MATH1132 Nov 15th, 2025

QUALITY	DIFFICULTY
1.0	5.0

For Credit: **Yes**

Attendance: **Mandatory**

Grade: **C+**

Textbook: **Yes**

I thought this guy would be chill because he is tiktok famous but he IS NOT. HE IS ONE OF THOSE CELEBRITIES THAT THINKS THEY ARE BETTER THAN EVERYONE ELSE LIKE HE IS THE WORST MATH TEACHER EVER AVOID HIM AT ALL COSTS.

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For Credit: **Yes**

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MATH2110Q Feb 3rd, 2026

QUALITY

1.0

DIFFICULTY

4.0

For Credit: **Yes**

Attendance: **Not Mandatory**

Grade: **Not sure yet**

Textbook: **Yes**

Not good lectures for calc 3, the concepts themselves aren't too bad but he doesn't provide clear steps to solving problems and just rushes through examples on the slides. Wastes time on things you don't need then says he cannot slow down, even though you can't copy fast enough to keep up with his slides. Often does algebra wrong.

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Day 8

Any Reminders? Any Questions?

- I will be away on
  - Monday 2/16 --- no office hours that day
  - Tuesday 2/17 --- I will send videos to watch instead of class

- I will be back teaching in-person on Thursday 2/19
- I will do some review for the midterm during Thursday's class
- I will have regular office hours 2/19 – 3:30-4:30
- I will have additional office hours 2/19 – 4:30-5:30
- Calc 3 Calc Night: MONT 104 at 6:30-8:30pm on Thursdays!

- Exam 1 is on Friday, Feb 20th

EXAM 1 -- Friday, February 20th

Exam Covers:

- **Chapter 12**
  - Sections 12.1 – 12.6
- **Chapter 14**
  - Sections 14.1, 14.3 – 14.8

(NEW) Exam Study Guide and Practice Problems in HuskyCT

ALVARO: Start the recording!

“Calculus 3”

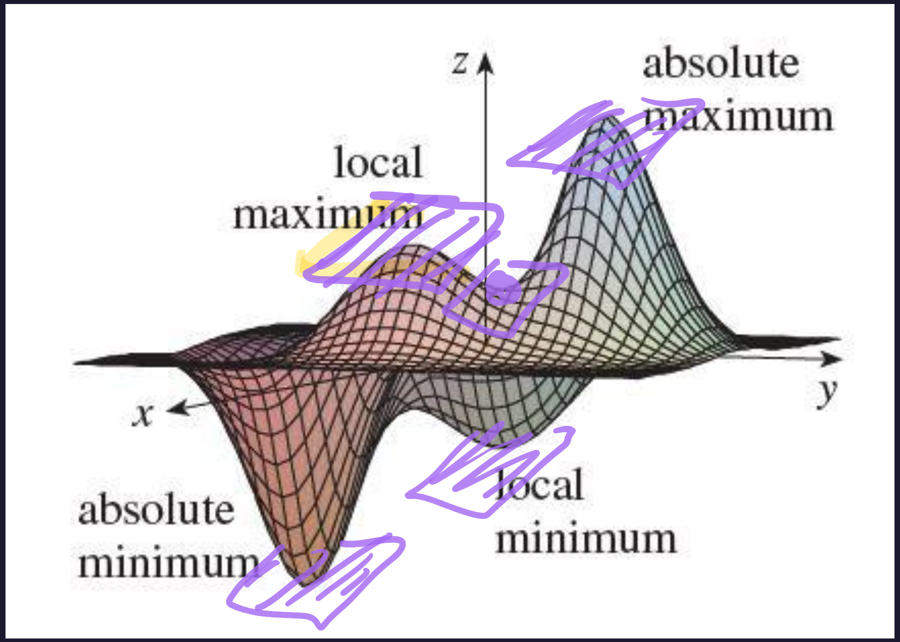
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

More on Maximum and Minimum Values

Today – Maximum and Minimum Values!

- Local Max and Min Values
- Second Derivative Test
- Absolute Max and Min Values



Local Max and Min Values

2 Theorem

If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x\,(a, b) = 0$ and $f_y\,(a, b) = 0$.

A **critical point** for a function $f(x, y)$ is a point (a, b) where

$$\nabla f(a, b) = \vec{0},$$

that is $f_x(a, b) = 0, f_y(a, b) = 0$.

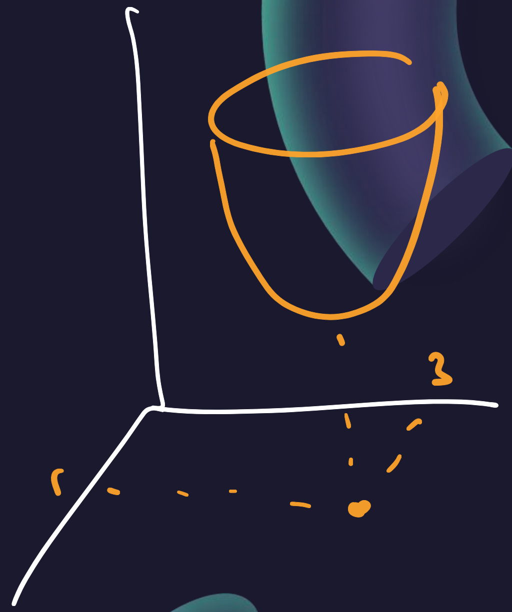
Example: Find all the critical points for the function

$$f(x, y) = x^2 + y^2 - 2x - 6y + 14$$

$$\frac{\partial f}{\partial x} = 2x - 2 = 0 \Rightarrow \boxed{x = 1}$$

$$\frac{\partial f}{\partial y} = 2y - 6 = 0 \Rightarrow \boxed{y = 3}$$

$$P = (1, 3)$$



Local Max and Min Values: Second Derivative Test

3 Second Derivatives Test

Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [so (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b)\, f_{yy}(a, b) - [f_{xy}(a, b)]^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$(f_{xy} = f_{yx})$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is a saddle point of f.

WARNING! IF D = 0 , THE TEST IS INCONCLUSIVE.

$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

Example: Find and classify the critical points for the function

$$f(x,y) = x^2 + y^2 - 2x - 6y + 14$$

$P = (1, 3)$ was critical.

$f_x = 2x - 2 \longrightarrow f_{xx} = 2$, $\quad f_{xy} = 0$

$f_y = 2y - 6 \longrightarrow f_{yy} = 2$, $\quad f_{yx} = 0$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

$$= 2 \cdot 2 - 0^2$$

$$= 4 > 0$$

$$f_{xx} = 2 > 0$$

LOCAL MIN AT (1, 3).

[Extra]

$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

Example: Find and classify the critical points for the function

$$f(x,y) = x^2 + y^2 - 2x - 6y + 14$$

$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

Example: Find and classify the critical points for the function

$$f(x,y) = x^2 + 4xy + y^2$$

$$\nabla f = 0 \quad \begin{cases} f_x = 2x + 4y = 0 \\ f_y = 4x + 2y = 0 \end{cases}$$

$y = -\frac{2x}{4}$ $\quad y = -\frac{x}{2}$

$\rightarrow 4x + 2\left(-\frac{x}{2}\right)$

$= 4x - x = 3x = 0$

$\Rightarrow x = 0$

$y = 0$

ONE CRITICAL PT.

$$\boxed{(0,0)}$$

$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

Example: Find and classify the critical points for the function

$$f(x,y) = x^2 + 4xy + y^2$$

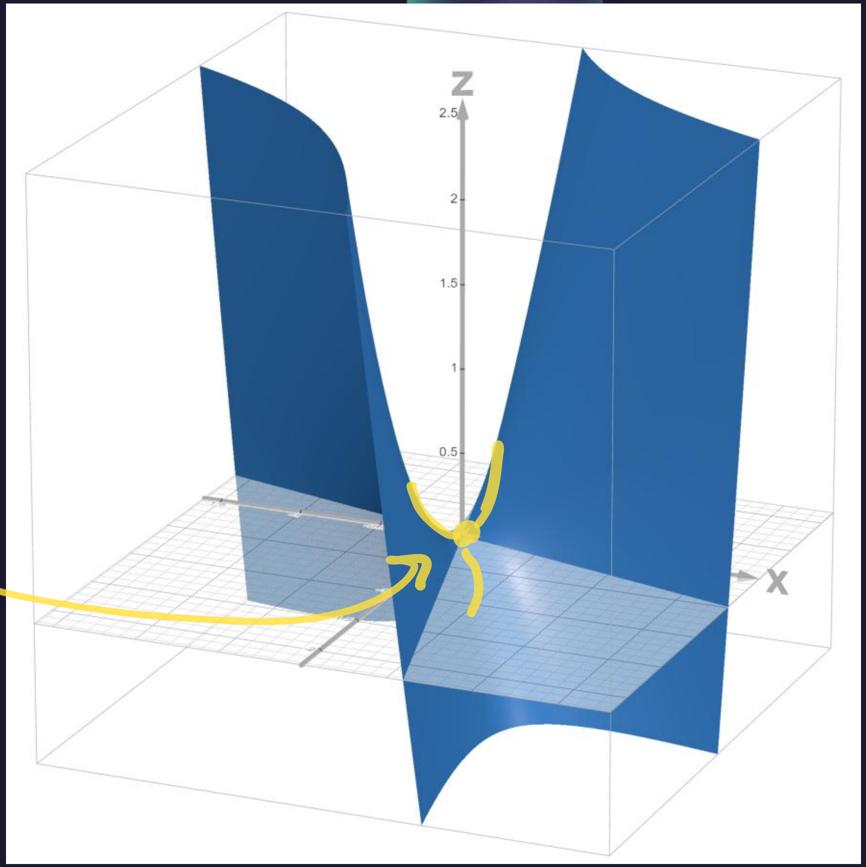
$$f_x = 2x + 4y \rightarrow f_{xx} = 2, \quad \rightarrow f_{xy} = 4$$

$$f_y = 4x + 2y \rightarrow f_{yy} = 2, \quad \rightarrow f_{yx} = 4$$

(0, 0)

SADDLE POINT!

$$D = 2 \cdot 2 - 4^2 = 4 - 16 = -12 < 0$$



Absolute Max and Min Values

Let (a, b) be a point in the domain D of a function f of two variables. Then $f(a, b)$ is the

- **absolute maximum** value of f on D if $f(a, b) \geqslant f(x, y)$ for all (x, y) in D.
- **absolute minimum** value of f on D if $f(a, b) \leqslant f(x, y)$ for all (x, y) in D.

8 Extreme Value Theorem for Functions of Two Variables

If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

Absolute Max and Min Values

Let (a, b) be a point in the domain D of a function f of two variables. Then $f(a, b)$ is the

- **absolute maximum** value of f on D if $f(a, b) \geqslant f(x, y)$ for all (x, y) in D.
- **absolute minimum** value of f on D if $f(a, b) \leqslant f(x, y)$ for all (x, y) in D.

$\boxed{9}$ To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

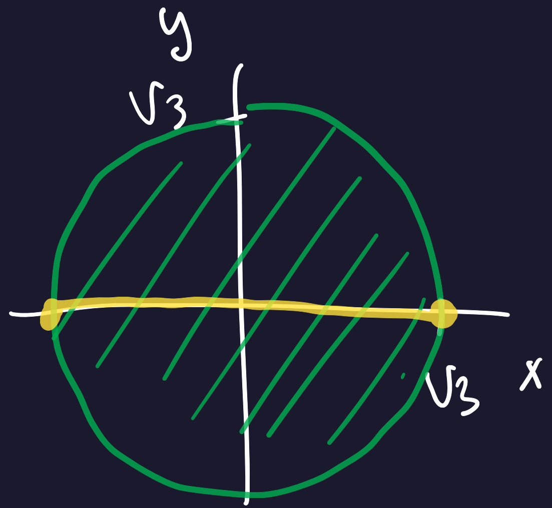
$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

Example: Find the absolute maximum and minimum values of

$$f(x,y) = xy^2$$

$R^2 \Rightarrow R = \sqrt{3}$

in the region $D = \{(x,y) : x^2 + y^2 \le 3\}$.



(1) CRIT. PTS.

$$\begin{cases} f_x = y^2 = 0 \\ f_y = 2xy = 0 \end{cases} \quad y = 0$$

FOR ANY x!

$\Rightarrow (x, 0)$ CRITICAL!

$$f(x,0) = x \cdot 0^2 = \boxed{0}$$

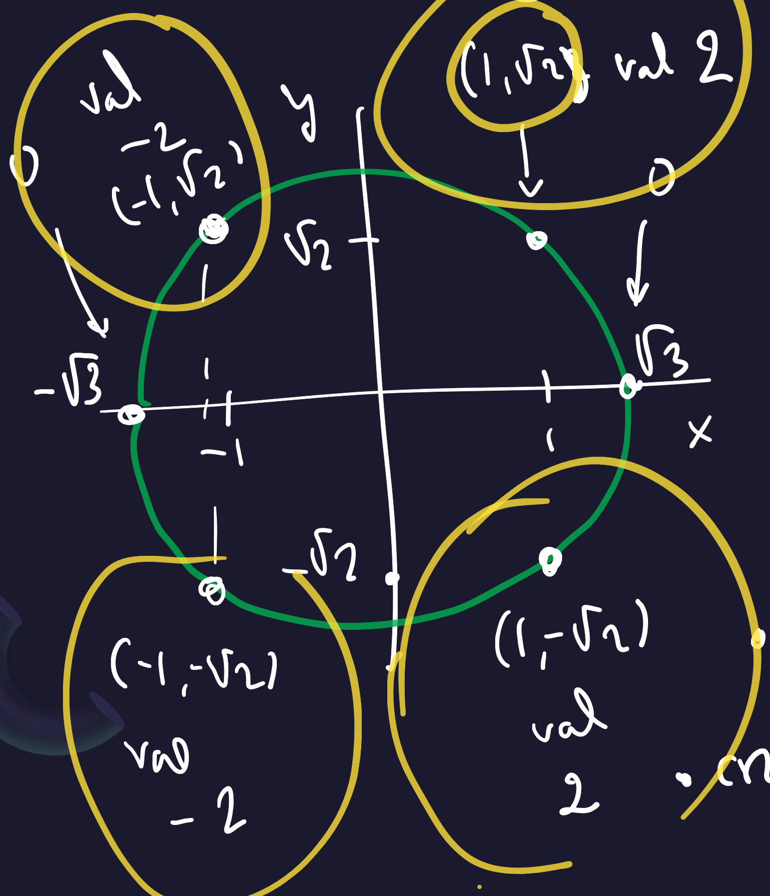
$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

[Extra]

Example: Find the absolute maximum and minimum values of

$$f(x,y) = xy^2$$

in the region $D = \{(x,y) : x^2 + y^2 \le 3\}$.



$3x - x^3$

$\shortparallel$

$f(x) = x(3 - x^2)$

$-\sqrt{3} \le x \le \sqrt{3}$

$f(x,y) = xy^2 = x(3-x^2)$

on $x^2 + y^2 = 3 \Rightarrow y^2 = 3 - x^2$

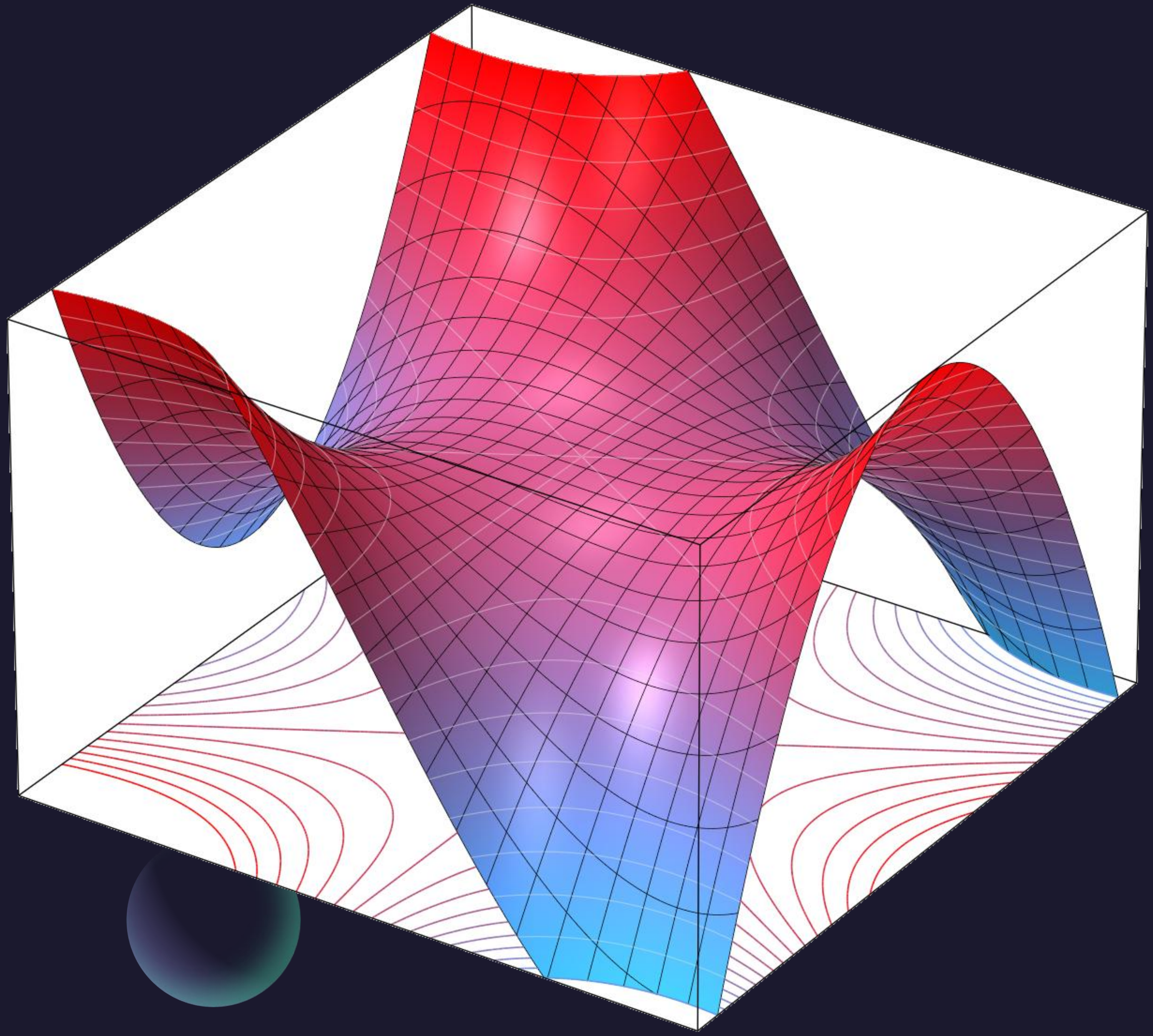
$f'(x) = 3 - 3x^2 = 0 \Rightarrow x = \pm 1 \Rightarrow y = \pm\sqrt{2}$

- BOUNDARY : $x = \pm\sqrt{3}$, $f(\pm\sqrt{3}) = 0$ MAX=2
- CRIT PTS : $x = \pm 1$, $y = \pm\sqrt{2}$, $\boxed{f(x,y) = \pm 2}$ MIN=-2

Questions?

Thank you

Until next time.



ALVARO: Start the recording!

"Calculus 3"

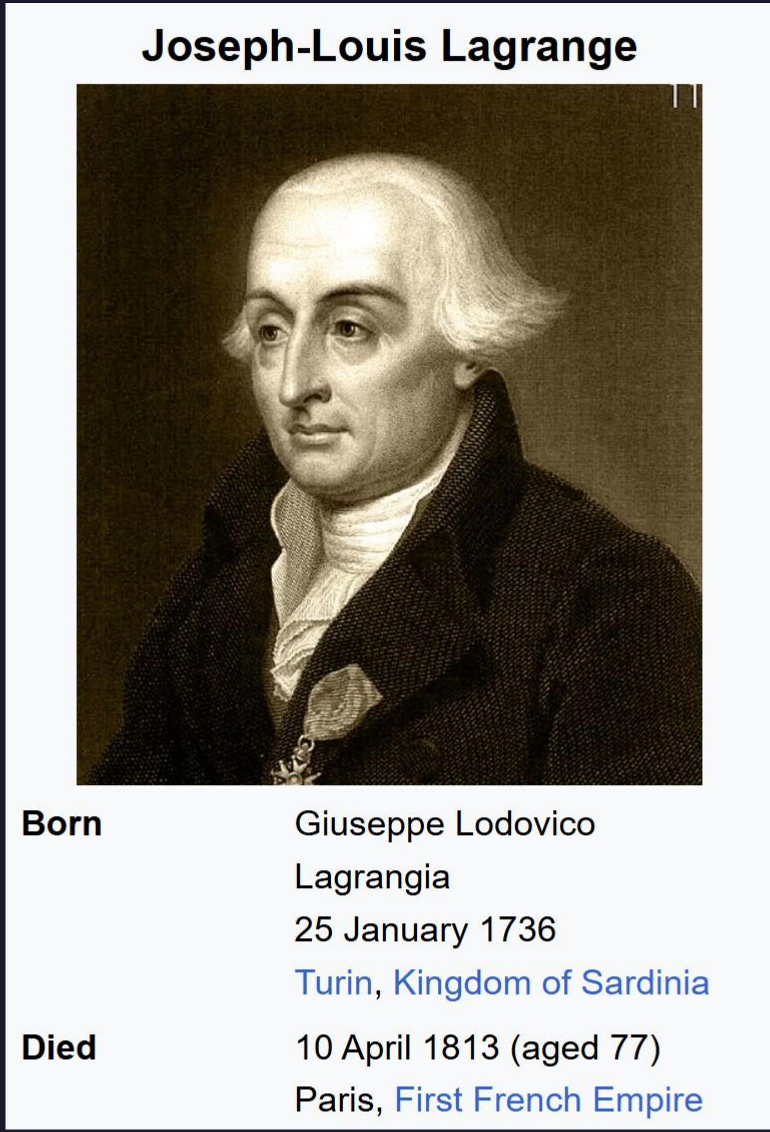
Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

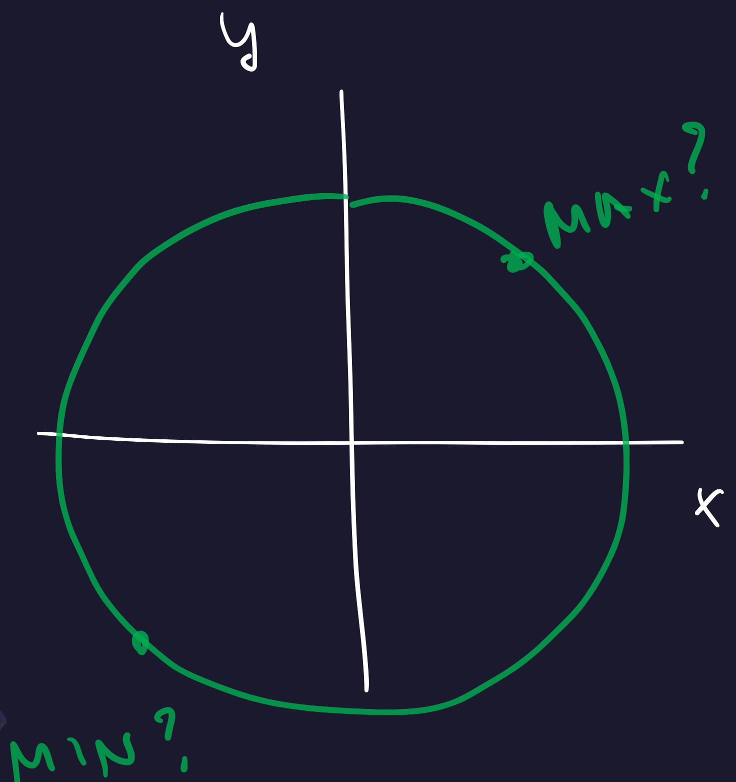
Lagrange Multipliers

Today – "Lagrange Multipliers!"

- The Method
- One Constraint
- Examples



Example: Find the extreme values of $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

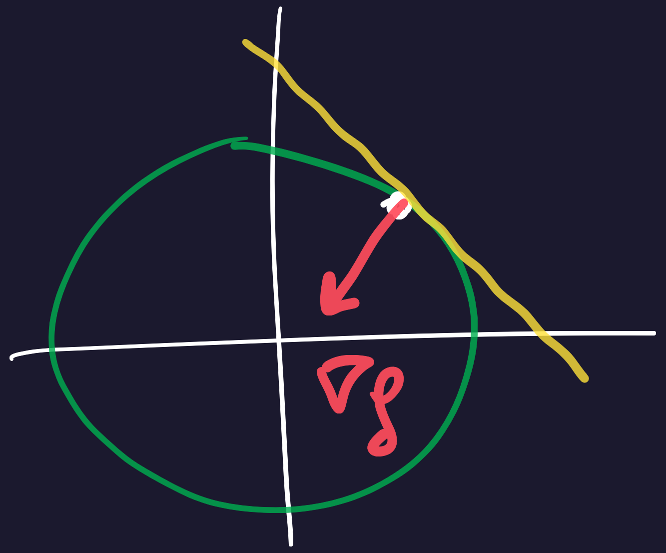


Recall: Properties of the Gradient Vector

$f(x,y) - z = 0$

Thus, the gradient vector for a surface $z = f(x, y)$ in three dimensions, $\nabla F = (f_x, f_y, -1)$ is normal to a surface at any point.

The gradient vector in two dimensions, $\cancel{\nabla F}$ ∇f $= (f_x, f_y)$ is normal to any level curves of $f(x,y)$ at any point, indicating the maximum rate of change.



Example: Let $f(x,y) = 4 - x^2 - y^2$

(a) Find the normal vector to the graph of $f(x,y)$ at $(1,1,2)$

(b) Find the tangent plane to the graph of $f(x,y)$ at $(1,1,2)$

(c) Find the normal vector to the cross section $z = 0$ at $\cancel{(1,1)}$ $(\sqrt{2}, \sqrt{2})$

$$f_x = -2x$$

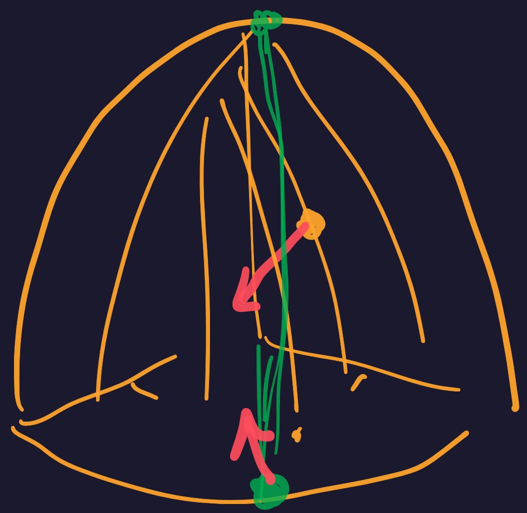
$$f_y = -2y$$

(a) $\nabla F = (f_x, f_y, -1)$

$$= (-2x, -2y, -1)\Big|_{(1,1,2)} = (-2,-2,-1)$$

(b) $f_x\big|_p \cdot (x - x_0) + f_y\big|_p \cdot (y - y_0) - (z - z_0) = 0$

$$-2(x-1) + (-2)\cdot(y-1) - (z-2) = 0$$



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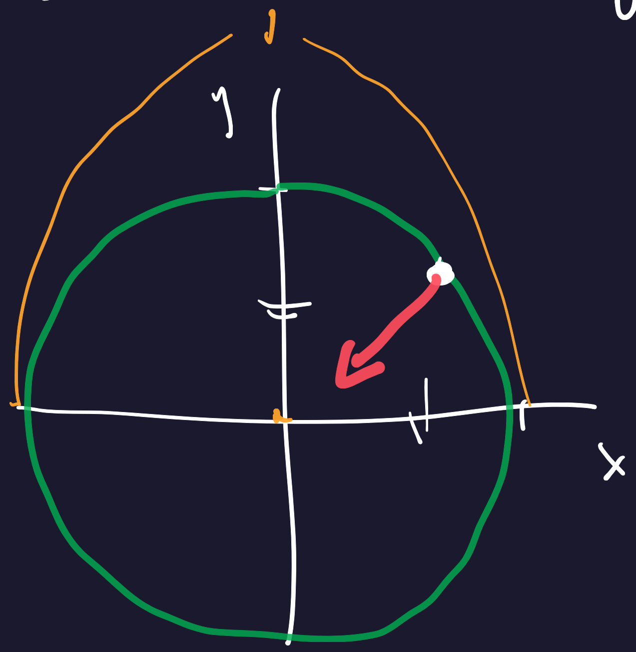
(c) Find the normal vector to the cross section $z = 0$ at $\cancel{(1,1)}$ $(\sqrt{2}, \sqrt{2})$

$z = 0 = 4 - x^2 - y^2 \Rightarrow x^2 + y^2 = 4$

$x^2 + y^2 = x^2 + x^2 = 4$

$= 2x^2 = 4$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$



$\nabla f = (f_x, f_y)\Big|_{(\sqrt{2},\sqrt{2})}$

$= (-2x, -2y)\Big|_{(\sqrt{2},\sqrt{2})} = (-2\sqrt{2}, -2\sqrt{2})$

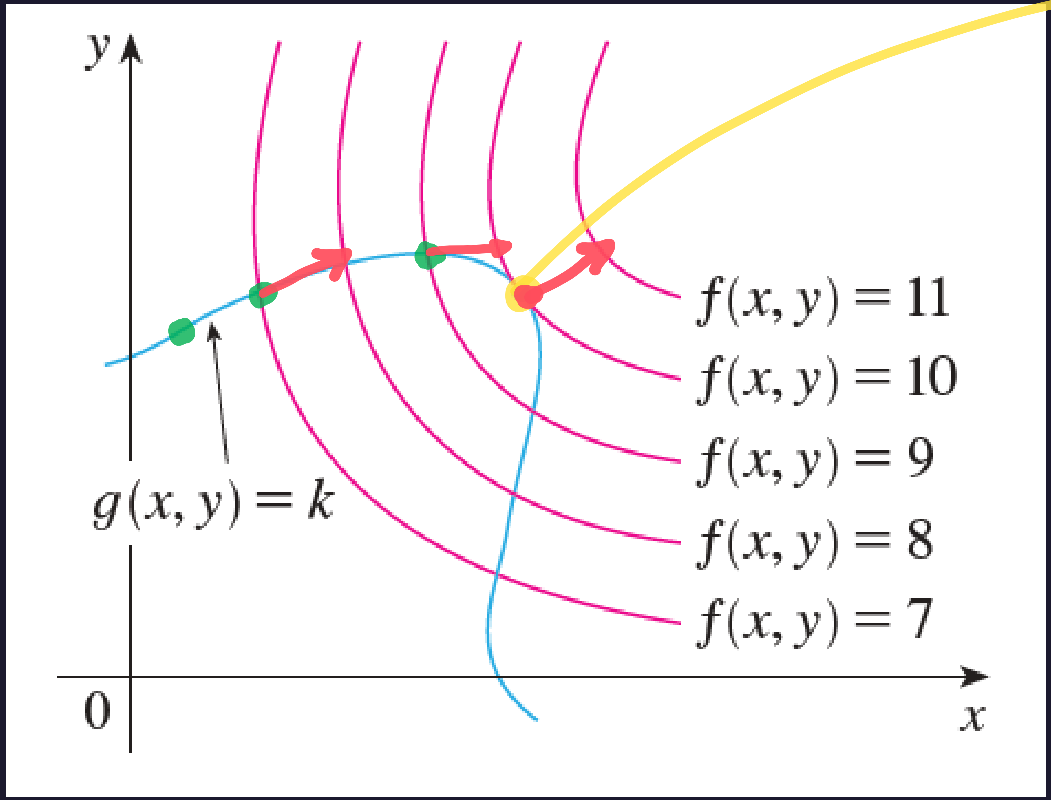
Example: Let $f(x,y) = 4 - x^2 - y^2$

(a) Find the normal vector to the graph of $f(x,y)$ at $(1,1,2)$
(b) Find the tangent plane to the graph of $f(x,y)$ at $(1,1,2)$
(c) Find the normal vector to the cross section $z = 0$ at $(1,1)$

The "Lagrange Multipliers" Method

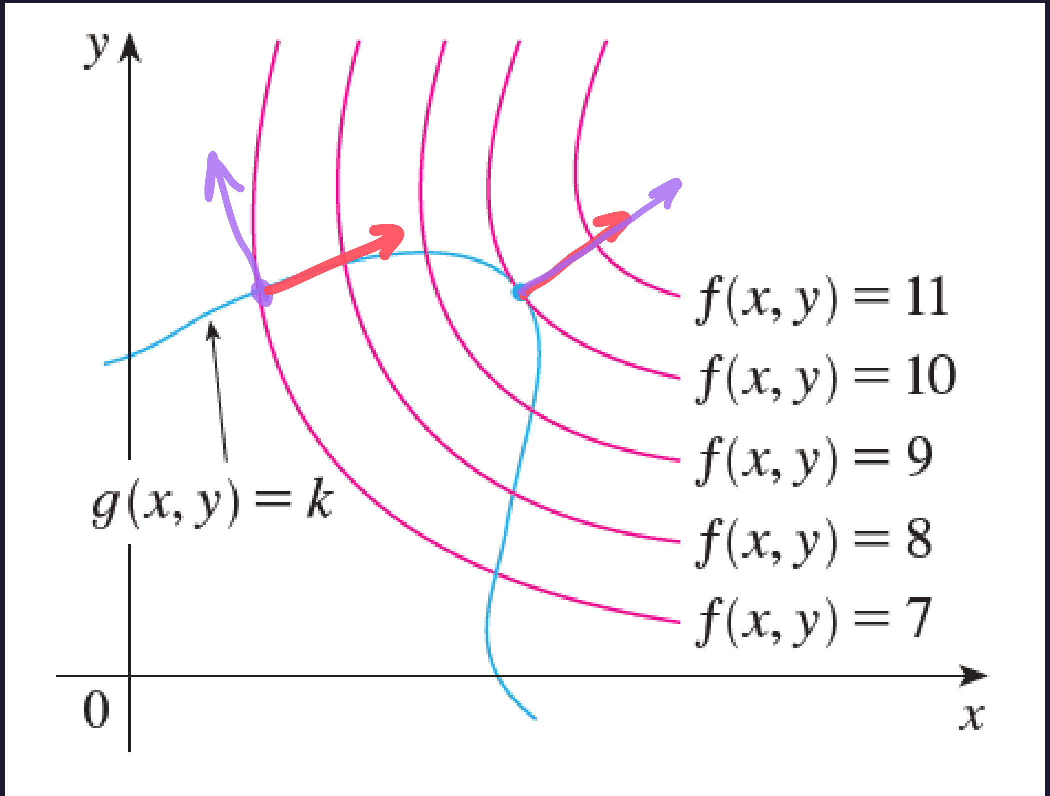
GOAL : Maximize $z = f(x,y)$ on the curve $g(x,y) = k$.

MAX



The "Lagrange Multipliers" Method

GOAL : Maximize $z = f(x,y)$ on the curve $g(x,y) = k$.



The value of f(x,y) on the curve g(x,y)=k will be maximized at some point (x_0,y_0) such that

$\nabla f\,(x_0,y_0)$ is parallel to $\nabla g\,(x_0,y_0)$

or equivalently a point (x_0,y_0) such that there is a constant λ with

$\nabla f\,(x_0,y_0) = \lambda \cdot \nabla g\,(x_0,y_0)$

and $g(x_0,y_0) = k$.

The “Lagrange Multipliers” Method

GOAL : Maximize $z = f(x,y)$ on the curve $g(x,y) = k$.

SOLVE:

$$\begin{cases} \nabla f\,(x_0,y_0) = \lambda \cdot \nabla g\,(x_0,y_0) \\ g(x_0,y_0) = k\,. \end{cases}$$

$\nabla f(x_0,y_0) = \lambda \cdot \nabla g(x_0,y_0)$ and $g(x_0,y_0) = k$.

Example: Find the extreme values of $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. $\leftarrow$ $g(x,y) = 1$, $g(x,y) = x^2 + y^2$

$\nabla f = (2x, 4y)$

$\nabla g = (2x, 2y)$

$$\begin{cases} \nabla f = \lambda \nabla g \\ x^2 + y^2 = 1 \end{cases} \iff \begin{cases} (2x, 4y) = \lambda (2x, 2y) \\ x^2 + y^2 = 1 \end{cases}$$

$$\begin{cases} 2x = 2x\lambda \\ 4y = 2y\lambda \\ x^2 + y^2 = 1 \end{cases} \iff \begin{cases} 2x(1-\lambda) = 0 \\ 2y(2-\lambda) = 0 \\ x^2 + y^2 = 1 \end{cases}$$

CASES

- $x = 0$, $y = \pm 1$, $\lambda = 2$
- $y = 0$, $x = \pm 1$, $\lambda = 1$
- $\lambda = 1$, $2y = 0 \Rightarrow y = 0$, $x = \pm 1$
- $\lambda = 2$, $2x = 0 \Rightarrow x = 0$, $y = \pm 1$

$(0, \pm 1)$ OR $(\pm 1, 0)$

$\nabla f(x_0,y_0) = \lambda \cdot \nabla g(x_0,y_0)$ and $g(x_0,y_0) = k$.

Example: Find the extreme values of $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

CANDIDATES: $(0, \pm 1), (\pm 1, 0)$

$\underbrace{f(0,\pm 1) = 2}_{\text{MAX}}$, $\underbrace{f(\pm 1, 0) = 1}_{\text{MIN}}$

MAX $(0, \pm 1)$

MIN $(\pm 1, 0)$

$\nabla f\,(x_0,y_0) = \lambda \cdot \nabla g\,(x_0,y_0)$ and $g(x_0,y_0) = k$.

Example: Find the extreme values of $f(x,y) = x^2 + y^2$ on the curve $xy = 1$.

$\nabla f(x_0,y_0) = \lambda \cdot \nabla g(x_0,y_0)$ and $g(x_0,y_0) = k$.

[Extra]

Example: Find the extreme values of $f(x,y) = x^2 + y^2$ on the curve $xy = 1$.

$$\nabla f(x_0,y_0) = \lambda \cdot \nabla g(x_0,y_0) \quad \text{and} \quad g(x_0,y_0) = k\,.$$

Example: Find the largest area of a rectangle with fixed perimeter equal to p.

The "Lagrange Multipliers" Method

Method of Lagrange Multipliers

To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ [assuming that these extreme values exist and $\nabla g \neq \mathbf{0}$ on the surface $g(x, y, z) = k$]:

1. Find all values of x, y, z, and λ such that

$$\nabla f(x, y, z) = \lambda \, \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

2. Evaluate f at all the points (x, y, z) that result from step 1. The largest of these values is the maximum value of f; the smallest is the minimum value of f.

$$\nabla f(x_0,y_0,z_0) = \lambda \cdot \nabla g(x_0,y_0,z_0) \quad \text{and} \quad g(x_0,y_0,z_0) = k\,.$$

Example: Find the dimensions of the closed box with the largest volume and fixed surface area S.

$$\nabla f(x_0,y_0,z_0) = \lambda \cdot \nabla g(x_0,y_0,z_0) \quad \text{and} \quad g(x_0,y_0,z_0) = k\ .$$

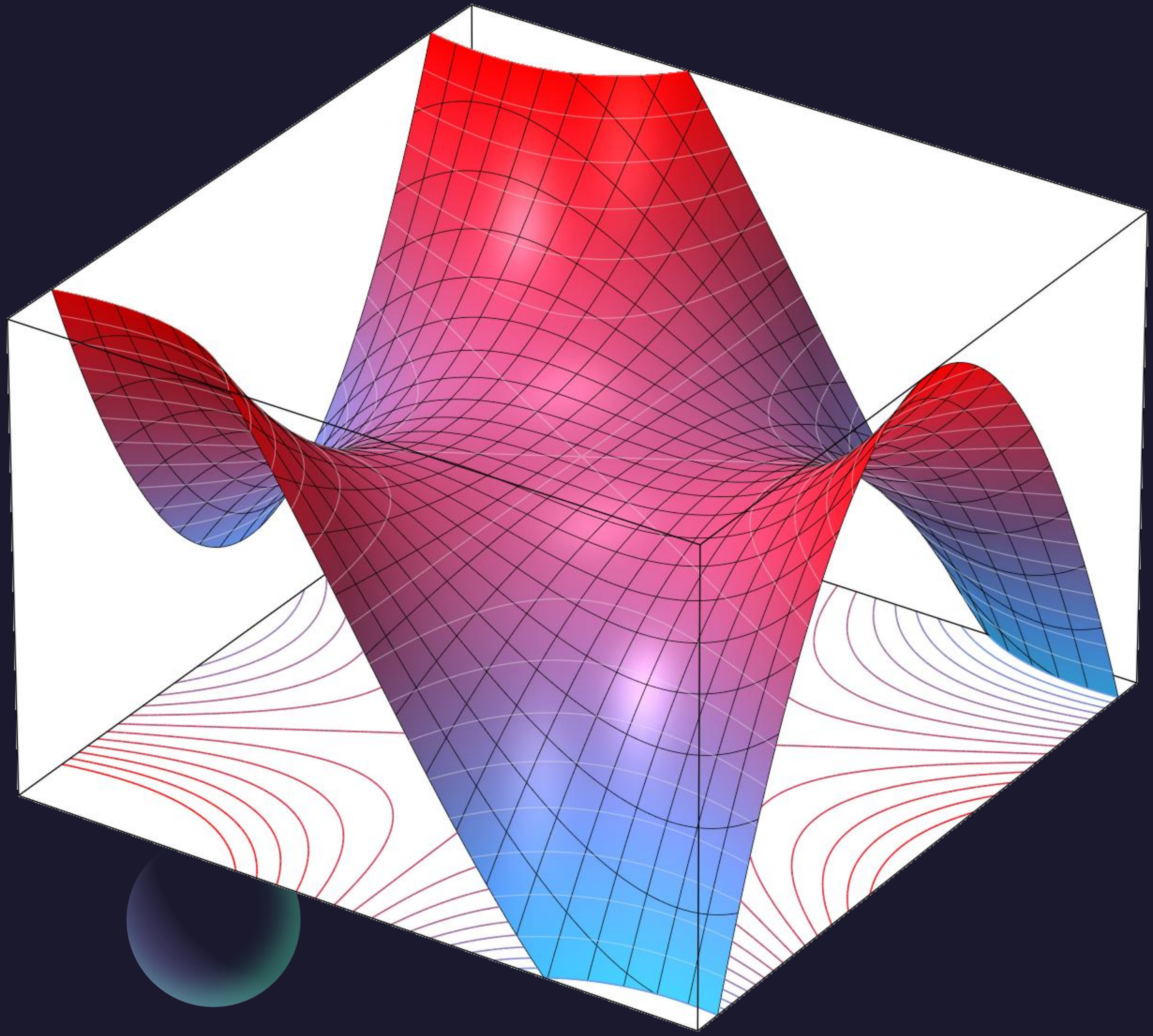
[Extra]

Example: Find the dimensions of the closed box with the largest volume and fixed surface area S.

Questions?

Thank you

Until next time.



ALVARO: Start the recording!

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Double Integrals over Rectangles

Today – Double Integrals!

- The Definite Integral
- The Riemann Integral
- Iterated Integrals
- Fubini's Theorem

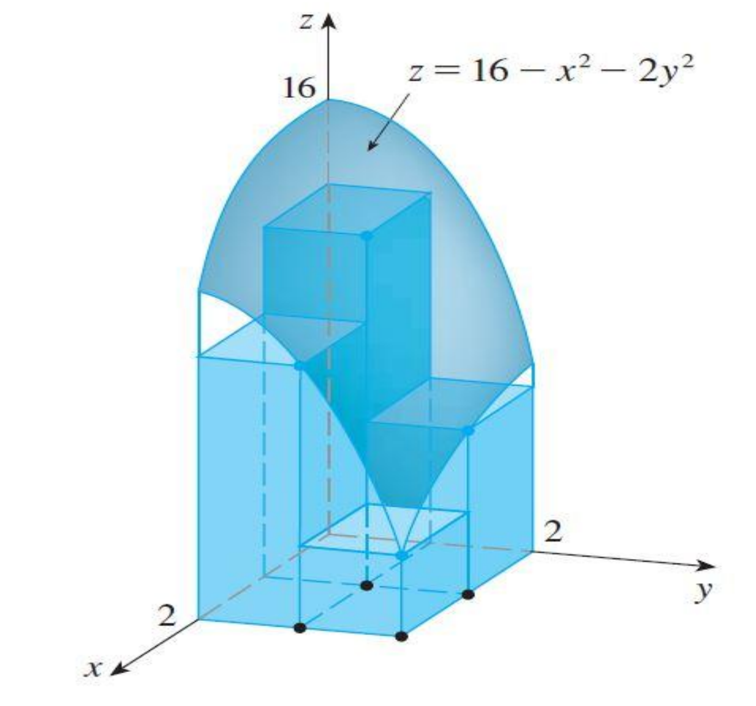
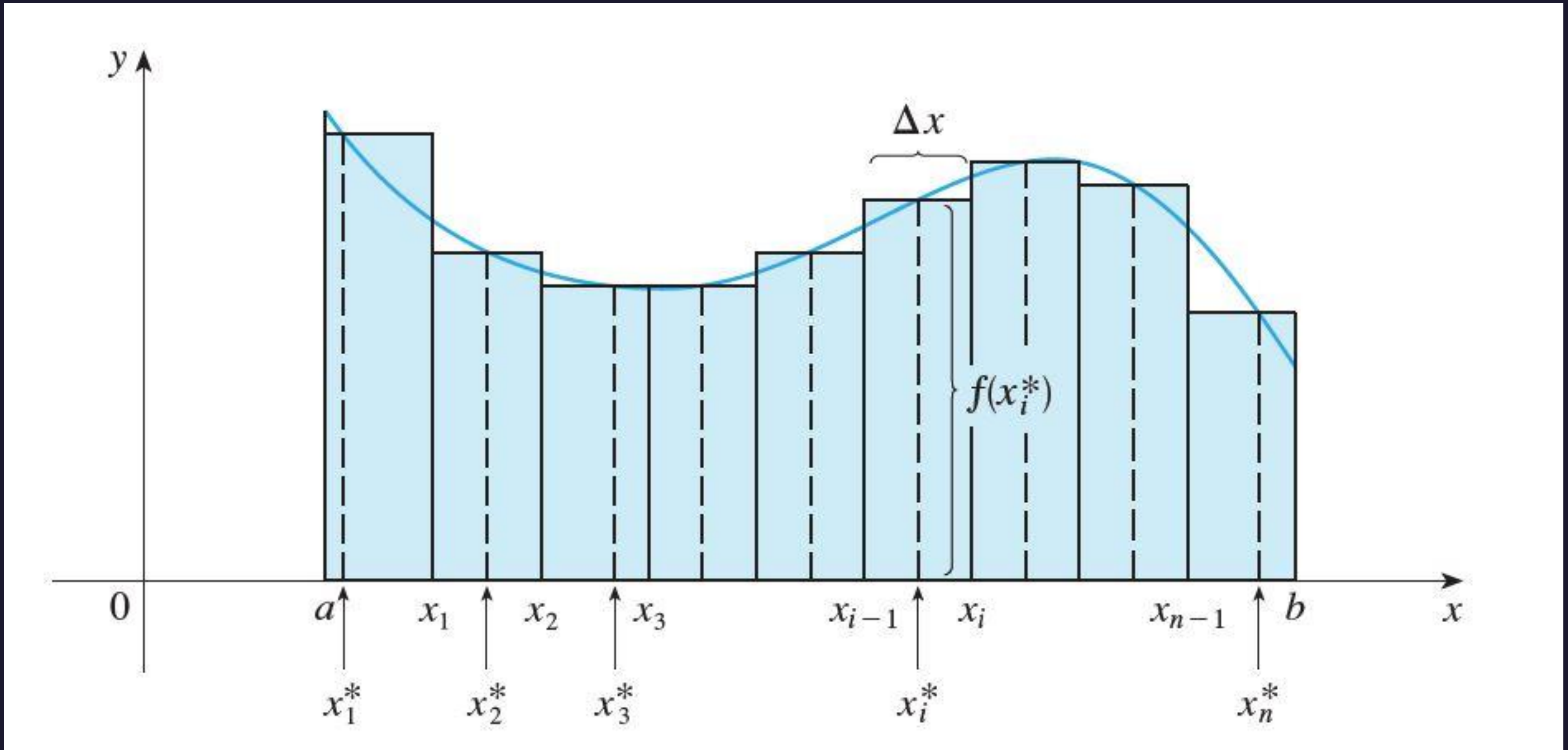
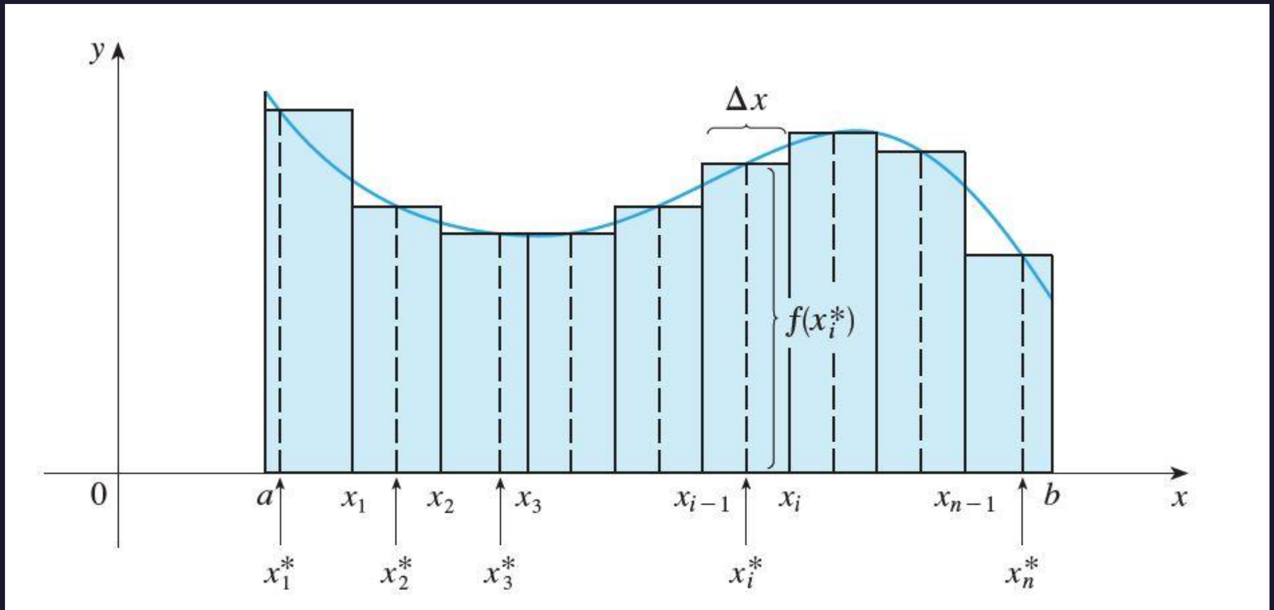


FIGURE 7

The Definite (Riemann) Integral



The Definite (Riemann) Integral



The Definite (Riemann) Integral

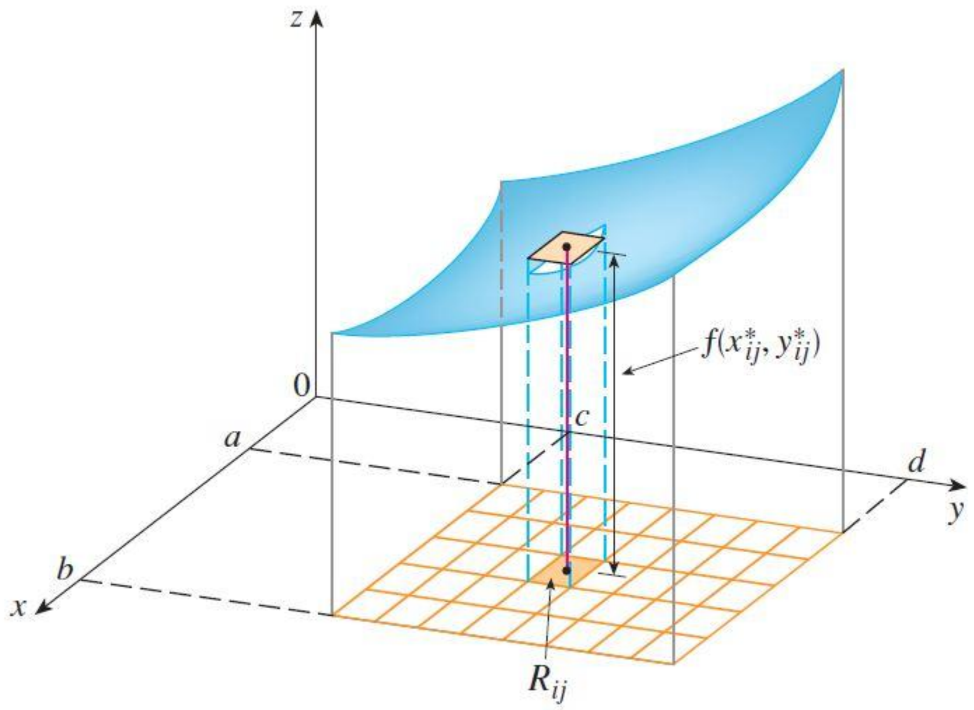


FIGURE 4

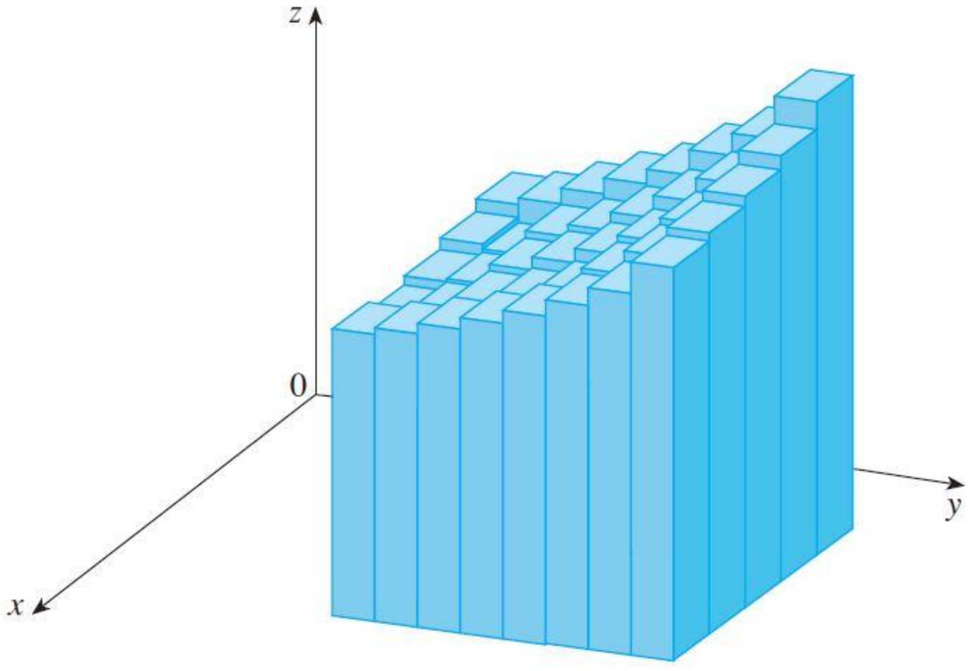


FIGURE 5

The Definite (Riemann) Integral

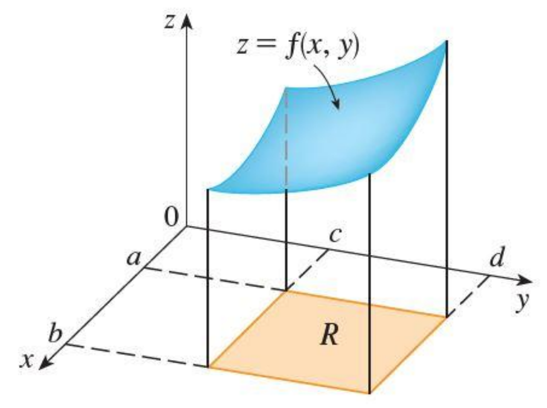


FIGURE 2

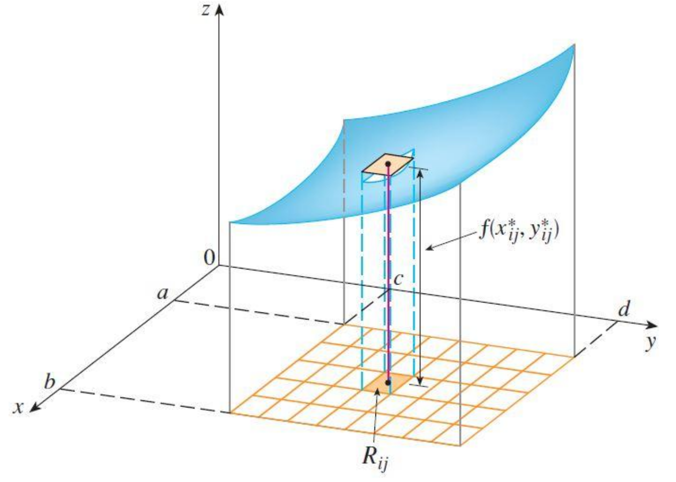


FIGURE 4

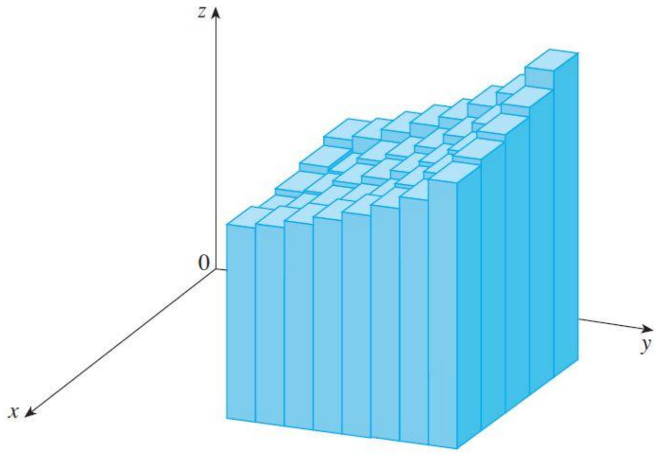


FIGURE 5

The Definite (Riemann) Integral

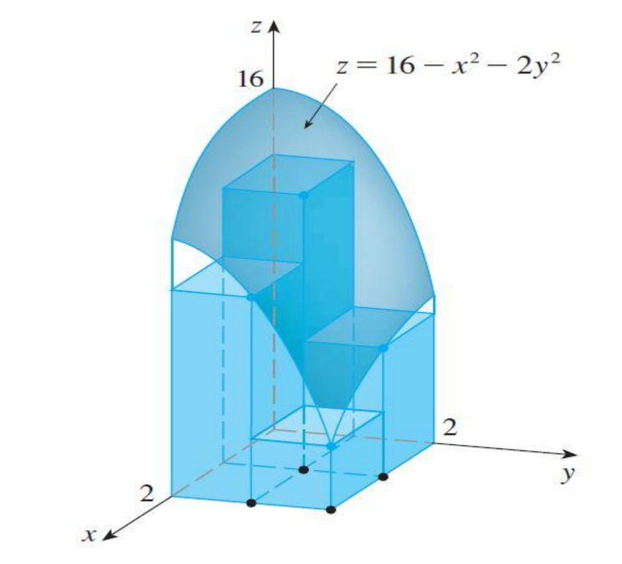
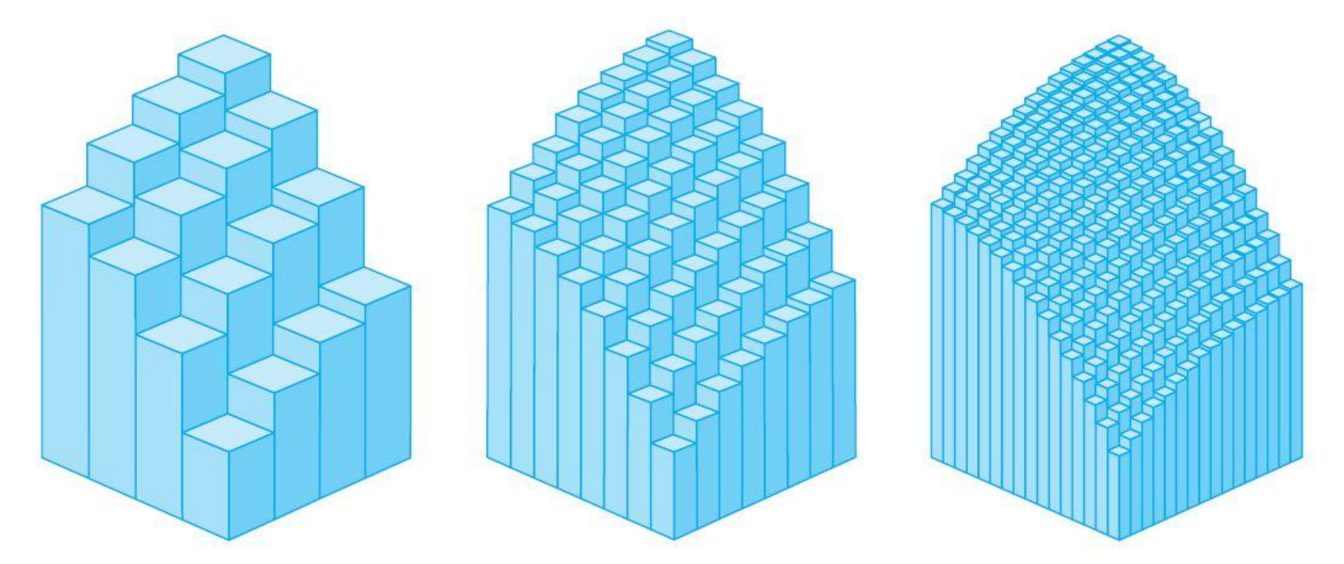


FIGURE 7



(a) $m = n = 4$, $V \approx 41.5$ (b) $m = n = 8$, $V \approx 44.875$ (c) $m = n = 16$, $V \approx 46.46875$

The usual properties of integration still hold for double integrals:

- $$\iint_R [f(x,y) \pm g(x,y)]dA = \iint_R f(x,y)dA \pm \iint_R g(x,y)dA.$$
- For any constant c,
$$\iint_R cf(x,y)dA = c\iint_R f(x,y)dA.$$
- If $f(x,y) \geq g(x,y)$ on the rectangle R, then
$$\iint_R f(x,y)dA \geq \iint_R g(x,y)dA.$$

And when letting $m, n \to \infty$, we have $\Delta A \to dA = dx \cdot dy$. Then

$$\iint_R f(x,y)dA = \int_c^d \int_a^b f(x,y)dxdy,$$

this is called an **iterated integral**, and we evaluate its value by computing the innermost integral first and then working the way out. Again, in the case this value represents a volume only if $f(x,y) \geq 0$ on R.

Example: Find the volume under the graph of $f(x,y) = 16 - x^2 - 2y^2$ above the square R = [0,2] x [0,2].

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Example: Calculate the following iterated integrals

$$\int_0^3 \int_1^2 x^2 y \, dy \, dx \quad \text{and} \quad \int_1^2 \int_0^3 x^2 y \, dx \, dy$$

[Extra]

Example: Calculate the following iterated integrals

$$\int_0^3 \int_1^2 x^2 y \, dy \, dx \quad \text{and} \quad \int_1^2 \int_0^3 x^2 y \, dx \, dy$$

Fubini's Theorem

If f is continuous on the rectangle

$$R = \{(x, y) \mid a \leqslant x \leqslant b, c \leqslant y \leqslant d\}$$

then

$$\iint\limits_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

Guido Fubini

Born	19 January 1879 Venice
Died	6 June 1943 (aged 64) New York

Example: Evaluate the double integral

$$\iint_R \left(x - 3y^2\right) dA$$

where $R = \{(x, y): 0 \le x \le 2, 1 \le y \le 2\}$.

[Extra]

Example: Evaluate the double integral

$$\iint_R \left(x - 3y^2\right) dA$$

where $R = \{(x, y) \colon 0 \le x \le 2, 1 \le y \le 2\}$.

Example: Evaluate the double integral

$$\iint_R y \sin(xy)\, dA$$

where $R = [1,2] \times [0, \pi]$.

[Extra]

Example: Evaluate the double integral

$$\iint_R y \sin(xy)\, dA$$

where $R = [1,2] \times [0, \pi]$.

When $f(x,y) = g(x) \cdot h(y)$, then

$$\iint_R f(x,y)dA = \int_c^d \int_a^b g(x)h(y)dxdy = \int_a^b g(x)dx \cdot \int_c^d h(y)dy$$

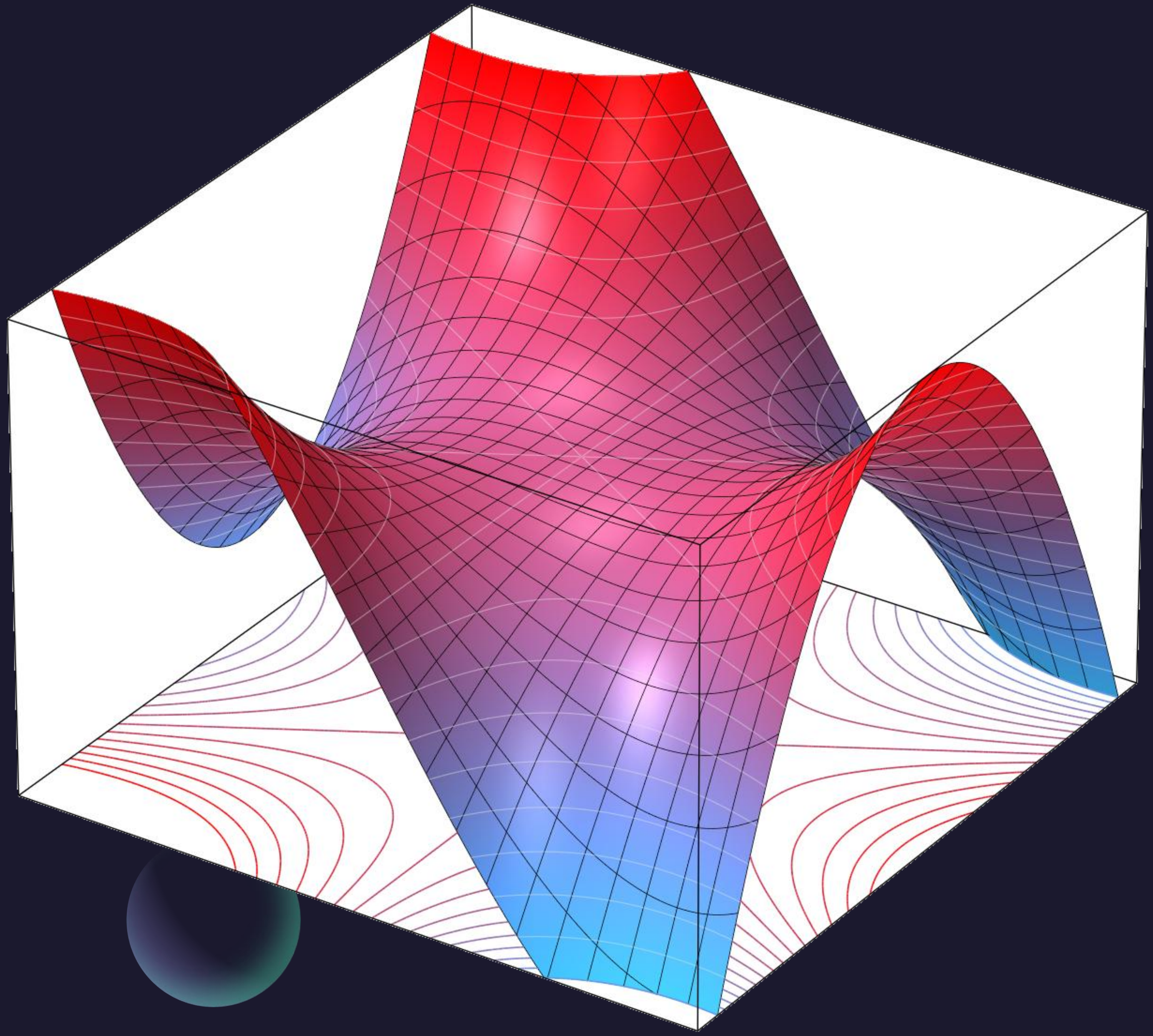
- Evaluate the iterated integral

$$\int_1^3 \int_1^5 \frac{\ln(y)}{xy} dxdy$$

Questions?

Thank you

Until next time.



“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo

Double Integrals over Regions

Today – Double Integrals in Regions!

- General Regions
- Regions of Type I and II
- Changing the Order of Integration
- Properties of Double Integrals

Regions of Type I and II

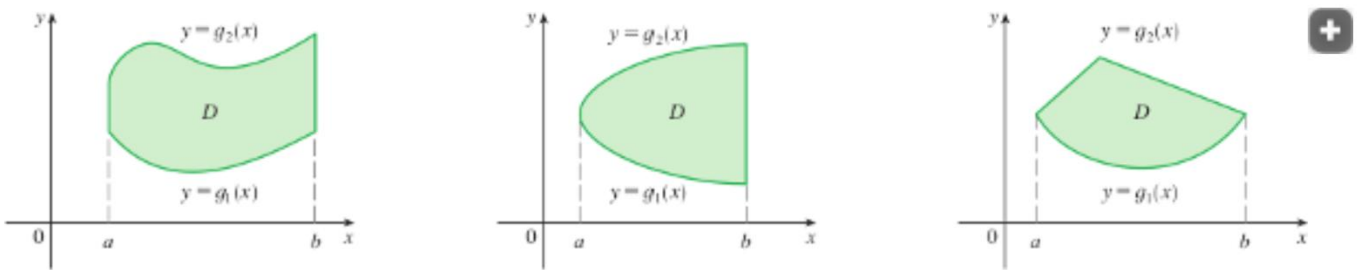
A plane region D is said to be of **type I** if it lies between the graphs of two continuous functions of x, that is,

$$D = \{(x, y) \mid a \leqslant x \leqslant b,\ g_1(x) \leqslant y \leqslant g_2(x)\}$$

where g_1 and g_2 are continuous on $[a, b]$. Some examples of type I regions are shown in Figure 5.

Figure 5

Some type I regions



Regions of Type I and II

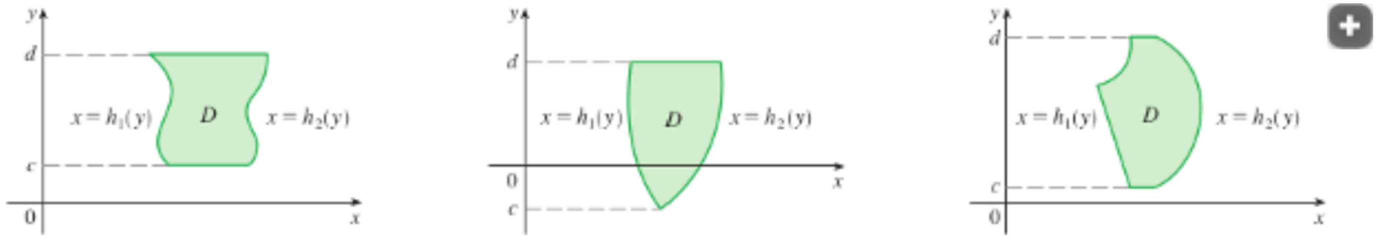
We also consider plane regions of **type II**, which can be expressed as

$$D = \{(x, y) \mid c \leqslant y \leqslant d,\ h_1(y) \leqslant x \leqslant h_2(y)\}$$

where h_1 and h_2 are continuous. Three such regions are illustrated in Figure 7.

Figure 7

Some type II regions



Integrals over Regions of Type I

3 If f is continuous on a type I region D described by

$$D = \{(x, y) \mid a \leqslant x \leqslant b,\ g_1(x) \leqslant y \leqslant g_2(x)\}$$

then

$$\iint\limits_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

Integrals over Regions of Type II

$\boxed{4}$ If f is continuous on a type II region D described by

$$D = \{(x, y) \mid c \leqslant y \leqslant d,\ h_1(y) \leqslant x \leqslant h_2(y)\}$$

then

$$\iint\limits_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$$

Example: Evaluate the double integral

$$\iint_R (x + 2y)\, dA$$

where R is the region bounded by the parabolas

$$y = 2x^2 \text{ and } y = 1 + x^2 .$$

Example: Evaluate the double integral

$$\iint_R (x + 2y)\, dA$$

where R is the region bounded by the parabolas

$$y = 2x^2 \text{ and } y = 1 + x^2 .$$

Example: Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$. (As a Type I integral.)

[Extra]

Example: Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

Example: Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$. (As a Type II integral.)

[Extra]

Example: Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

Example: Evaluate the iterated integral

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

[Extra]

Example: Evaluate the iterated integral

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

Properties of Double Integrals

$$\iint_D [f(x,y) + g(x,y)]\, dA = \iint_D f(x,y)\, dA + \iint_D g(x,y)\, dA$$

$$\iint_D cf(x,y)\, dA = c\iint_D f(x,y)\, dA \qquad \text{where } c \text{ is a constant}$$

If $f(x,y) \geqslant g(x,y)$ for all (x,y) in D, then

7

$$\iint_D f(x,y)\, dA \geqslant \iint_D g(x,y)\, dA$$

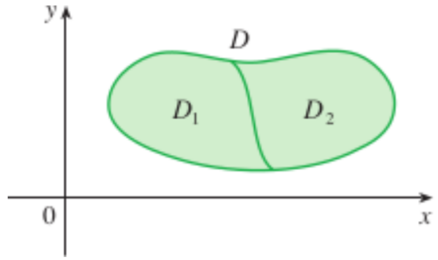
Properties of Double Integrals

If $D = D_1 \cup D_2$, where D_1 and D_2 don't overlap except perhaps on their boundaries (see Figure 17), then

$\boxed{8}$

$$\iint_D f(x, y)\, dA = \iint_{D_1} f(x, y)\, dA + \iint_{D_2} f(x, y)\, dA$$

Figure 17



Properties of Double Integrals

$$\iint_D 1\, dA = A(D)$$

10 If $m \leqslant f(x, y) \leqslant M$ for all (x, y) in D, then

$$m \cdot A(D) \leqslant \iint_D f(x, y)\, dA \leqslant M \cdot A(D)$$

[Extra]

Example: Estimate the value of the double integral

$$\iint_R e^{-(x^2+y^2)} dA$$

where $R = \{(x, y): x^2 + y^2 \leq 1\}$ is the circle of radius 1.

Questions?

Thank you

Until next time.