Introduction

Numerical Methods for Deep Learning

Course Overview

- Module 1: Linear Models
  2. Linear Models and Least-Squares
  3. Iterative Methods for Least-Squares
  4. Linear Models for Classification
  5. Newton's Method for Classification
  6. Regularization for Image Classification

Course Overview

- Module 2: Neural Networks
  7. Introduction to Nonlinear Models
  8. Single Layer Neural Networks
  9. Training Algorithms for Single Layer Neural Networks
  10. Introduction to Deep Neural Networks
  11. Differentiating Deep Neural Networks
  12. Stochastic Gradient Descent and Variants
- Module 3: Parametric Models/Convolution Neural Networks
  13. Introduction to Parametric Models
  14. Application of CNN: Image Segmentation
  15. CNN and their relation to PDEs

Neural Networks - A Quick Overview

- Neural Networks with a particular (deep) architecture
- Exist for a long time (70's and even earlier) [10, 11, 8]
- Recent revolution - computational power and lots of data [1, 9, 7]
- Can perform very well for large amounts of data
- Applications
  - Image recognition [4, 6, 7], segmentation, natural language processing [2, 3, 5]

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- A few recent news articles:
  - Apple Is Bringing the AI Revolution to Your iPhone, WIRED 2016
  - Why Deep Learning Is Suddenly Changing Your Life, FORTUNE 2016
  - Data Scientist: Sexiest Job of the 21st Century, Harvard Business Rev 17

NN - A Quick Overview

Neural Networks is a data interpolator/classifier when the underlying model is unknown.

A generic way to write it is

$$\mathbf{c} = f(\mathbf{y}, \boldsymbol{\theta}).$$

- The function f is the computational model.
- $\mathbf{y} \in \mathbb{R}^{n_f}$ is the input data (e.g., an image)
- $\mathbf{c} \in \mathbb{R}^{n_c}$ is the output (e.g. class the image)
- $\boldsymbol{\theta} \in \mathbb{R}^{n_p}$ are parameters of the model f

In learning we have examples $\{(\mathbf{y}_j, \mathbf{c}_j) \ : \ j = 1, \dots, n\}$ and the goal is to estimate or "learn" the parameters $\boldsymbol{\theta}$

Learning From Data: The Core of Science

How to choose f?

Option 1 (Fundamental(?) understanding): For example, Newton's formula

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with unknown parameter g.
To estimate g observe falling object

t	x
0	0
1	4.9
2	20.1
3	44.1

What is the optimal value for g?

Learning From Data: The Core of Science

How to choose f?

Option 2 (Phenomenological models): For example, Archie's law - what is the electrical resistivity of a rock and how it relates to its porosity, ϕ and saturation, S_w?

$$\rho(\phi, S_w) = a\phi^{n/2} S_w^p$$

a, n, p unknown parameters

Obtaining parameters from observed data and lab experiments on rocks

Phenomenological vs. Fundamental

Fundamental laws come from understanding(?) the underlying process. They are **assumed invariant** and can therefore be predictive(?).

Phenomenological models are data driven. They “work” on some given data. Hard to know what are the limitations.

But ...

- models based on understanding can do poorly - weather, economics ...
- models based on data can sometimes do better
- how do we quantify understanding?

Generalization

Suppose that we have examples $\{\mathbf{y}_j, \mathbf{c}_j\}, \quad j = 1, \ldots, n$, a model $f(\mathbf{y}, \boldsymbol{\theta})$ and some optimal parameter $\boldsymbol{\theta}^*$.
Let $\{(\mathbf{y}_j^t, \mathbf{c}_j^t) \;:\; j = 1, \ldots, s\}$ be some test set, that was not used to compute $\boldsymbol{\theta}^*$.

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For phenomenological models, there is no reason why the model should generalize, but in practice it often does.

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Why would a model generalize poorly?

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Two common reasons:

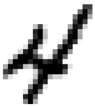
1. Our "optimal" $\boldsymbol{\theta}^*$ was optimal for the training but is less so for other data
2. The chosen computational model f is poor (e.g. linear model for a nonlinear function).

Example 1: Classification of Hand-written Digits

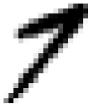
- Let $\mathbf{y}_j \in \mathbb{R}^{n_f}$ and let $\mathbf{c}_j \in \mathbb{R}^{n_c}$.
- The vector $\mathbf{c}$ is the probability of $\mathbf{y}$ belonging to a certain class. Clearly, $0 \leq \mathbf{c}_j \leq 1$ and $\sum_{j=1}^{n_c} \mathbf{c}_j = 1$.

Examples (MNIST):

$\mathbf{y}_1$



$\mathbf{y}_2$



$\mathbf{c}_1 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0]^\top$ $\quad$ $\mathbf{c}_2 = [0, 0.3, 0, 0, 0, 0, 0, 0.7, 0, 0]^\top$

Example 2: Classification of Natural Images

Same problem but images are natural images

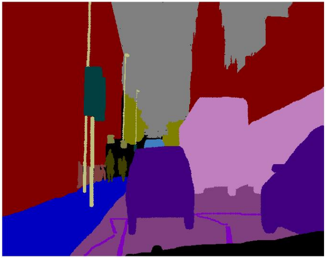
Examples (CIFAR-10):

Example 3: Semantic Segmentation

- let $\mathbf{y}_j \in \mathbb{R}^n$ be an RGB or grey valued image.
- let the pixels in $\mathbf{c}_j \in \{1, 2, 3, \ldots\}^k$ denote the labels.

$\mathbf{y}$, input image

$\mathbf{c}$, segmentation (labeled image)



Goal: Find map $\mathbf{c} = f(\mathbf{y}, \boldsymbol{\theta})$

Example 3: Semantic Segmentation

Problem: Given image $\mathbf{y}$ and label $\mathbf{c}$ find a map $f(\cdot, \boldsymbol{\theta})$ such that $\mathbf{c} \approx f(\mathbf{y}, \boldsymbol{\theta})$

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First step: Reduce the dimensionality of problem.

- extract features from the image
- classify in the feature space

Reduce the problem of learning from the image to feature detection and classification

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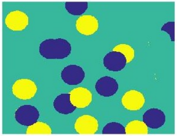
Possible features: Color, neighbors, edges ...

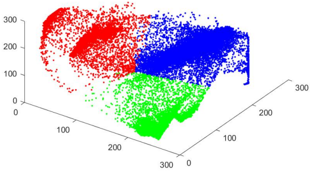
Example 3 - Semantic Segmentation

Simpler setup

- data, $\mathbf{y}$ is the RGB value of the pixel (and its neighbors?)
- $\mathbf{c}$ is a labeled pixel
- The map $\mathbf{c} = f(\mathbf{y}, \boldsymbol{\theta})$



input image and segmentation



3D representation of RGB values

Coding: Download Data and Setup MATLAB

The following data sets will be used throughout the course (and homework projects).

The following are ordered from small and easy to large and challenging:

- MNIST
- CIFAR-10
- CamVid: download from Mathworks web page.

References

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