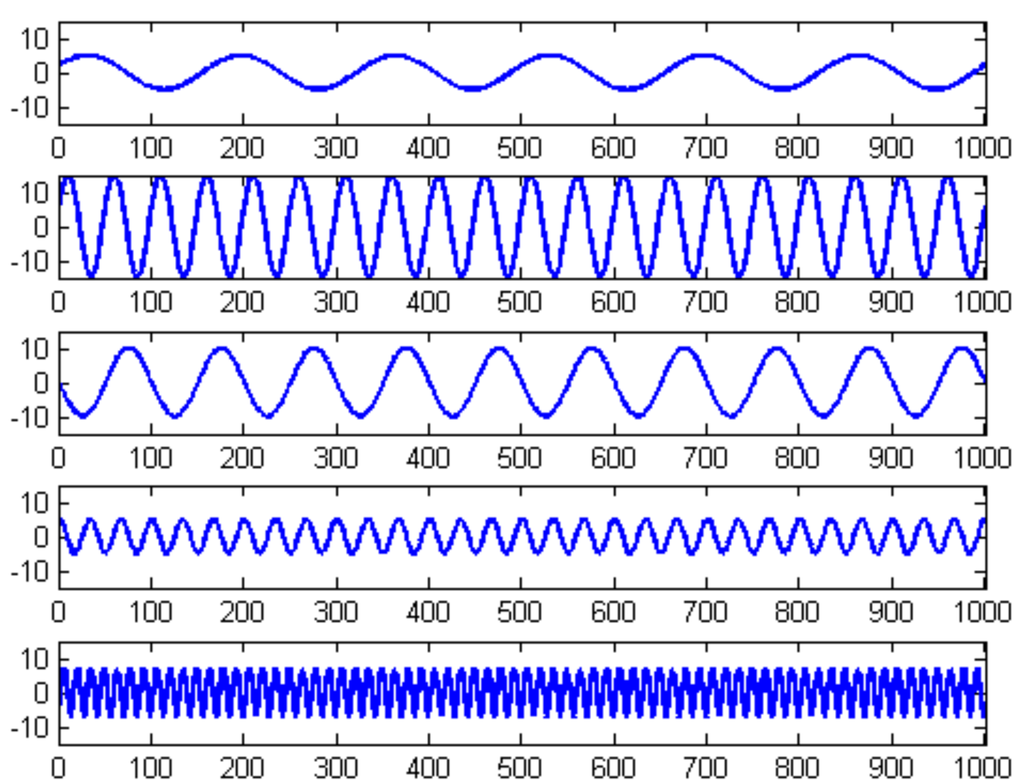
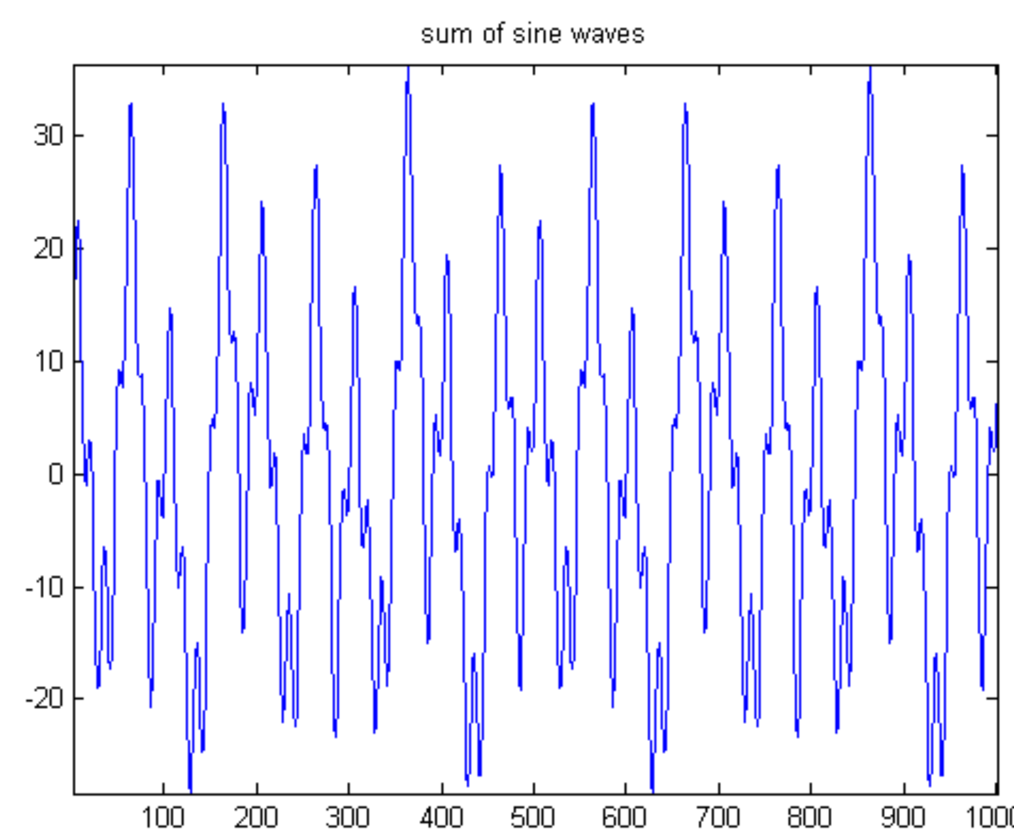
Fourier Transform

- signals can be expressed as the sum of a series of sine/cosine waves of specific amplitude and phase
- computing the Fourier transform of a signal allows us to extract the ‘amount’ of signal at different frequencies





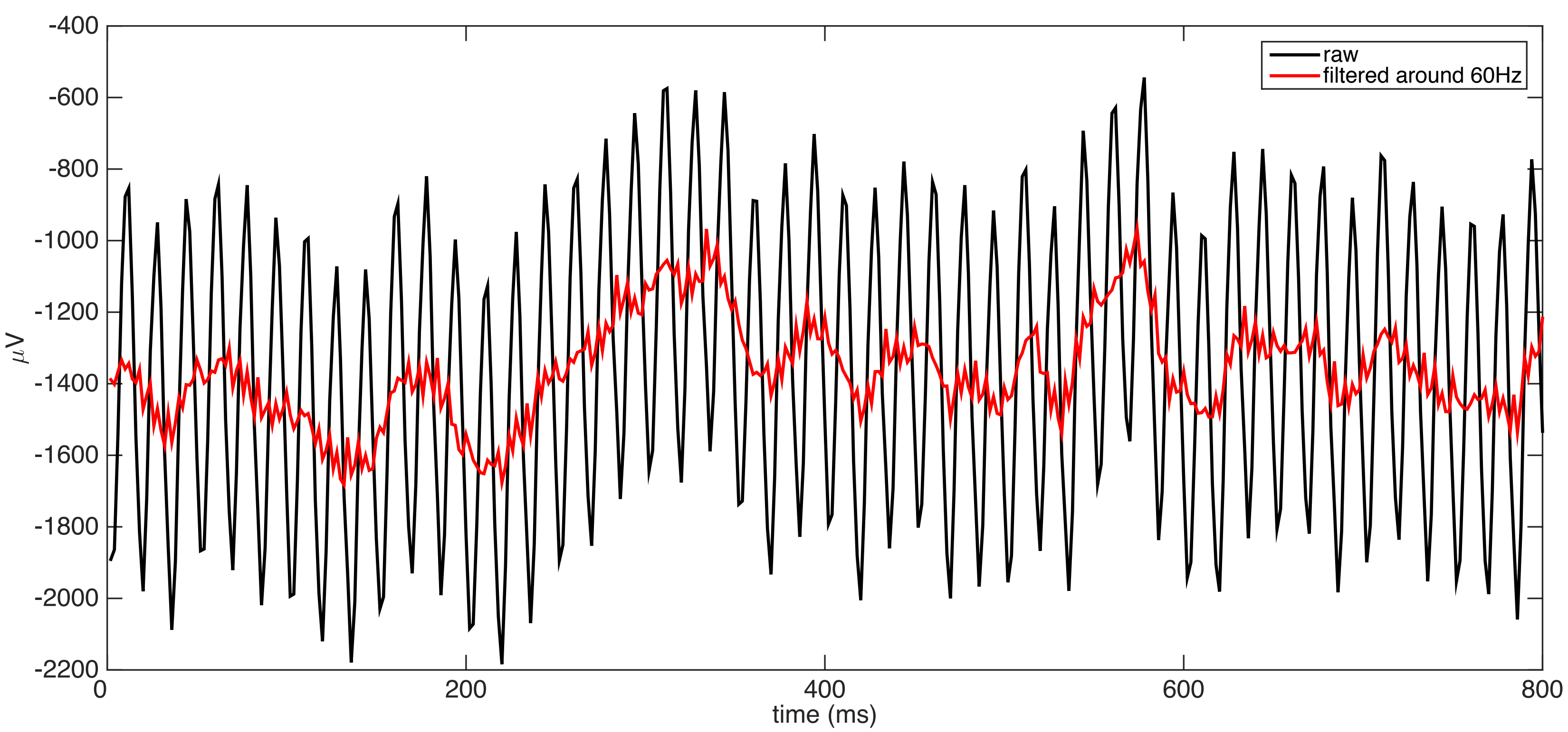
Fourier Transform

$$F(\omega) = \int_{-\infty}^{\infty} \underbrace{f(t)}_{\text{your timeseries signal}} \underbrace{e^{-i\omega t}}_{\text{sinusoid (more about this later)}} dt$$

Fourier transform (FT) is the product of a **signal** with a **sinusoid** at a particular frequency, summed (integrated) across all time…should sound familiar

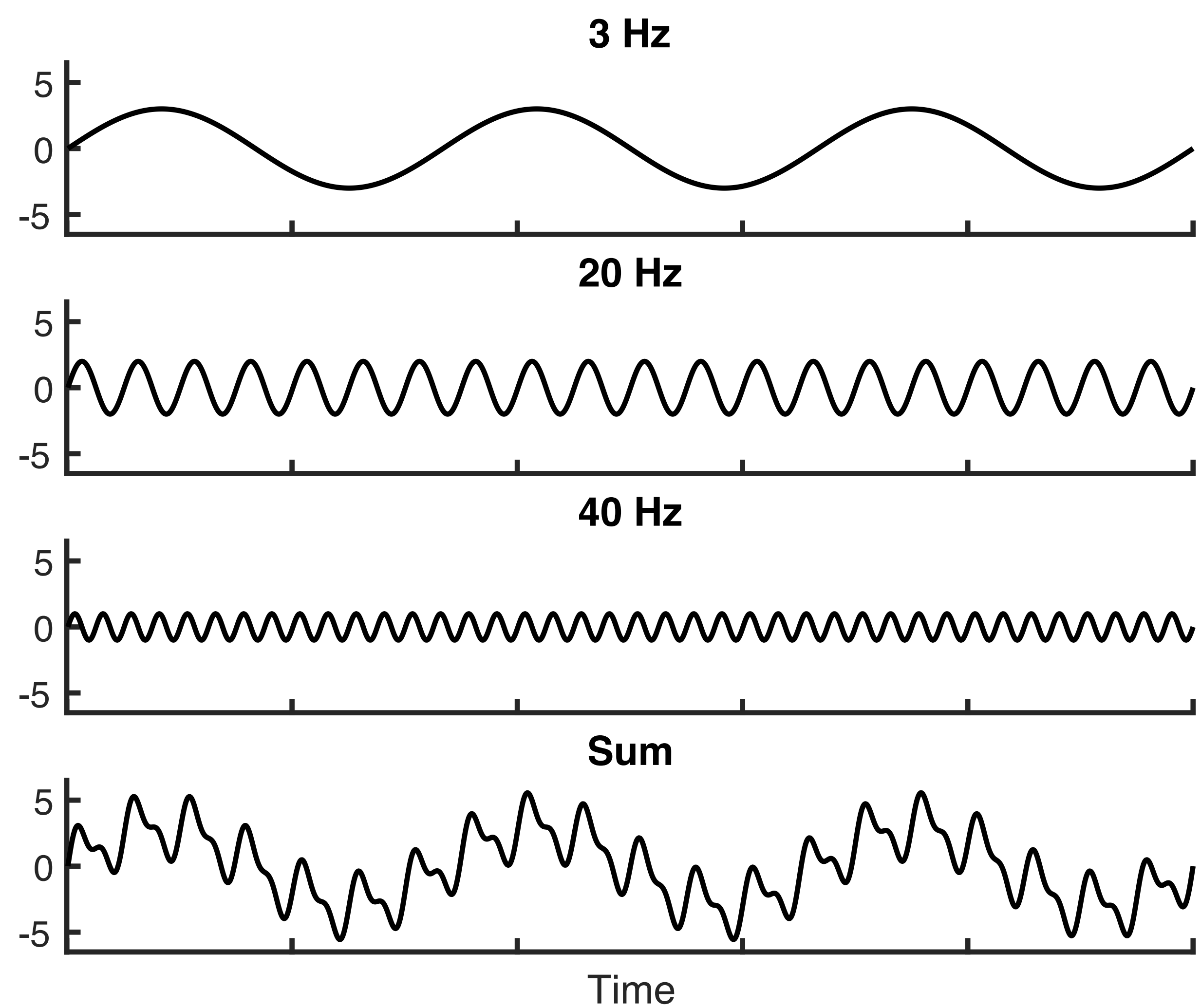
Fourier Transform

- Fourier analysis is the foundation of all signal processing
  - Analyze components of a signal (what are the frequencies at which the brain oscillates?)
  - Filter out components you don't want (electrical noise from the environment)



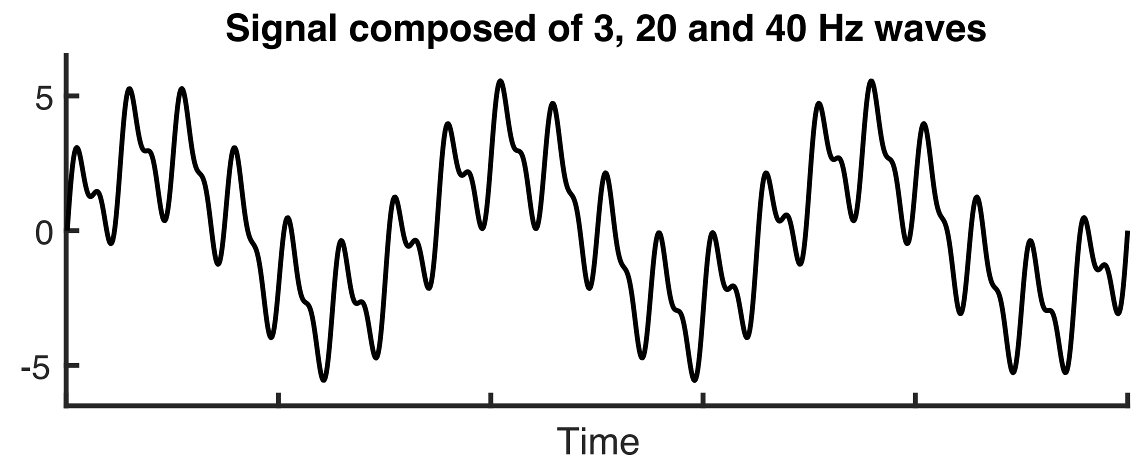
Fourier Transform

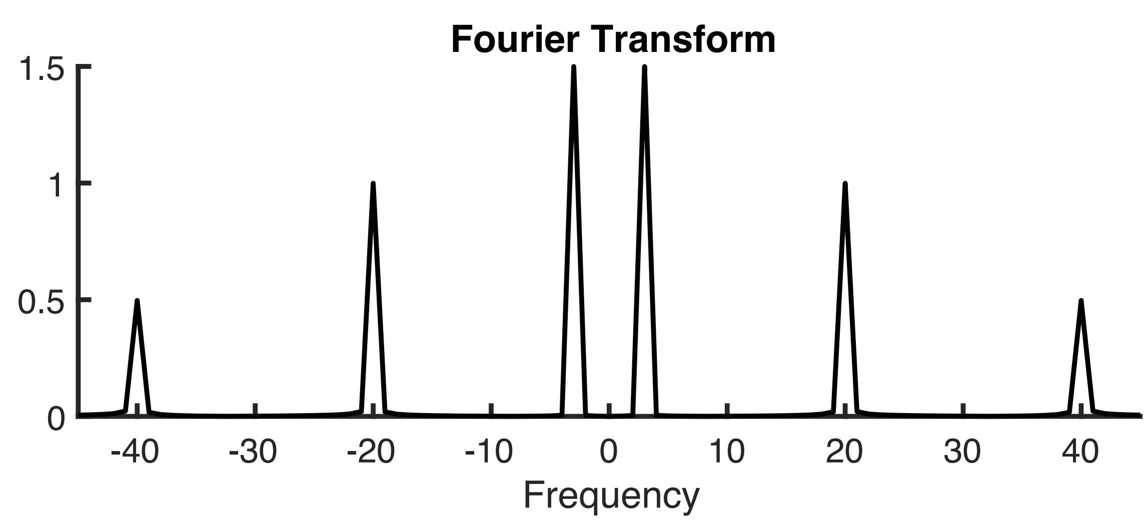
sum of 3, 20 and 40 Hz
sine waves



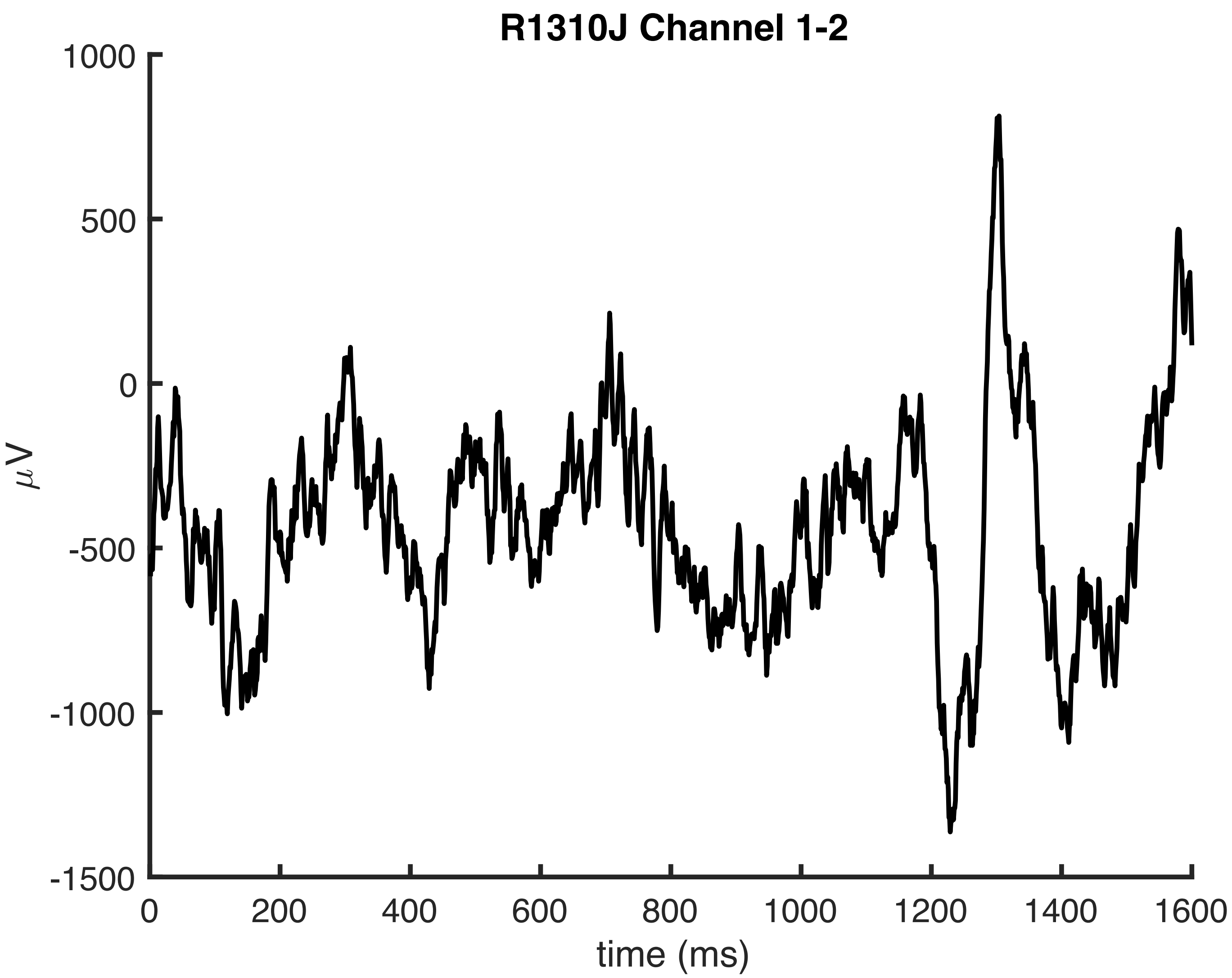
Fourier Transform

FT of the sum (same as
sum of the FTs)

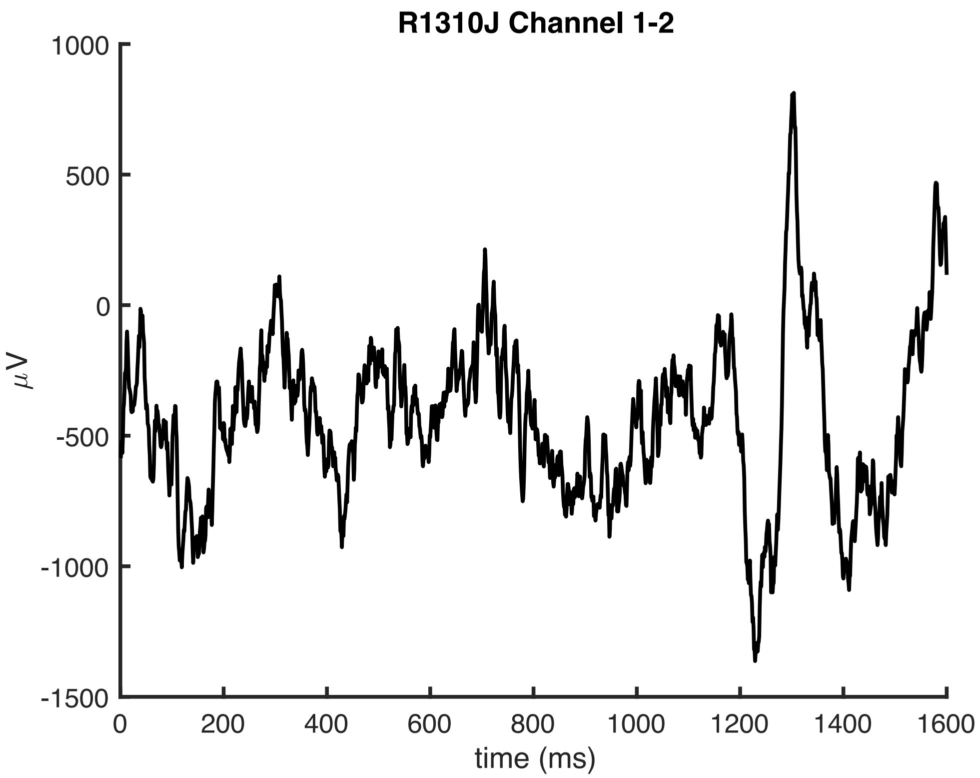




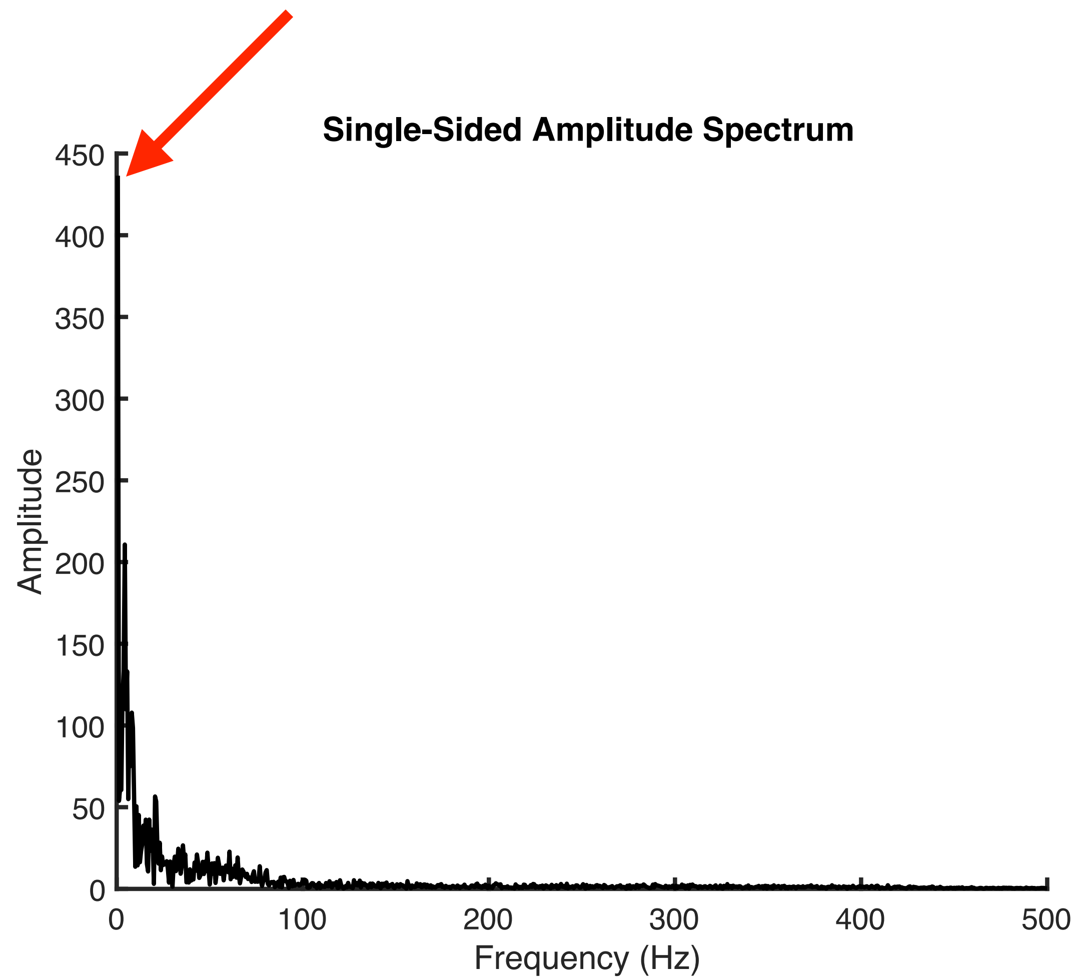
Fourier Transform



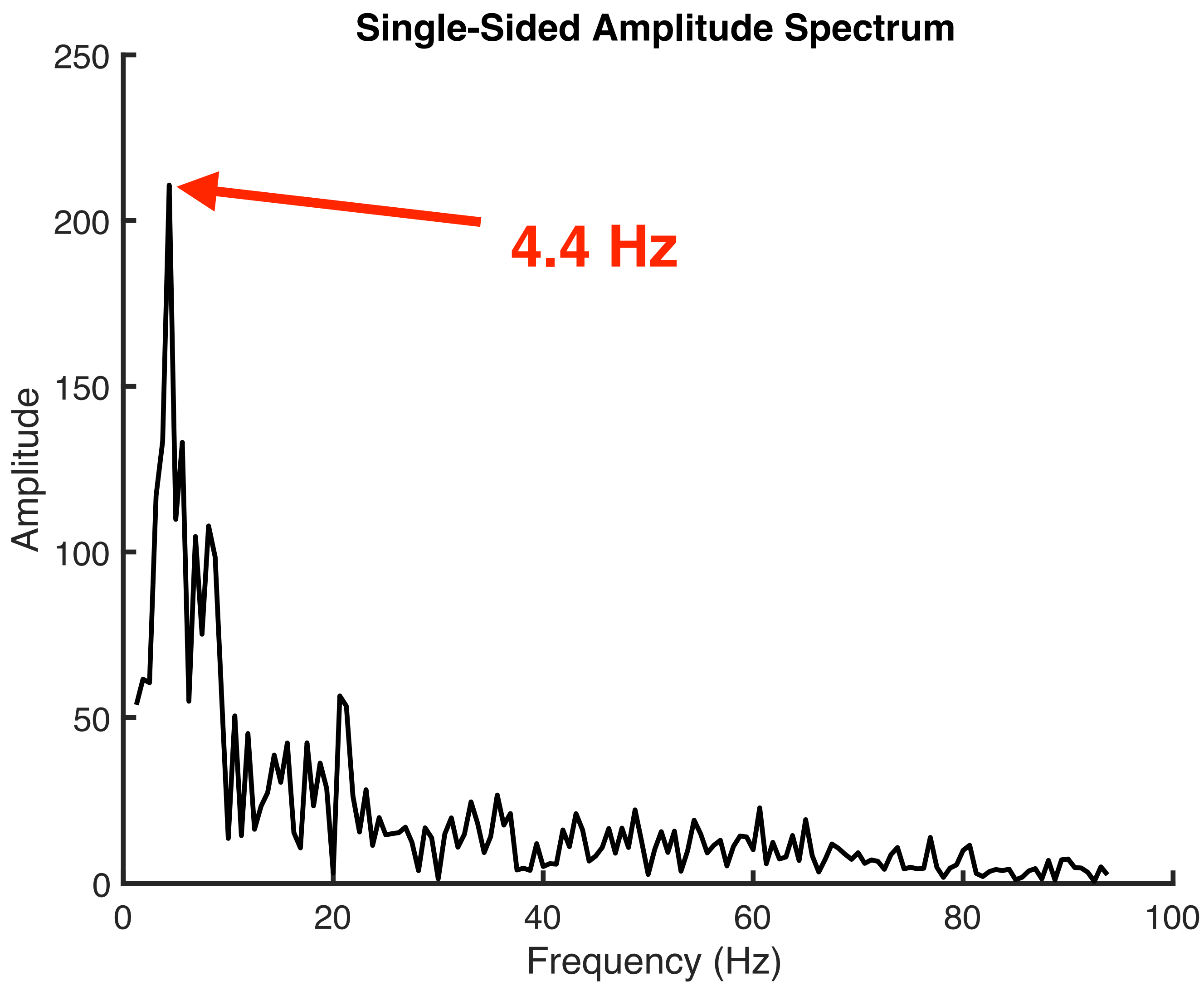
Fourier Transform



zero-frequency component = 436.4

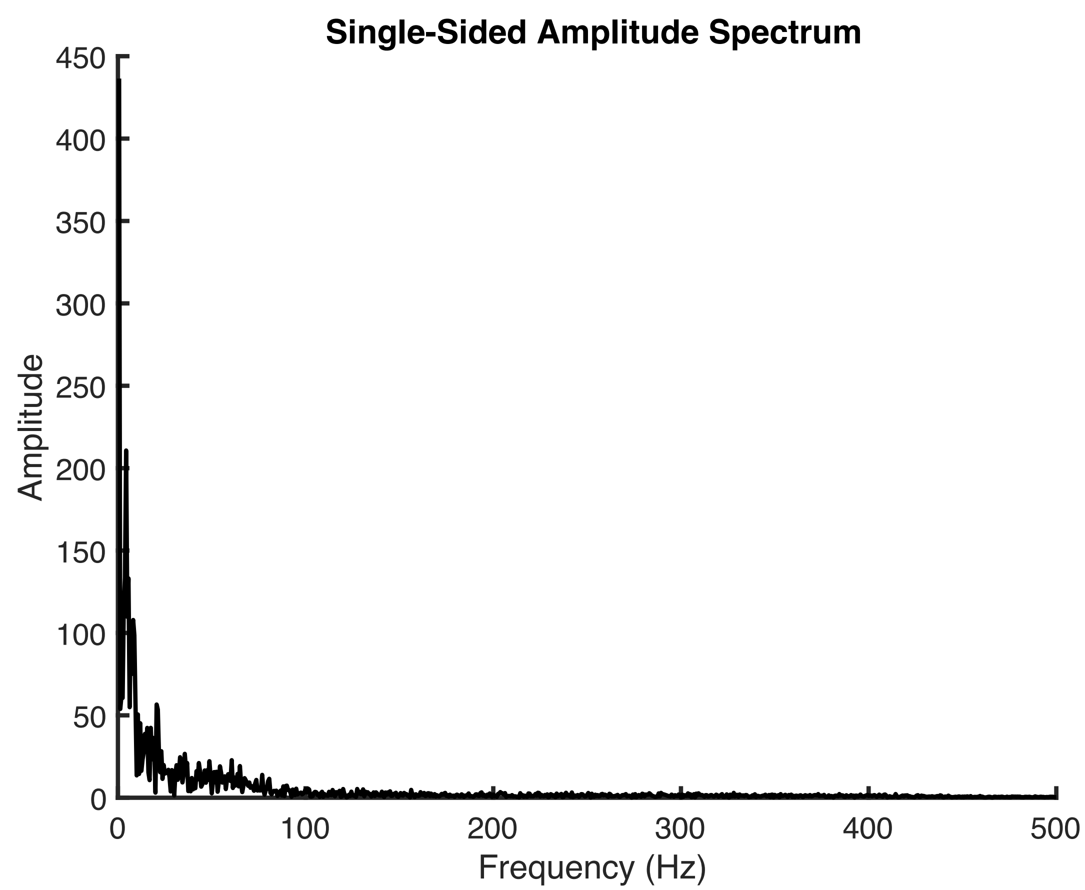


Fourier Transform



Fourier Transform

- Why is the maximum frequency of the Fourier transform for this particular EEG time series 500 Hz?
- Sampling rate of this time series was 1000 Hz
- What does this mean?



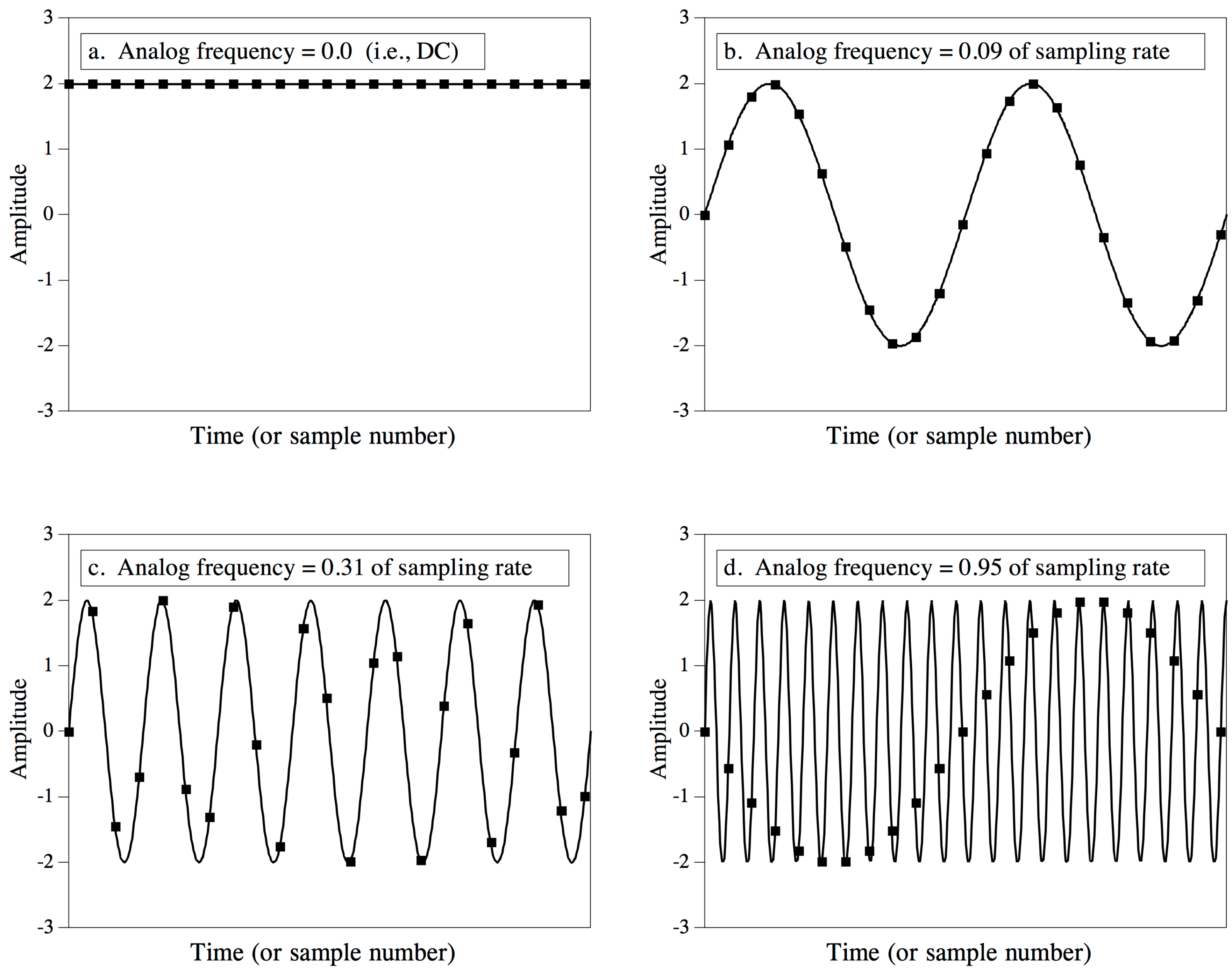
Sampling Theorem

- The signals that we are interested in are continuous—voltage fluctuations over time
- Can take on an infinite number of values
- Digital computers cannot do infinite
- Continuous signals must therefore be *sampled*

Sampling Theorem

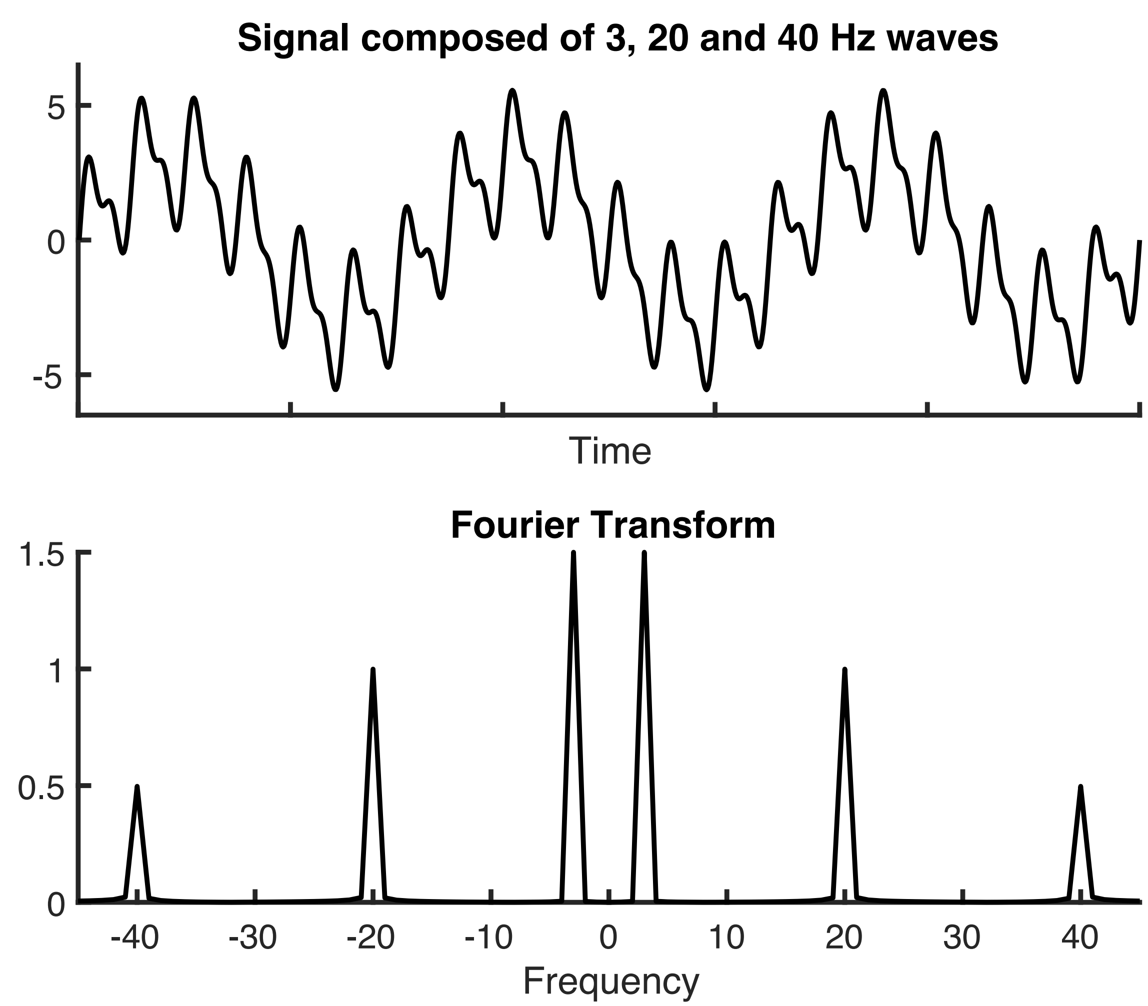
- Digital signals are therefore representations of the continuous quantities we are actually interested
- What is the proper sampling rate?
- Allows perfect reconstruction of the original signal
- Nyquist rate: twice the frequency of the highest sinusoid contained in the signal
- In the case of a signal sampled 1000 times/second (1000 Hz), it is not possible to extract information about frequencies higher than 500 Hz

Sampling Theorem



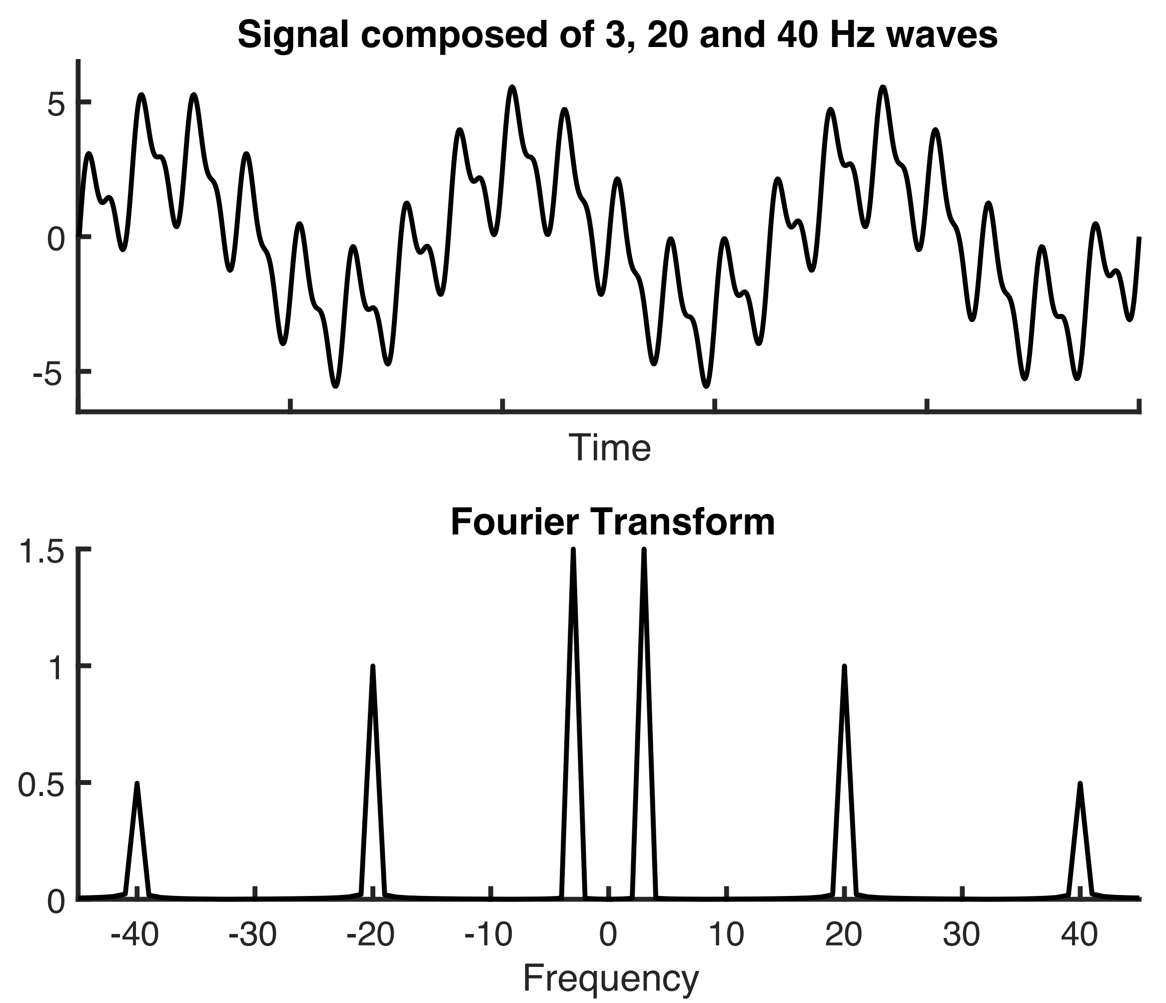
Fourier Transform

- so we'll use Fourier analysis?

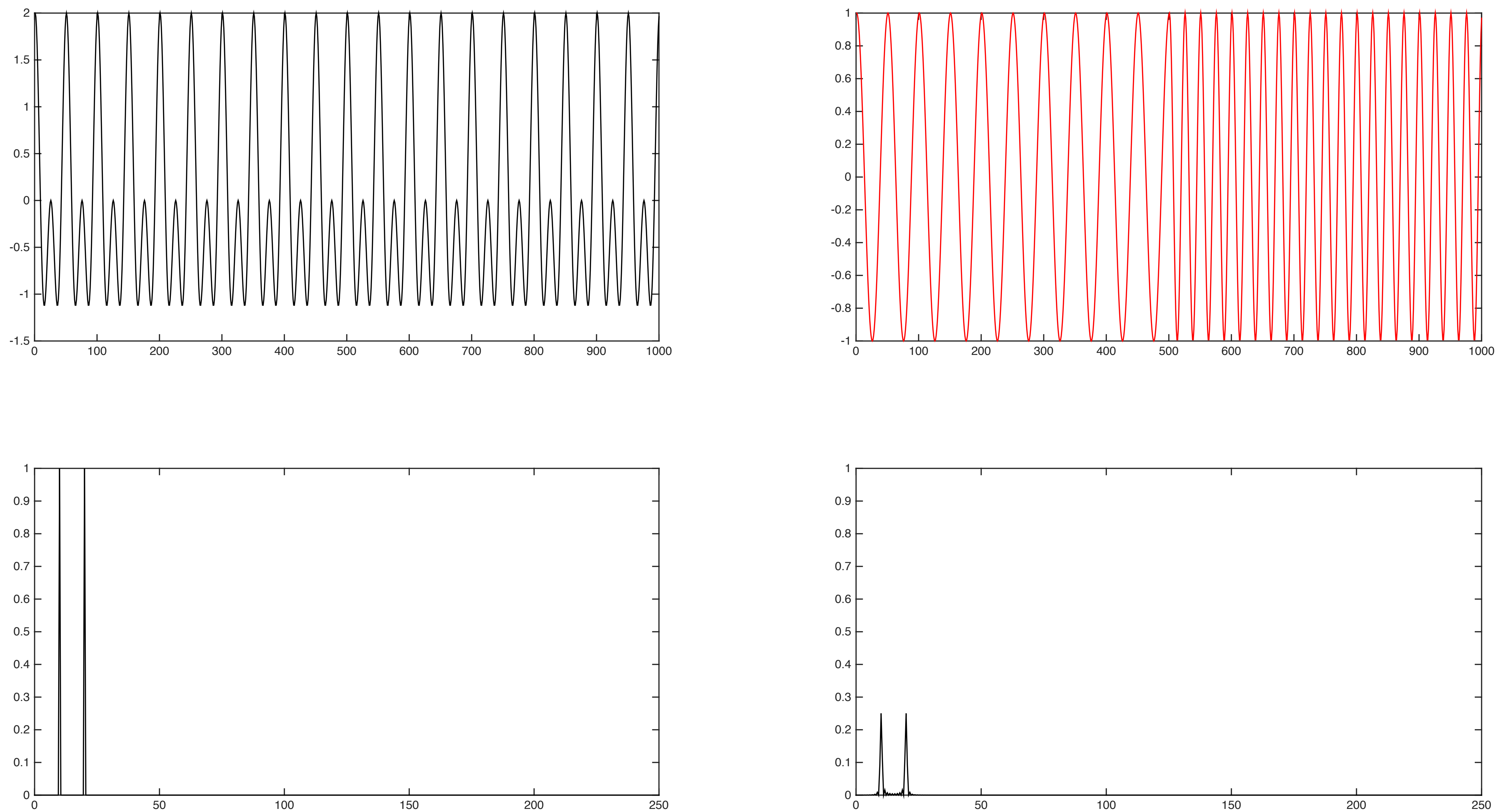


Fourier Transform

- FT has some limitations that make it not perfectly suited for analysis of neural EEG data

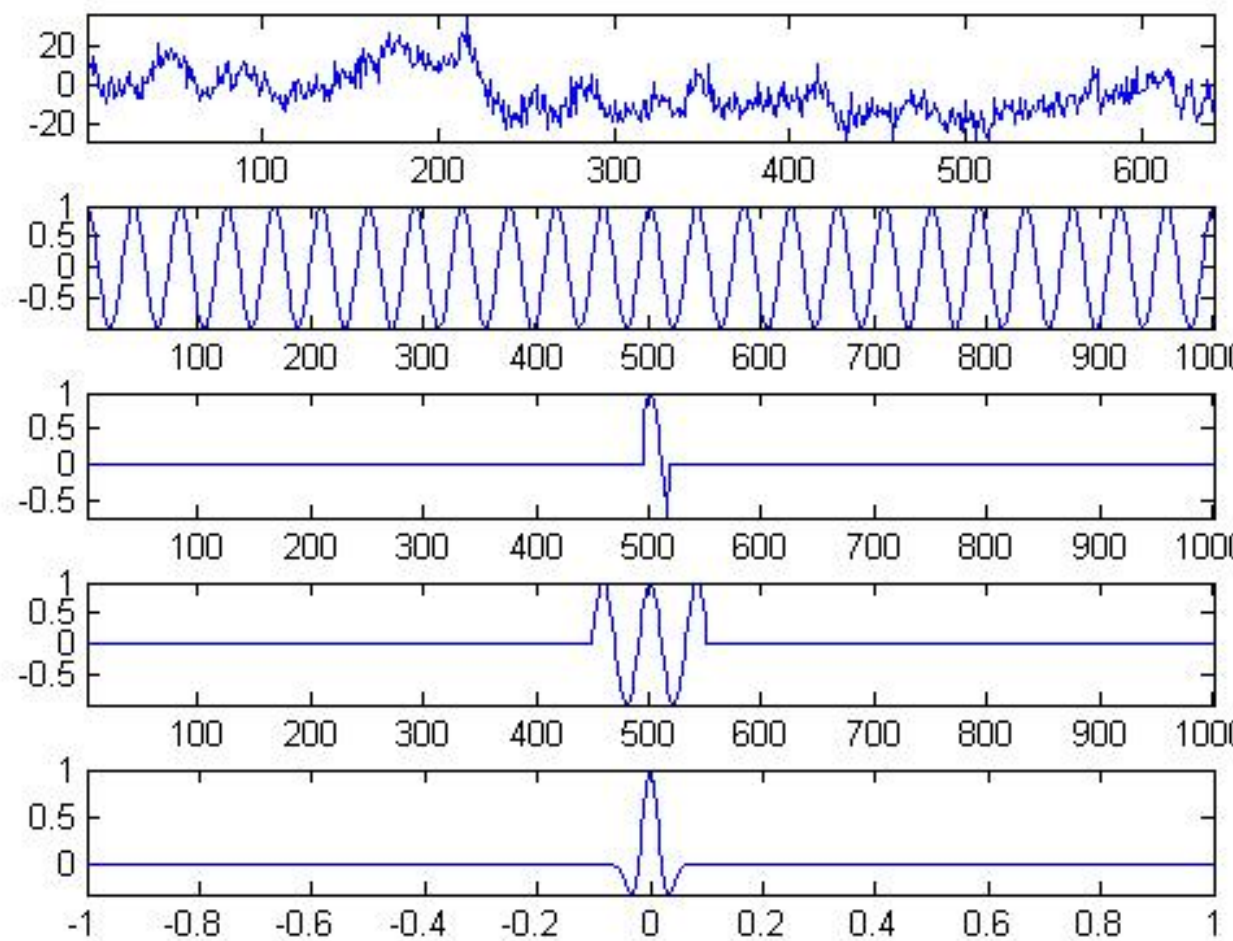


Limitation of FFT: If signal composition changes over time (**stationarity**)



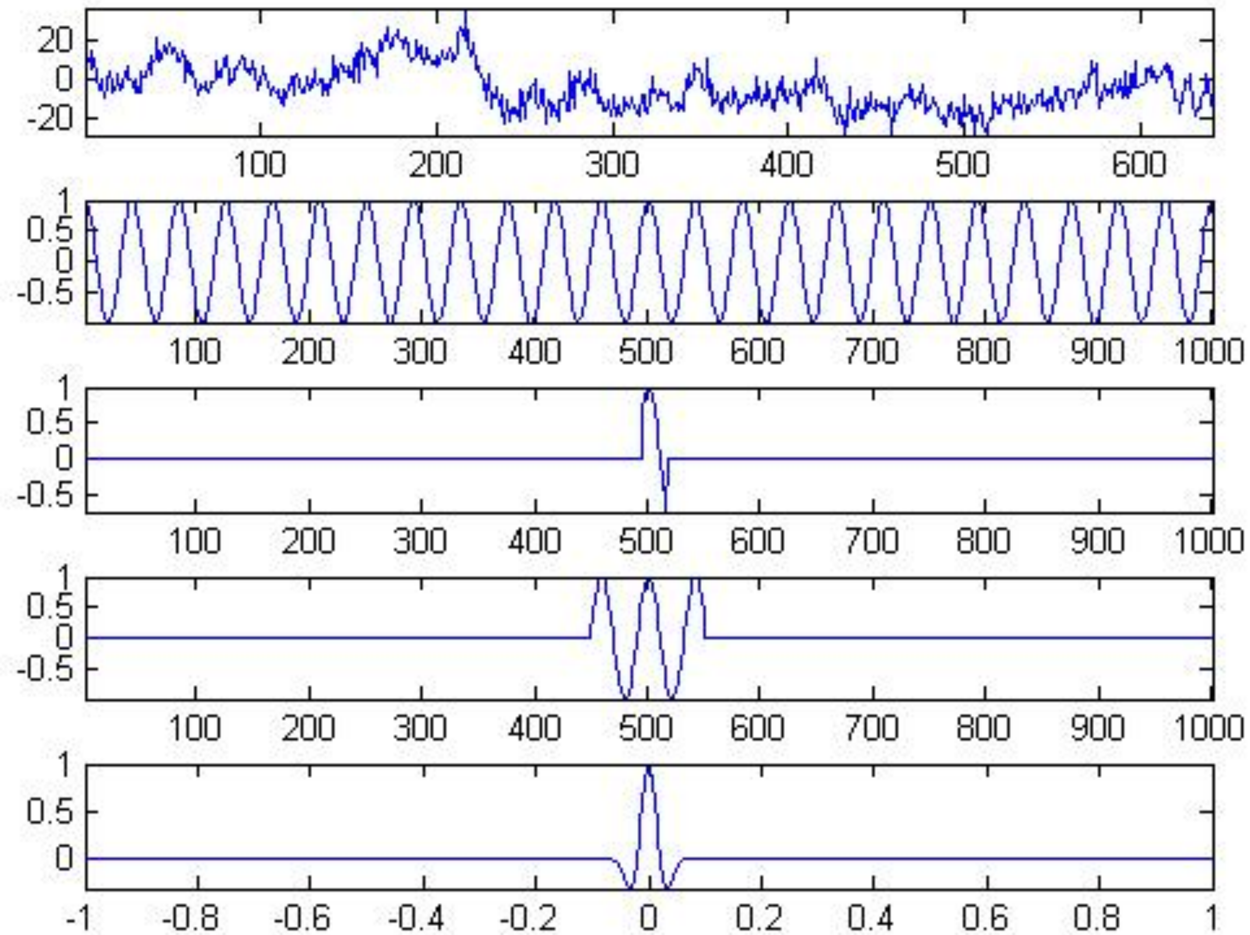
Limitation of FFT: If signal composition changes over time (**stationarity**)

- could we just do the FT on many small segments of the data that are assumed to be stationary?
- so, take a few cycles of a sinusoid and compute the dot product with each small segment of data?



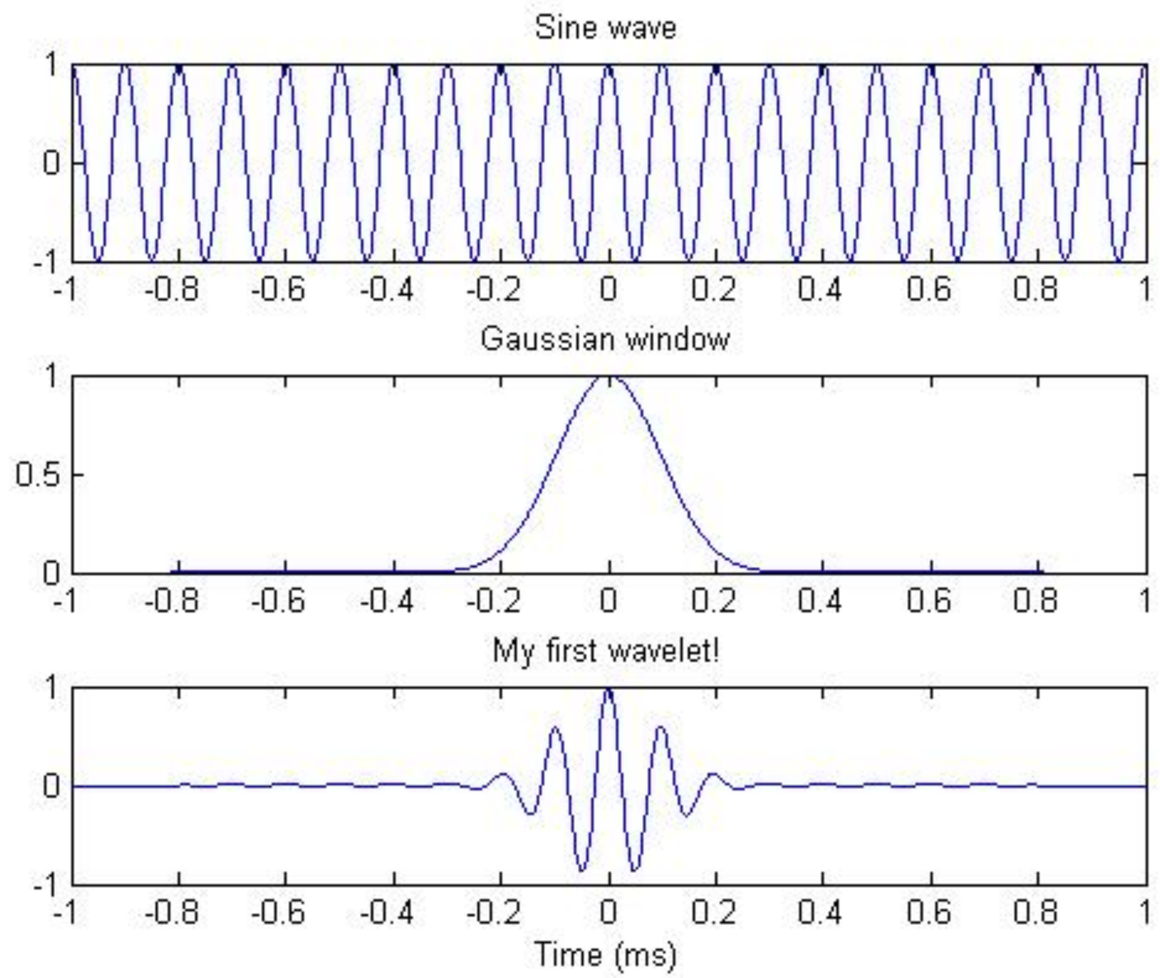
Limitation of FFT: If signal composition changes over time (**stationarity**)

- can't literally take just one/a few cycles of a sinusoid because of the sharp transition at the edges
- solution: wavelets



Wavelets

- wavelet = small wave
- take a sinusoid, multiply it by a Gaussian that 'windows' it to be (mostly) limited to brief time period



Wavelets

Gaussian

$$x(t, \mu, \sigma) = Ae^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

$$A = \frac{1}{\sqrt{\sigma\sqrt{\pi}}}$$

$$\sigma = \frac{1}{2\pi\frac{f}{n}}$$

$$\mu = mean$$

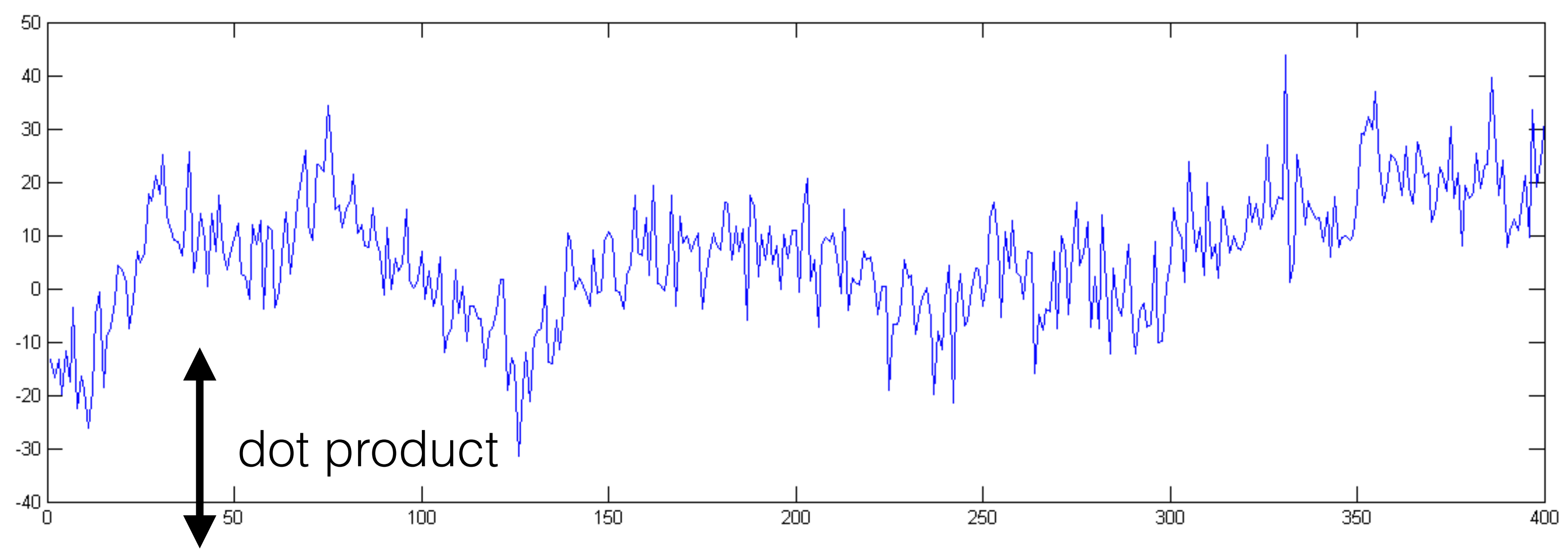
$$\sigma = SD$$

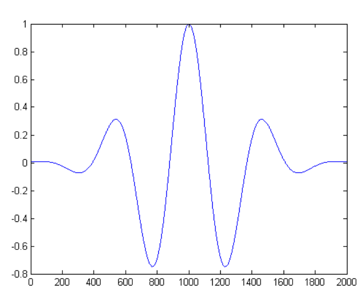
$$f = frequency$$

$$n = cycles$$

Wavelet convolution

- now we can take our wavelet and compute the dot product with the signal across time
- computing the dot product between two signals **over time** is called **convolution**

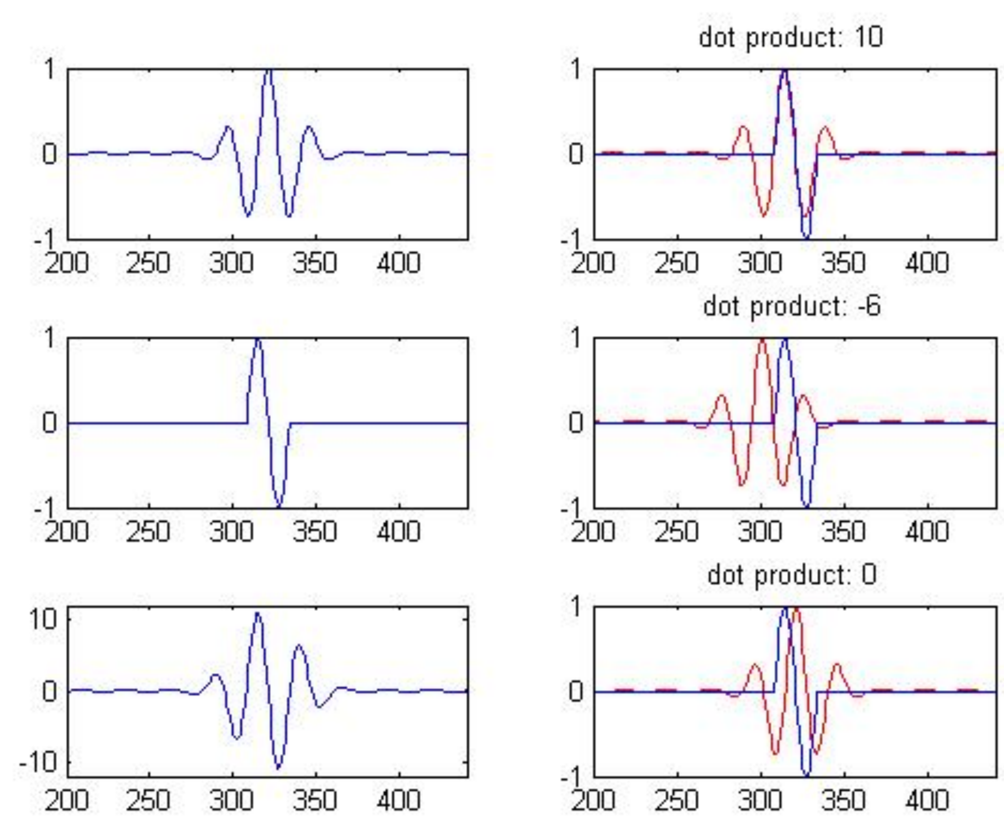




slide the wavelet along the signal
and compute at each point

Wavelet dot products

- problem: at any point in time, the dot product between the wavelet and the signal will be influenced by the **phase offset** between the wavelet (kernel) and the data (signal)
- this is why we need the complex wavelet



Complex wavelets

- complex sinusoid is the sum of a real and imaginary part
- cos = real part, sin = imaginary part

$$x(t, f) = M[\cos(2\pi f t) + i \sin(2\pi f t)]$$

Euler's formula

- a complex sinusoid can also be written as a complex exponential

$$x(t, f) = M[\cos(2\pi f t) + i\sin(2\pi f t)]$$

$$e^{ix} = \cos(x) + i\sin(x) \longleftarrow$$

greatly simplifies the math of dealing with sinusoids