K) = S(001) = 01$$

How to find the key?

We know,

$$X_1 = 110, X_2 = 010$$

$$Sbox(X_1 \oplus K) = 10$$

$$Sbox(X_2 \oplus K) = 01$$

$$(X_1 \oplus K) \in \{000, 101\}$$

$$(X_2 \oplus K) \in \{001, 110\}$$

$$K \in \{110, 011\} \cap \{011, 100\}$$

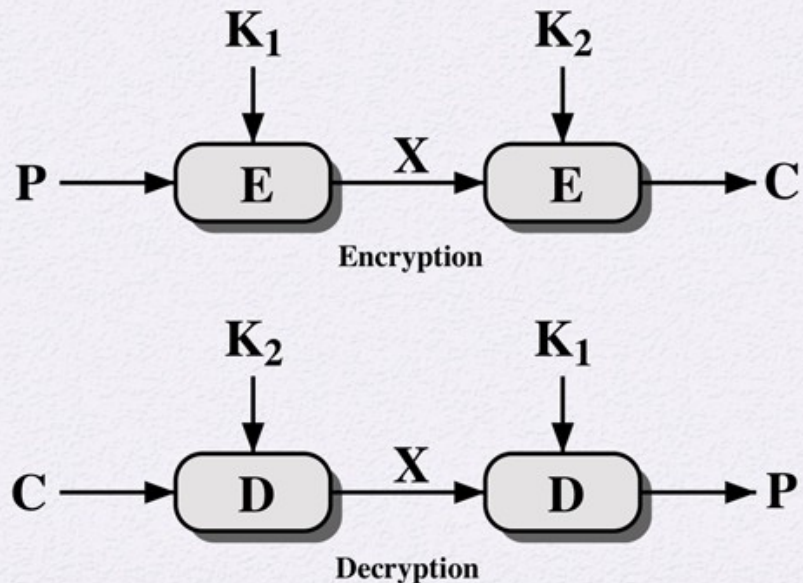
$$\mathbf{K = 011}$$

Decorative diagonal lines in blue and black on the left side of the slide.A horizontal blue arrow pointing from the left towards the BRAC University logo.

# Thank you

	00 00	00 01	00 10	00 11	01 00	01 01	01 10	01 11	10 00	10 01	10 10	10 11	11 00	11 01	11 10	11 11
0 0	11 10	01 00	11 01	00 01	00 10	11 11	10 11	10 00	00 11	10 10	01 10	11 00	01 01	10 01	00 00	01 11
0 1	00 00	11 11	01 11	01 00	11 10	00 10	11 01	00 01	10 10	01 10	11 00	10 11	10 01	01 01	00 11	10 00
1 0	01 00	11 01	11 10	10 00	11 01	01 10	00 10	10 11	11 11	11 00	10 01	01 11	00 11	10 10	01 01	00 00
1 1	11 11	11 00	10 00	00 10	01 00	10 01	00 01	01 11	01 11	10 11	00 11	11 10	10 10	00 00	01 10	11 01

# Double DES and meet in the middle attack



For DES, Key length is 56.

Hacker needs to check  $2^{56}$  combination in brute force attack.

In 2DES, Key length  $56 + 56 = 112$

It only works when there is a known plaintext/ciphertext pair.

1. Encrypt the plaintext with all  $2^{56}$  possible keys and write down the results
2. Decrypt the ciphertext with all  $2^{56}$  possible keys and write down the results
3. Check where the results are the same. That is your key.

Note that all you had to do to recover the key was using DES  $2 \times 2^{56}$  times, which makes  $2^{57}$ .

# Block Cipher: AES

# Advanced Encryption Standard (AES ) History



Inspiring Excellence

- Needs for replacement for DES
  - DES had outlived its usefulness
    - Attacked by exhaustive key search: Special purpose DES crackers and distributed attack at internet
- 3DES is very resistant to crypto analysis but
  - No efficient software code
  - Too slow: 3 times as many rounds as DES
  - 3DES use 64-bit block size: for reasons of both efficient and security, a larger blk size desirable
  - **So, 3DES is not solution for long-term use**
- In 1997, NIST made a formal call for advanced encryption standard algorithms

# Advanced Encryption Standard (AES ) History

- GOAL: replace DES for both government and private sector encryption.
- Requirement of AES
  - **Unclassified**, publicly disclosed encryption algorithm, **available royalty-free**, worldwide.
  - The algorithm must implement **symmetric key**
  - Cryptography as a block cipher and (at a minimum) support **block sizes of 128-bits** and **key sizes of 128- , 192-, and 256-bits**.
- In 1998, NIST announced a group of 15 AES candidate algorithms.

# AES History



Inspiring Excellence

- Criteria for selecting AES:
  - Security, Robustness, Speed
- In 1999, out of 15, the selection was narrowed to 5 candidates:
  - MARS, RC6, Rijndael, Serpent, and Twofish.
- All the five protocols were thought to be secure
- On October 2, 2000, NIST has selected **Rijndael** to propose for the AES.
  - Pronounced like “Rain Doll” or “Rhine Doll”
  - Invented by Joan Daemen and Vincent Rijmen



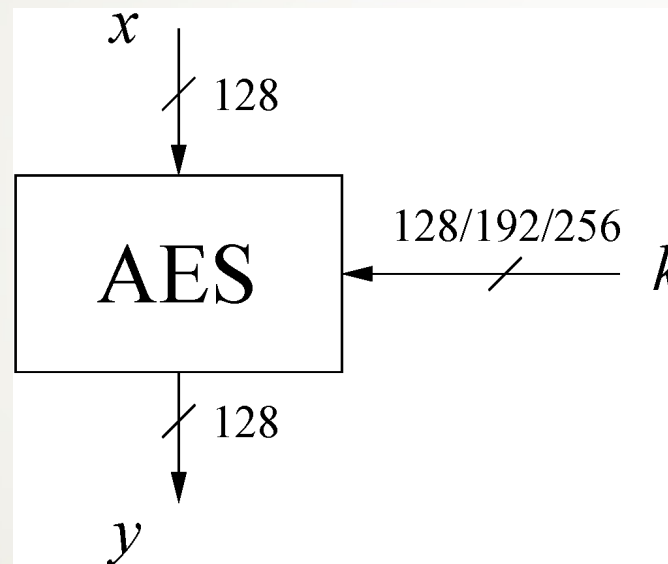
# AES Features

- Designed to be efficient in both hardware and software across a variety of platforms.
- Not a Feistel Network
  - Iterated block cipher (like DES)
  - Not a Feistel cipher (unlike DES)
  - "Secure forever" – Shamir

# AES Overview

- **Block size:** 128 bits (others in Rijndael)
- **Key length:** 128, 192 or 256 bits (independent of block size in Rijndael)
- 10 to 14 rounds (depends on key length)
- Each round uses 4 functions (3 "layers")
  - ByteSub (nonlinear layer)
  - ShiftRow (linear mixing layer)
  - MixColumn (nonlinear layer)
  - AddRoundKey (key addition layer)

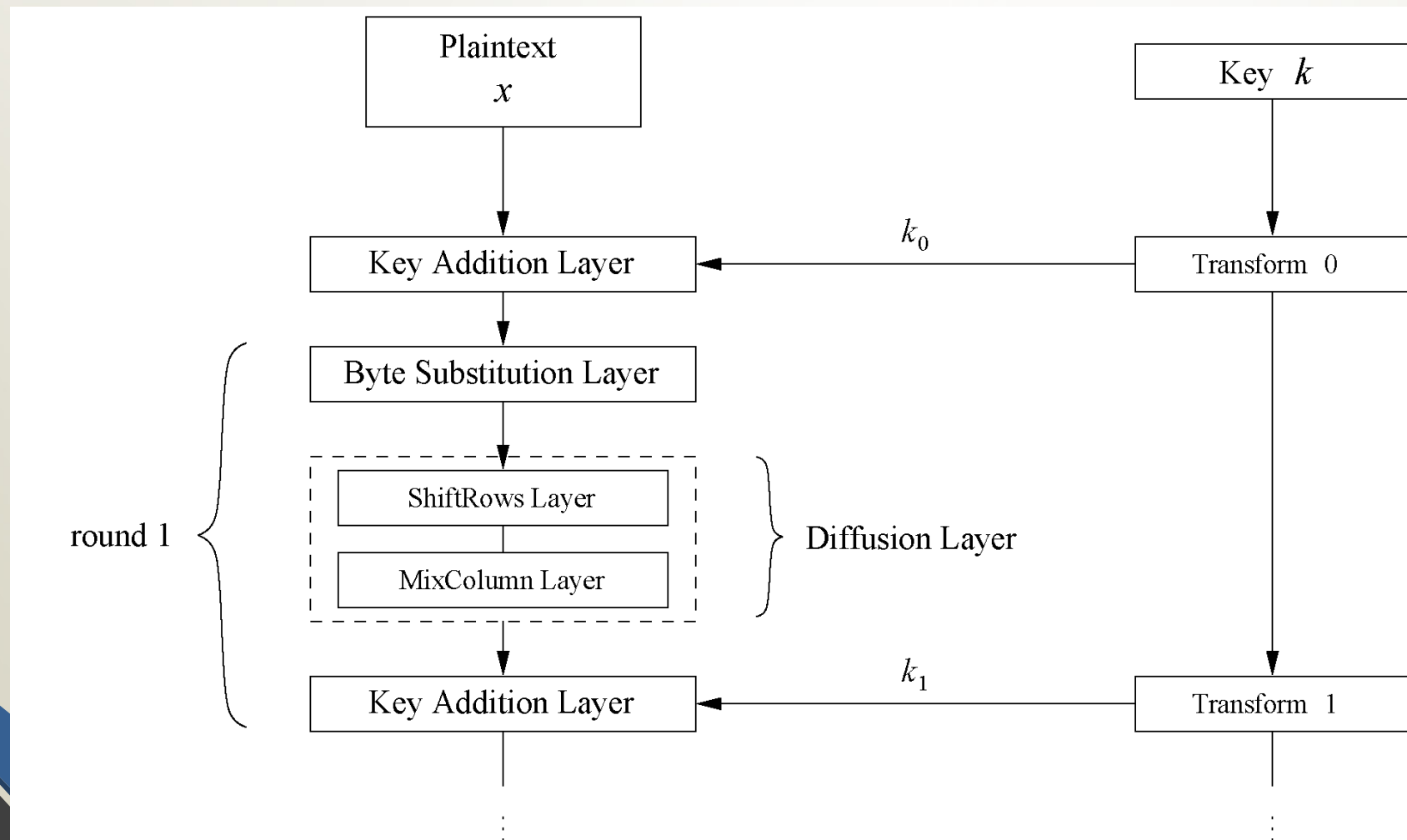
# AES: Overview

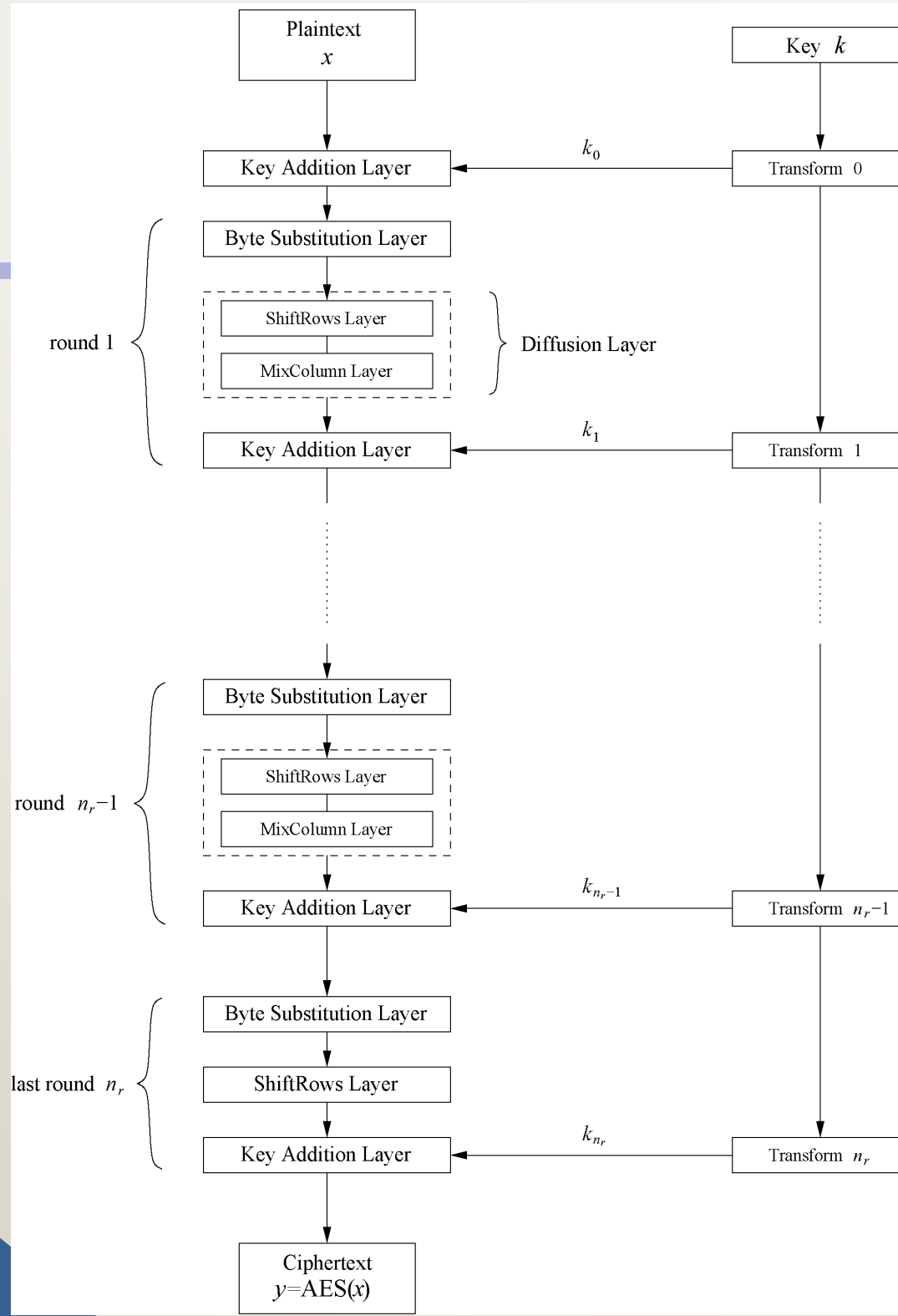


The number of rounds depends on the chosen key length:

Key length (bits)	Number of rounds
128	10
192	12
256	14

# AES: Overview





# Internal Structure of AES

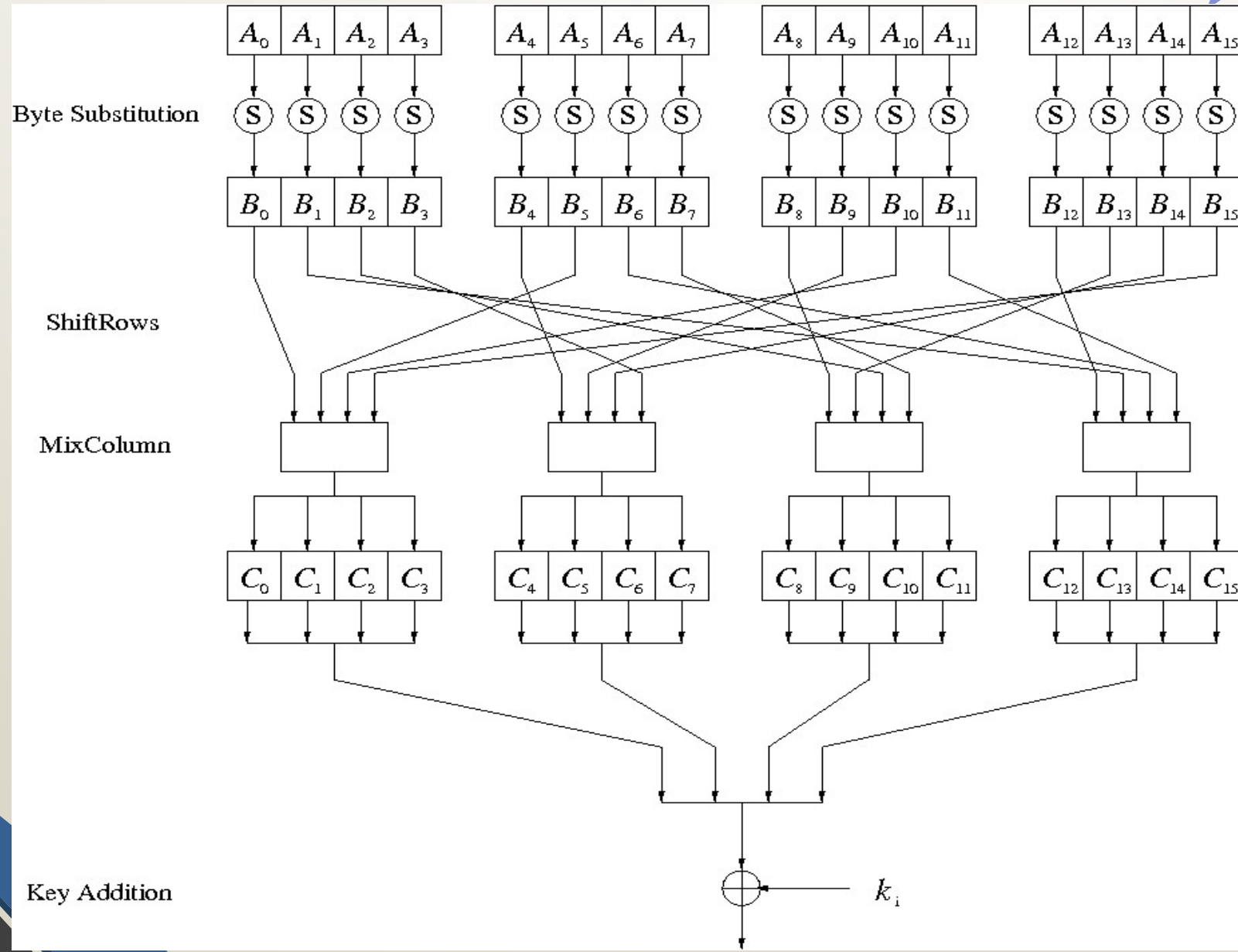
- AES is a byte-oriented cipher
- The state  $A$  (i.e., the 128-bit data path) can be arranged in a 4x4 matrix:

$A_0$	$A_4$	$A_8$	$A_{12}$
$A_1$	$A_5$	$A_9$	$A_{13}$
$A_2$	$A_6$	$A_{10}$	$A_{14}$
$A_3$	$A_7$	$A_{11}$	$A_{15}$

$A_0, \dots, A_{15}$  Contain HEX number.  
For example:  $A_0=C2$ ,  $A_2=EA \dots$

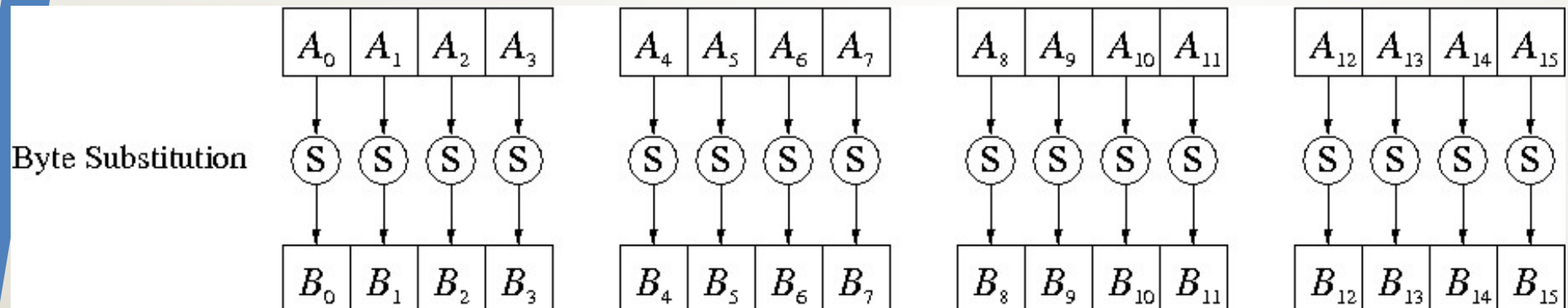
with  $A_0, \dots, A_{15}$  denoting the 16-byte input of AES

# Internal Structure of AES





# Byte Substitution

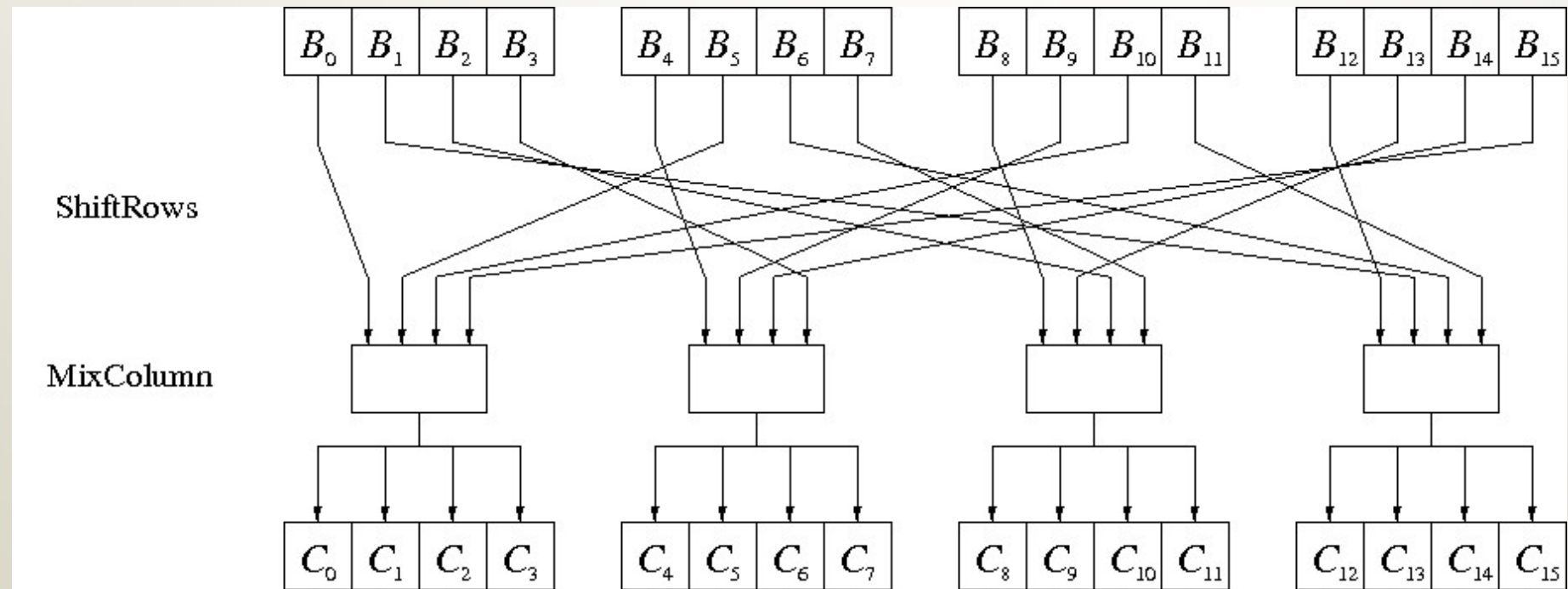


Let's assume the input byte to the S-Box is  $A_i = (C2)_{\text{hex}}$ , then the substituted value is  $S((C2)_{\text{hex}}) = (25)_{\text{hex}}$ .

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16



# Shiftrows and MixColumn



# ShiftRows Sublayer

- Rows of the state matrix are shifted cyclically:

Input matrix

$B_0$	$B_4$	$B_8$	$B_{12}$
$B_1$	$B_5$	$B_9$	$B_{13}$
$B_2$	$B_6$	$B_{10}$	$B_{14}$
$B_3$	$B_7$	$B_{11}$	$B_{15}$

Output matrix

$B_0$	$B_4$	$B_8$	$B_{12}$
$B_5$	$B_9$	$B_{13}$	$B_1$
$B_{10}$	$B_{14}$	$B_2$	$B_6$
$B_{15}$	$B_3$	$B_7$	$B_{11}$

no shift

← one position left shift

← two positions left shift

← three positions left shift

# MixColumn Sublayer

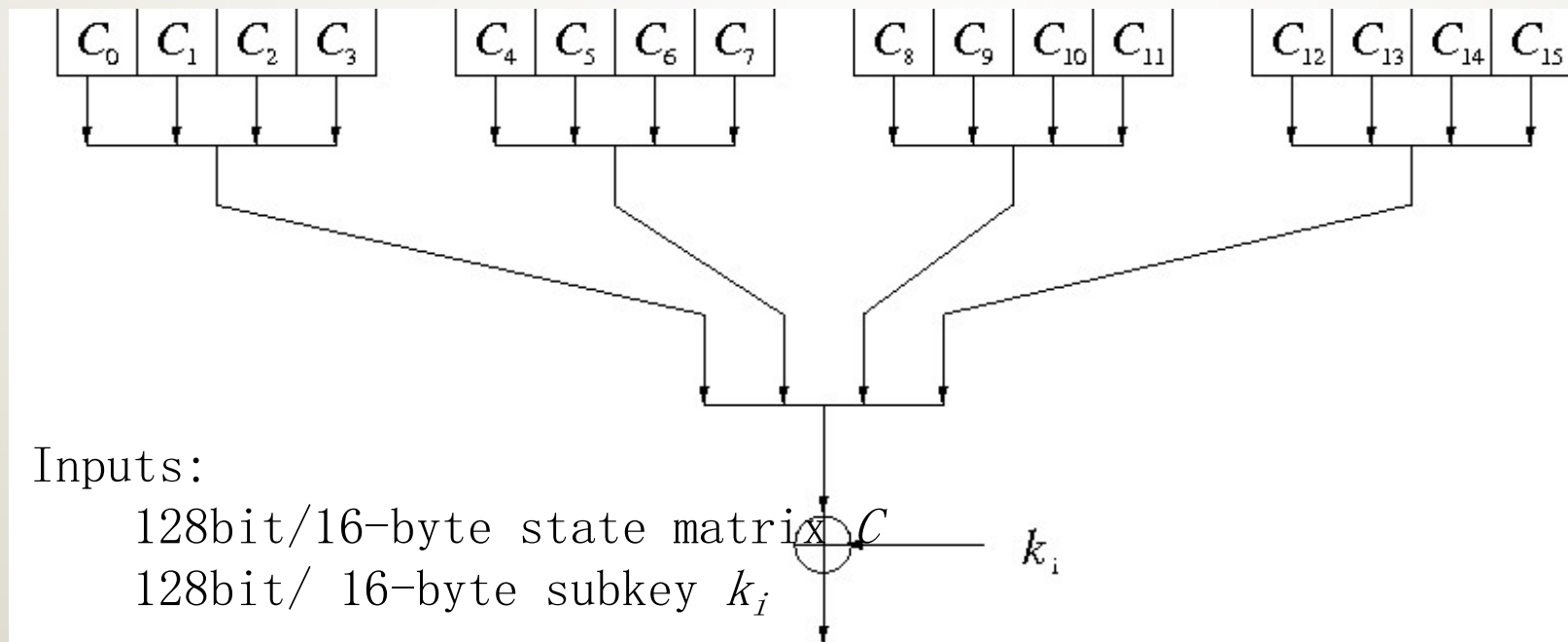
Each 4-byte column is considered as a vector and multiplied by a fixed 4x4 matrix, e.g.,

$B_0$	$B_4$	$B_8$	$B_{12}$
$B_5$	$B_9$	$B_{13}$	$B_1$
$B_{10}$	$B_{14}$	$B_2$	$B_6$
$B_{15}$	$B_3$	$B_7$	$B_{11}$

Output matrix

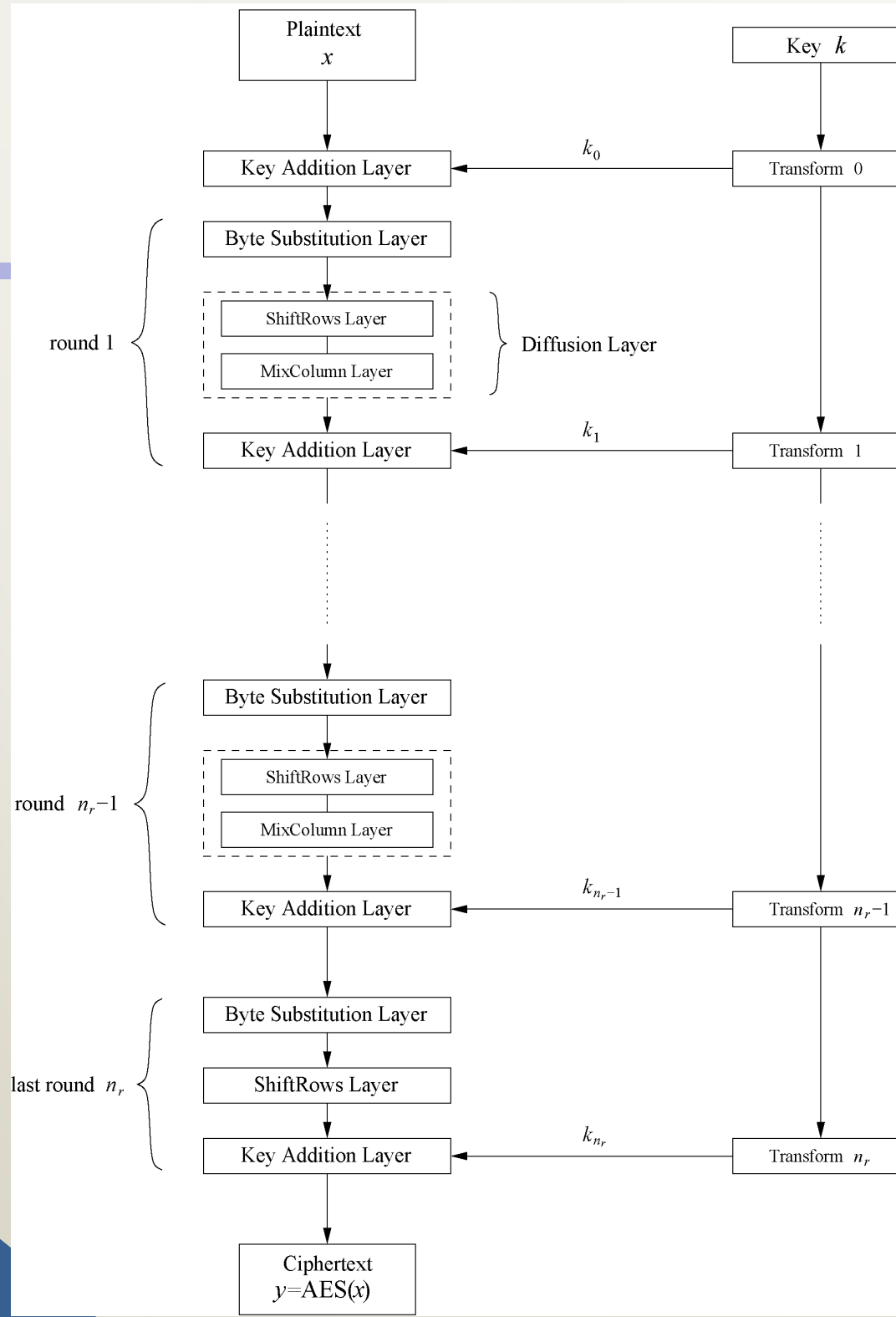
$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} B_0 \\ B_5 \\ B_{10} \\ B_{15} \end{pmatrix}$$

# Key Addition Layer



Output:  $C \oplus k_i$

The subkeys are generated in the key schedule



## Key Schedule

# Example

Plain text

T	w	o		O	n	e		N	i	n	e		T	w	o
54	77	6F	20	4F	6E	65	20	43	69	6E	25	20	54	77	6F

Key

T	h	a	t	s		m	y		K	u	n	g		F	u
54	68	61	74	73	20	6D	79	20	4B	75	6E	67	20	46	75



### Keys generated for every round

- Round 0: 54 68 61 74 73 20 6D 79 20 4B 75 6E 67 20 46 75
- Round 1: E2 32 FC F1 91 12 91 88 B1 59 E4 E6 D6 79 A2 93
- Round 2: 56 08 20 07 C7 1A B1 8F 76 43 55 69 A0 3A F7 FA
- Round 3: D2 60 0D E7 15 7A BC 68 63 39 E9 01 C3 03 1E FB
- Round 4: A1 12 02 C9 B4 68 BE A1 D7 51 57 A0 14 52 49 5B
- Round 5: B1 29 3B 33 05 41 85 92 D2 10 D2 32 C6 42 9B 69
- Round 6: BD 3D C2 B7 B8 7C 47 15 6A 6C 95 27 AC 2E 0E 4E
- Round 7: CC 96 ED 16 74 EA AA 03 1E 86 3F 24 B2 A8 31 6A
- Round 8: 8E 51 EF 21 FA BB 45 22 E4 3D 7A 06 56 95 4B 6C
- Round 9: BF E2 BF 90 45 59 FA B2 A1 64 80 B4 F7 F1 CB D8
- Round 10: 28 FD DE F8 6D A4 24 4A CC C0 A4 FE 3B 31 6F 26

# Step 1: Key Addition

54	4F	4E	20
77	6E	69	54
6F	65	6E	77
20	20	65	6F



XOR

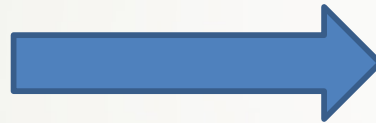
54	73	20	67
68	20	4B	20
61	6D	75	46
74	79	6E	75

00	3C	63	47
1F	4E	22	74
0E	08	1B	31
54	59	0B	1A



# Step 2: Byte substitution

00	3C	63	47
1F	4E	22	74
0E	08	1B	31
54	59	0B	1A



63	EB	9F	A0
C0	2F	93	92
AB	30	AF	C7
20	CB	2B	A2

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
x 8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

# Step 3: Shift Row

63	EB	9F	A0	→	63	EB	9F	A0
C0	2F	93	92		2F	93	92	C0
AB	30	AF	C7		AF	C7	AB	30
20	CB	2B	A2		A2	20	CB	2B

# Step 4: Mix column

02	03	01	01	X	63	EB	9F	A0	→	BA	84	E8	1B
01	02	03	01		2F	93	92	C0		75	A4	8D	40
01	01	02	03		AF	C7	AB	30		F4	8D	06	7D
03	01	01	02		A2	20	CB	2B		7A	32	0E	5D

# Step 5: Add round key

BA	84	E8	1B
75	A4	8D	40
F4	8D	06	7D
7A	32	0E	5D

$\oplus$

E2	91	B1	D6
32	12	59	79
FC	91	E4	A2
F1	88	E6	93

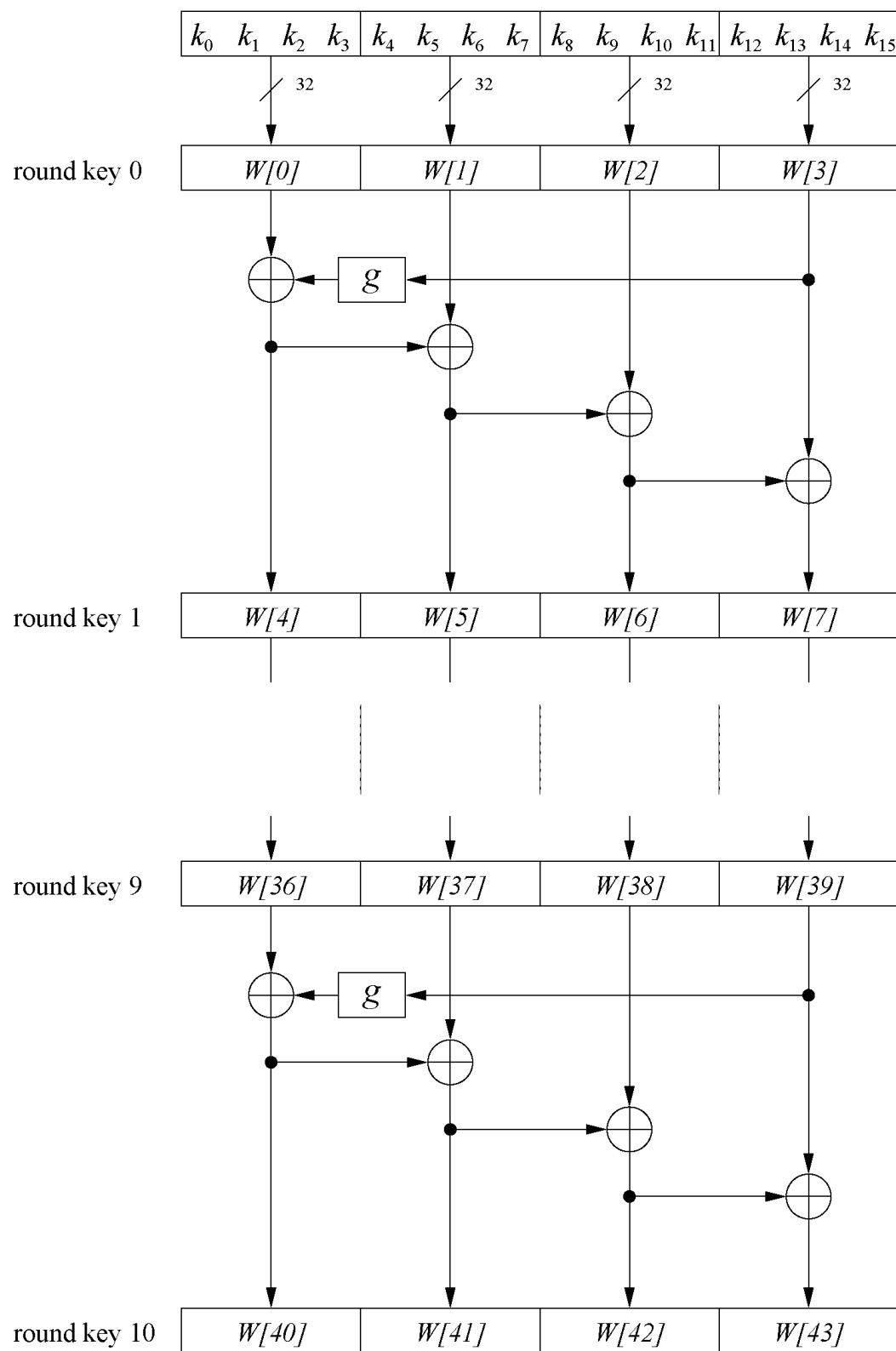
=

58	15	59	CD
47	B6	D4	39
08	1C	E2	DF
8B	BA	E8	CE



Input of next round

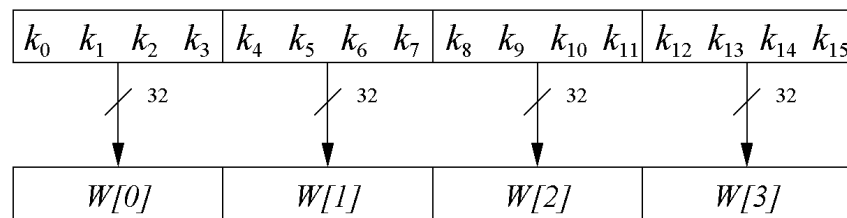
# Key Generation



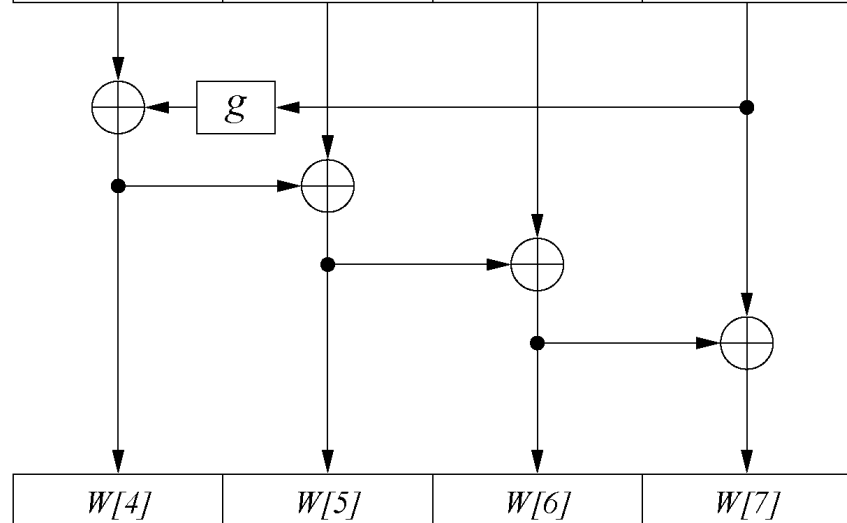
- Word-oriented: 1 word = 32 bits
- 11 subkeys are stored in  $W[0]...W[3]$ ,  $W[4]...W[7]$ , ...,  $W[40]...W[43]$
- First subkey  $W[0]...W[3]$  is the original AES key

Example: Key schedule for 128-bit key AES

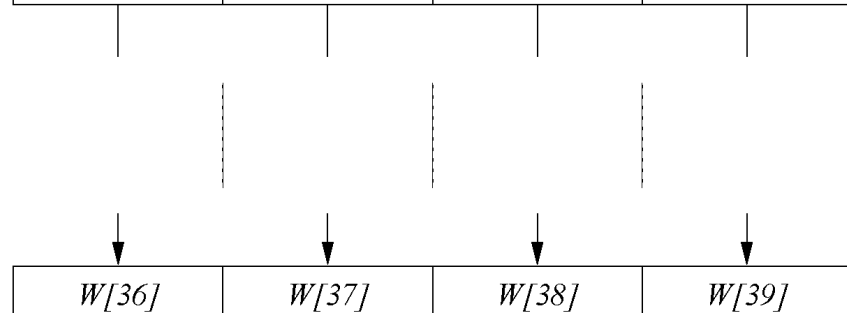
round key 0



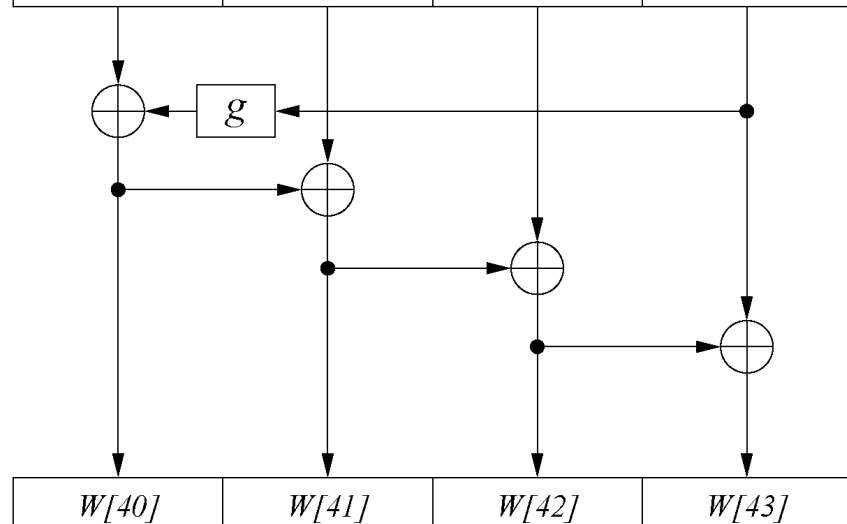
round key 1



round key 9



round key 10



## • G-Function

- **RotWord** is quite simple. It takes a 4-byte word  $[a_0, a_1, a_2, a_3]$  and returns  $[a_1, a_2, a_3, a_0]$
- **SubWord** is a little bit more complex. It takes a 4-byte word  $[a_1, a_2, a_3, a_0]$  and applies the AES S-Box to each of the bytes to produce a new 4-byte word  $[b_0, b_1, b_2, b_3]$ .
- The result of steps 1 and 2 is XORed with a round constant, **Rcon[j]**.



This question is about the Key Schedule of AES algorithm.

Let's assume the initial key is:  $(W[0] \ W[1] \ W[2] \ W[3]) = (4A \ C6 \ 9E \ 45)$  and the Round constant is  $00 \ 01 \ 10 \ 11$ .

You need to use the S-box of AES.

$$W[4] = W[0] \oplus g(W[3])$$

$$W[5] = W[1] \oplus W[4]$$

$$W[6] = W[2] \oplus W[5]$$

$$W[7] = W[3] \oplus W[6]$$

$$\begin{aligned} W[4] &= 4A \oplus g(W[3]) = 01001010 \oplus 00111011 \\ &= 01110001 = 71 \end{aligned}$$

$$\begin{aligned} W[5] &= C6 \oplus 71 = 11000110 \oplus 01110001 \\ &= 10110111 = B7 \end{aligned}$$

And so on...

$$g(W[3]) = g(45)$$

Step 1:

Rotate  $W[3]$  i.e 45 becomes 54

Step 2:

$$S\text{-box}(54) = 20$$

Step 3:

$$20 \oplus RC$$

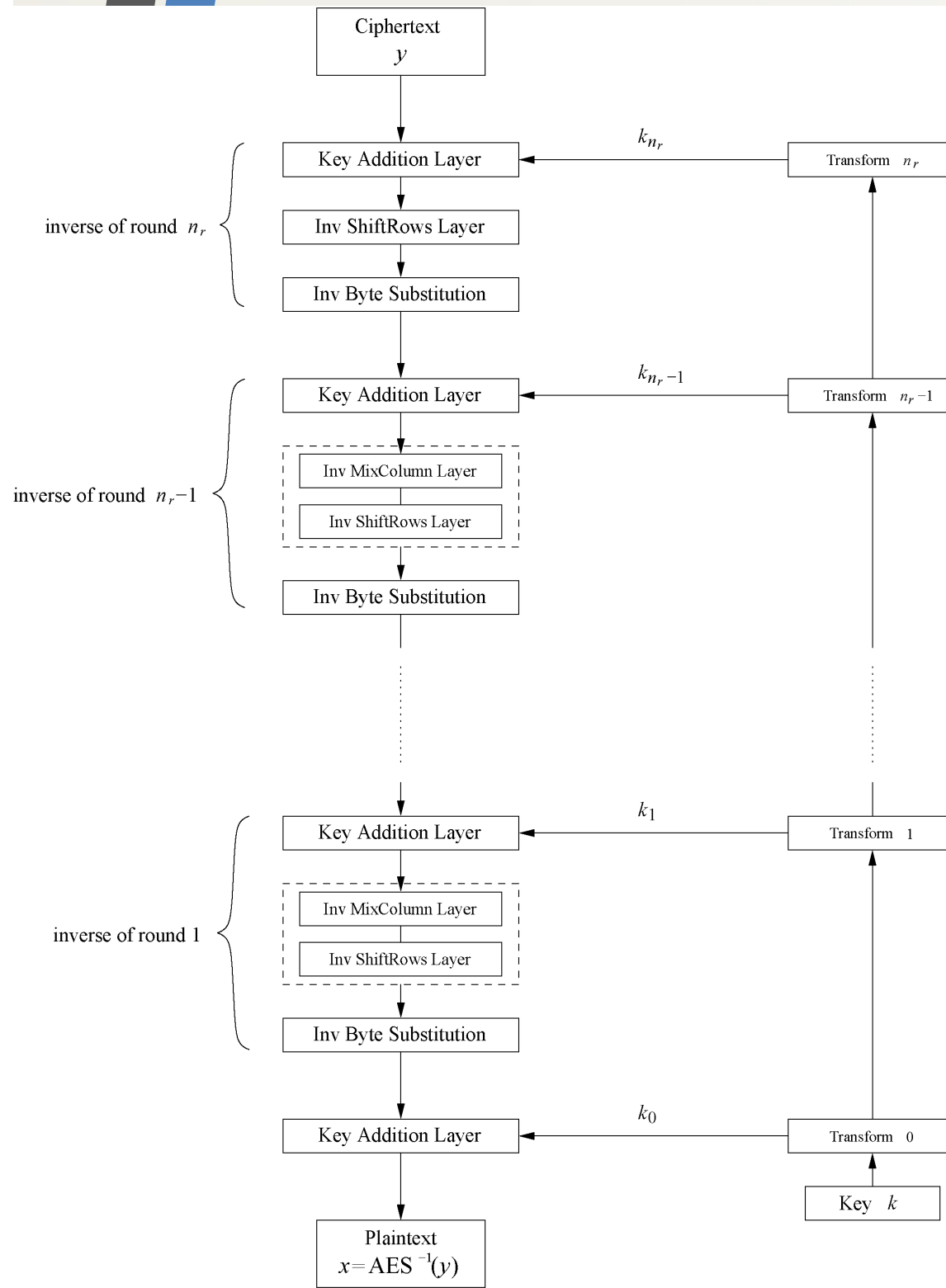
$$\begin{aligned} &= 00100000 \oplus 00011011 \\ &= 00111011 = g(W[3]) \end{aligned}$$



# AES Decryption

- To decrypt, process must be invertible
- Inverse of MixAddRoundKey is easy, since " $\oplus$ " is its own inverse
- MixColumn is invertible (inverse is also implemented as a lookup table)
- Inverse of ShiftRow is easy (cyclic shift the other direction)
- ByteSub is invertible (inverse is also implemented as a lookup table)

# AES Decryption



- AES is not based on a Feistel network
- ⇒ All layers must be inverted for decryption:
- MixColumn layer → Inv MixColumn layer
- ShiftRows layer → Inv ShiftRows layer
- Byte Substitution layer → Inv Byte Substitution layer
- Key Addition layer is its own inverse

		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

(b) Inverse S-box

# A Few Other Block Ciphers

- Briefly...
  - IDEA
  - Blowfish
  - RC6
- More detailed...
  - TEA

End of segment

# Modes of Operation

- Many modes — we discuss 3 most popular
- Electronic Codebook (**ECB**) mode
  - Encrypt each block independently
  - Most obvious approach, but a bad idea
- Cipher Block Chaining (**CBC**) mode
  - Chain the blocks together
  - More secure than ECB, virtually no extra work
- Counter Mode (**CTR**) mode
  - Block ciphers acts like a stream cipher
  - Popular for random access

# Data Integrity

- **Integrity** — detect unauthorized writing (i.e., detect unauthorized mod of data)
- Example: Inter-bank fund transfers
  - Confidentiality may be nice, integrity is *critical*
- Encryption provides **confidentiality** (prevents unauthorized disclosure)
- Encryption alone does **not** provide integrity
  - One-time pad, ECB cut-and-paste, etc., etc.

# MAC

- Message Authentication Code (MAC)
  - Used for data **integrity**
  - Integrity **not** the same as confidentiality
- MAC is computed as **CBC residue**
  - That is, compute CBC encryption, saving only final ciphertext block, the MAC
  - The MAC serves as a cryptographic checksum for data



# MAC Computation

- MAC computation (assuming N blocks)

$$C_0 = E(IV \oplus P_0, K),$$

$$C_1 = E(C_0 \oplus P_1, K),$$

$$C_2 = E(C_1 \oplus P_2, K), \dots$$

$$C_{N-1} = E(C_{N-2} \oplus P_{N-1}, K) = \text{MAC}$$

- Send IV,  $P_0, P_1, \dots, P_{N-1}$  and MAC
- Receiver does same computation and verifies that result agrees with MAC
- Both sender and receiver must know K

# Does a MAC work?


- Suppose Alice has 4 plaintext blocks
- Alice computes  
 $C_0 = E(IV \oplus P_0, K)$ ,  $C_1 = E(C_0 \oplus P_1, K)$ ,  
 $C_2 = E(C_1 \oplus P_2, K)$ ,  $C_3 = E(C_2 \oplus P_3, K) = \text{MAC}$
- Alice sends IV,  $P_0$ ,  $P_1$ ,  $P_2$ ,  $P_3$  and **MAC** to Bob
- Suppose Trudy changes  $P_1$  to X
- Bob computes  
 $C_0 = E(IV \oplus P_0, K)$ ,  $C_1 = E(C_0 \oplus X, K)$ ,  
 $C_2 = E(C_1 \oplus P_2, K)$ ,  $C_3 = E(C_2 \oplus P_3, K) = \text{MAC} \neq \text{MAC}$
- It works since error propagates into **MAC**
- Trudy can't make **MAC** == **MAC** without K

# Confidentiality and Integrity

- Encrypt with one key, MAC with another key
- Why not use the same key?
  - Send last encrypted block (MAC) twice?
  - This cannot add any security!
- Using different keys to encrypt and compute MAC works, even if keys are related
  - But, twice as much work as encryption alone
  - Can do a little better — about 1.5 “encryptions”
- Confidentiality *and* integrity with same work as one encryption is a research topic

# Uses for Symmetric Crypto

Decorative blue and grey diagonal stripes on the left side of the slide.

- 
- A horizontal purple arrow pointing from the title area towards the list.
- Confidentiality
    - Transmitting data over insecure channel
    - Secure storage on insecure media
  - Integrity (MAC)
  - Authentication protocols (later...)
  - Anything you can do with a hash function (upcoming chapter...)

- End