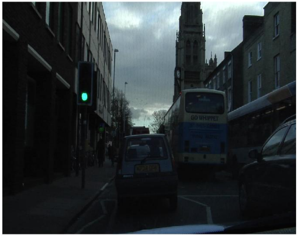
Application of CNN - Image segmentation

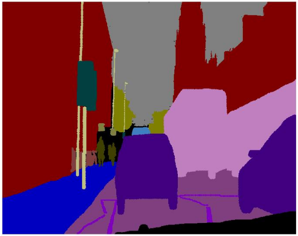
Numerical Methods for Deep Learning

Image Segmentation

The problem: Given an image $\mathbf{Y} : R^2 \to R$ compute probability maps for n_c classes, i.e., for $j = 1, \ldots, n_c$

$$\mathbf{P}_j : R^2 \to [0,1], \quad \sum_{j=1}^{n_c} \mathbf{P}_j = 1$$





$\mathbf{P}_j$ is the probability of a pixel to belong to class j

Setup of Image Segmentation Problem

- Use a neural network: original image → n_c “images”
- Output image represents class probabilities for each pixel
- Loss: average cross entropy for **each pixel**
- Optimize over network parameters

Simple example: a single layer and linear classification

Image Segmentation with Single-Layer NN

Let $\mathbf{Y}_0 \in \mathbb{R}^{n \times n_f}$ with $n_f = n_1 \cdot n_2 \cdot 3$ ($n_1 \times n_2$ RGB data)

$$\mathbf{Y}_1 = \sigma(\mathbf{Y}_0 \mathbf{K}(\theta) + \mathbf{B}\mathbf{b}).$$

Use 2D convolutions for $\mathbf{K}(\theta)$ and assume k output channels, i.e.,

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_{2D}(\theta_{11}) & \mathbf{K}_{2D}(\theta_{12}) & \cdots & \mathbf{K}_{2D}(\theta_{1k}) \\ \mathbf{K}_{2D}(\theta_{21}) & \mathbf{K}_{2D}(\theta_{22}) & \cdots & \mathbf{K}_{2D}(\theta_{2k}) \\ \mathbf{K}_{2D}(\theta_{31}) & \mathbf{K}_{2D}(\theta_{32}) & \cdots & \mathbf{K}_{2D}(\theta_{3k}) \end{pmatrix}.$$

Then we have to classify each pixel in $\mathbf{Y}_1$ using $\mathbf{W} \in \mathbb{R}^{k \times n_c}$. This gives the problem

$$\min_{\mathbf{W},\mathbf{K},\mathbf{b}} E(\mathrm{reshape}(\mathbf{Y}_1)\mathbf{W}, \mathbf{P}), \qquad \mathbf{Y}_1 = \sigma(\mathbf{Y}_0 \mathbf{K}(\theta) + \mathbf{B}\mathbf{b})$$

Here $\mathrm{reshape}(\mathbf{Y}_1) \in \mathbb{R}^{(n \cdot n_1 \cdot n_2) \times k}$ and E is our cross entropy

References