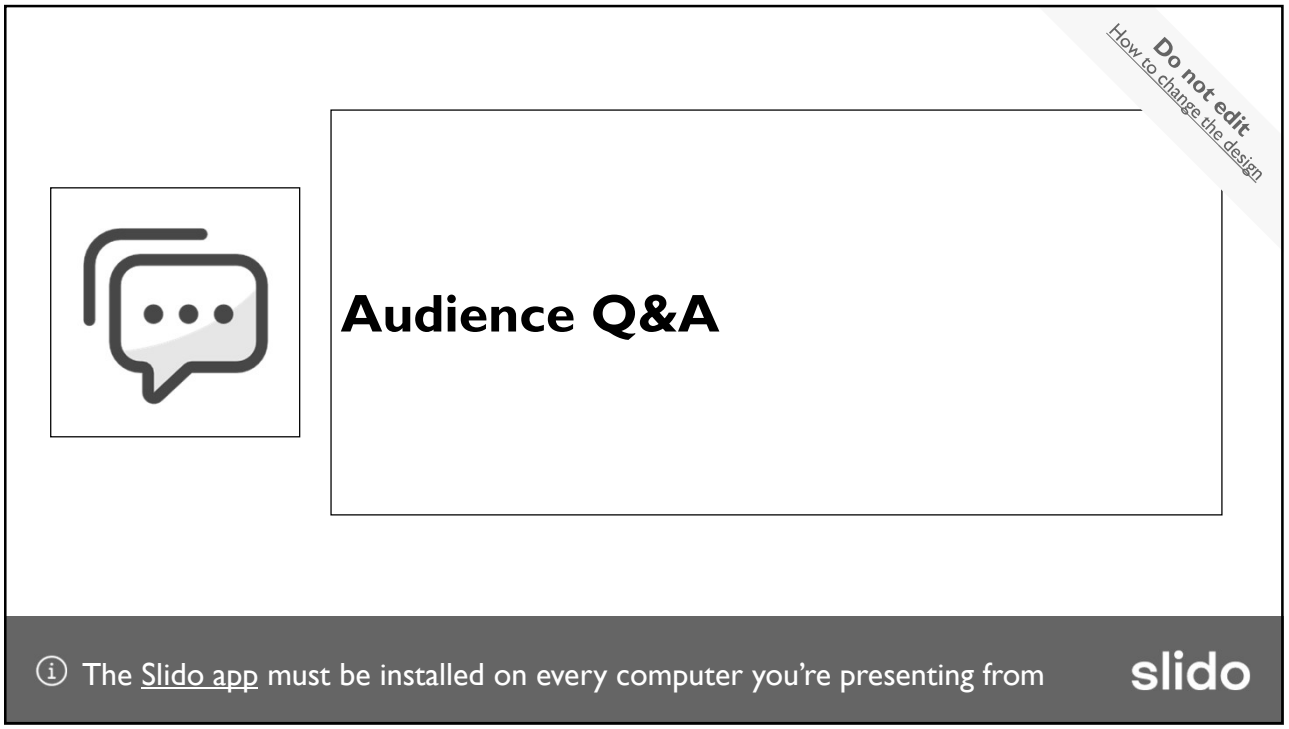
Do not edit
How to change the design
Audience Q&A
The Slido app must be installed on every computer you're presenting from
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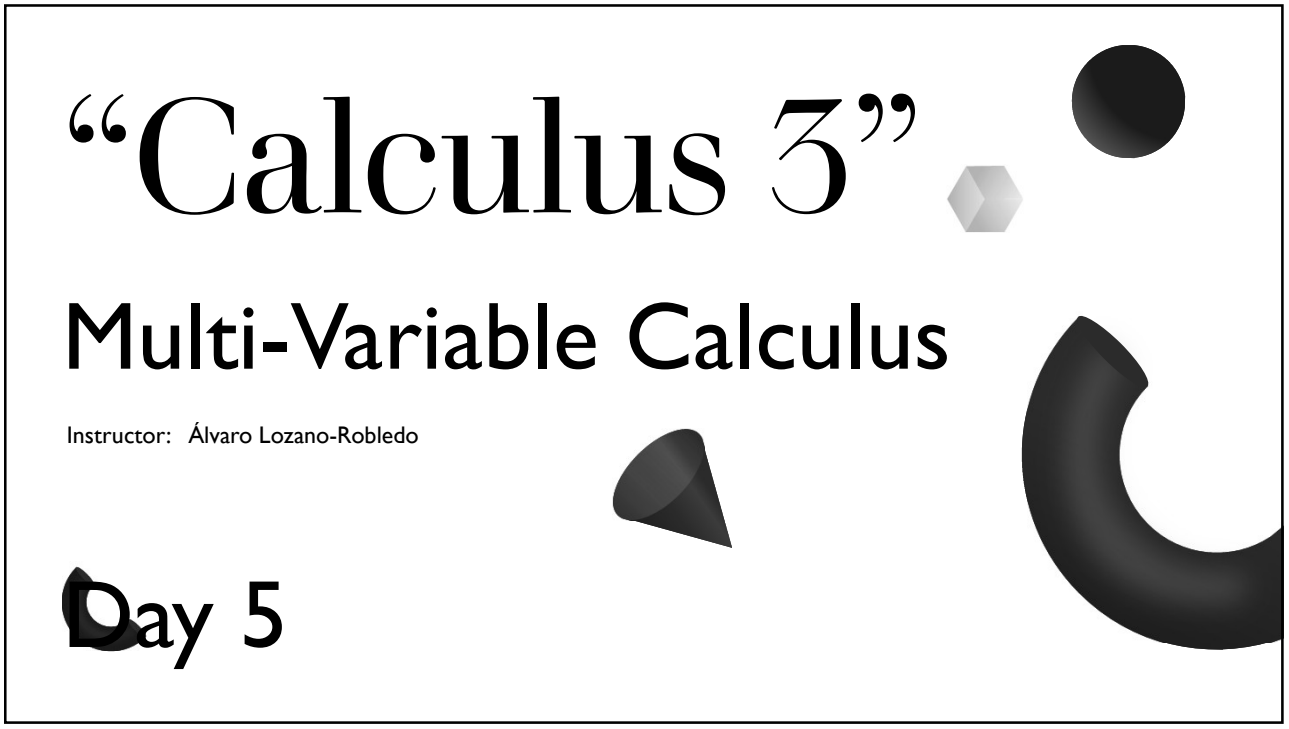

"Calculus 3"
Multi-Variable Calculus
Instructor: Álvaro Lozano-Robledo
Day 5

2

Any Reminders? Any Questions?

- Class ends at 3:15.
- Slides are being posted on GitHub!
  https://github.com/alozanoroble/MATH-2110Q-Spring-2026
- Videos will be posted on YouTube… but they may lag!
- All requests for make-up quizzes need to go to your TA
- Second quiz (Friday) will be on previous week's material

3

Questions?

4

ALVARO: Start the recording!

5

"Calculus 3"

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

More on Quadrics

6

How to sketch a quadric surface?

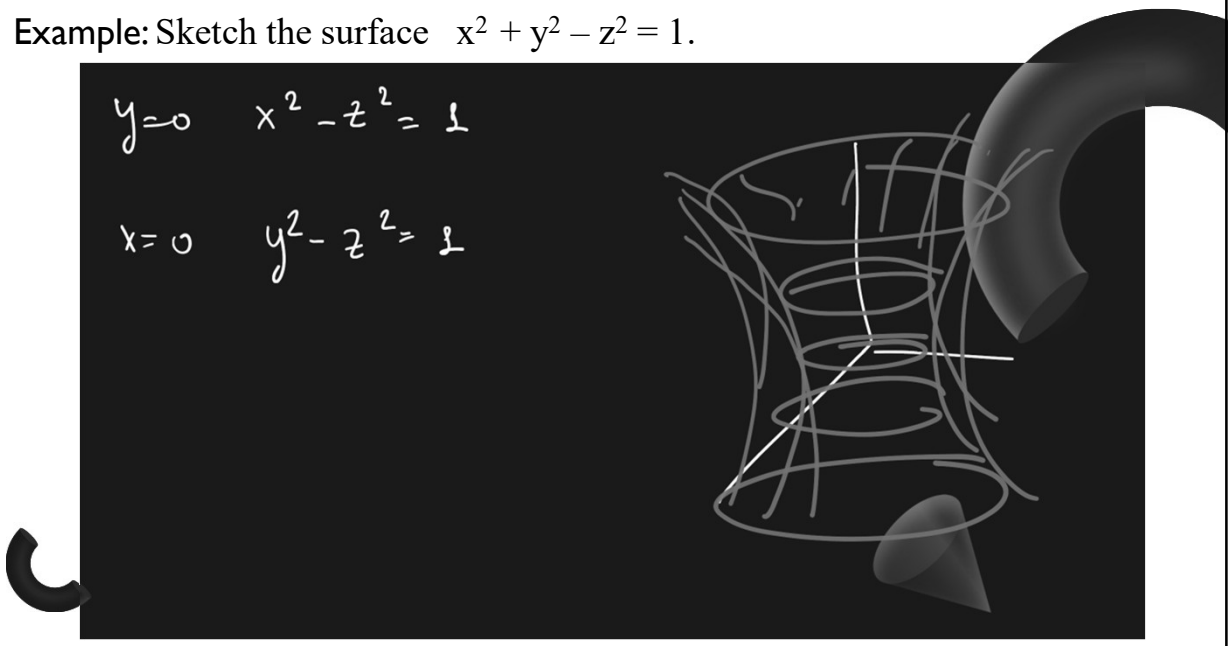
Traces or Cross Sections of a Surface

7

Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.

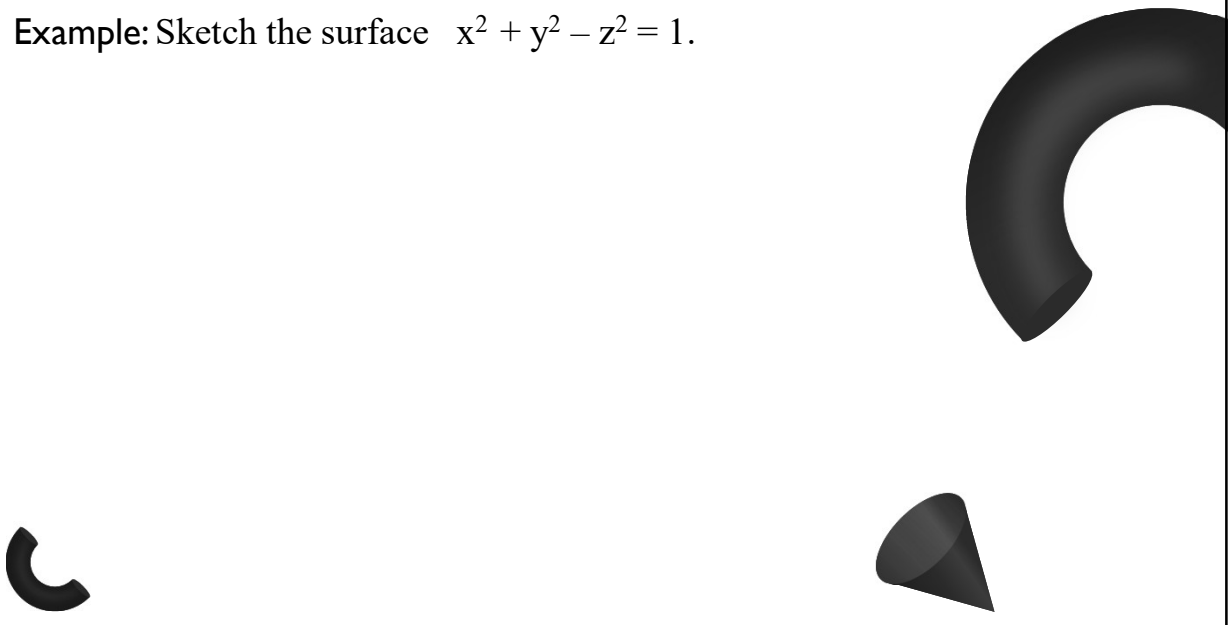
8

Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.



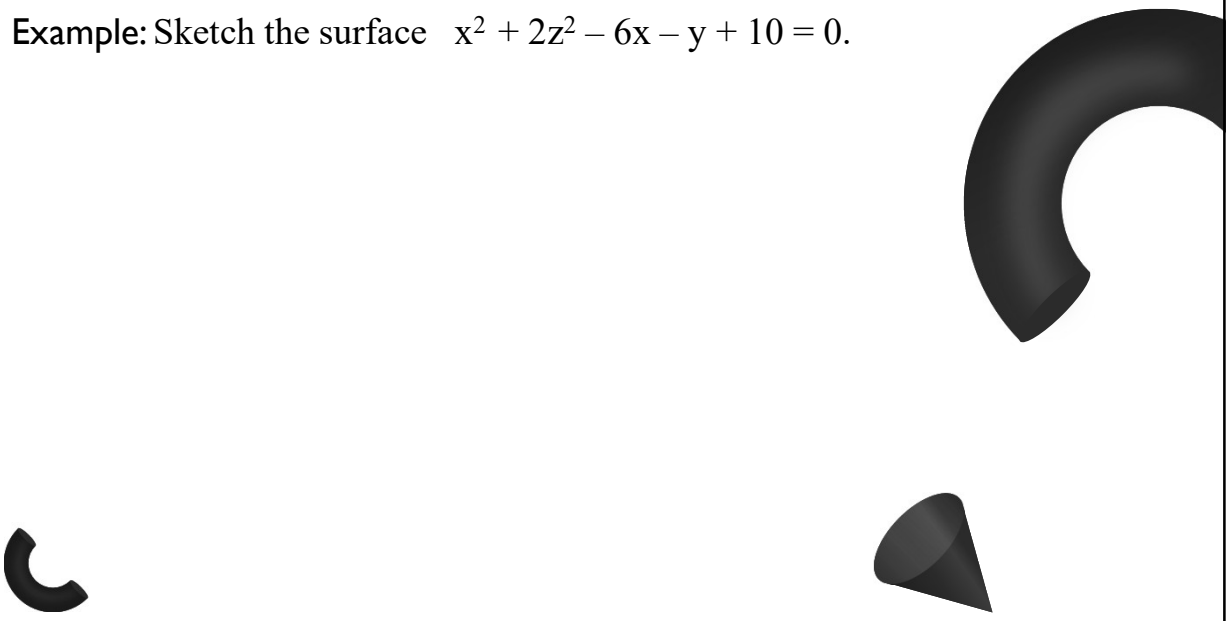
9

Example: Sketch the surface $x^2 + y^2 - z^2 = 1$.



10

Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.



11

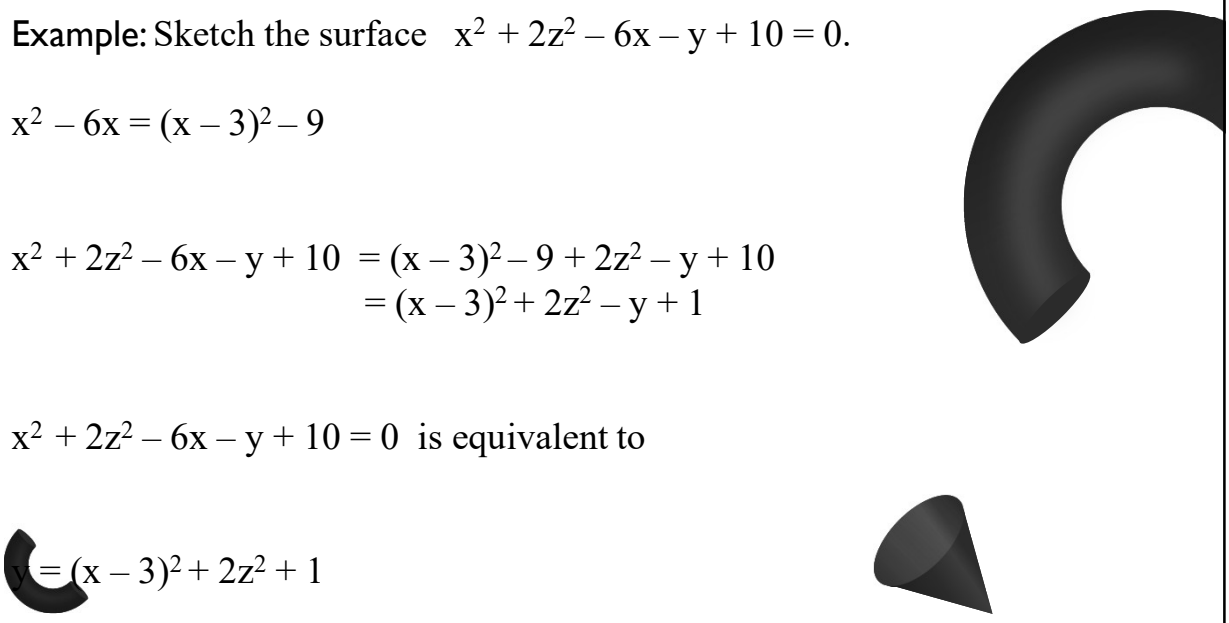
Example: Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.

$x^2 - 6x = (x - 3)^2 - 9$

$x^2 + 2z^2 - 6x - y + 10 = (x - 3)^2 - 9 + 2z^2 - y + 10$
$= (x - 3)^2 + 2z^2 - y + 1$

$x^2 + 2z^2 - 6x - y + 10 = 0$ is equivalent to

$y = (x - 3)^2 + 2z^2 + 1$



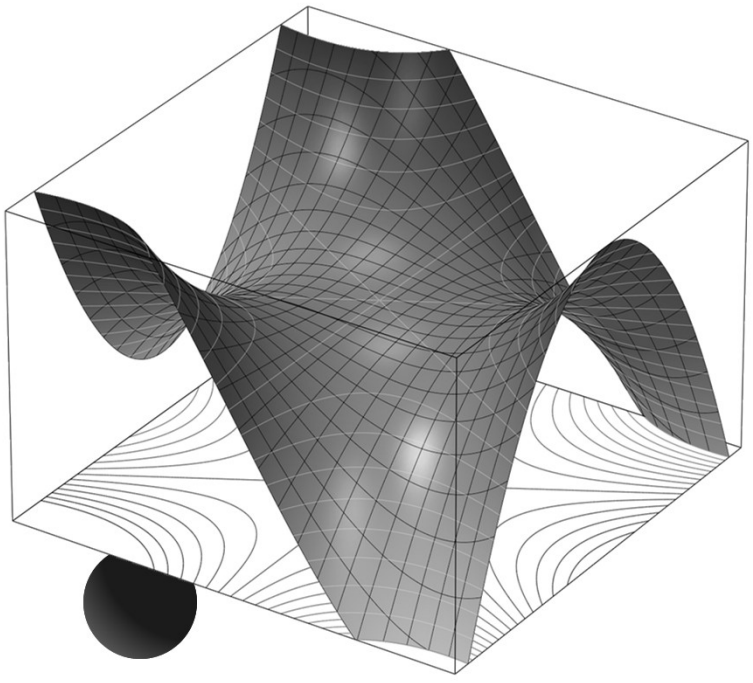
12

Questions?

13

Thank you

Until next time.



14

ALVARO: Start the recording!

15

"Calculus 3"

Multi-Variable Calculus

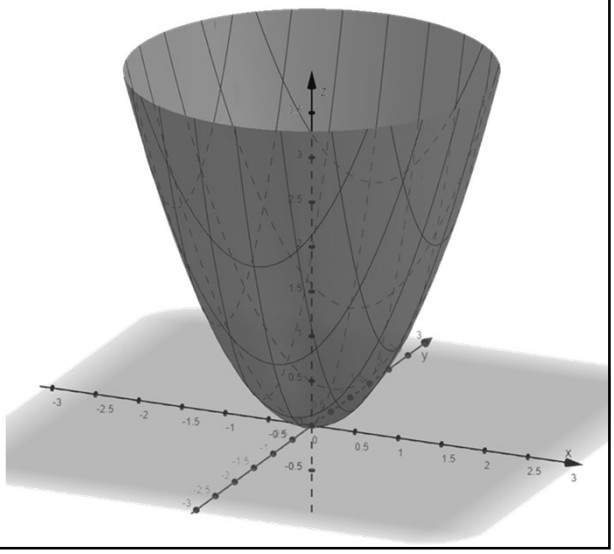
Instructor: Álvaro Lozano-Robledo

Functions of Several Variables

16

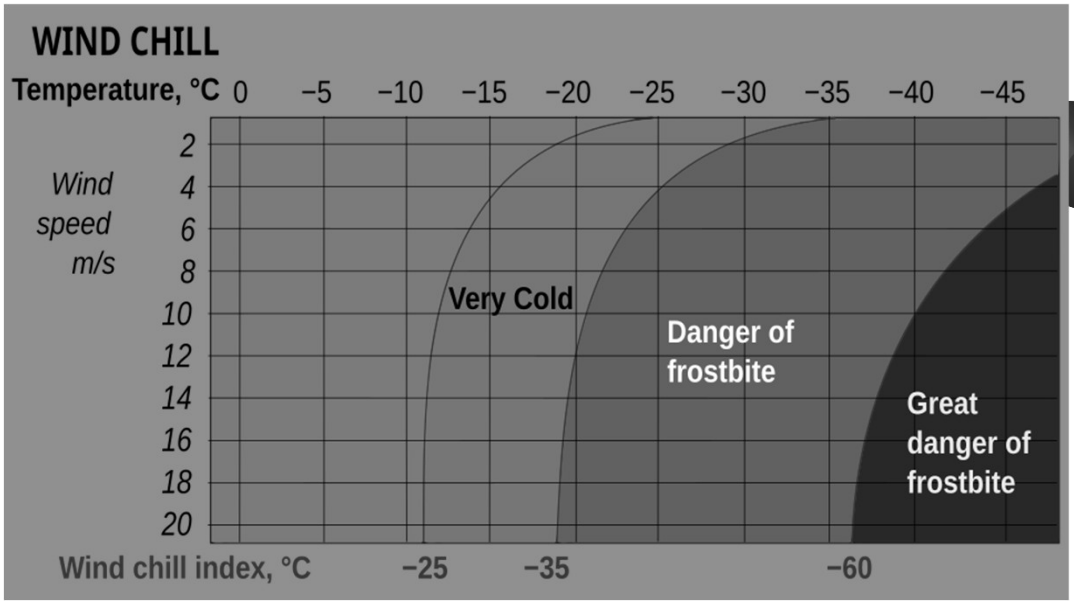
Today – Functions!

- Functions of Two Variables
- Domain and Range
- Graphs
- Level Curves
- Functions of More Than Two Variables



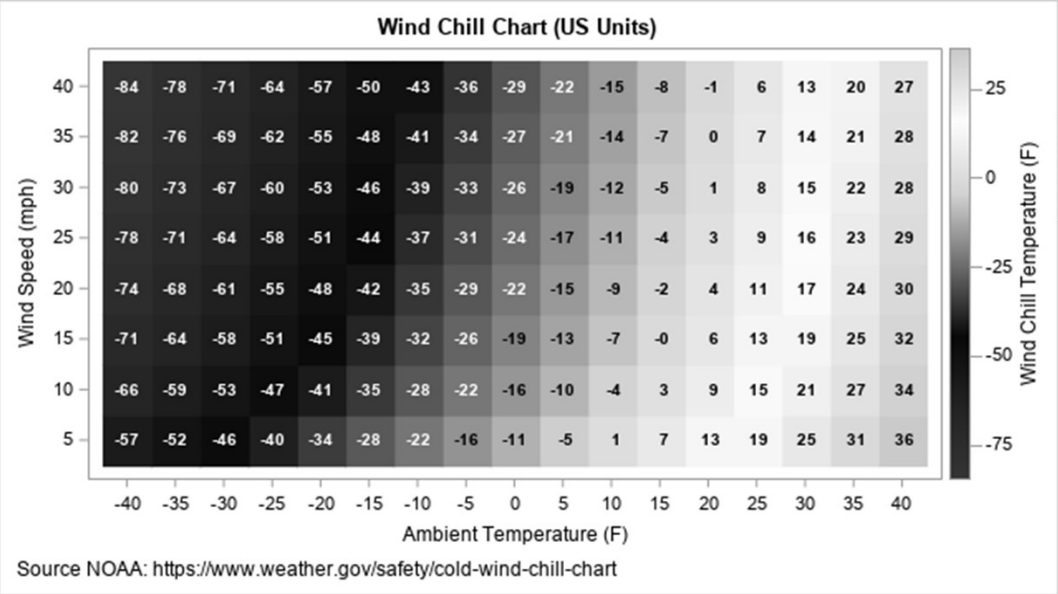
17

Functions of Two Variables



18

Functions of Two Variables



19

Functions of Two Variables

The standard wind chill formula for Environment Canada is:[3]

$$T_{\mathrm{wc}} = 13.12 + 0.6215T_{\mathrm{a}} - 11.37v^{+0.16} + 0.3965T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Celsius temperature scale; T_{a} is the air temperature in degrees Celsius; and v is the wind speed at 10 m (33 ft) standard anemometer height, in kilometres per hour.[11]

When the temperature is −20 °C (−4 °F) and the wind speed is 5 km/h (3 mph), the wind chill index is −24. If the temperature remains at −20 °C and the wind speed increases to 30 km/h (19 mph), the wind chill index falls to −33.

The equivalent formula in US customary units is:[12][3]

$$T_{\mathrm{wc}} = 35.74 + 0.6215T_{\mathrm{a}} - 35.75v^{+0.16} + 0.4275T_{\mathrm{a}}v^{+0.16},$$

where T_{wc} is the wind chill index, based on the Fahrenheit scale; T_{a} is the air temperature in degrees Fahrenheit; and v is the wind speed in miles per hour.[13]

20

Functions of Two Variables, Domain, and Range

Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is,

$\{f(x, y) \mid (x, y) \in D\}$.

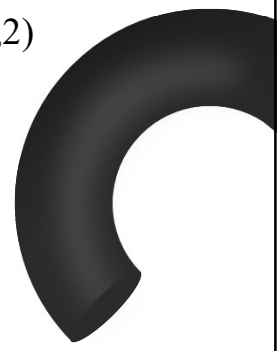
21

Functions of Two Variables, Domain, and Range

22

Example: Sketch the domain of the function, and evaluate at (3,2)

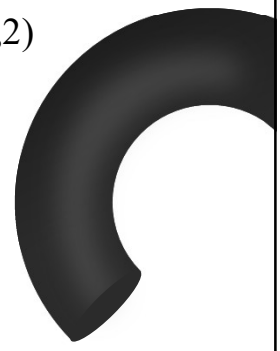
$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$



23

Example: Sketch the domain of the function, and evaluate at (3,2)

$$f(x,y) = x \cdot \ln(y^2 - x)$$



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Example: Find the domain and range of the function

$$f(x, y) = \sqrt{9 - x^2 - y^2}$$

25

Graphs of Functions of Two Variables

Example: Sketch a graph of the function f(x,y) = 2 – x – y

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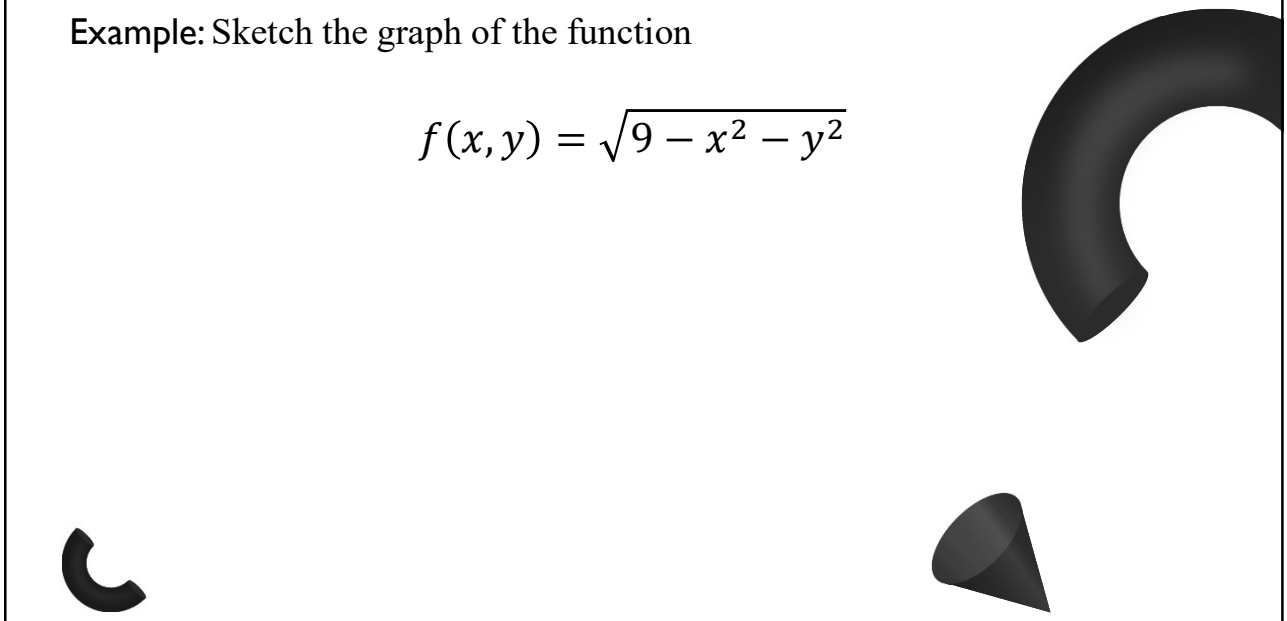
Example: Sketch the graph of the function

$$f(x, y) = \sqrt{9 - x^2 - y^2}$$



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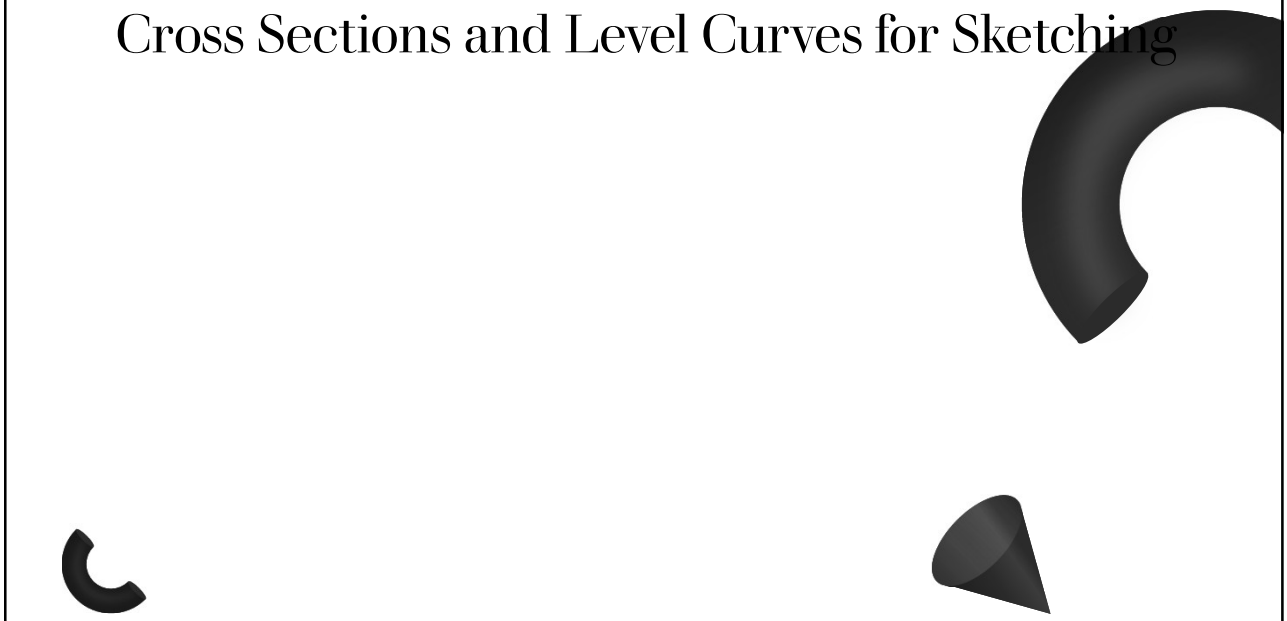
Cross Sections and Level Curves for Sketching



28

Cross Sections and Level Curves for Sketching

29

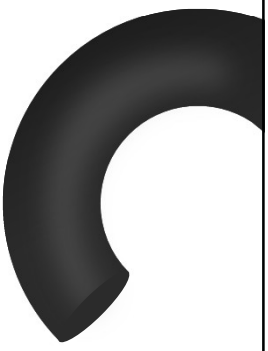
Example: Sketch the graph of the function

$$f(x,y) = e^{-(x^2+y^2)}$$

30

Example: Find the level surfaces of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$



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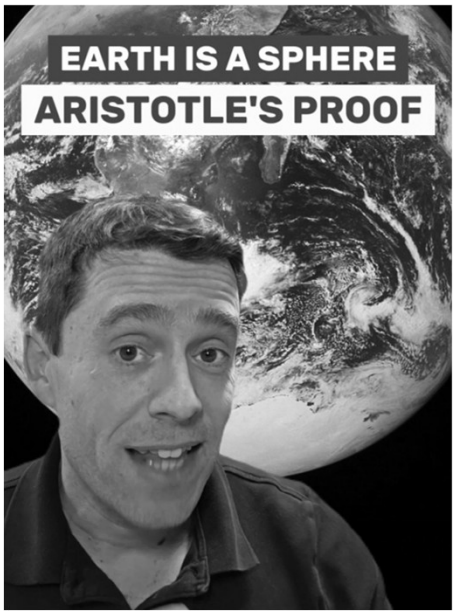
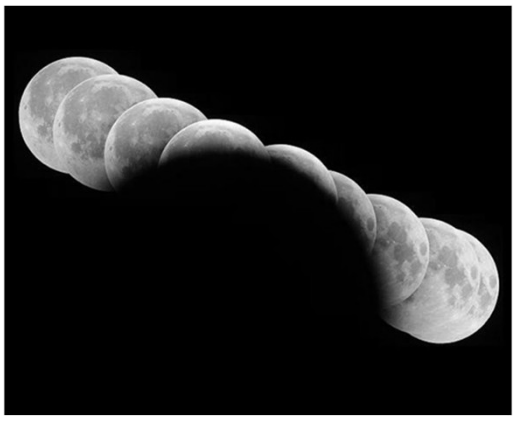


Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

32

Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

33



Using Cross Sections to "Prove" that Earth is Spherical: Aristotle's Proof

34

Questions?

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ALVARO: Start the recording!

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"Calculus 3"

Multi-Variable Calculus

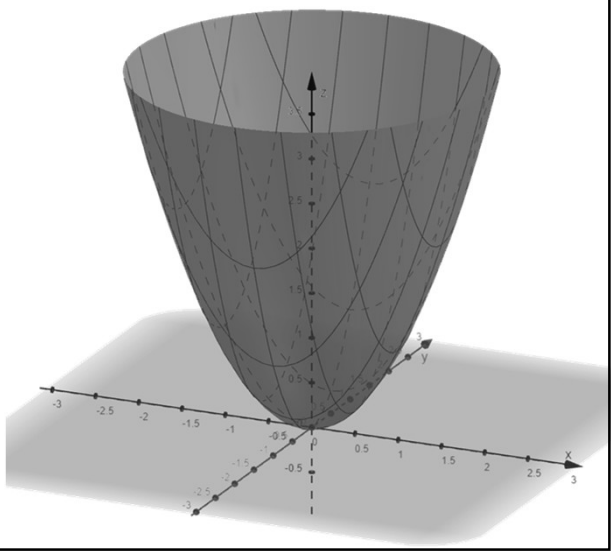
Instructor: Álvaro Lozano-Robledo

Partial Derivatives

37

Today – Derivatives!

- Partial Derivatives
- Interpretation
- Higher Derivatives
- PDEs



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Partial Derivatives

39

Example: Partial derivatives of $f(x, y) = x^2 + y^2$

40

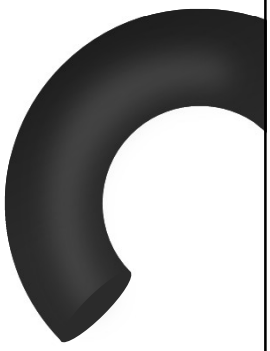
Partial Derivatives – The Limit Definition

41

Partial Derivatives – Notation

42

Example: Find the partial derivatives of $f(x,y) = 4 - x^2 - y^2$ at (1,1) and interpret those as slopes.



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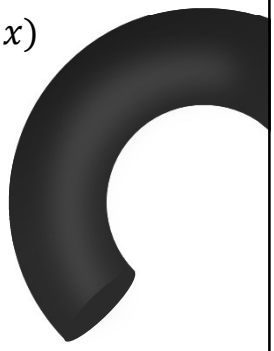
Example: Find the partial derivatives of $f(x,y) = x \cdot \ln(y^2 - x)$ at (3,2).



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Example: Find the partial derivatives of the Body-Mass-Index formula $B(m,h) = m/\text{h}^2$ at m = 64kg and h = 1.68m.

45

Example: Find the partial derivatives of $f(x,y,z) = \sin(\text{x}^2 + \text{y}^2 + \text{z}^2)$ at (1,2,3).

46

Higher Partial Derivatives

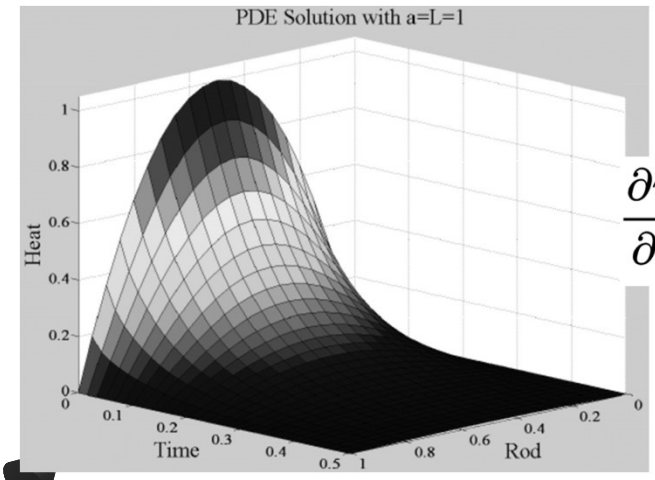
47

Example: Find the second partial derivatives of

$$f(x,y) = 4x^2y - x^3 - y^2$$

48

Partial Differential Equations



Example: The Heat Equation

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

49

Example: Show that the function $u(x,t) = \sin(x - a \cdot t)$ satisfies the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

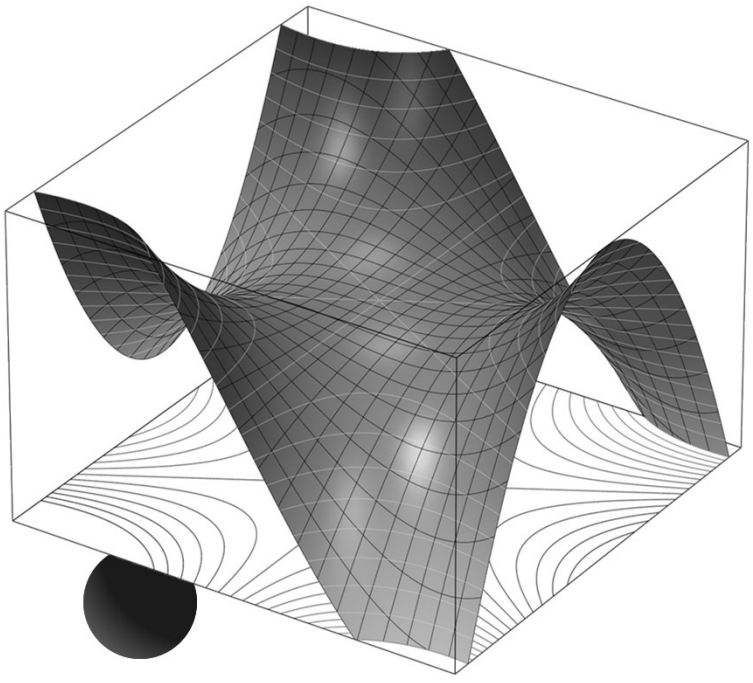
50

Questions?

51

Thank you

Until next time.



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