Audience Q&A

Do not edit
How to change the design

The Slido app must be installed on every computer you're presenting from

slido

"Calculus 3"

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 5

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week's material

Questions?

ALVARO: Start the recording!

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More on Quadrics

How to sketch a quadric surface?

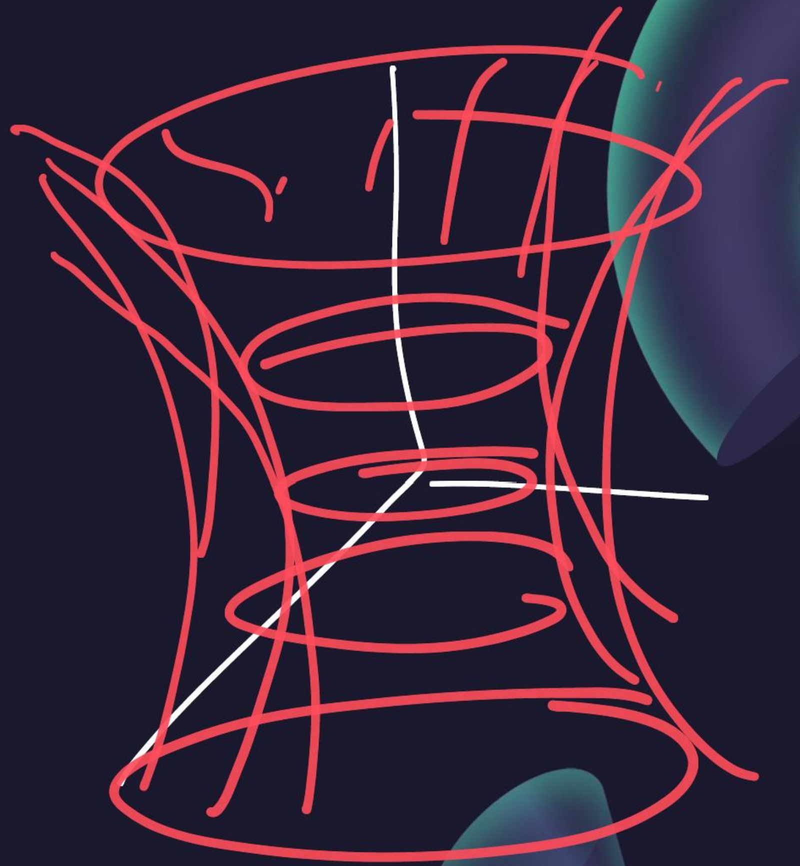
Traces or Cross Sections of a Surface

Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.

Example: Sketch the surface $x^2 + y^2 - z^2 = 1.$

$y=0 \quad x^2 - z^2 = 1$

$x=0 \quad y^2 - z^2 = 1$



Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.

Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0.$

Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.

$$x^2 - 6x = (x - 3)^2 - 9$$

$$\begin{aligned} x^2 + 2z^2 - 6x - y + 10 &= (x - 3)^2 - 9 + 2z^2 - y + 10 \\ &= (x - 3)^2 + 2z^2 - y + 1 \end{aligned}$$

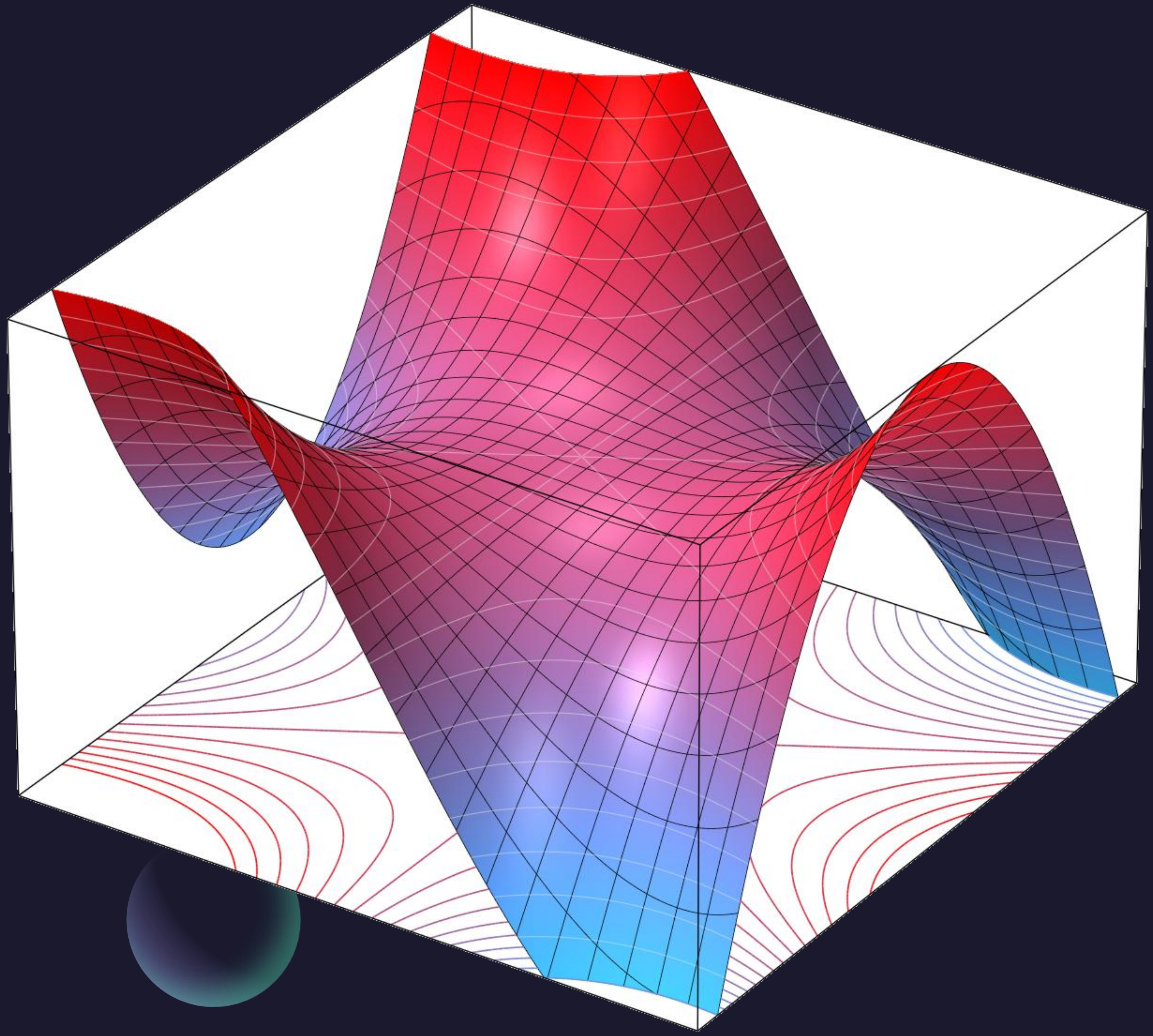
$x^2 + 2z^2 - 6x - y + 10 = 0$ is equivalent to

$$y = (x - 3)^2 + 2z^2 + 1$$

Questions?

Thank you

Until next time.



ALVARO: Start the recording!

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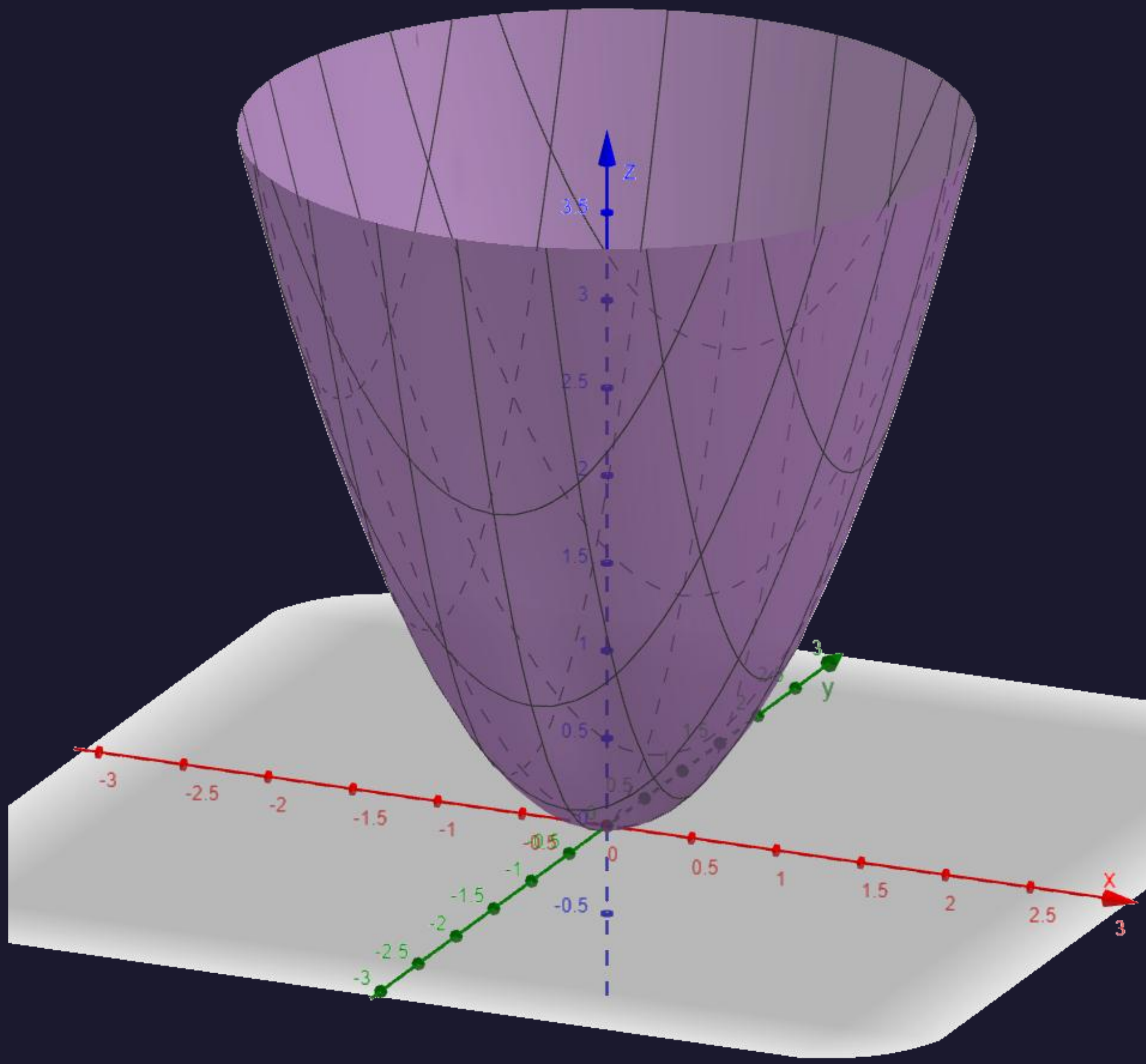
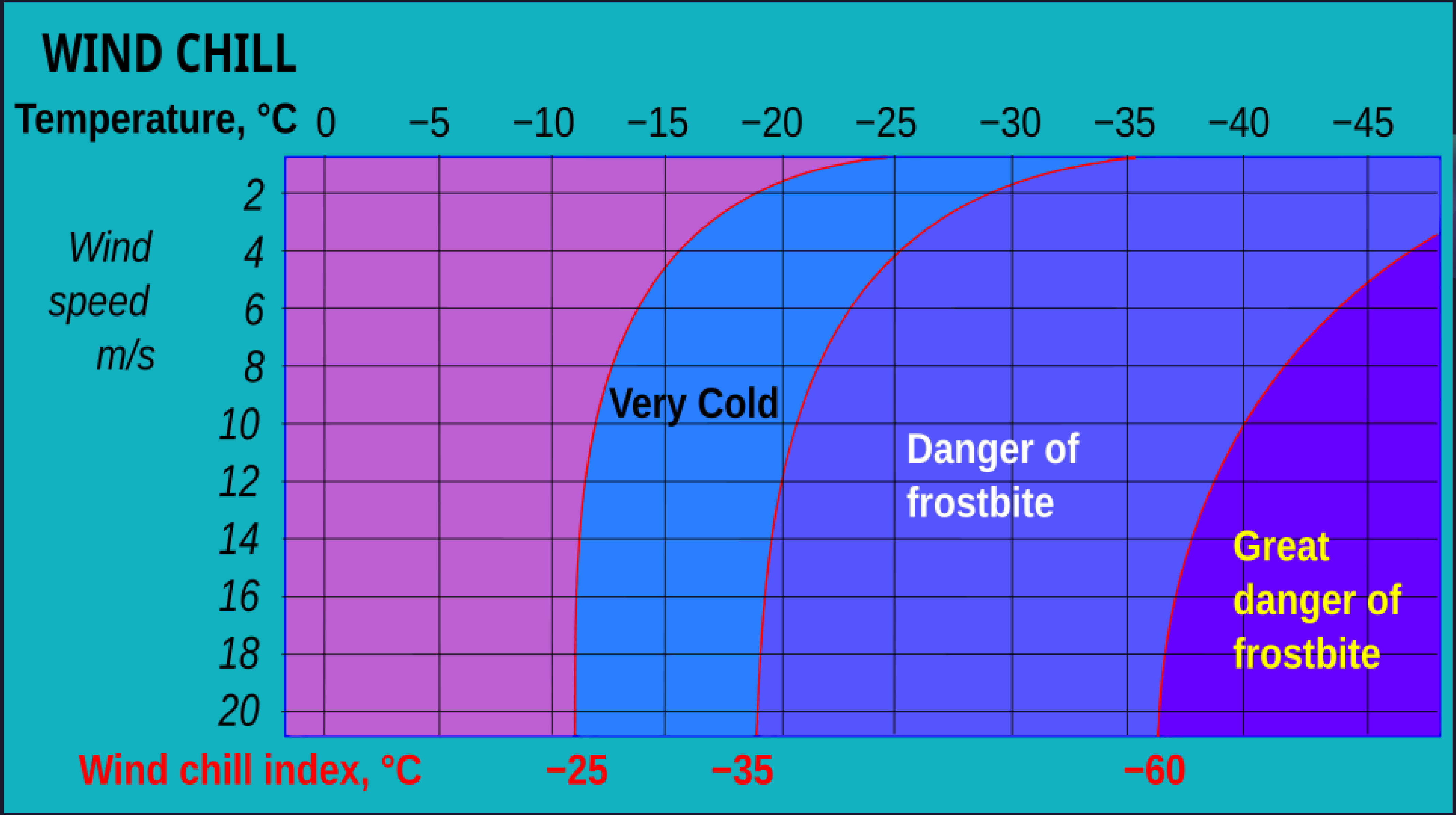
Functions of Several Variables

Today – Functions!

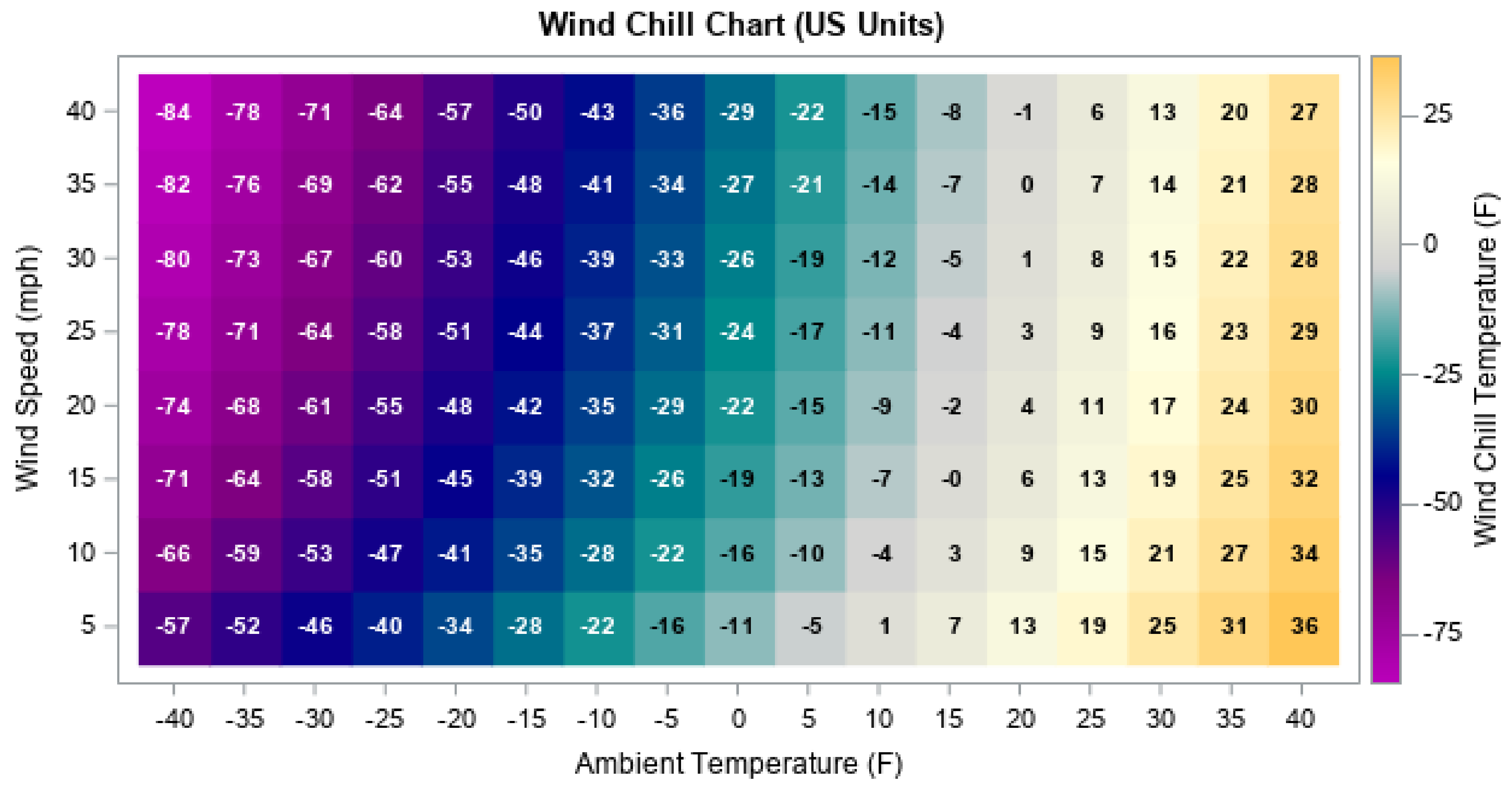
- Functions of Two Variables
- Domain and Range
- Graphs
- Level Curves
- Functions of More Than Two Variables



Functions of Two Variables



Functions of Two Variables



Source NOAA: https://www.weather.gov/safety/cold-wind-chill-chart

Functions of Two Variables

The standard wind chill formula for Environment Canada is:[3]

$$T_{\mathrm{wc}} = 13.12 + 0.6215 T_{\mathrm{a}} - 11.37 v^{+0.16} + 0.3965 T_{\mathrm{a}} v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Celsius temperature scale; T_a is the air temperature in degrees Celsius; and v is the wind speed at 10 m (33 ft) standard anemometer height, in kilometres per hour.[11]

When the temperature is −20 °C (−4 °F) and the wind speed is 5 km/h (3 mph), the wind chill index is −24. If the temperature remains at −20 °C and the wind speed increases to 30 km/h (19 mph), the wind chill index falls to −33.

The equivalent formula in US customary units is:[12][3]

$$T_{\mathrm{wc}} = 35.74 + 0.6215 T_{\mathrm{a}} - 35.75 v^{+0.16} + 0.4275 T_{\mathrm{a}} v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Fahrenheit scale; T_a is the air temperature in degrees Fahrenheit; and v is the wind speed in miles per hour.[13]

Functions of Two Variables, Domain, and Range

Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is,

$$\{f(x, y) \mid (x, y) \in D\}.$$

Functions of Two Variables, Domain, and Range

Example: Sketch the domain of the function, and evaluate at (3,2)

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1}$$

Example: Sketch the domain of the function, and evaluate at (3,2)

$$f(x, y) = x \cdot \ln(y^2 - x)$$

Example: Find the domain and range of the function

$$f(x, y) = \sqrt{9 - x^2 - y^2}$$

Graphs of Functions of Two Variables

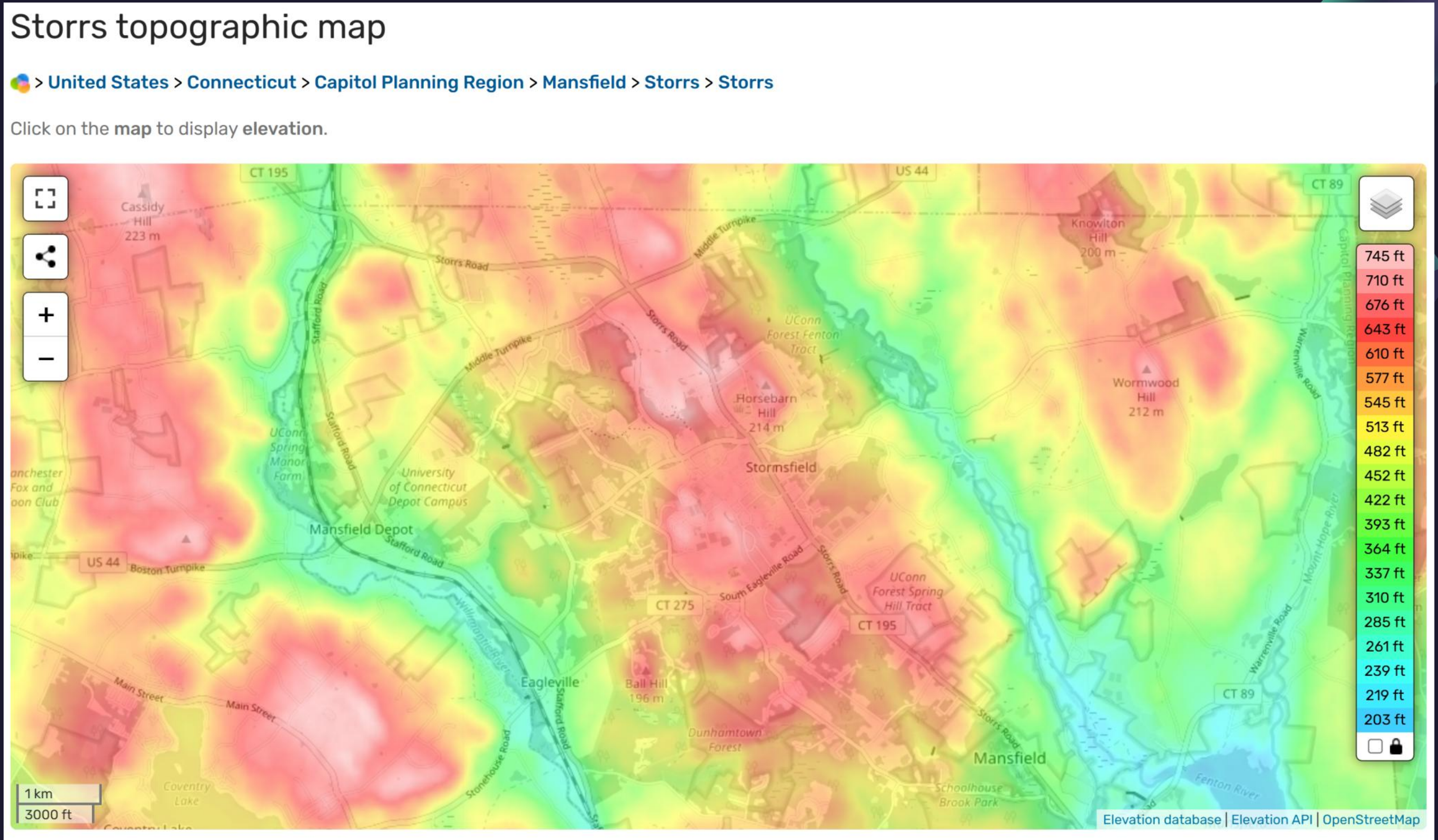
Example: Sketch a graph of the function $f(x,y) = 2 - x - y$

Example: Sketch the graph of the function

$$f(x, y) = \sqrt{9 - x^2 - y^2}$$

Cross Sections and Level Curves for Sketching

Cross Sections and Level Curves for Sketching



Example: Sketch the graph of the function

$$f(x,y) = e^{-(x^2+y^2)}$$

Example: Find the level surfaces of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

Using Cross Sections to “Prove” that Earth is Spherical: Aristotle’s Proof

Questions?

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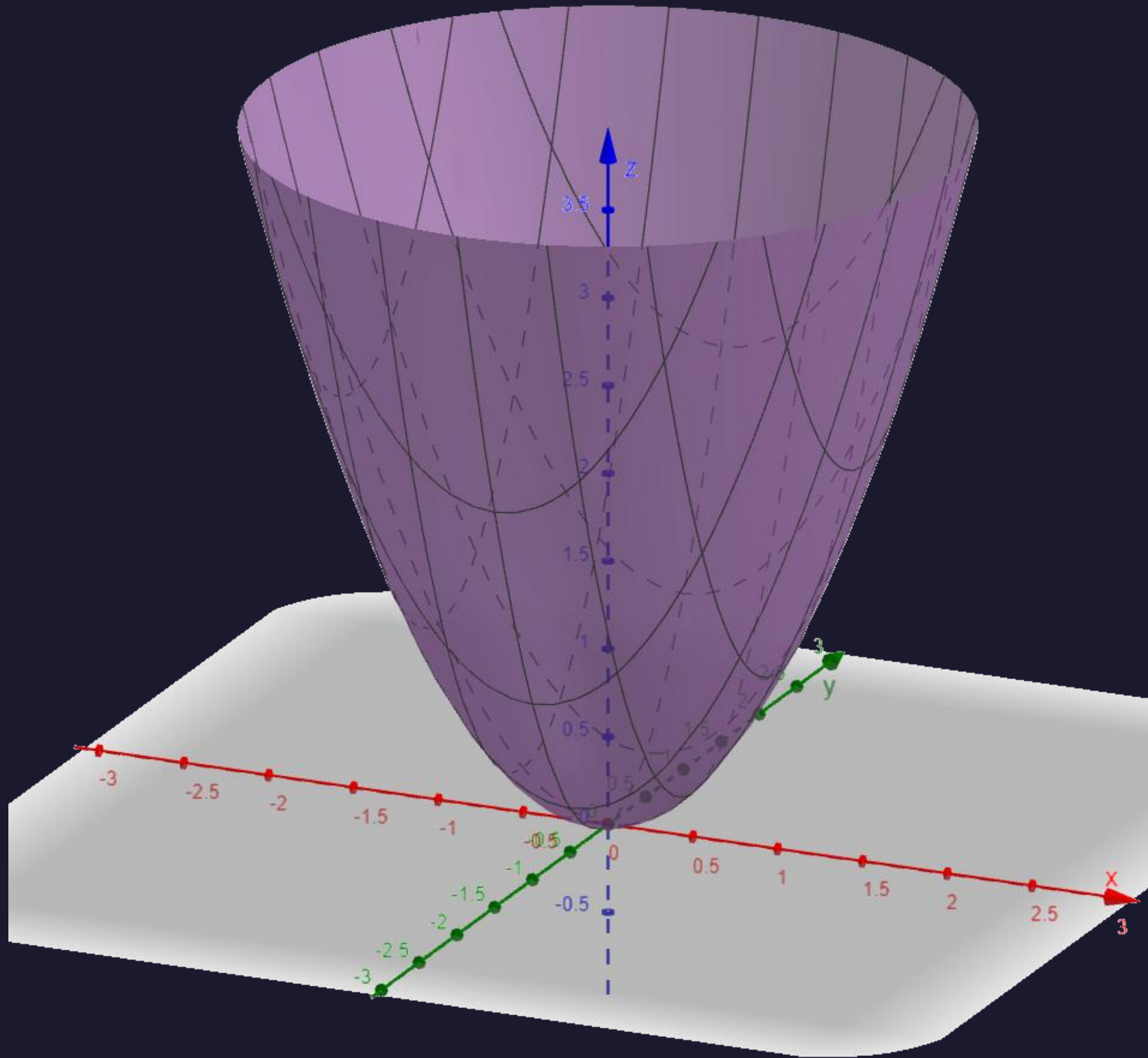
Partial Derivatives

Today – Derivatives!

- Partial Derivatives
- Interpretation
- Higher Derivatives
- PDEs



Partial Derivatives

Example: Partial derivatives of $f(x, y) = x^2 + y^2$

Partial Derivatives – The Limit Definition

Partial Derivatives – Notation

Example: Find the partial derivatives of $f(x, y) = 4 - \mathrm{x}^2 - \mathrm{y}^2$ at (1,1) and interpret those as slopes.

Example: Find the partial derivatives of $f(x, y) = x \cdot \ln(y^2 - x)$ at $(3,2)$.

Example: Find the partial derivatives of the Body-Mass-Index formula

$$B(m,h) = m/\mathrm{h}^2 \quad \text{at} \quad \mathrm{m} = 64\mathrm{kg} \quad \text{and} \quad \mathrm{h} = 1.68\mathrm{m}.$$

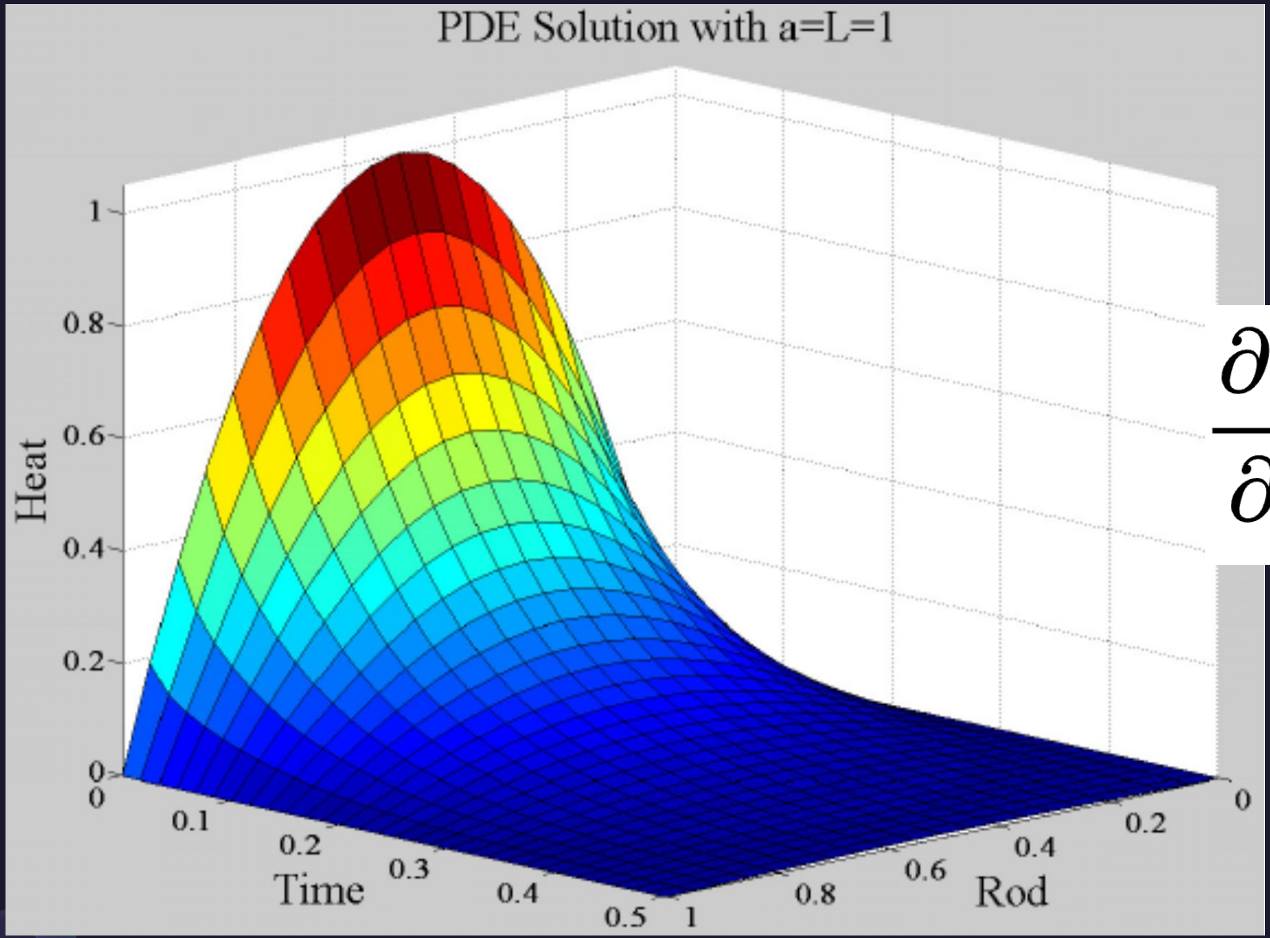
Example: Find the partial derivatives of $f(x, y, z) = \sin(x^2 + y^2 + z^2)$ at (1,2,3).

Higher Partial Derivatives

Example: Find the second partial derivatives of

$$f(x, y) = 4x^2y - x^3 - y^2$$

Partial Differential Equations



Example: The Heat Equation

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

Example: Show that the function u(x,t) = sin(x – a·t) satisfies the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Questions?

Thank you

Until next time.