Point-of-Care Tomographic Imaging

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Emory University
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Acknowledgements

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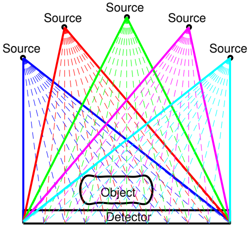
- This work is supported by the National Science Foundation.

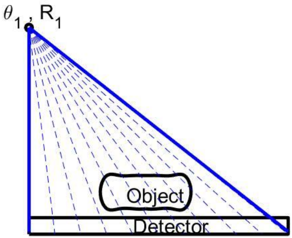
Introduction



Question to Consider

- How do we estimate these geometry parameters to obtain a reconstructed image?

Mathematical Problem



Linear Algebra Problem

$$\boldsymbol{Ax} = \boldsymbol{b}$$

Problem Set-up

The Optimization Problem

$$\min_{\boldsymbol{p},\boldsymbol{x}} \left\{ \|\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x} - \boldsymbol{b}\|_2^2 + \lambda^2 \|\boldsymbol{x}\|^2 \right\}$$

$\boldsymbol{A}(\boldsymbol{p})$ is a matrix $\boldsymbol{A}$ created as a function of $\boldsymbol{p}$.
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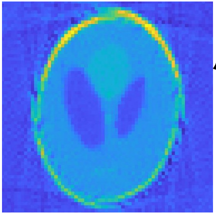
Block Coordinate Descent (BCD)

Given an initial guess p_0.

$$\boldsymbol{x}_k = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\boldsymbol{p}_k)\boldsymbol{x} - \boldsymbol{b}\|_2^2 + \lambda_k^2 \|\boldsymbol{x}\|^2$$

$$\boldsymbol{p}_{k+1} = \arg\min_{\boldsymbol{p}} \|\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}_k - \boldsymbol{b}\|_2^2$$

Block Coordinate Descent



$\boldsymbol{x}_0$

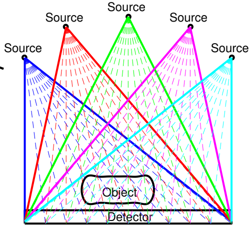


Figure: Use initial parameters to take the image.

$$\boldsymbol{x}_0 = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\boldsymbol{p}_0)\boldsymbol{x} - \boldsymbol{b}\|_2^2 + \lambda_0^2\|\boldsymbol{x}\|^2$$

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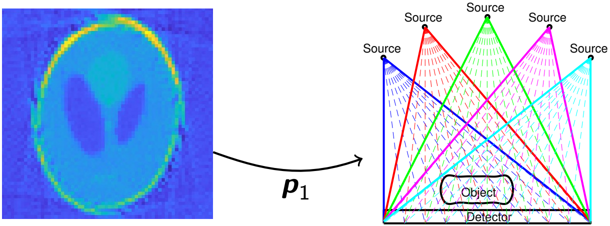

Figure: Use the image to find better parameters.

$$\boldsymbol{p}_1 = \arg\min_{\boldsymbol{p}} \|\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}_0 - \boldsymbol{b}\|_2^2$$

Block Coordinate Descent

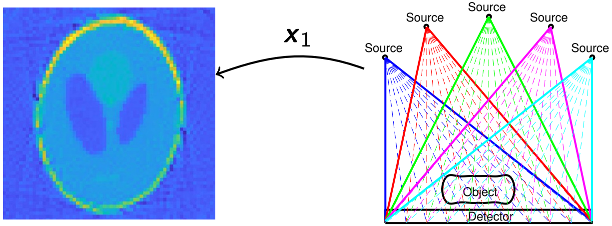

Figure: Use the better parameters to find a better image.

$$\boldsymbol{x}_1 = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}(\boldsymbol{p}_1)\boldsymbol{x} - \boldsymbol{b}\|_2^2 + \lambda_1^2 \|\boldsymbol{x}\|^2$$

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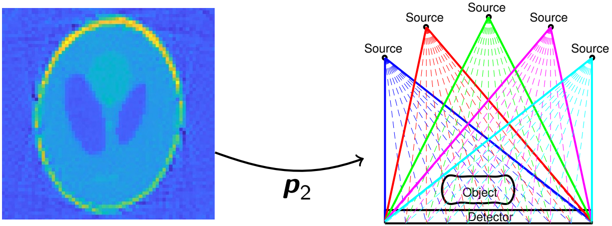

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$$\boldsymbol{p}_2 = \arg\min_{\boldsymbol{p}} \|\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}_1 - \boldsymbol{b}\|_2^2$$

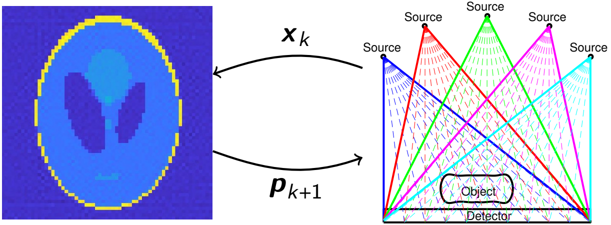

Figure: Continue until you get a good image

$$x_k = \arg\min_{x} \|A(p_k)x - b\|_2^2 + \lambda_k^2 \|x\|^2$$

$$p_{k+1} = \arg\min_{p} \|A(p)x_k - b\|_2^2$$

Block Coordinate Descent

(a) Solution after BCD

(b) True Solution

Stimulating Perturbations

- The "perfect" problem

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$

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$$\boldsymbol{Ax} = \boldsymbol{b}$$

- Reality

$$\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x} = \widehat{\boldsymbol{b}}$$

where

$$\widehat{\boldsymbol{b}} = \boldsymbol{b} + \text{noise}$$

$$\boldsymbol{p} = \begin{bmatrix} R_1 & \cdots & R_m & \theta_1 & \cdots & \theta_m \end{bmatrix}^T$$

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Q: Should the perturbations be randomly or constantly stimulated?

Random Perturbations

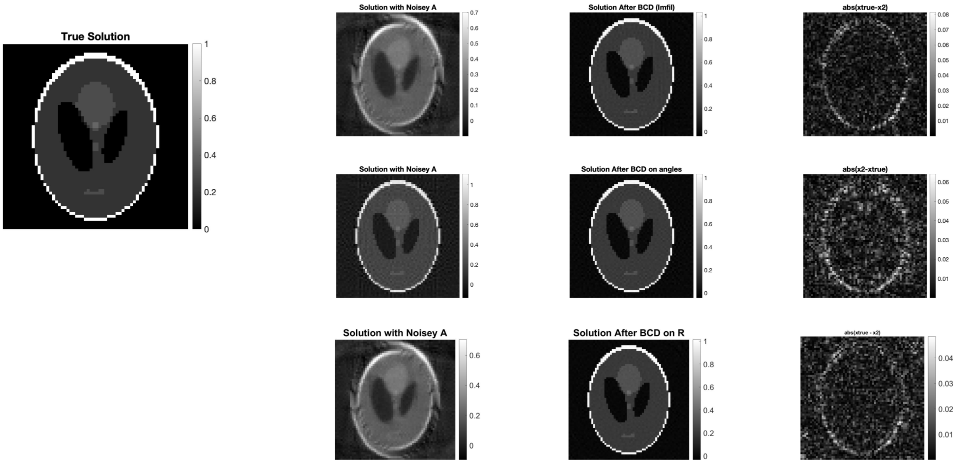


Figure: First row: Both random, Second row: θ, Third row: R

Constant Perturbations

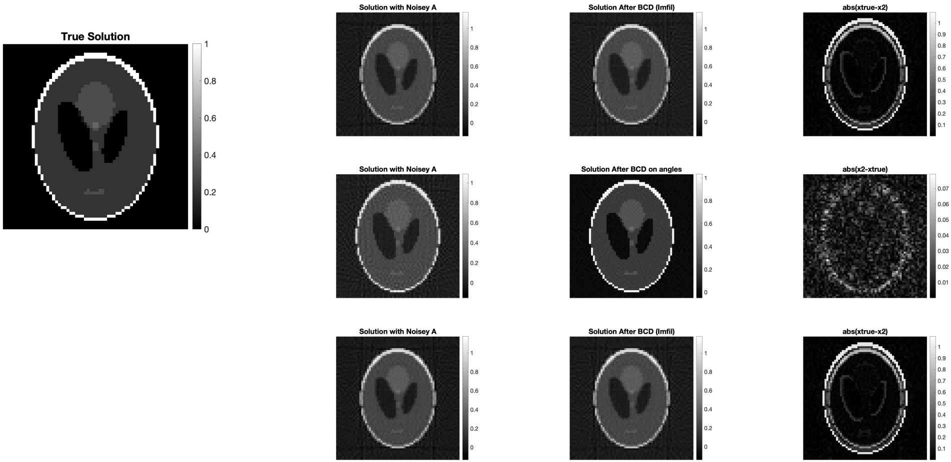


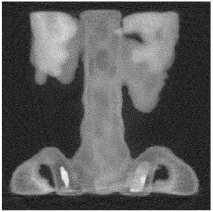
Figure: First row: Both constant, Second row: θ, Third row: R

Preliminary Results

True Solution

Original Image Source: Matlab Image Processing Toolbox

Preliminary Results



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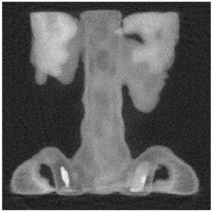
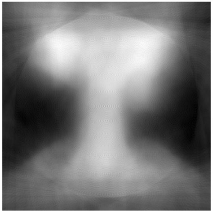
Solution With True Parameters



Solution With Initial Parameters



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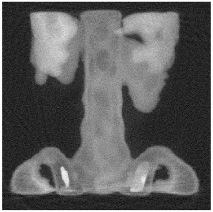

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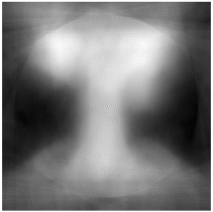
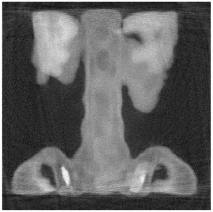


Solution After BCD



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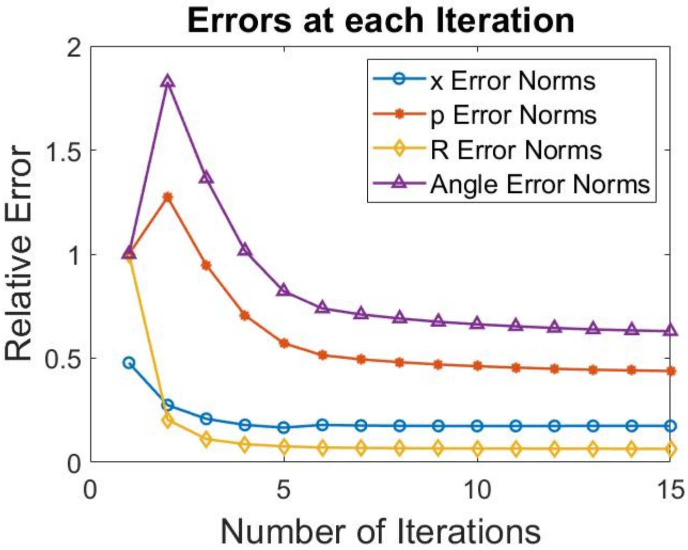
Preliminary Results



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- Figure out how close of an initial guess we need for convergence.
- Consider different forms of Tikhonov Regularization.
- Explore acceleration techniques for block coordinate descent.
- Time code-speed ups due to parallelization.
- Find other applications of this algorithm.

References

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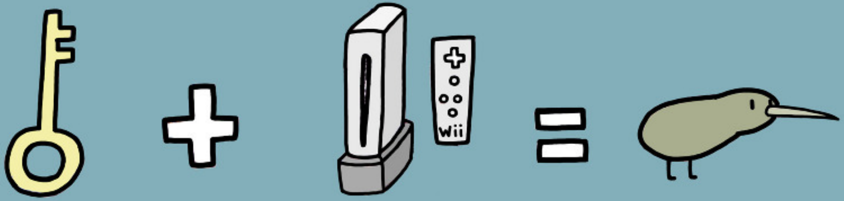
THANK YOU!



Math. It explains everything.