CHAPTER 4
PUBLIC KEY CRYPTO

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Inspiring Excellence

APPENDIX

DIFFIE-HELLMAN KEY EXCHANGE

ELLIPTIC CURVE CRYPTOGRAPHY

USES FOR PUBLIC KEY CRYPTO

INFORMATION HIDING

STEGANOGRAPHY

HASH FUNCTION

Diffie-Hellman Key exchange

DIFFIE-HELLMAN

- Invented by Williamson (GCHQ) and, independently, by D and H (Stanford)
- A "key exchange" algorithm
  - Used to establish a shared symmetric key
- **Not for encrypting or signing**

- Security rests on difficulty of discrete log problem: (Not known: NP-complete)

  given g, p, and $g^k \bmod p \rightarrow$ find k

DIFFIE-HELLMAN

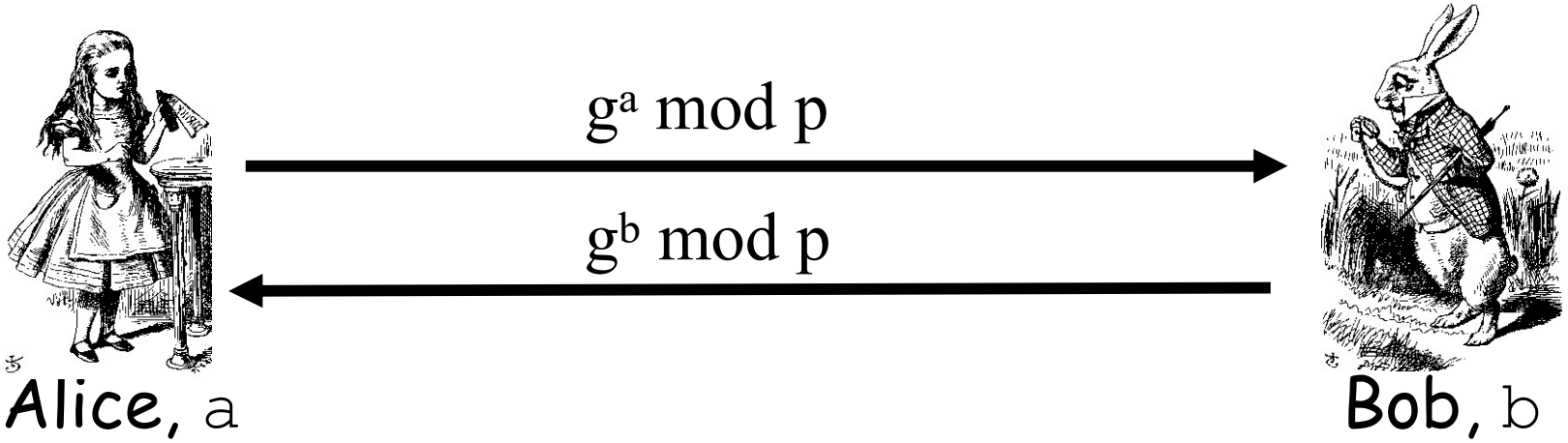
- Let p be prime, let g be a **generator** (p,g are public)
  - For any $x \in \{1,2,\ldots,p-1\}$ there is n s.t. $x = g^n \bmod p$
- Alice selects secret value a
- Bob selects secret value b
- Alice sends $g^a \bmod p$ to Bob
- Bob sends $g^b \bmod p$ to Alice
- Both compute shared secret $g^{ab} \bmod p$
  - $(g^b)^a = g^{ba} = g^{ab} \bmod p$
- Shared secret can be used as symmetric key

DIFFIE-HELLMAN

- Suppose that Bob and Alice use $g^{ab} \bmod p$ as a symmetric key
- Trudy can see $g^a \bmod p$ and $g^b \bmod p$
- Note $g^a g^b \bmod p = g^{a+b} \bmod p \neq g^{ab} \bmod p$
- If Trudy can find a or b, system is broken
- If Trudy can solve **discrete log** problem, then she can find a or b

DIFFIE-HELLMAN

- **Public:** g and p
- **Secret:** Alice's exponent a, Bob's exponent b



- Alice computes $(g^b)^a = g^{ba} = g^{ab} \bmod p$
- Bob computes $(g^a)^b = g^{ab} \bmod p$
- Could use $K = g^{ab} \bmod p$ as symmetric key

DIFFIE-HELLMAN KEY EXCHANGE: EXAMPLE

Domain parameters p=29, g=2

Alice		Bob
Choose random private key $a = 5$		Choose random private key $b = 12$
Compute corresponding public key $A = g^a = 2^5 = 3 \bmod 29$	A →	
		Compute correspondig public key $B = g^b = 2^{12} = 7 \bmod 29$
	← B	
Compute common secret $k_{AB} = B^a = g^{ba} = 7^5 = 16 \bmod 29$		Compute common secret $k_{AB} = A^b = g^{ab} = 3^{12} = 16 \bmod 29$

Proof of correctness:

Alice computes: $B^a = (g^b)^a \mod p$
Bob computes: $A^b = (g^a)^b \mod p$

i.e., Alice and Bob compute the same key k_{AB}!

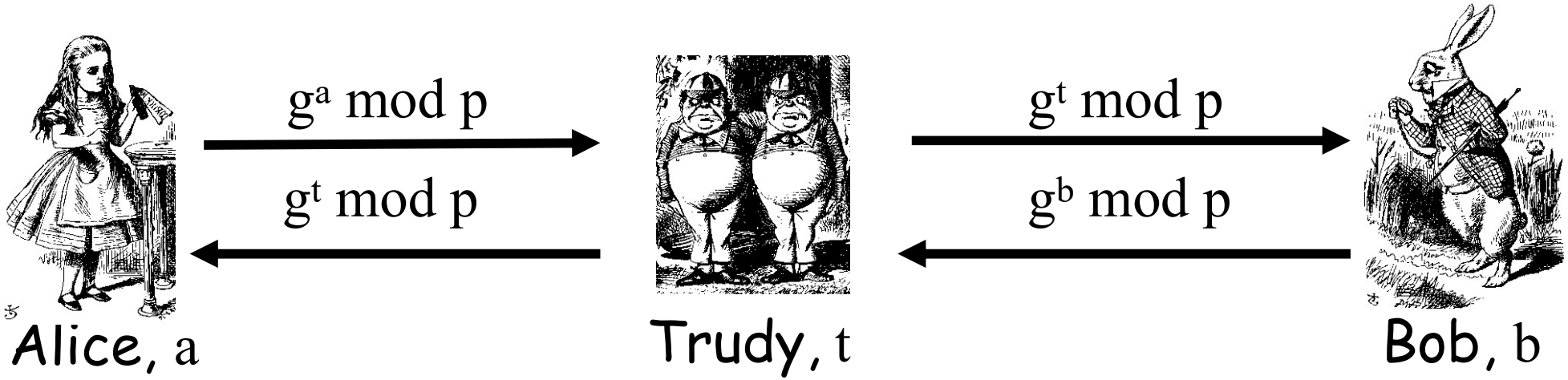
Alice		Bob
Choose random private key $a \in \{1,2,\ldots,p\text{-}1\}$		Choose random private key $b \in \{1,2,\ldots,p\text{-}1\}$
Compute corresponding public key $A = \alpha^a \bmod p$	A →	
	← B	Compute correspondig public key $B = \alpha^b \bmod p$
Compute common secret $k_{AB} = B^a = (g^a)^b \bmod p$		Compute common secret $k_{AB} = A^b = (g^b)^a \bmod p$

We can now use the joint key k_{AB} for encryption, e.g., with AES

$$y = AES_{kAB}(x) \quad \xrightarrow{\;y\;} \quad x = AES^{-1}_{kAB}(y)$$

DIFFIE-HELLMAN

- Subject to **man-in-the-middle (MiM)** attack



- Trudy shares secret $g^{at} \bmod p$ with Alice
- Trudy shares secret $g^{bt} \bmod p$ with Bob
- Alice and Bob don't know Trudy exists!

DIFFIE-HELLMAN

- How to prevent MiM attack?
- Solutions
  1. Encrypt DH exchange with symmetric key
  2. Encrypt DH exchange with public key
  3. Sign DH values with private key
  4. Other?
- You **MUST** be aware of MiM attack on Diffie-Hellman

ECC:
ELLIPTIC CURVE CRYPTOGRAPHY

ELLIPTIC CURVE CRYPTO (ECC)

- "Elliptic curve" is **not** a cryptosystem
  - Elliptic curves are a different way to do the math in public key system
- Elliptic curve versions of DH, RSA, etc.
- Elliptic curves may be more efficient
  - Fewer bits needed for same security
  - But the operations are more complex

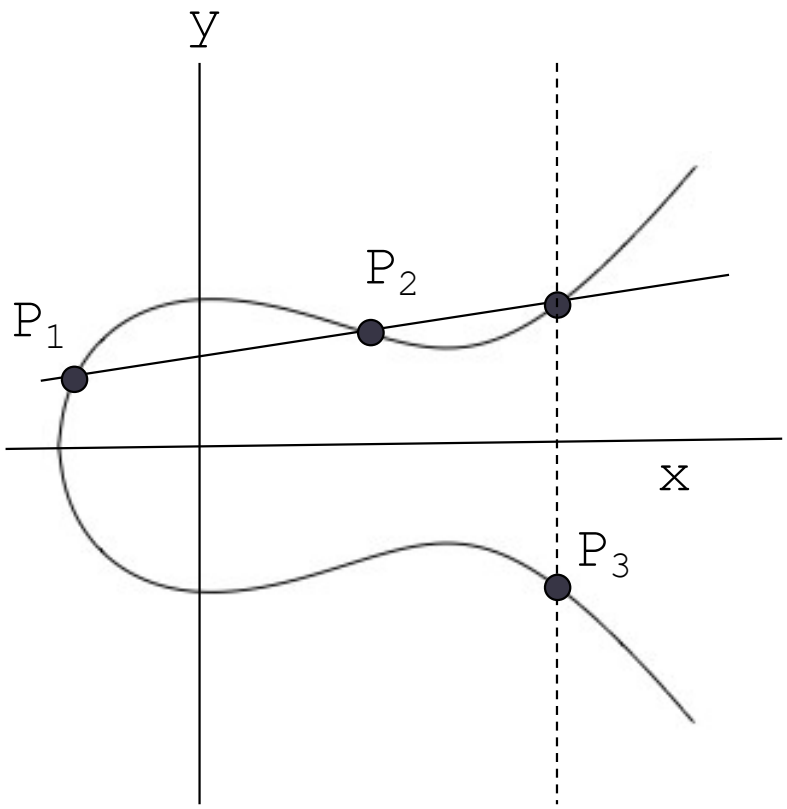
WHAT IS AN ELLIPTIC CURVE?

- An elliptic curve E is the graph of an equation of the form

$$y^2 = x^3 + ax + b$$

- Also includes a "point at infinity" : ∞
- What do elliptic curves look like?
- See the next slide!

ELLIPTIC CURVE PICTURE



- Consider elliptic curve

  E: $y^2 = x^3 - x + 1$

- If P_1 and P_2 are on E, we can define

  $P_3 = P_1 + P_2$

  as shown in picture

- **Addition is all we need**

- For discrete points, we add "mod p" to the EC

KEY SIZE COMPARISON

Symmetric	*ECC*	*RSA, DL*	*Remark*
64 Bit	128 Bit	≈ 700 Bit	Only short term security (a few hours or days)
80 Bit	160 Bit	≈ 1024 Bit	Medium security (except attacks from big governmental institutions etc.)
128 Bit	256 Bit	≈ 3072 Bit	Long term security (without quantum computers)

USES FOR PUBLIC KEY CRYPTO

USES FOR PUBLIC KEY CRYPTO

- Confidentiality
  - Transmitting data over insecure channel
  - Secure storage on insecure media
- Authentication (later)
- Digital signature provides integrity and **non-repudiation**
  - **No** non-repudiation with symmetric keys
  - Who has the secret key is the key for non-repudiation.

NON-NON-REPUDIATION

- Alice orders 100 shares of stock from Bob
- Alice computes **MAC** using symmetric key
- Stock drops, Alice claims she did not order
- Can Bob prove that Alice placed the order?
- **No!** Since Bob also knows symmetric key, he could have forged message
- **Problem:** Bob knows Alice placed the order, but he can't prove it

NON-REPUDIATION

- Alice orders 100 shares of stock from Bob
- Alice **signs** order with her private key
- Stock drops, Alice claims she did not order
- Can Bob prove that Alice placed the order?
- **Yes!** Only someone with Alice's private key could have signed the order
- This assumes Alice's private key is not stolen (revocation problem)

PUBLIC KEY NOTATION

- **Sign** message M with Alice's **private key:** $[M]_{Alice}$
- **Encrypt** message M with Alice's **public key:** $\{M\}_{Alice}$
- Then

$$\{[M]_{Alice}\}_{Alice} = M$$

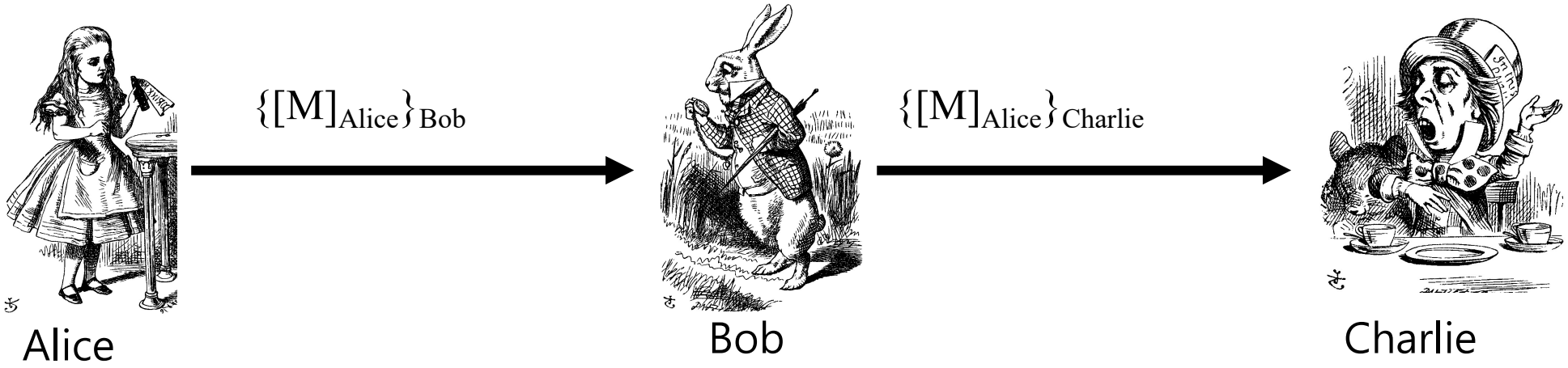
$$[\{M\}_{Alice}]_{Alice} = M$$

CONFIDENTIALITY AND NON-REPUDIATION?

- Suppose that we want confidentiality and integrity/non-repudiation
- Can public key crypto achieve both?
- Alice sends message to Bob
  - **Sign and encrypt:** $\{[M]_{Alice}\}_{Bob}$
  - **Encrypt and sign:** $[\{M\}_{Bob}]_{Alice}$
- Can the order possibly matter?

SIGN AND ENCRYPT

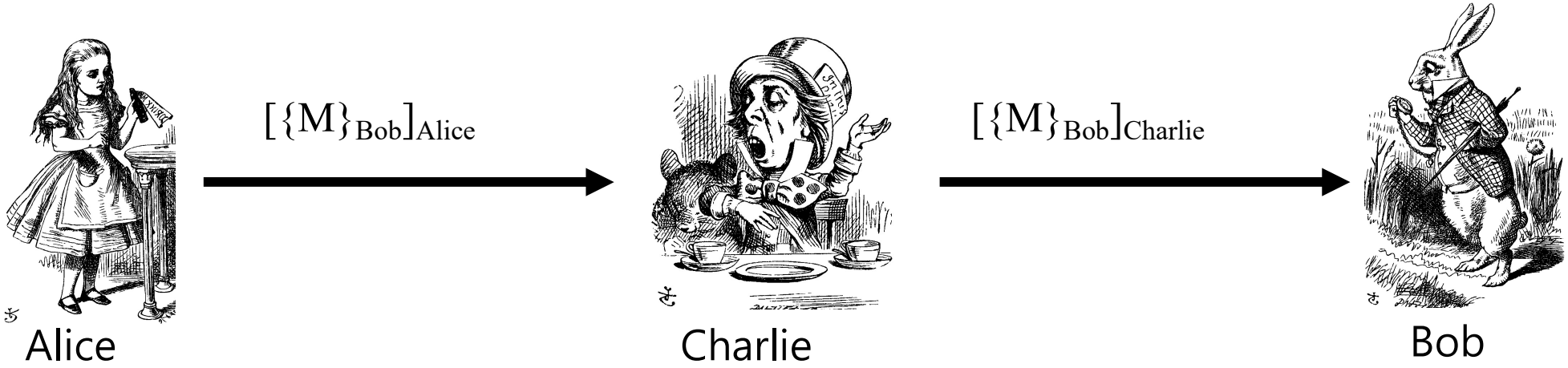
- M = "I love you"



- **Q:** What's the problem?
- **A:** No problem — public key is public

ENCRYPT AND SIGN

- M = "My theory, which is mine…."



- **Note** that Charlie cannot decrypt M
- **Q:** What is the problem?
- **A:** No problem — public key is public

PUBLIC KEY CERTIFICATE

- Digital **certificate** contains name of user and user's public key (possibly other info too)
- It is ***signed*** by the issuer, a ***Certificate Authority*** (CA), such as VeriSign

  M = (Alice, Alice's public key), $S = [M]_{CA}$

  Alice's Certificate = (M, S)
- Signature on certificate is verified using CA's public key

  Must verify that $M = \{S\}_{CA}$

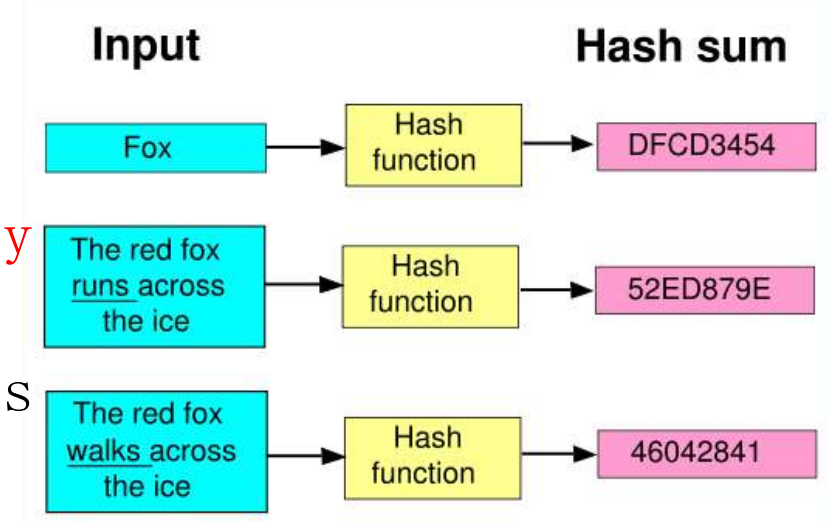
CERTIFICATE AUTHORITY

- Certificate authority (CA) is a trusted 3rd party (TTP) — creates and signs certificates
- Verify signature to verify **integrity** & identity of **owner of corresponding private key**
  - Does **not** verify the identity of the **sender** of certificate — certificates are public!
- Big problem if CA makes a mistake
  - CA once issued Microsoft cert. to someone else
- A common format for certificates is X.509

HASH FUNCTION?

- Hash function
  - a reproducible method of turning some kind of data into a (relatively) small number that may serve as a digital "fingerprint" of the data.

- Crypto Hash function
  - a hash function with certain additional security properties to make it suitable for use as various info security applications



HASH FUNCTION MOTIVATION

- Suppose Alice signs **M**
  - Alice sends M and S = $[M]_{Alice}$ to Bob
  - Bob verifies that M = $\{S\}_{Alice}$
  - Aside: Is it OK to just send S?
- If M is big, $[M]_{Alice}$ is costly to compute
- Suppose instead, Alice signs h(M), where h(M) is much smaller than M
  - Alice sends M and S = $[h(M)]_{Alice}$ to Bob
  - Bob verifies that h(M) = $\{S\}_{Alice}$

CRYPTO HASH FUNCTION

- Crypto hash function h(x) must provide the following properties
- **Compression**
  - output length is small
- **Efficiency**
  - h(x) easy to computer for any x
- **One-way**
  - given a value y it is infeasible to find an x such that h(x) = y

CRYPTO HASH FUNCTION

- **Weak collision resistance**
  - given x and h(x), infeasible **to find y** with $y \neq x$ such that $h(y) = h(x)$
- **Strong collision resistance**
  - infeasible **to find any x and y,** with $x \neq y$ such that $h(x) = h(y)$
- Lots of collisions exist, but hard to find one

POPULAR CRYPTO HASHES

- **MD(Message Digest) 5**
  - invented by Rivest
  - 128 bit output
  - MD2 → MD4 → MD5
  - MD2 and MD4 are no longer secure, due to collision found
  - Note: even MD5, collision recently found

POPULAR CRYPTO HASHES

- **SHA(Secure Hash Algorithm)-1**
  - A US government standard (similar to MD5)
  - "The world's most popular hash function"
  - 180 bit output
  - SHA-0 → SHA-1
- Many others hashes, but MD5 and SHA-1 most widely used

Hash usages

HASH USES

- Authentication (HMAC)
- Message integrity (HMAC)
- Message fingerprint
- Data corruption detection
- Digital signature efficiency
- Anything you can do with symmetric crypto ???

ONLINE AUCTION

- Suppose Alice, Bob and Charlie are bidders
- Alice plans to bid A, Bob B and Charlie C
- They don't trust that bids will stay secret
- Solution?
  - Alice, Bob, Charlie submit **hashes** h(A), h(B), h(C)
  - All hashes received and posted online
  - Then bids A, B and C revealed
- Hashes don't reveal bids (one way)
- Can't change bid after hash sent (collision)

INFORMATION HIDING DIGITAL WATERMARK

- Digital Watermarks
  - The "Digital Watermarking" name from watermarking of paper or money as a security measure
  - A technique which allows an individual to add hidden copyright notices to digital audio, video, or image signals and documents
  - Defense against music or software piracy
  - Example: Add "invisible" identifier to data
  - Digital watermarking can be a form of steganography

INFORMATION HIDING DIGITAL WATERMARK

- Add a "mark" to data
- Several types of watermarks
- Type 1
  - Invisible — Not obvious the mark exists
  - Visible — Such as **TOP SECRET** stamp
- Type 2
  - Robust — Readable even if attacked
  - Fragile — Mark destroyed if attacked

INFORMATION HIDING DIGITAL WATERMARK

- Add robust invisible mark to digital music
  - If pirated music appears on Internet, can trace it back to original source
- Add fragile invisible mark to audio file
  - If watermark is unreadable, recipient knows that audio has been tampered (integrity)
- Combinations of several types are sometimes used
  - E.g., visible plus robust invisible watermarks

INFORMATION HIDING STEGANOGRAPHY

- Steganography – "Hidden writing"
  - The art and science of writing hidden messages
    - recipient does not know of the existence of the mssg
  - Hide the fact that information is being transmitted – a kind of covert channel (Ch8)
    - Secret communication channel
    - Cryptography, where the existence of the message itself is not disguised, but the content is obscured.
  - Example: Hide data in image or music file

INFORMATION HIDING STEGANOGRAPHY

- According to Herodotus (Greece 440BC)
  - Shaved slave's head
  - Wrote message on head
  - Let hair grow back
  - Send slave to deliver message
  - Shave slave's head to expose message (warning of Persian invasion)
- Historically, Steganography has been used more than cryptography!

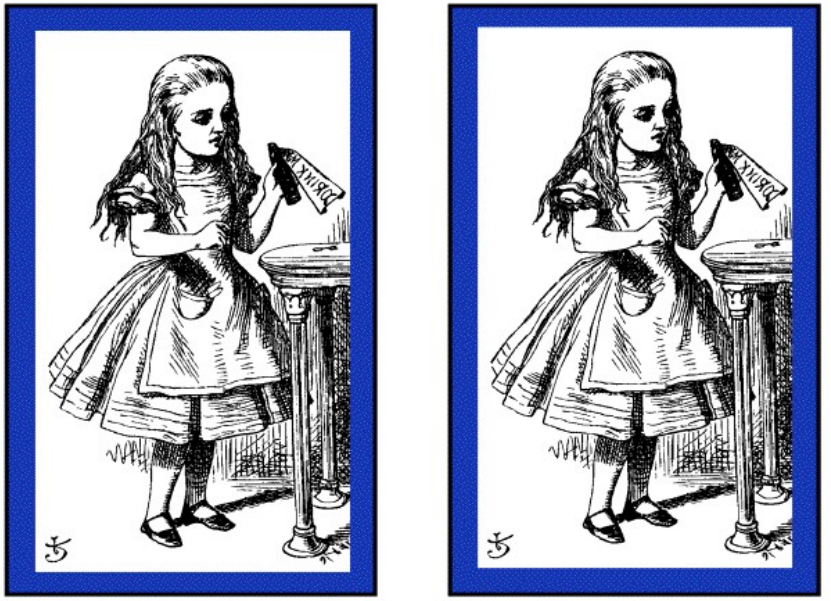
INFORMATION HIDING STEG EXAMPLE

- Images use 24 bits for color: **RGB**
  - 8 bits for red, 8 for green, 8 for blue
- For example
  - **0x7E 0x52 0x90** is this color
  - **0xFE 0x52 0x90** is this color
- While
  - **0xAB 0x33 0xF0** is this color
  - **0xAB 0x33 0xF1** is this color
- Low-order bits are unimportant!

INFORMATION HIDING STEG EXAMPLE

- Given an uncompressed image file
  - For example, BMP format
- Then we can insert any information into low-order RGB bits
- Since low-order RGB bits don't matter, result will be "invisible" to human eye
- But a computer program can "see" the bits

INFORMATION HIDING STEG EXAMPLE 1



- Left side: plain Alice image
- Right side: Alice with entire *Alice in Wonderland* (pdf) "hidden" in image

INFORMATION HIDING STEG EXAMPLE 2

Non-Steganography Example

- Walrus.html in web browser

"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- View source

```
<font color="#000000">"The time has come," the Walrus said,</font><br>
<font color="#000000">"To talk of many things:</font><br>
<font color="#000000">Of shoes and ships and sealing wax</font><br>
<font color="#000000">Of cabbages and kings</font><br>
<font color="#000000">And why the sea is boiling hot</font><br>
<font color="#000000">And whether pigs have wings."</font><br>
```

INFORMATION HIDING STEG EXAMPLE 2

- stegoWalrus.html in web browser

"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- View source

```
<font color="#010100">"The time has come," the Walrus said,</font><br>
<font color="#000100">"To talk of many things:</font><br>
<font color="#010100">Of shoes and ships and sealing wax</font><br>
<font color="#000101">Of cabbages and kings</font><br>
<font color="#000000">And why the sea is boiling hot</font><br>
<font color="#010001">And whether pigs have wings."</font><br>
```

- "Hidden" message: **110 010 110 011 000 101**

INFORMATION HIDING STEGANOGRAPHY

- Some formats (jpg, gif, wav, etc.) are more difficult (than html) for humans to read
- Easy to hide information in **unimportant bits**
- Easy to **destroy** or remove info stored in unimportant bits!

INFORMATION HIDING STEGANOGRAPHY

- To be robust, information must be stored in **important bits**
- But stored information must not damage data!
- Collusion attacks also a major concern
  - The original and watermarked object can be compared
- Robust steganography is trickier than it seems

THE BOTTOM LINE OF INF HIDING

- If information hiding is suspected
  - Attacker can probably make information/watermark unreadable
  - Attacker may be able to read the information, given the original document (image, audio, etc.)

CHAPTER 5
HASH FUNCTIONS

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- SHA -Secure Hash Algorithm
- MD5 – Message Digest
- The process
  - Sender use MD5
  - Append Message Digest to plain text
  - Send it to receiver
  - Receiver compute with MD5
  - Receiver compare MD5, MD5

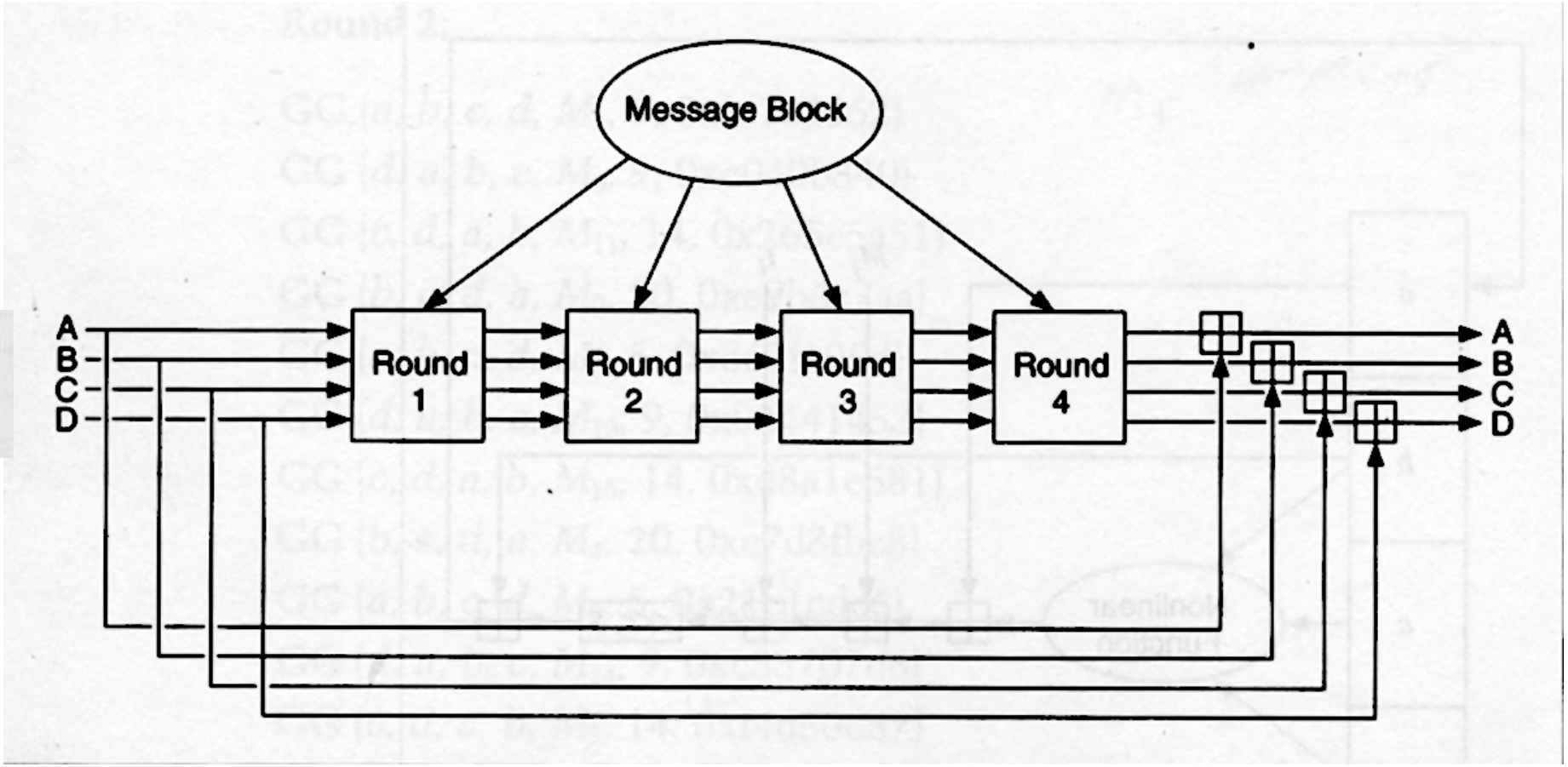
MD5 ALGORITHM (128 BIT)

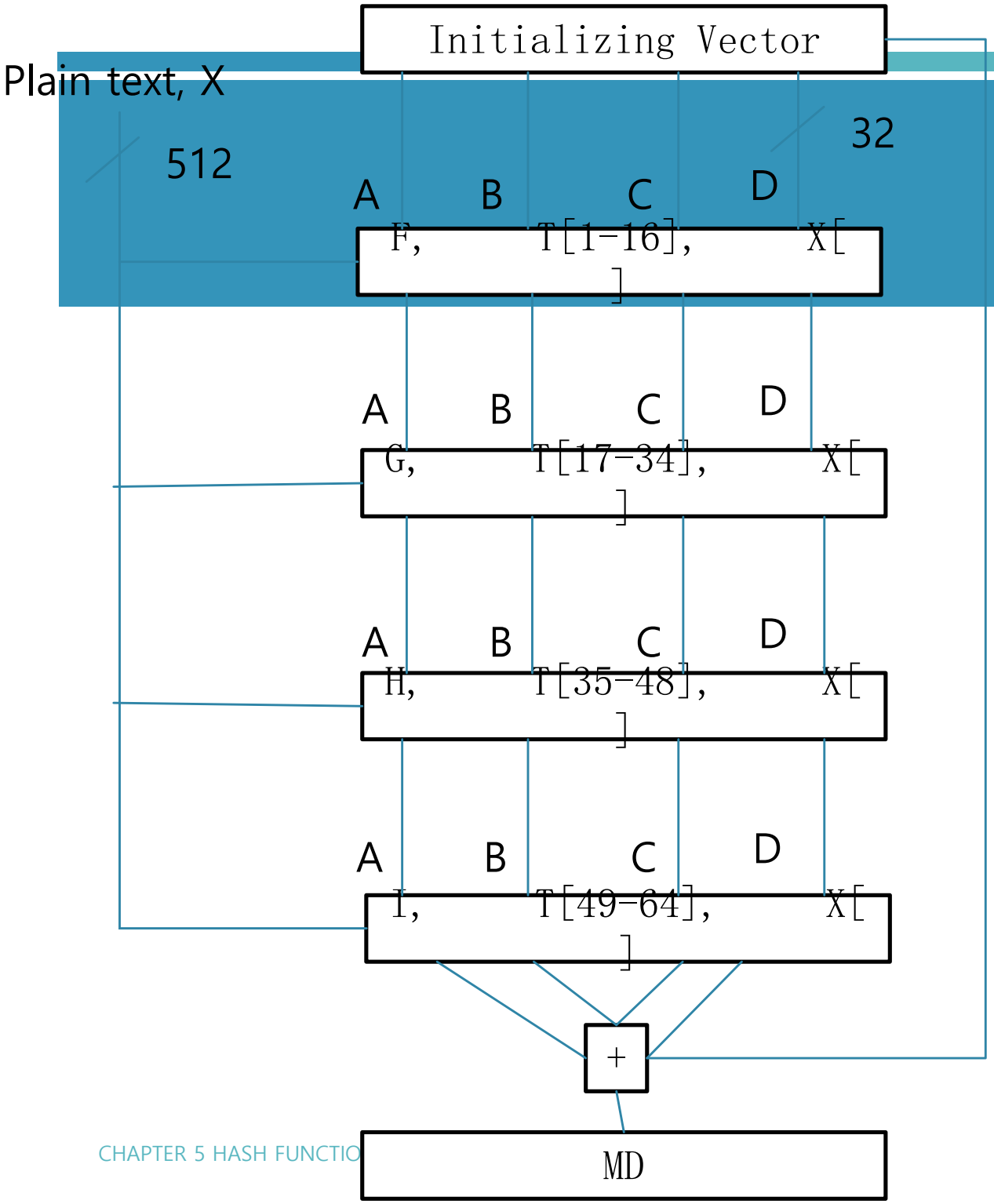
1. The Message is padded to an exact multiple of 512 bit blocks.
    a. Append 64 bit representation
2. Initiate the MD buffers (32-bit, 4 buffer, A, B, C, D)
3. Process the each block (512)
4. Output (message digest in buffers)

MAIN MD5 LOOP

IVs

$A = 0x01234567$
$B = 0x89abcdef$
$C = 0xfedcba98$
$D = 0x76543210$





Each round have 16 constraints and 16 steps. Each constraint will use in each step.

Uses 4 32-bit inputs: *a, b, c, d*
Also uses a 32-bit sub-block of the message block
Generates 4 32-bit outputs for the next round step or the next round

DIFFICULTY IN CRACKING

- Md5, with its 128bit encryption algorithm has 1,280,000,000,000,000,000 possible combinations.
- Even if the exact same hash value found, possible other string combination could have created it.
- It is considered that the md5 message digest would take an unrealistic time to crack via brute force attack.

PROS/CONS MD5

- Easy to use
- Widely used
- Considered secure
- Difficult to crack

- Is susceptible to brute force attacks
- Hash collisions is a known flaw
- Quantum computers would make such an algorithm worthless