Audience Q&A

Do not edit
How to change the design

The Slido app must be installed on every computer you're presenting from

slido

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 5

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week’s material

Questions?

ALVARO: Start the recording!

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

More on Quadrics

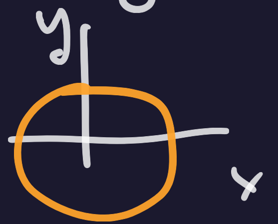
How to sketch a quadric surface?

Traces or Cross Sections of a Surface

$$f(x,y,z)=0$$

$z=0$

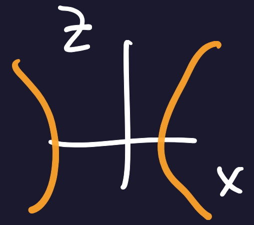
$f(x,y,0)=0$

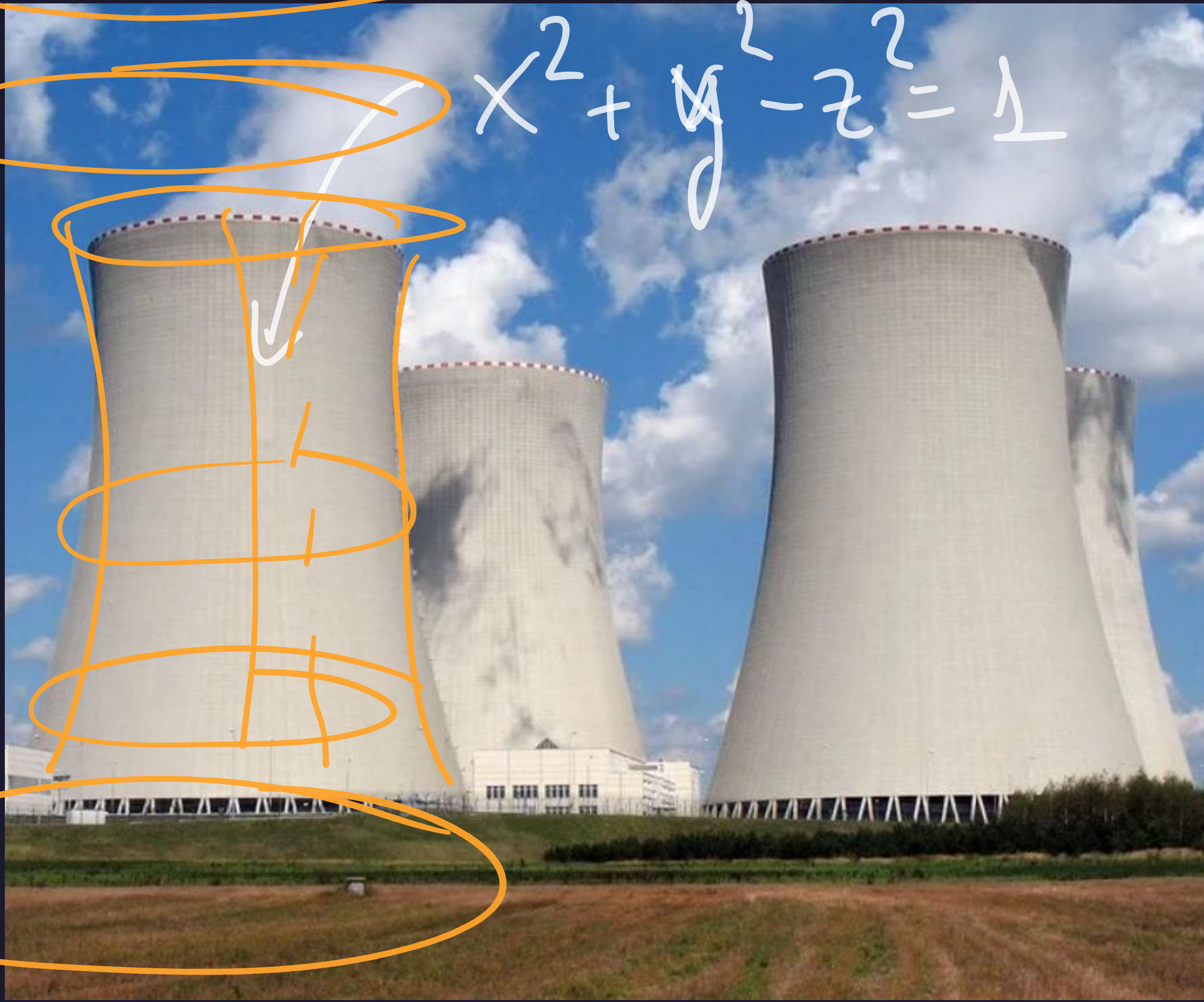


$x=0$

$y=0$

$f(x,0,z)=0$



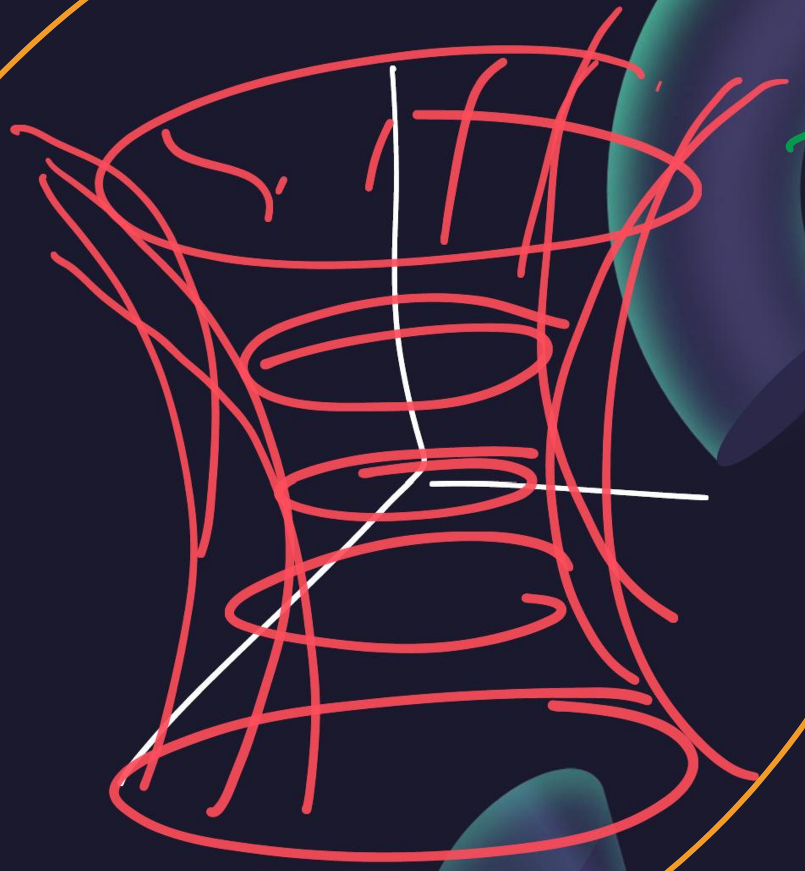


Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.

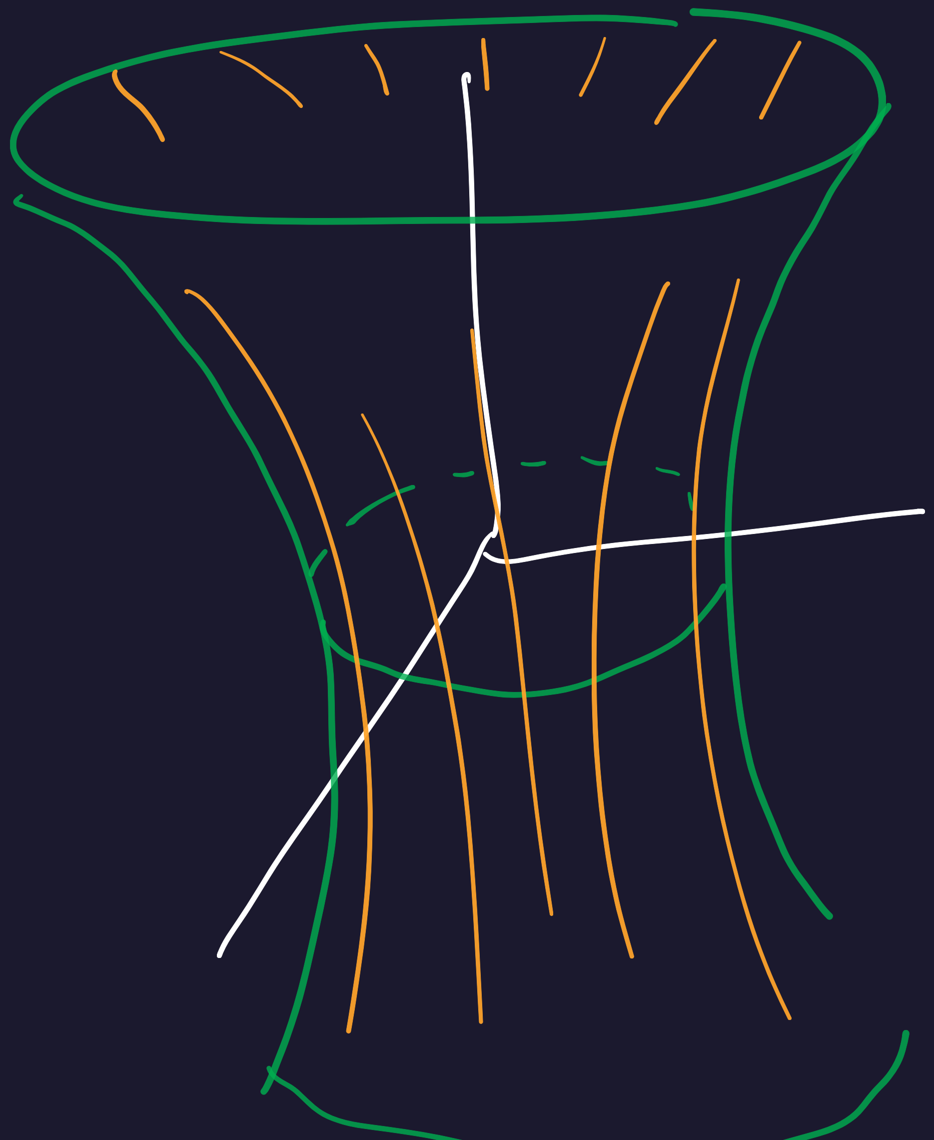
Example: Sketch the surface $x^2 + y^2 - z^2 = 1.$

$y = 0 \quad x^2 - z^2 = 1$

$x = 0 \quad y^2 - z^2 = 1$



Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.



Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0.$

$$x^2 - 6x = (x-3)^2 - 9$$

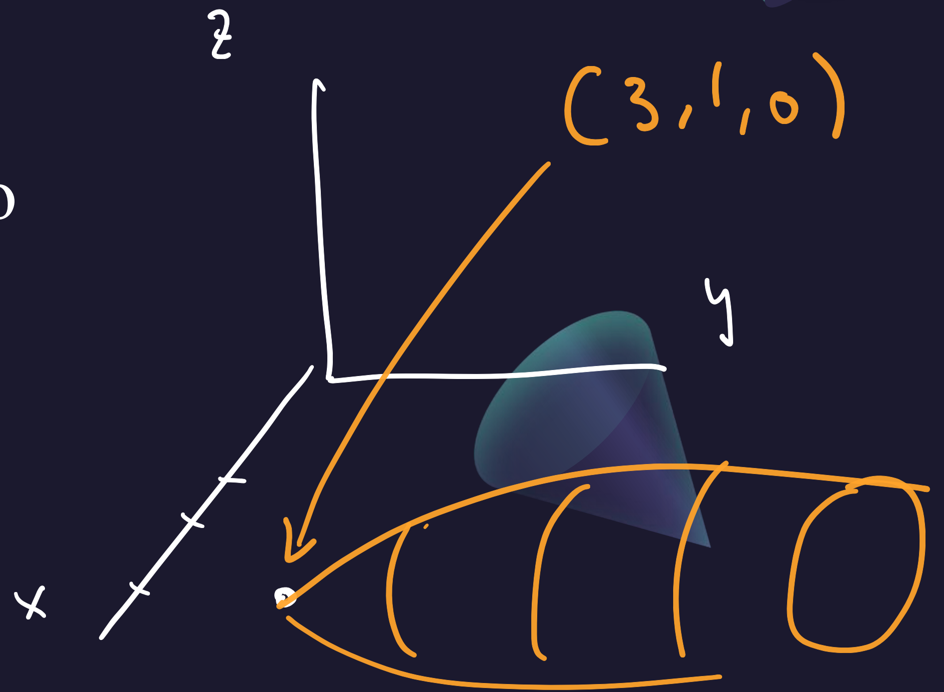
Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0.$

$$x^2 - 6x = (x-3)^2 - 9$$

$$\begin{aligned} x^2 + 2z^2 - 6x - y + 10 &= (x-3)^2 - 9 + 2z^2 - y + 10 \\ &= (x-3)^2 + 2z^2 - y + 1 \end{aligned}$$

$x^2 + 2z^2 - 6x - y + 10 = 0$ is equivalent to

$$y = (x-3)^2 + 2z^2 + 1$$



Questions?

ALVARO: Start the recording!

“Calculus 3”

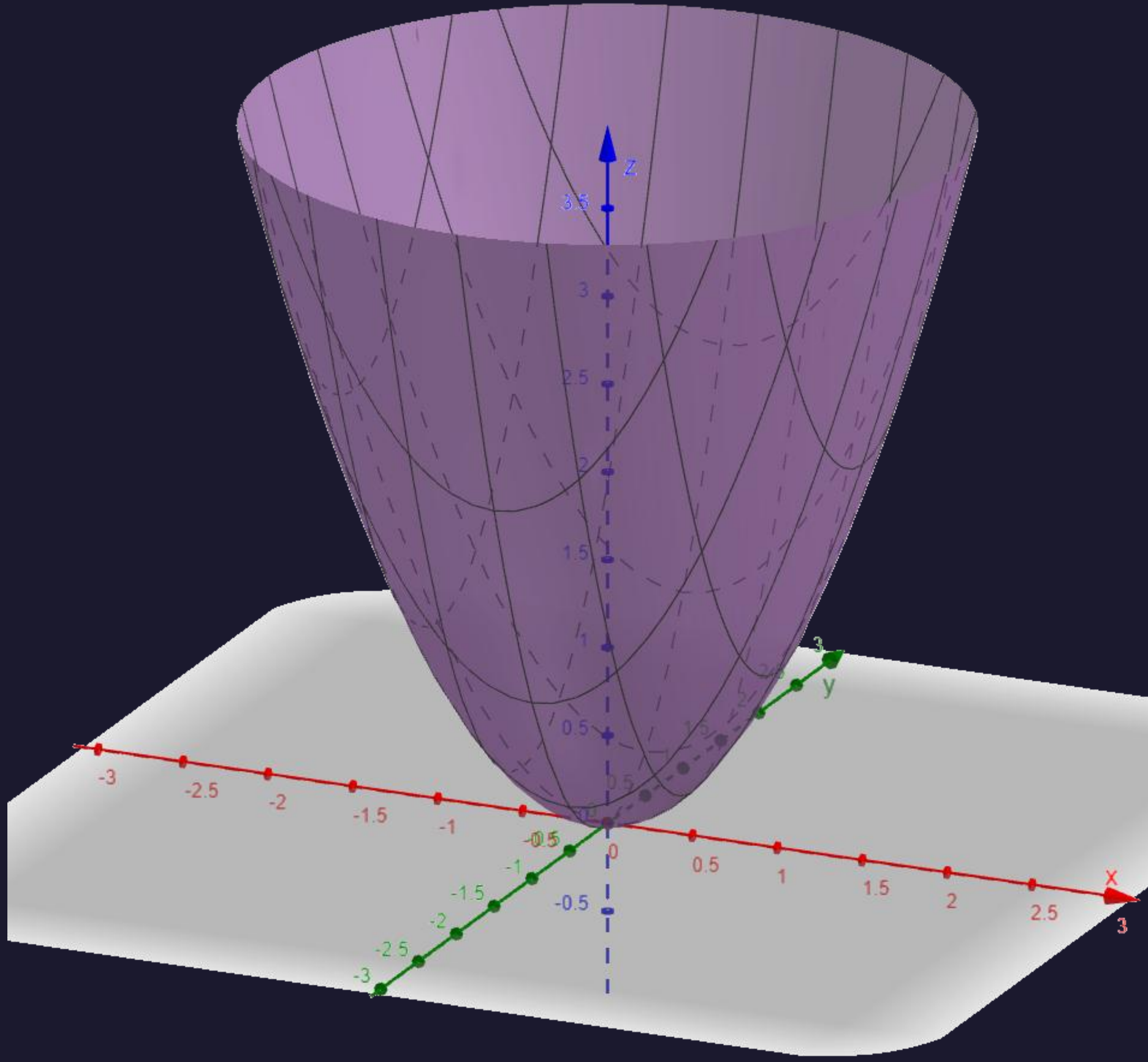
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

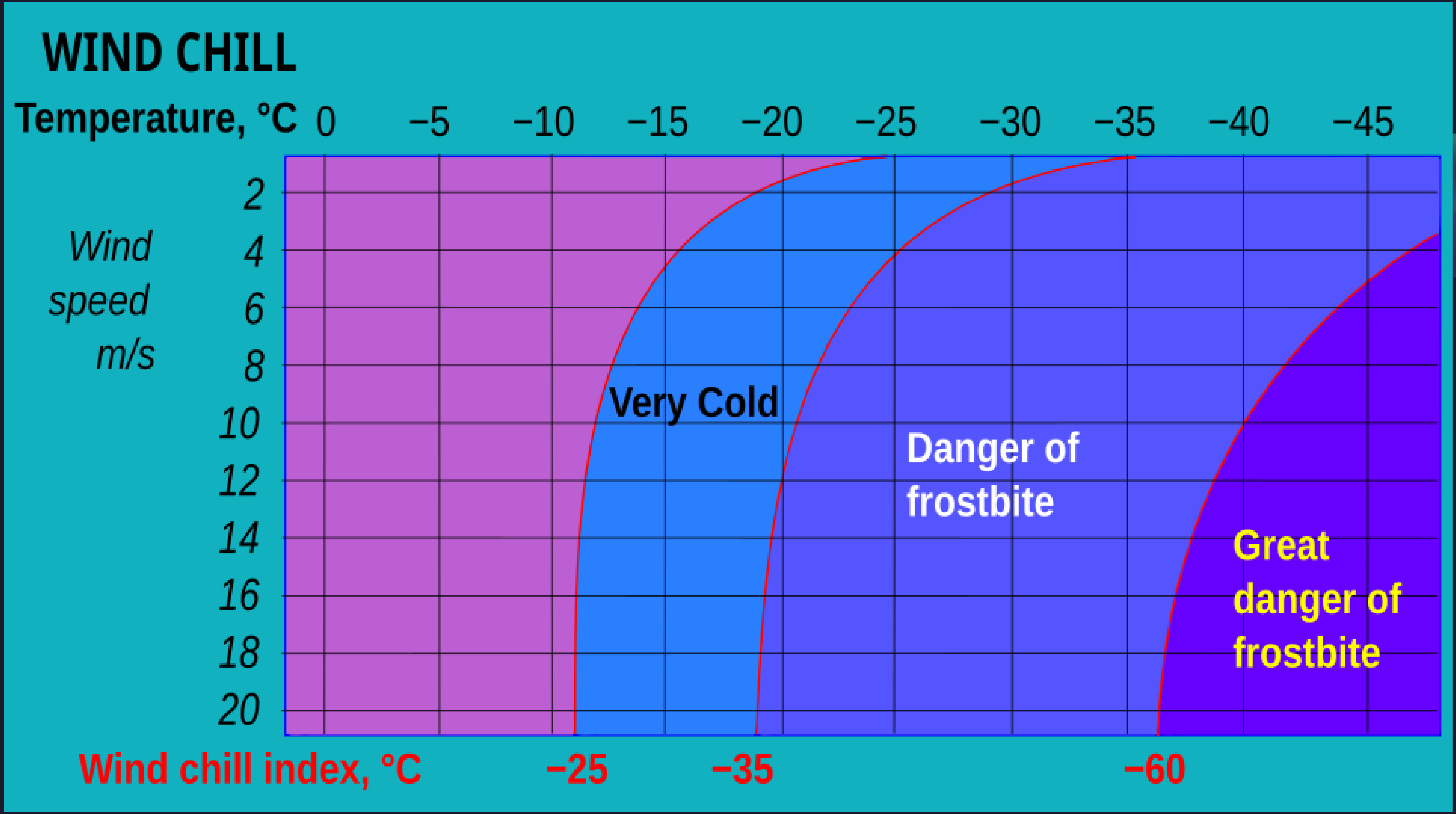
Functions of Several Variables

Today – Functions! $f(x,y) = x^2 + y^2$

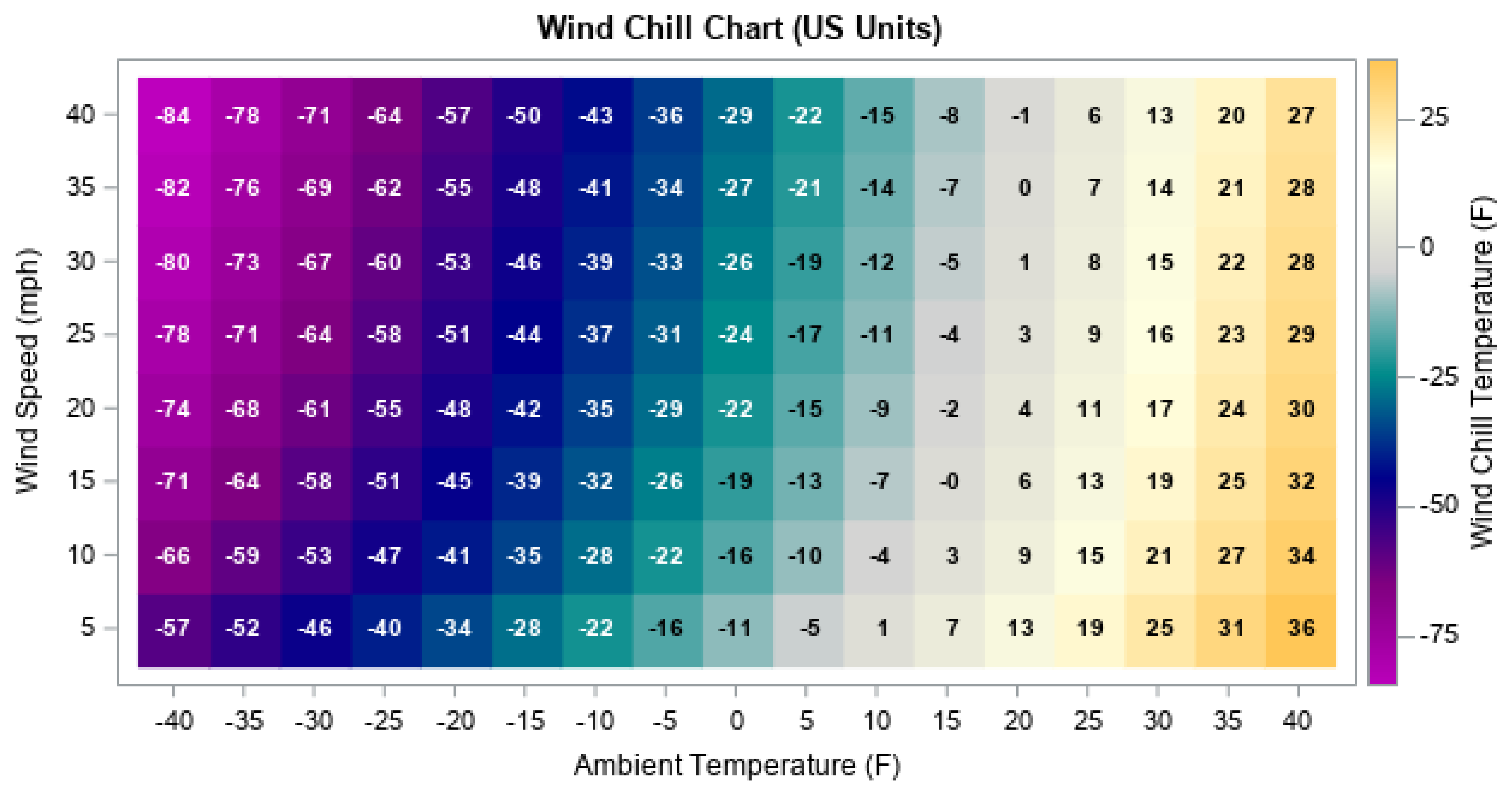
- Functions of Two Variables
- Domain and Range
- Graphs
- Level Curves
- Functions of More Than Two Variables



Functions of Two Variables



Functions of Two Variables



Source NOAA: https://www.weather.gov/safety/cold-wind-chill-chart

Functions of Two Variables

The standard wind chill formula for Environment Canada is:[3]

$$T_{\mathrm{wc}} = 13.12 + 0.6215T_{\mathrm{a}} - 11.37v^{+0.16} + 0.3965T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Celsius temperature scale; T_{a} is the air temperature in degrees Celsius; and v is the wind speed at 10 m (33 ft) standard anemometer height, in kilometres per hour.[11]

When the temperature is −20 °C (−4 °F) and the wind speed is 5 km/h (3 mph), the wind chill index is −24. If the temperature remains at −20 °C and the wind speed increases to 30 km/h (19 mph), the wind chill index falls to −33.

The equivalent formula in US customary units is:[12][3]

$$T_{\mathrm{wc}} = 35.74 + 0.6215T_{\mathrm{a}} - 35.75v^{+0.16} + 0.4275T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Fahrenheit scale; T_{a} is the air temperature in degrees Fahrenheit; and v is the wind speed in miles per hour.[13]

Functions of Two Variables, Domain, and Range

Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is,

$\{f(x, y) \mid (x, y) \in D\}$.

Functions of Two Variables, Domain, and Range

domain

$$f : D \to \mathbb{R}$$

$$(x, y) \mapsto f(x, y)$$

$$\text{Range} = \{ \alpha \in \mathbb{R} \text{ s.t. } \alpha = f(x_0, y_0) \text{ for some } (x_0, y_0) \in D \}$$

Example: Sketch the domain of the function, and evaluate at (3,2)

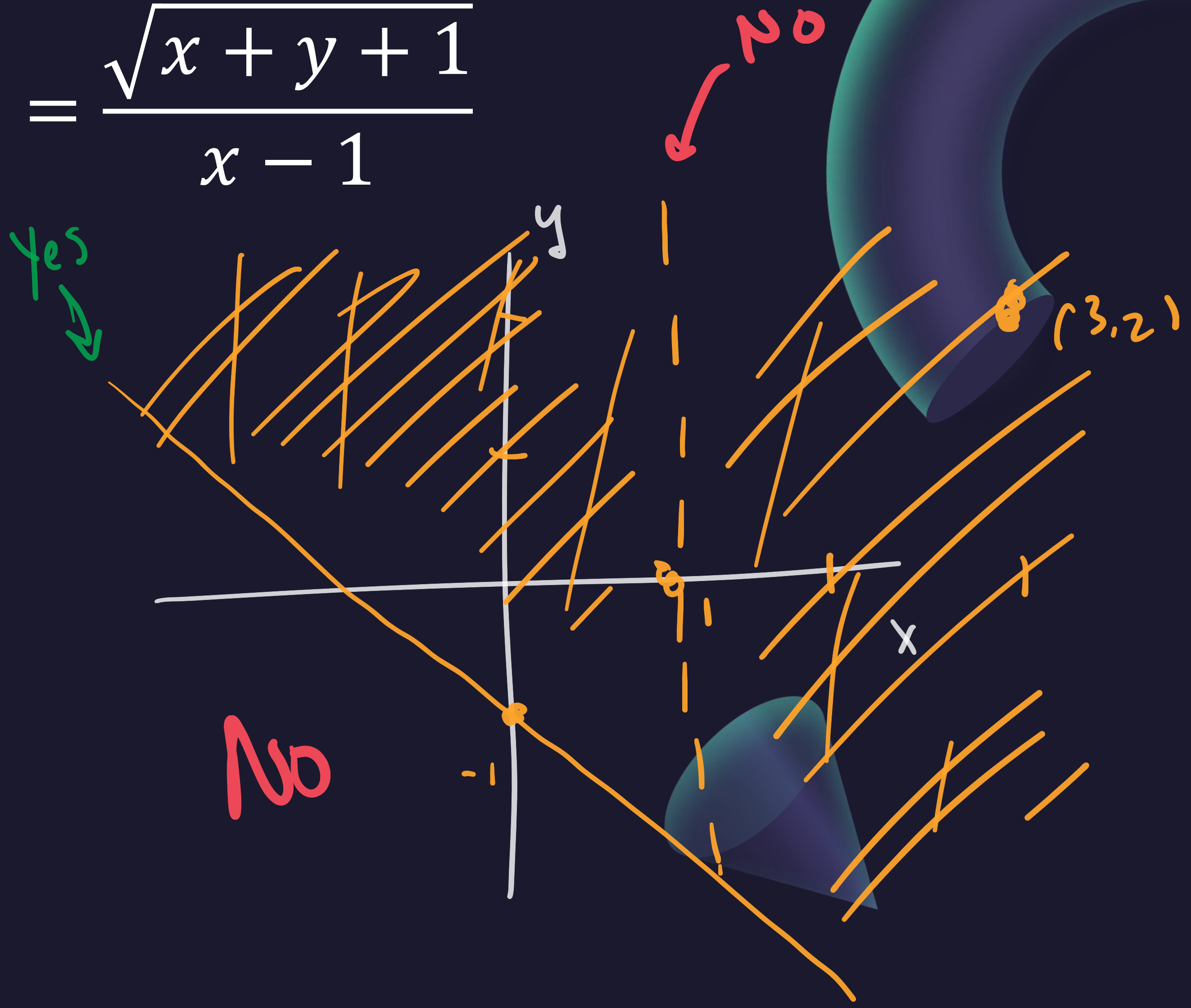
$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

$$f(3,2) = \frac{\sqrt{3+2+1}}{3-1} = \frac{\sqrt{6}}{2}$$

$$\begin{cases} x \neq 1 \\ x+y+1 \geq 0 \end{cases}$$

$$x+y+1 = 0$$

$$y = -x-1$$



Example: Sketch the domain of the function, and evaluate at (3,2)

$$f(x,y) = x \cdot \ln(y^2 - x)$$

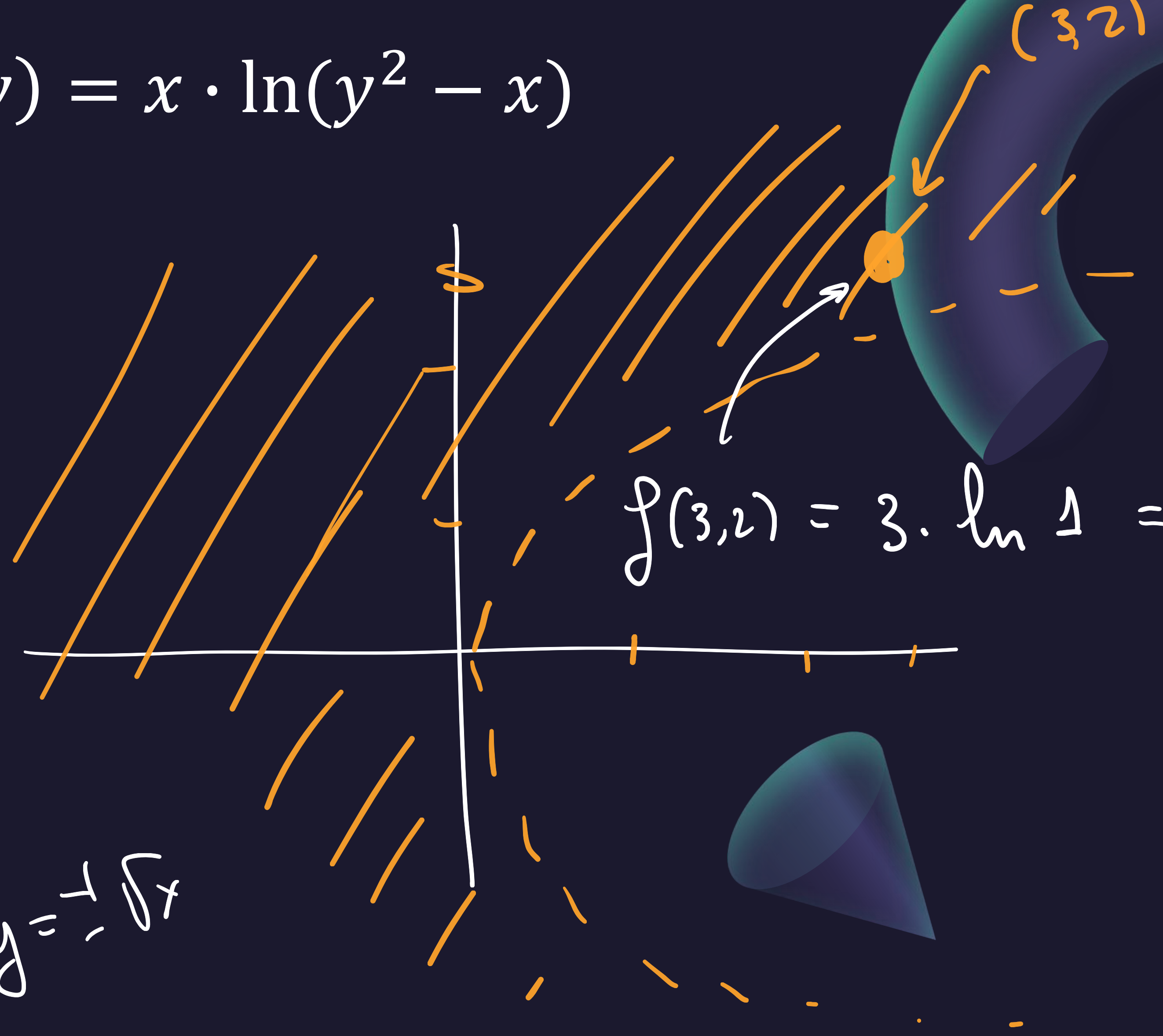
$\ln(x) \quad x > 0$

$y^2 - x > 0$

$y^2 > x$

$y > \pm\sqrt{x}$

$y^2 = x \Rightarrow y = \pm\sqrt{x}$



$f(3,2) = 3 \cdot \ln 1 = 0$

Example: Find the domain and range of the function

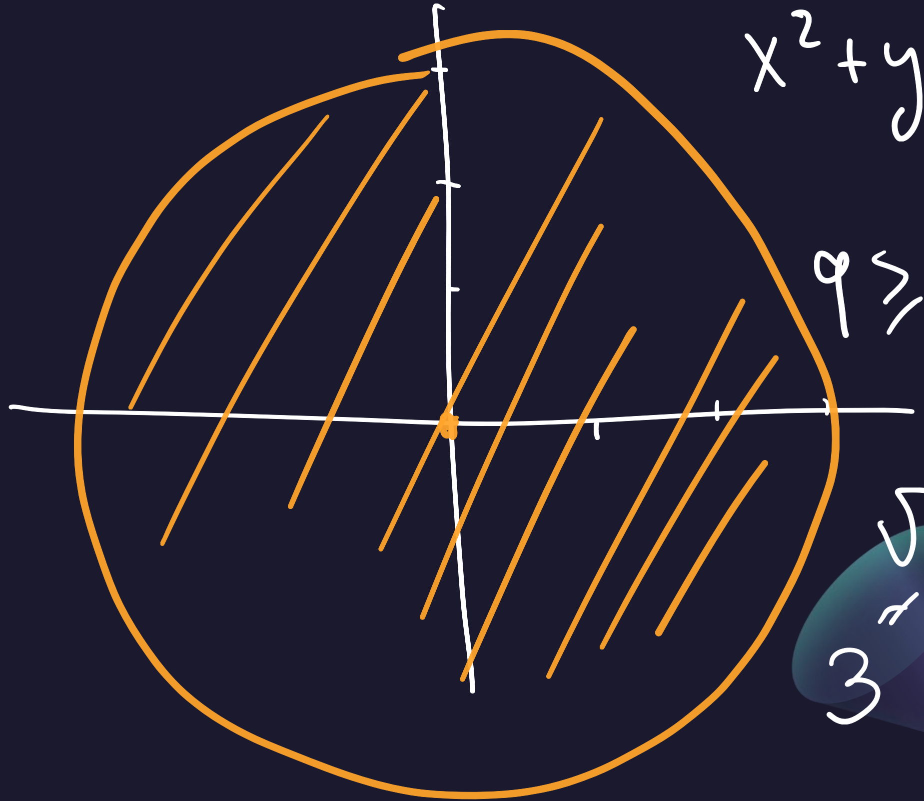
$$f(x, y) = \sqrt{9 - x^2 - y^2}$$

$$9 - x^2 - y^2 \geq 0$$

$$9 \geq x^2 + y^2$$

$$9 = x^2 + y^2$$

$$\text{Range} = [0, 3]$$



$$z = \sqrt{9 - x^2 - y^2}$$

$$z^2 = 9 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 9$$

$$9 \geq 9 - x^2 - y^2 \geq 0$$

$$\Downarrow$$

$$\sqrt{9} \geq \sqrt{9 - x^2 - y^2} \geq 0$$

$$3$$

Graphs of Functions of Two Variables

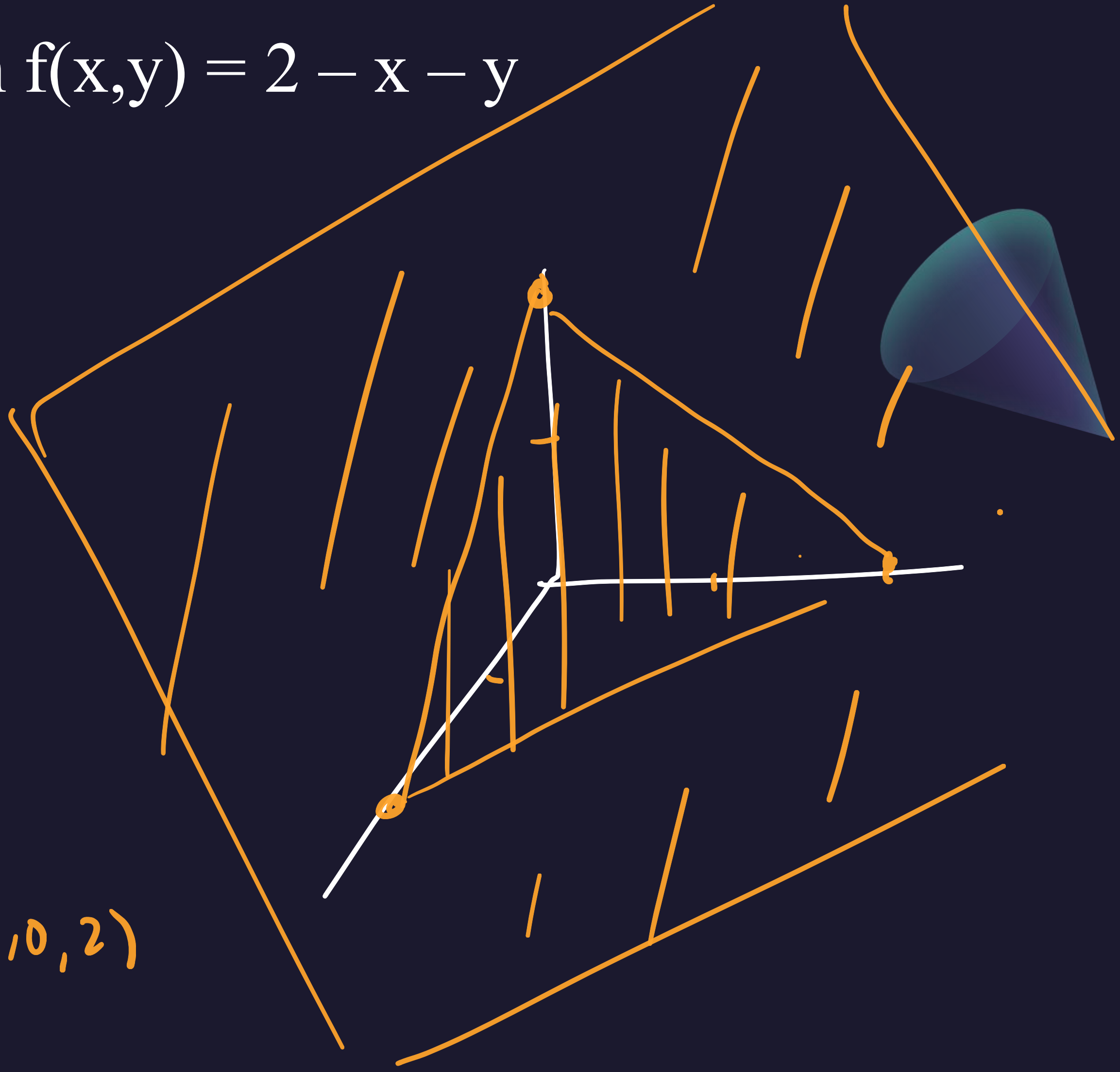
Example: Sketch a graph of the function f(x,y) = 2 – x – y

$$z = 2 - x - y$$

$$\boxed{x + y + z = 2}$$

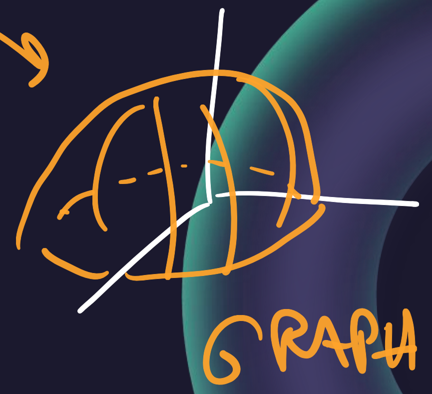
$$n = (1,1,1)$$

pts: $(2,0,0), (0,2,0), (0,0,2)$



Example: Sketch the graph of the function

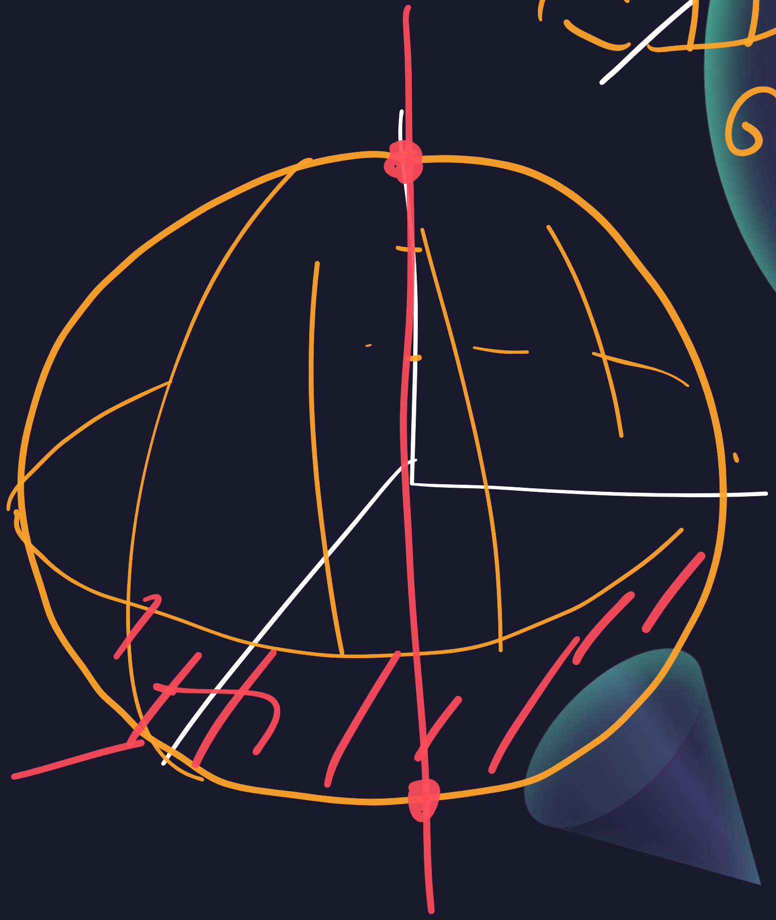
$$f(x, y) = \sqrt{9 - x^2 - y^2}$$



$$z = \sqrt{9 - x^2 - y^2}$$

$$z^2 = 9 - x^2 - y^2$$

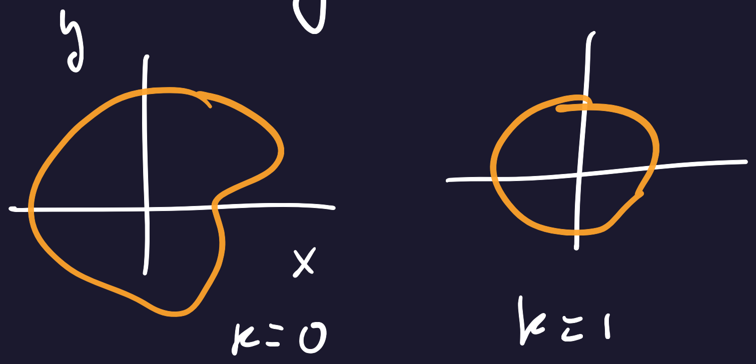
$$\boxed{x^2 + y^2 + z^2 = 9}$$



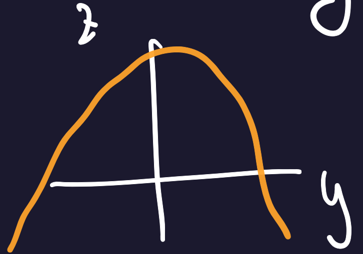
Cross Sections and Level Curves for Sketching

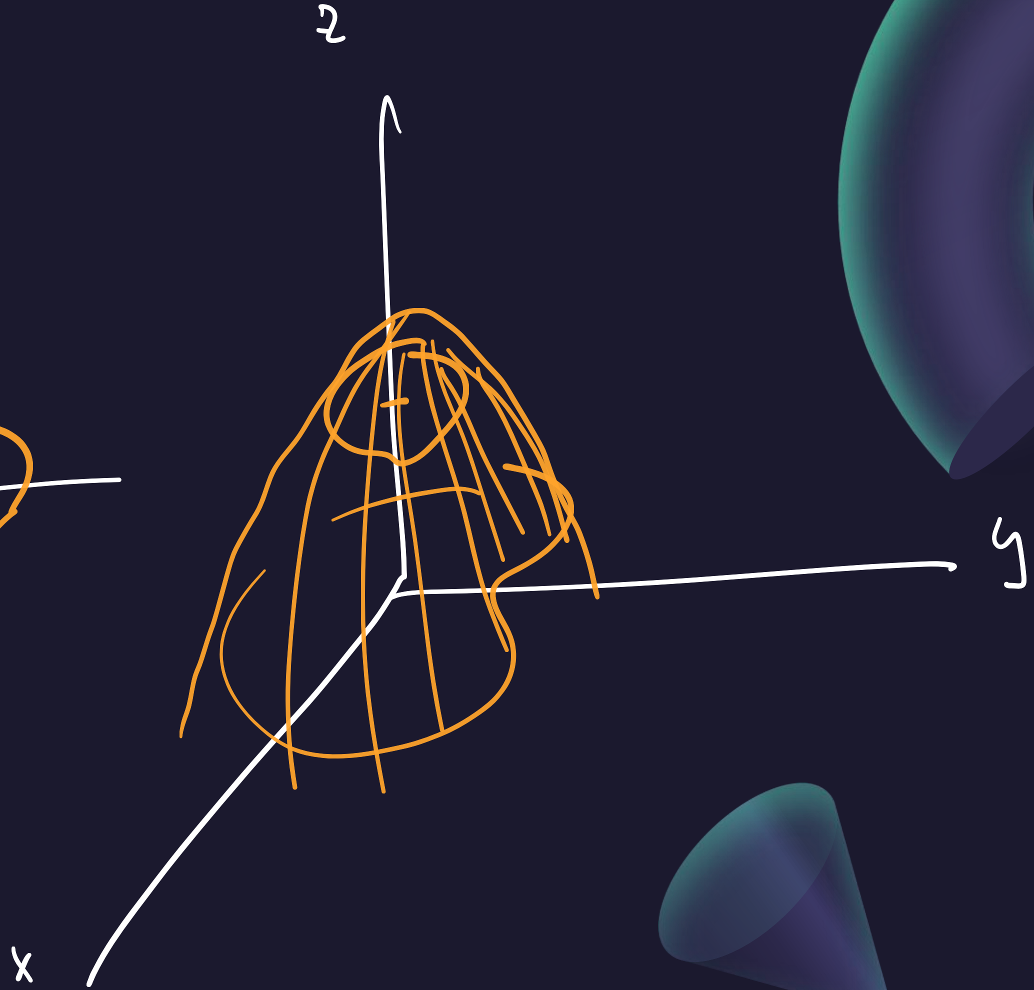
$$z = f(x, y)$$

$$z = k \Rightarrow k = f(x, y)$$



$$x = 0 \quad z = f(0, y)$$



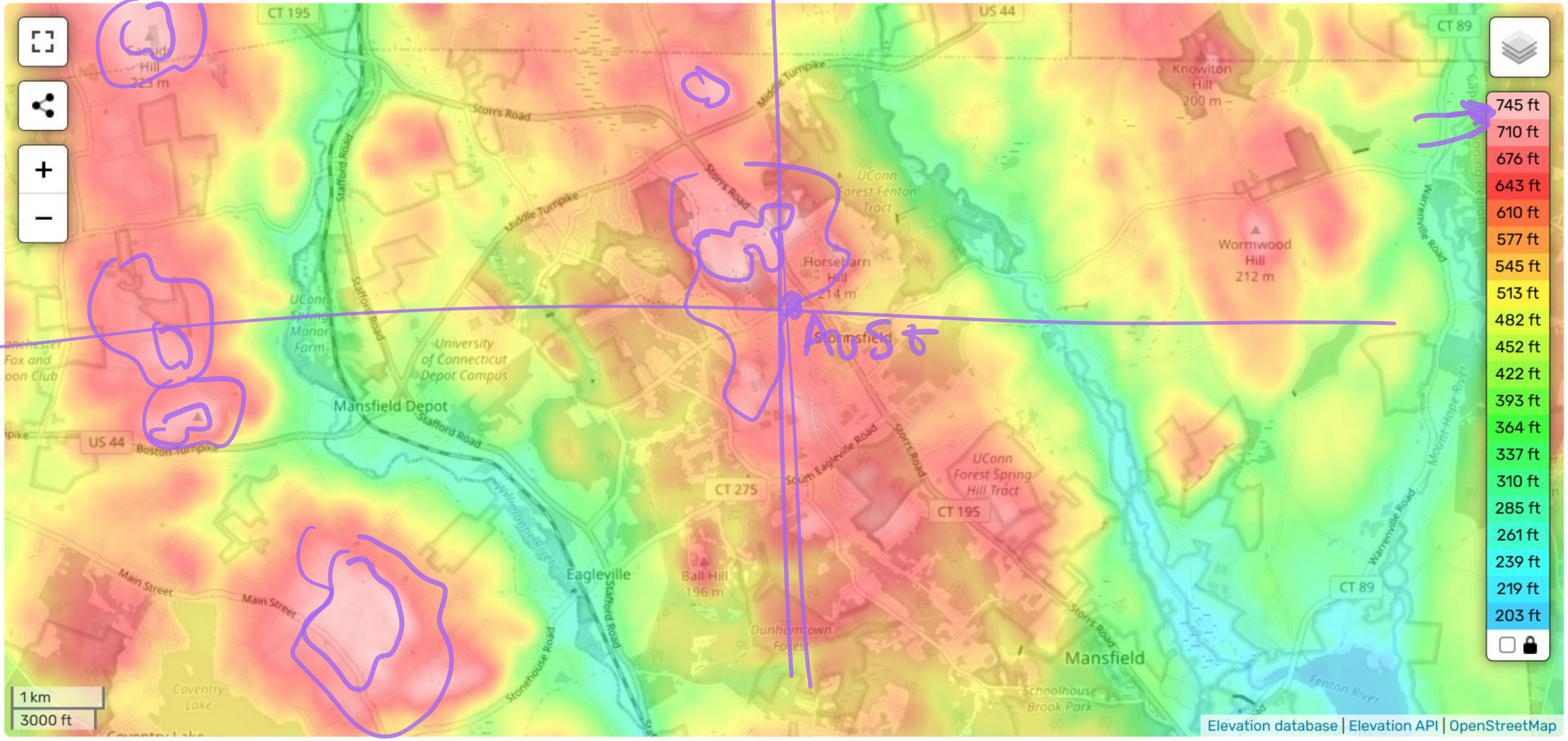


Cross Sections and Level Curves for Sketching

Storrs topographic map

> United States > Connecticut > Capitol Planning Region > Mansfield > Storrs > Storrs

Click on the **map** to display **elevation**.



Example: Sketch the graph of the function

$$f(x,y) = e^{-(x^2+y^2)} > 0$$

$k = 1$

$x^2 + y^2 = 0$

$\Rightarrow (0,0)$

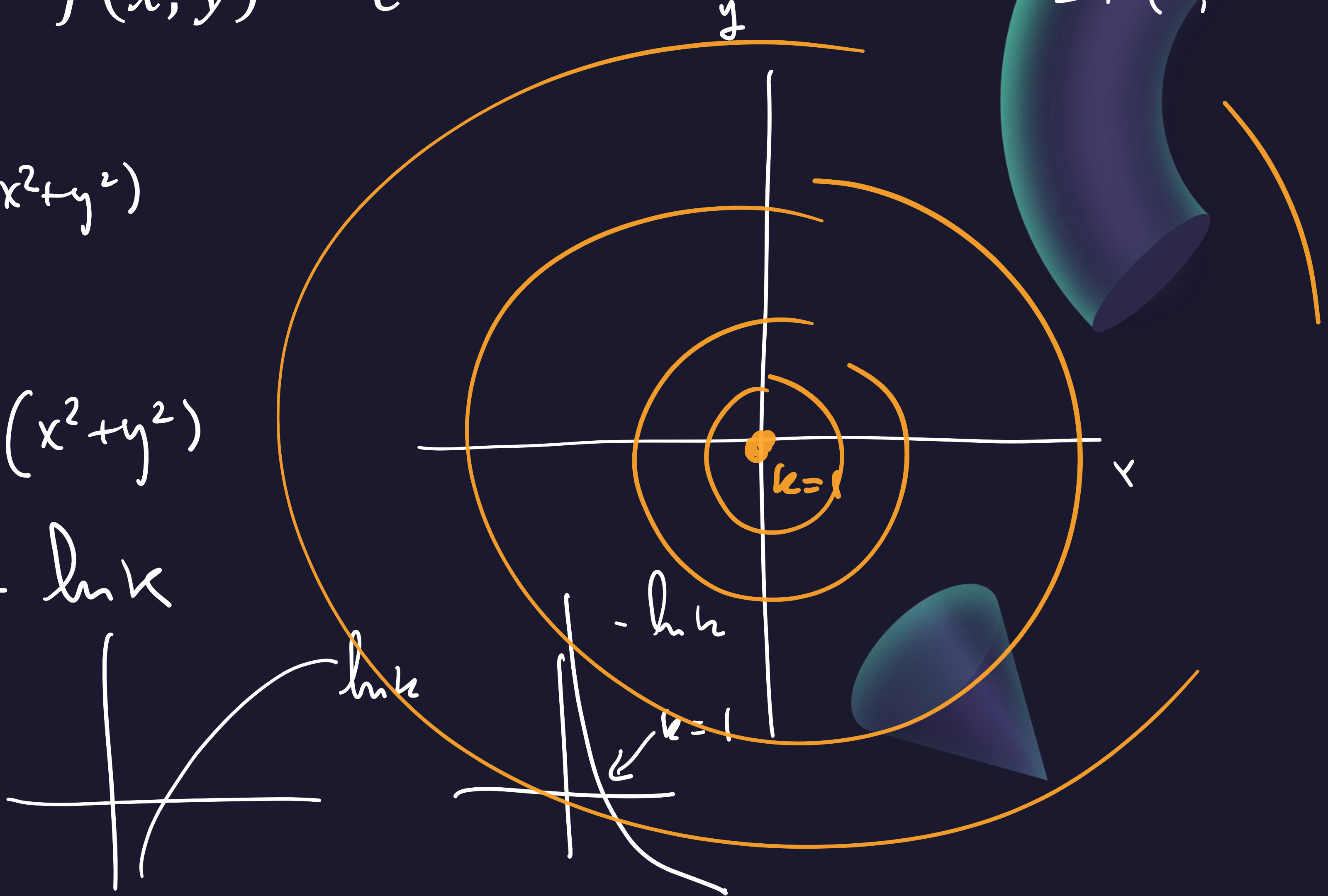
$$z = e^{-(x^2+y^2)}$$

$z = k \qquad k = e^{-(x^2+y^2)}$

$$\ln k = -(x^2+y^2)$$

$$x^2+y^2 = -\ln k$$

$k > 0$



Example: Find the level surfaces of the function

$$t = f(x, y, z) = x^2 + y^2 + z^2$$

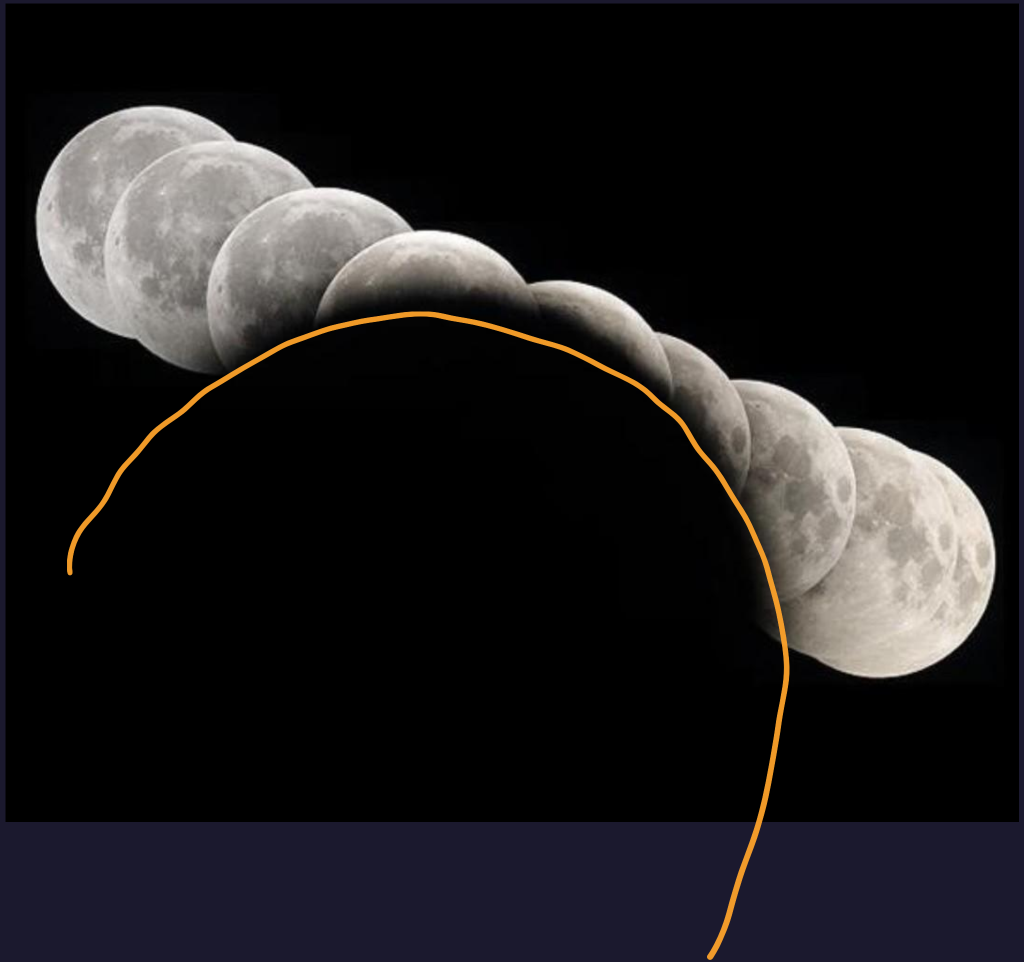
$$t = k = x^2 + y^2 + z^2$$

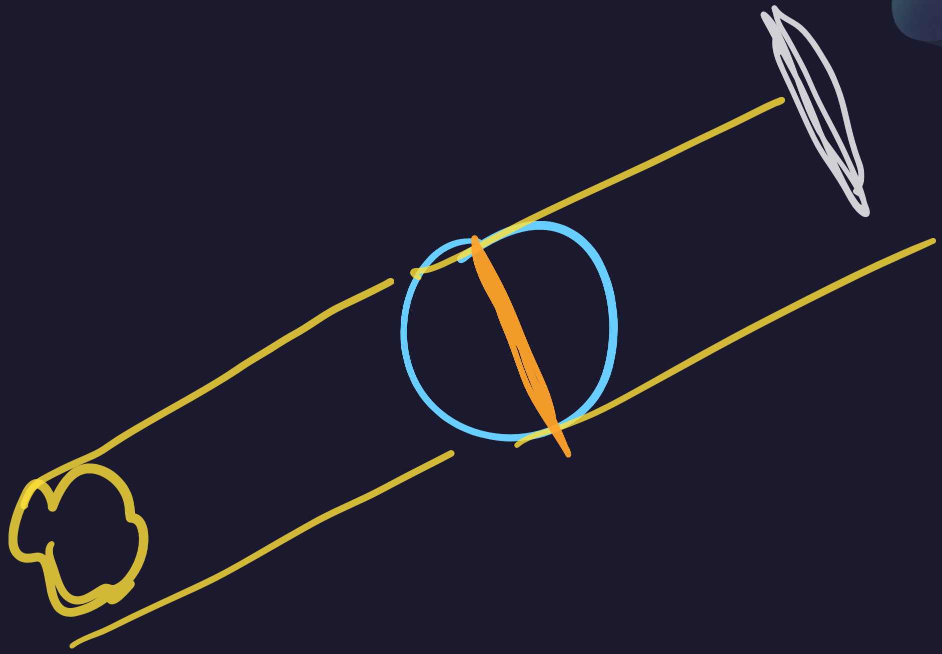
$t=0$ $t=1$ $t=4$ $t=9$

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof





Questions?

ALVARO: Start the recording!

“Calculus 3”

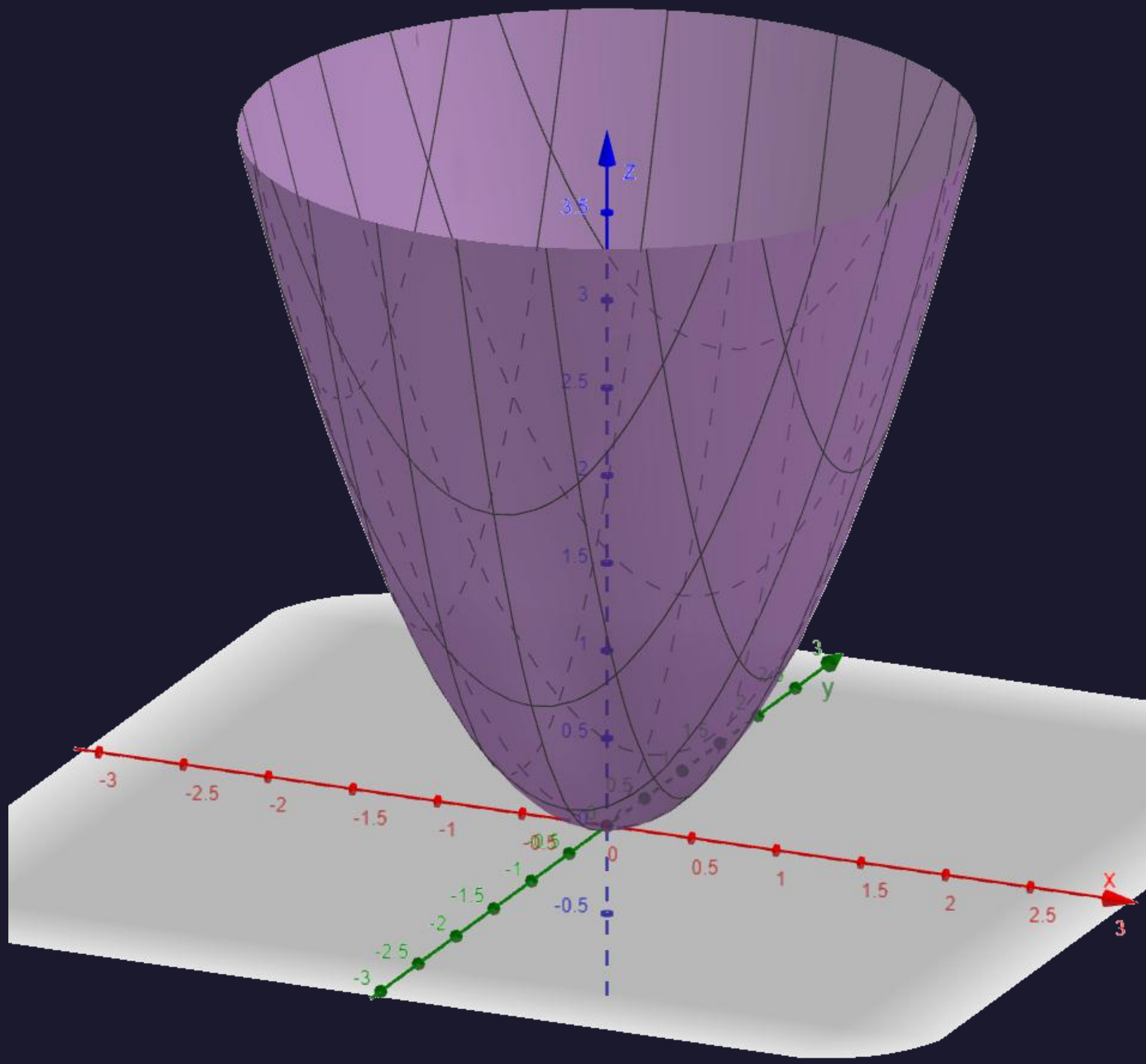
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

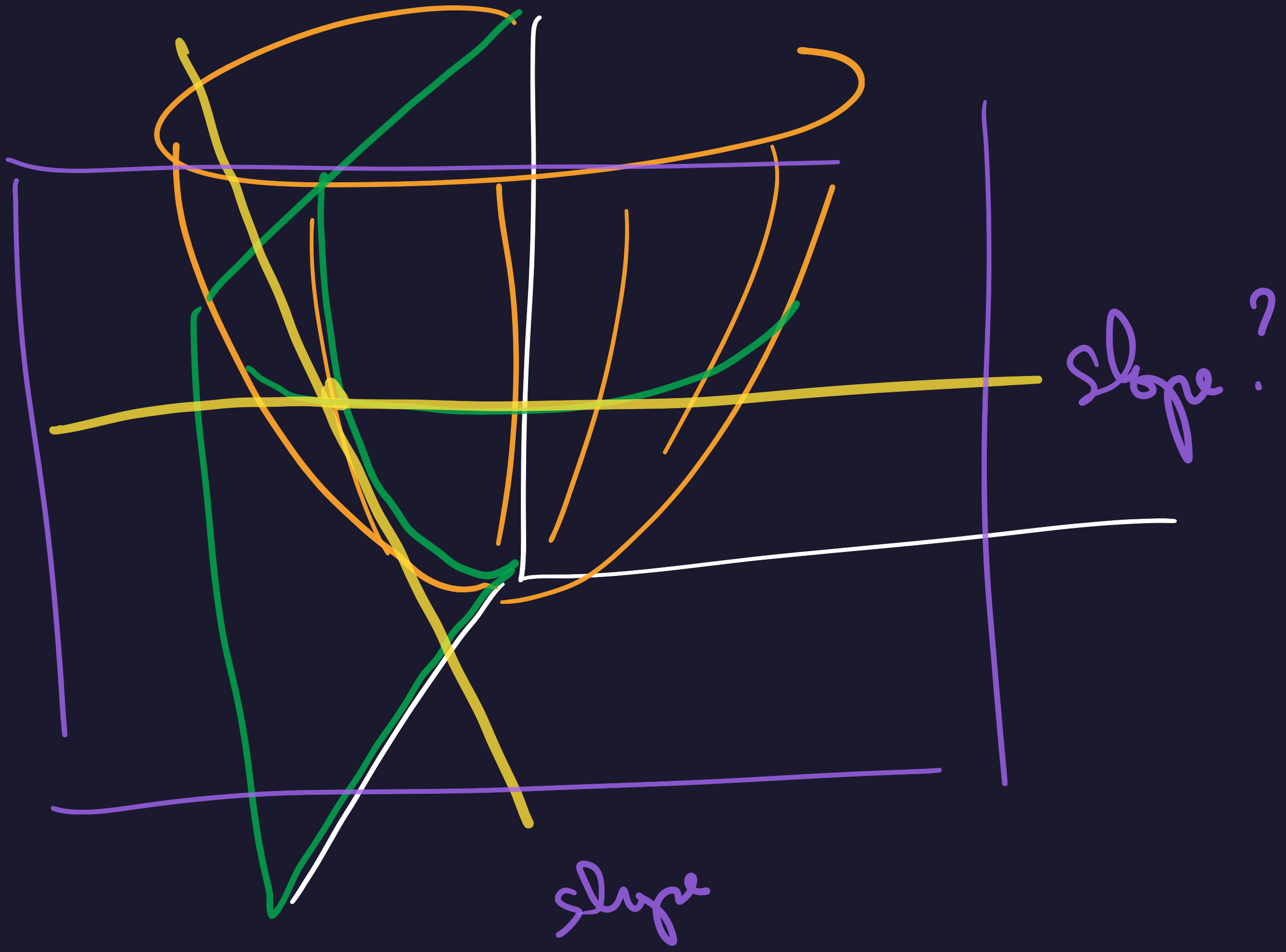
Partial Derivatives

Today – Derivatives!

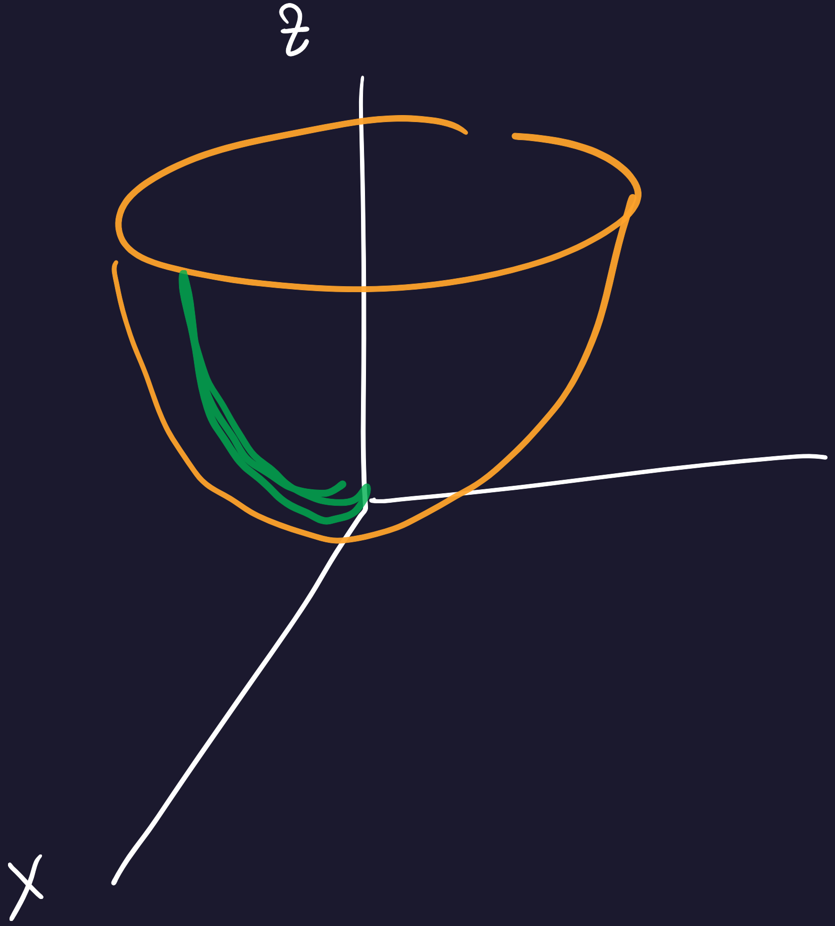
- Partial Derivatives
- Interpretation
- Higher Derivatives
- PDEs



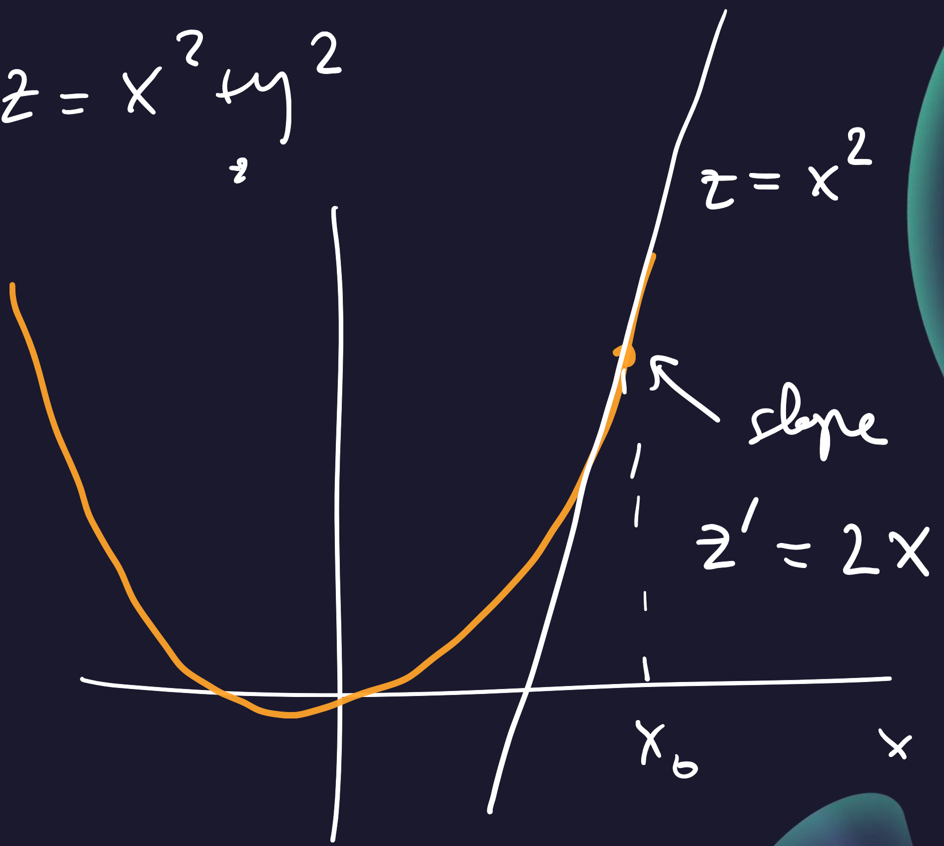
Partial Derivatives



Example: Partial derivatives of $f(x, y) = x^2 + y^2$



$z = x^2 + y^2$



$z_0 = x_0^2$

$z - z_0 = m(x - x_0)$

$2x_0$

Partial Derivatives – The Limit Definition

$$y = f(x) \qquad \left.\frac{df}{dx}\right|_{x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$z = f(x, y)$

(x_0, y_0)

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0, y_0)} = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

$z = f(x, y_0)$

Partial Derivatives – Notation

Example: Find the partial derivatives of $f(x,y) = 4 - x^2 - y^2$ at $(1,1)$ and interpret those as slopes.

Example: Find the partial derivatives of $f(x, y) = x \cdot \ln(y^2 - x)$ at $(3,2)$.

Example: Find the partial derivatives of the Body-Mass-Index formula

$$B(m,h) = m/\mathrm{h}^2 \quad \text{at} \quad \mathrm{m} = 64\mathrm{kg} \quad \text{and} \quad \mathrm{h} = 1.68\mathrm{m}.$$

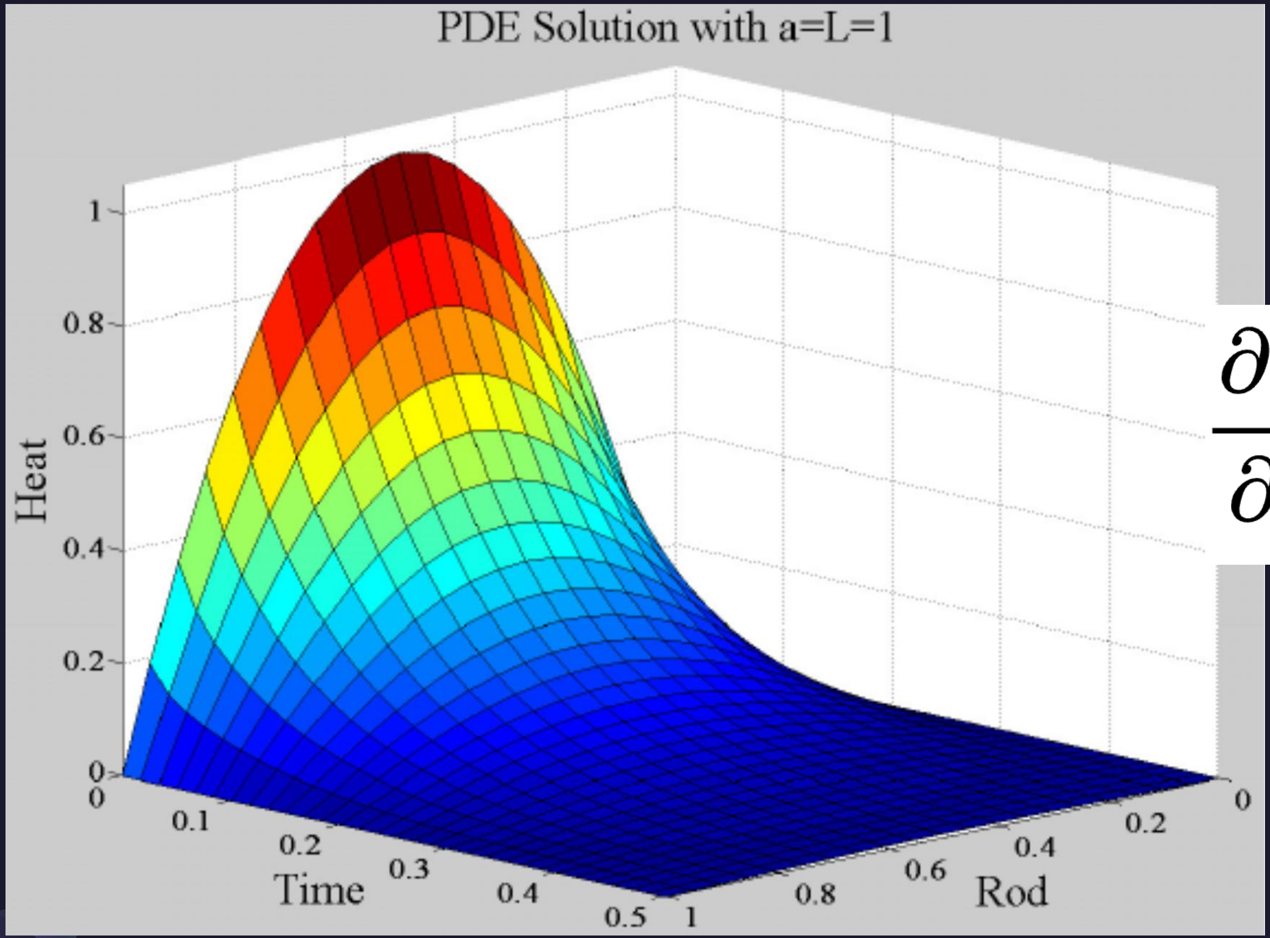
Example: Find the partial derivatives of $f(x, y, z) = \sin(x^2 + y^2 + z^2)$ at (1,2,3).

Higher Partial Derivatives

Example: Find the second partial derivatives of

$$f(x, y) = 4x^2y - \mathrm{x}^3 - \mathrm{y}^2$$

Partial Differential Equations



Example: The Heat Equation

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

Example: Show that the function $u(x,t) = \sin(x - a \cdot t)$ satisfies the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Questions?

Thank you

Until next time.