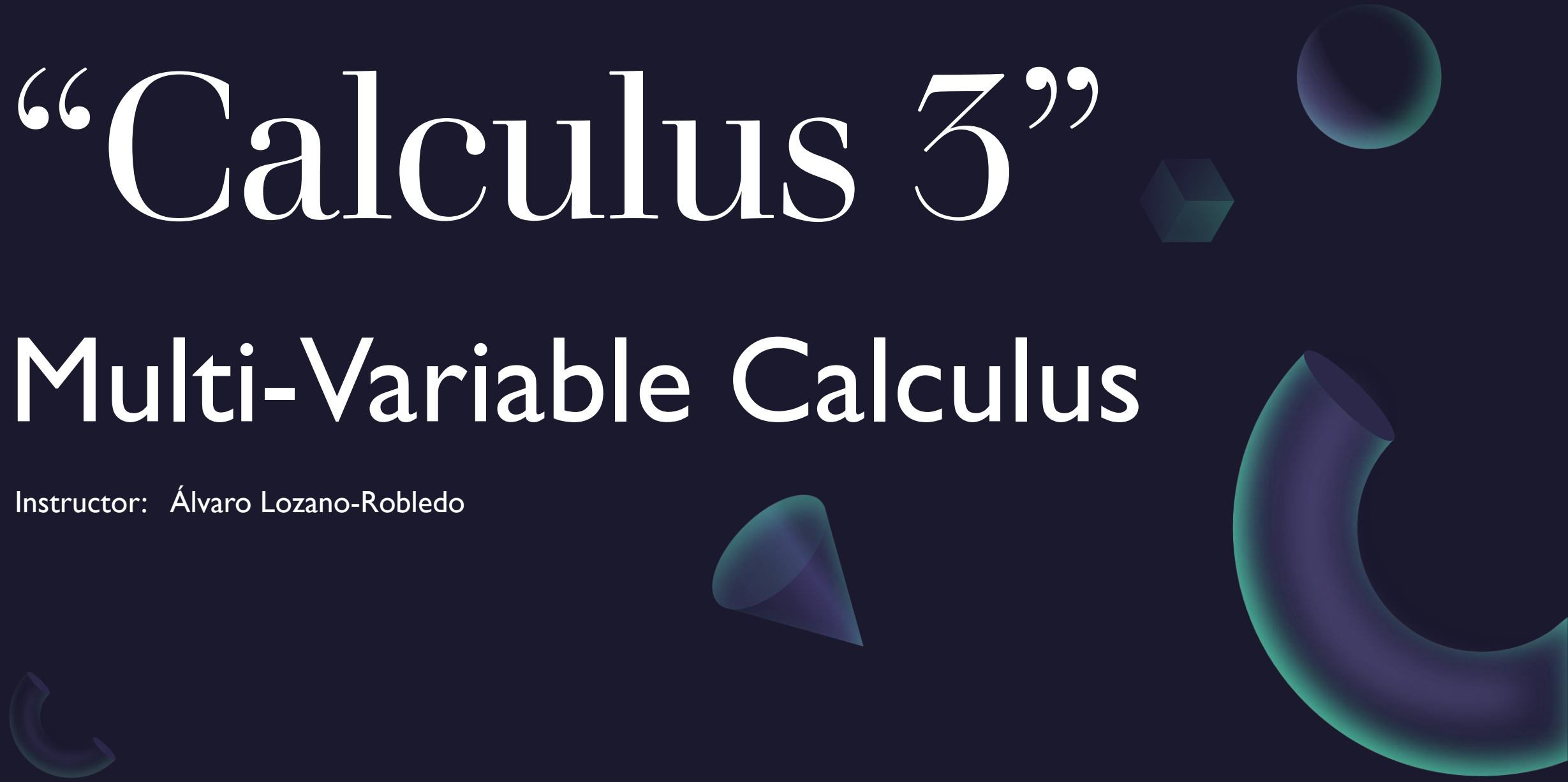
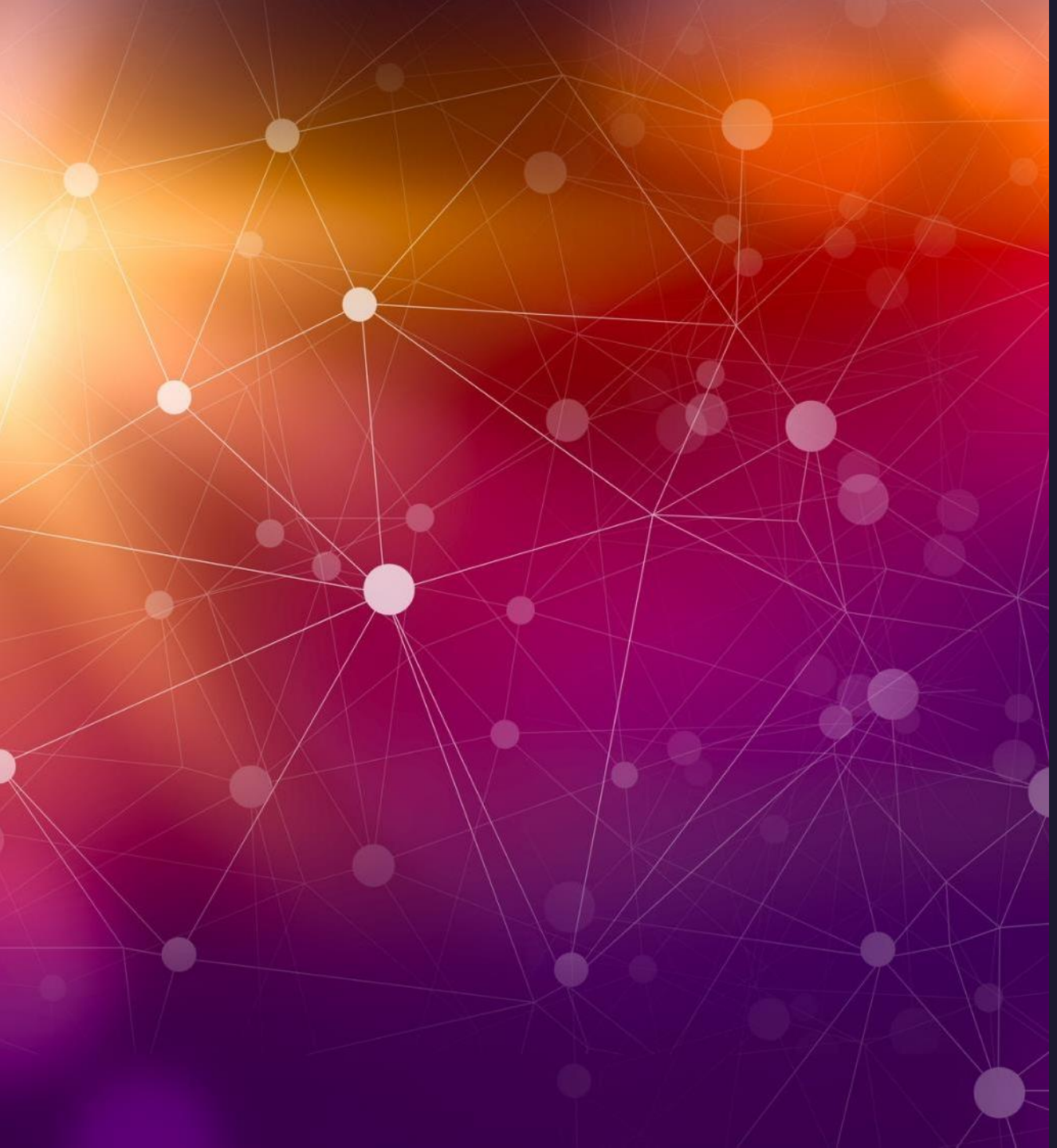
How do you feel about math?

“Calculus 3”

Multi-Variable Calculus

Instructor: Álvaro Lozano-Robledo





Agenda

Introductions

Syllabus

Outline

Let's get started!

MATH

About Me

Professor of Mathematics

At UConn since 2008

Research Area: Number Theory

Book Author and Journal Assoc. Editor

About Me

Office: MONT 233

Email: alvaro@uconn.edu

Subject line **must** start: [MATH2110-14x] …

Office hours:

Mondays (online) – 1-2pm
Thursdays (in-person) – 3:30-4:30pm

The Other Me

@mathandcobb

Videos will be posted on YouTube

Shorter videos also on Instagram and TikTok

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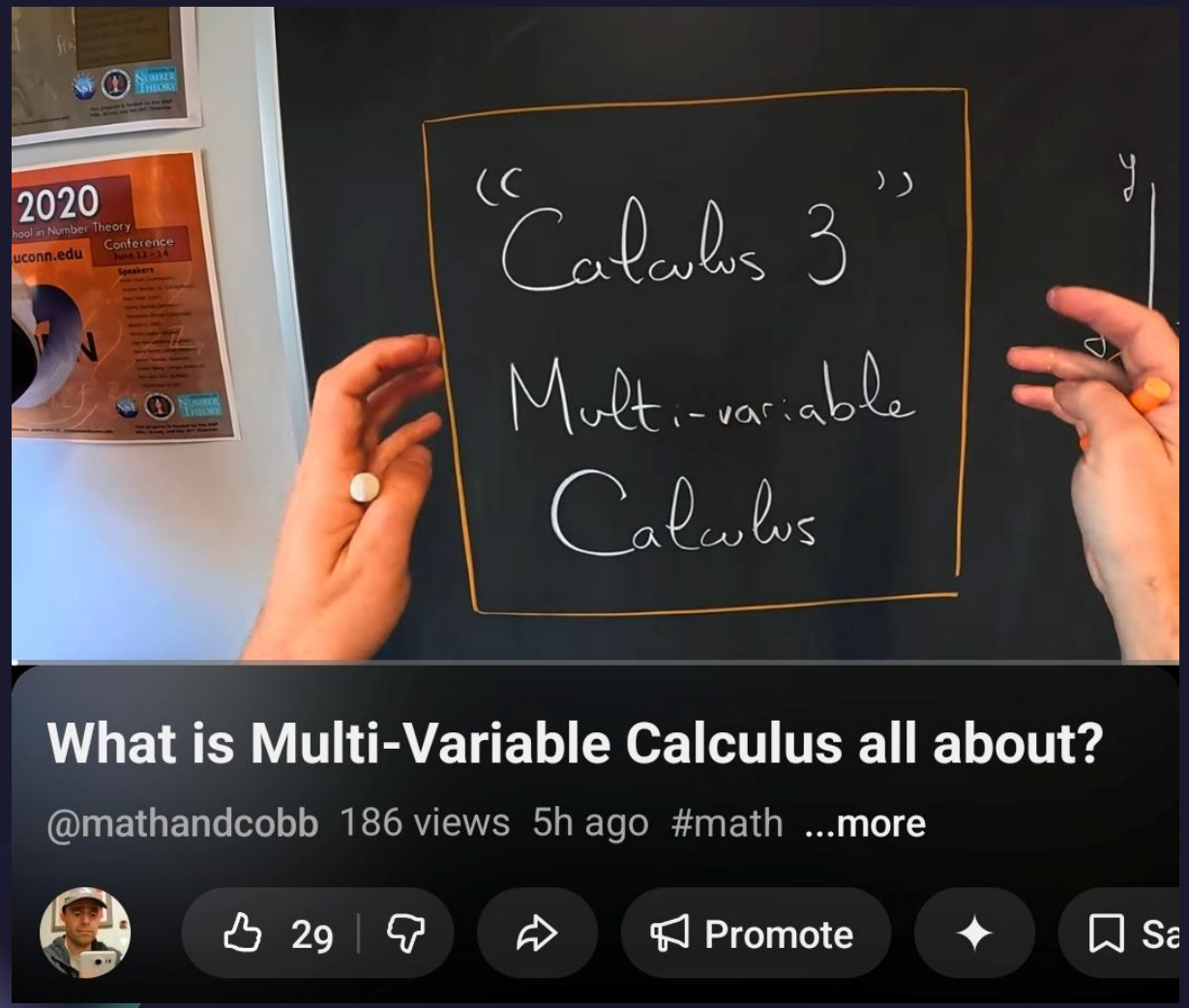

About Your Course – The Syllabus...

Please check HuskyCT often!

All content, announcements, assignments, etc.,
will be posted on HuskyCT

Grading Summary

- Three Exams (20%, 25%, and 25%)
- 10 Quizzes/Honors Assignments (15%)
- WebAssign (15%)

- *Extra Credit! (up to an extra 7%)*

Questions?

ALVARO: Start the recording!

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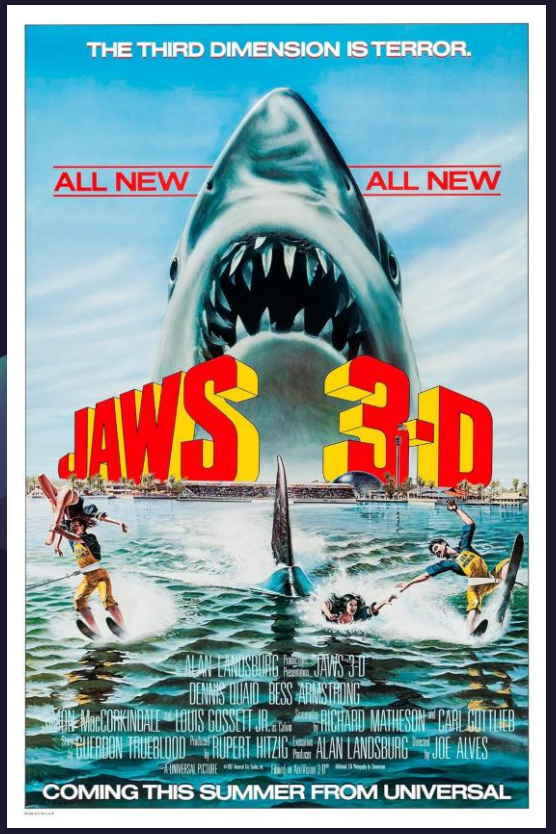
Day 1

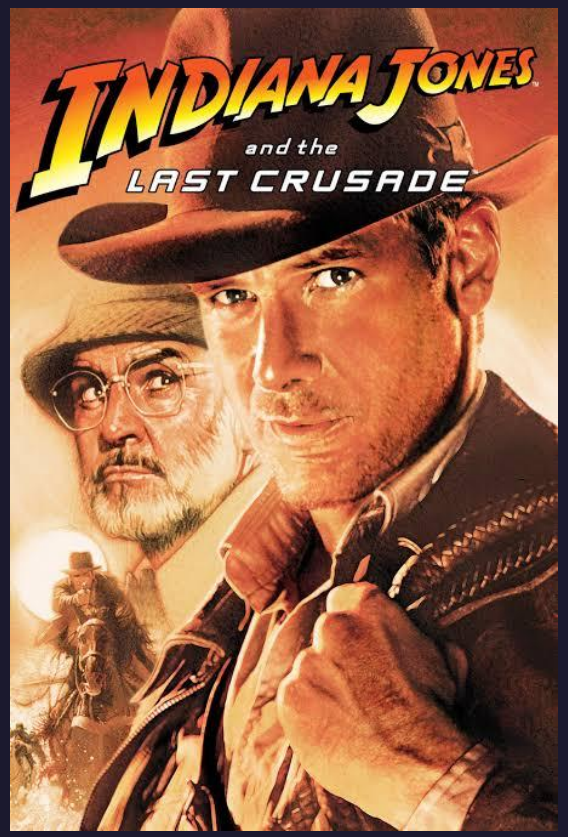
Calculus 3

The “Third Part” Curse

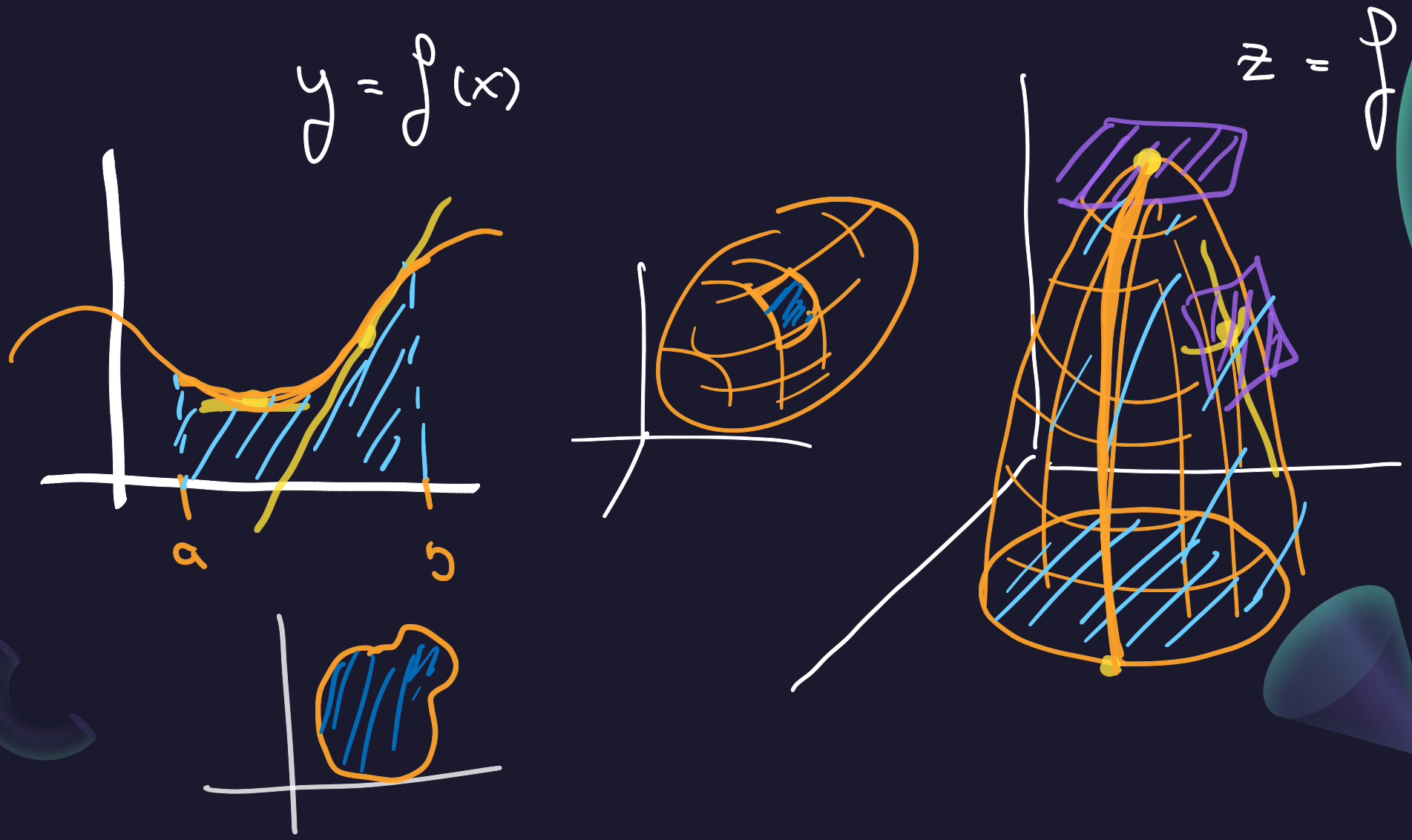


Calculus 3

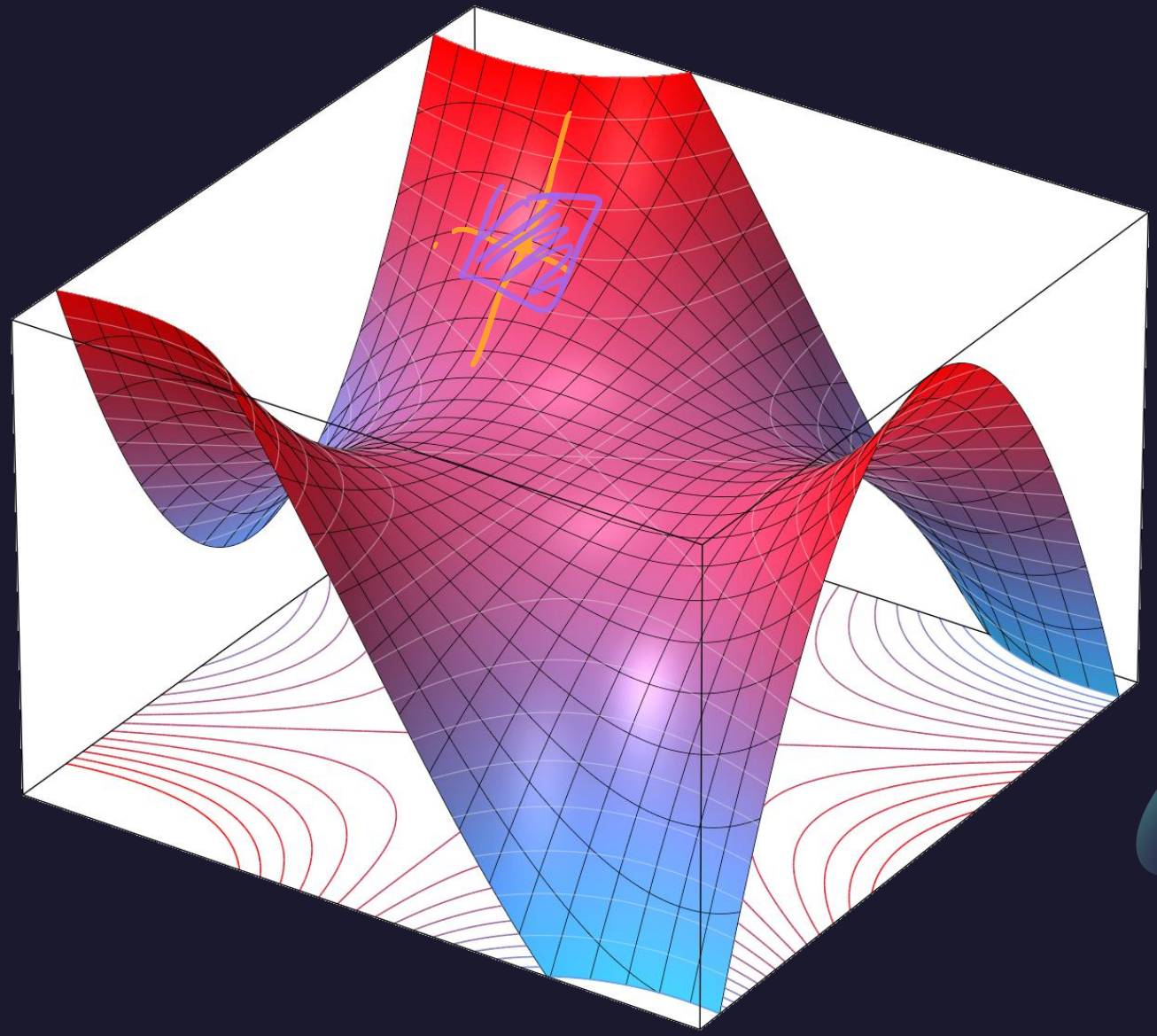
The “Third Part” Curse?



What is Multi-Variable Calculus all about?

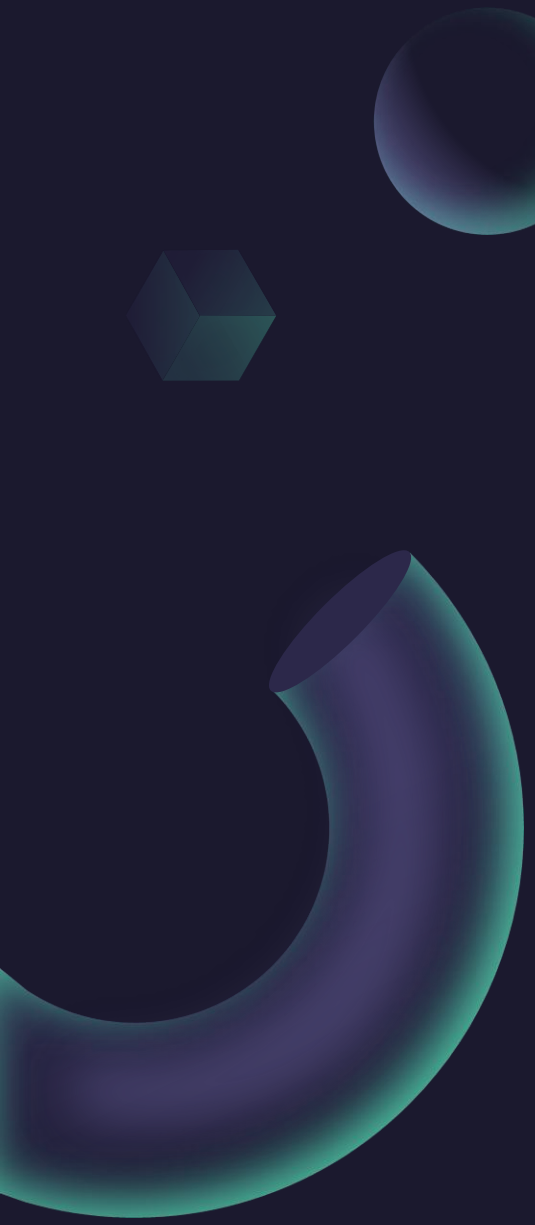


What is Multi-Variable Calculus all about?



Content

- Three-dimensional space
- Differential calculus in 2 or more variables
- Double and triple integrals
- Vector functions
- Parametric curves and surfaces
- Line and surface integrals



Today!

- Three-dimensional space
  - Coordinate systems
  - Surfaces and solids
  - Distance formula and spheres

Notation

$\mathbb{R}$ - real numbers (all decimal expansion)

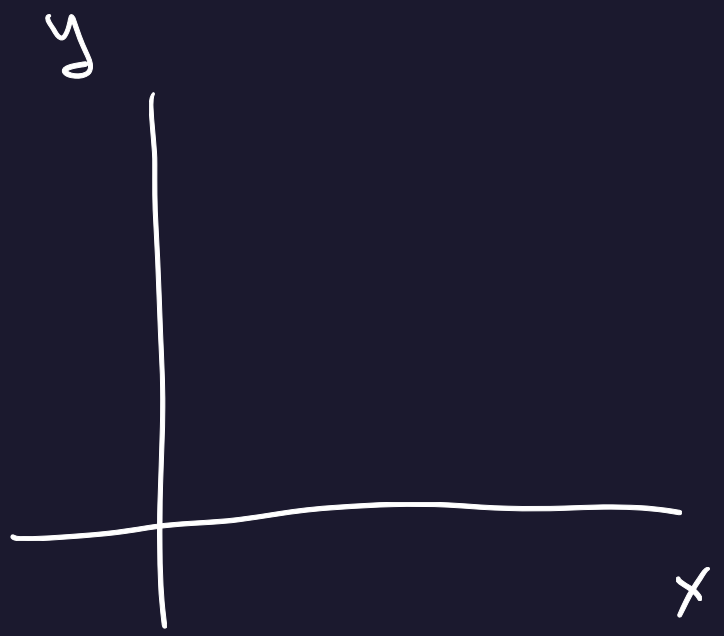
$\mathbb{R}^2$ - real euclidean plane $= \{(x, y) : x, y \text{ are in } \mathbb{R}\}$

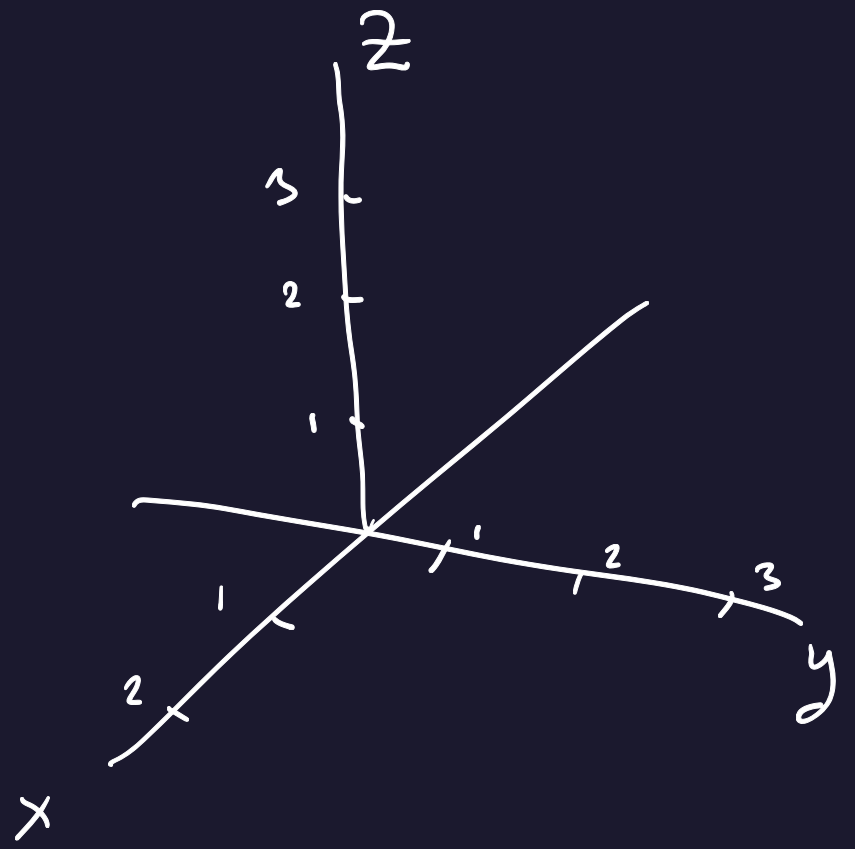
$\mathbb{R}^3$ - real euclidean space $= \{(x, y, z) : x, y, z \in \mathbb{R}\}$

$\mathbb{R}^n$ - real euclidean n-dimensional space

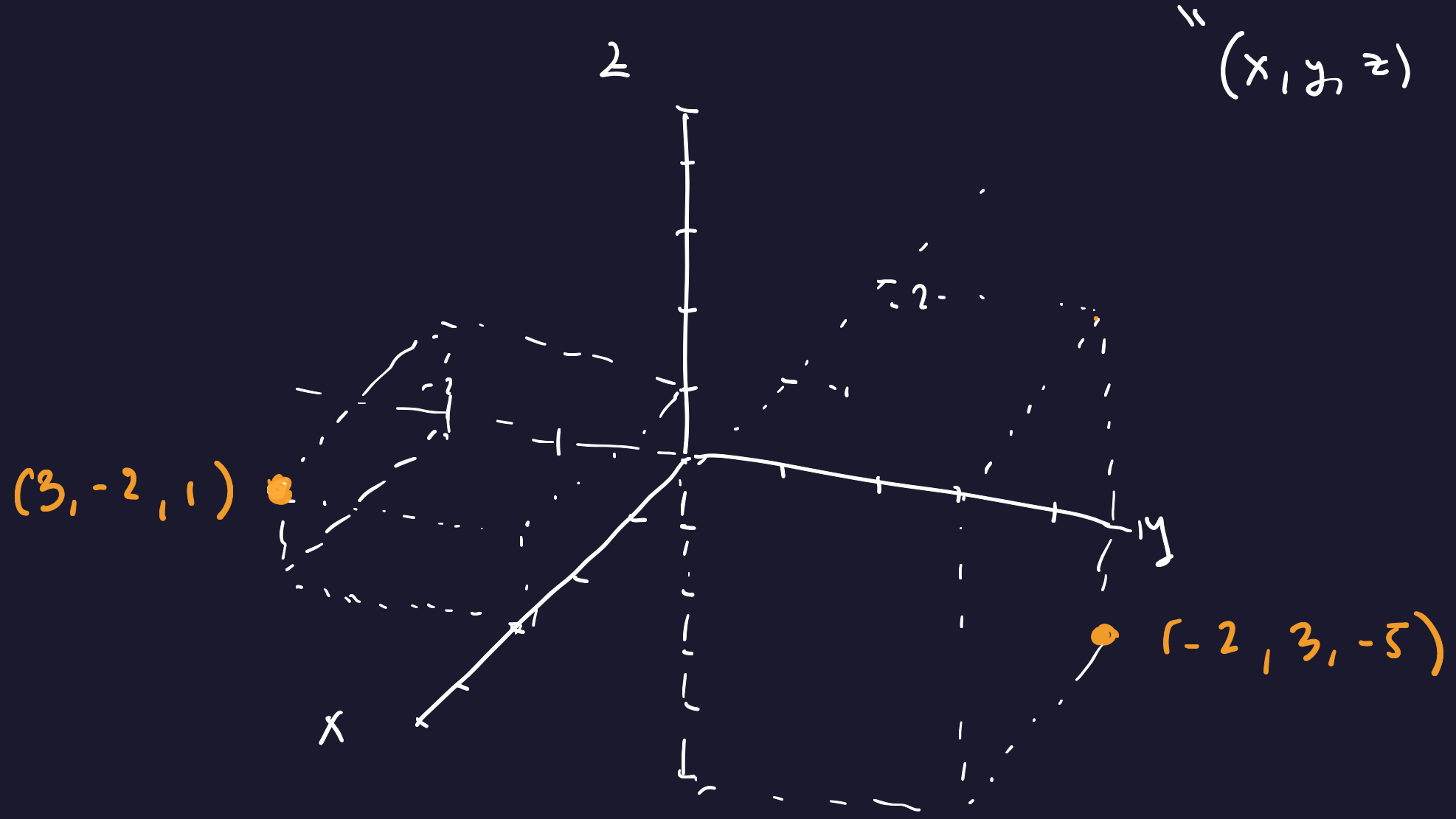
$= \{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}\}$

Meet your new axes (2D vs 3D)





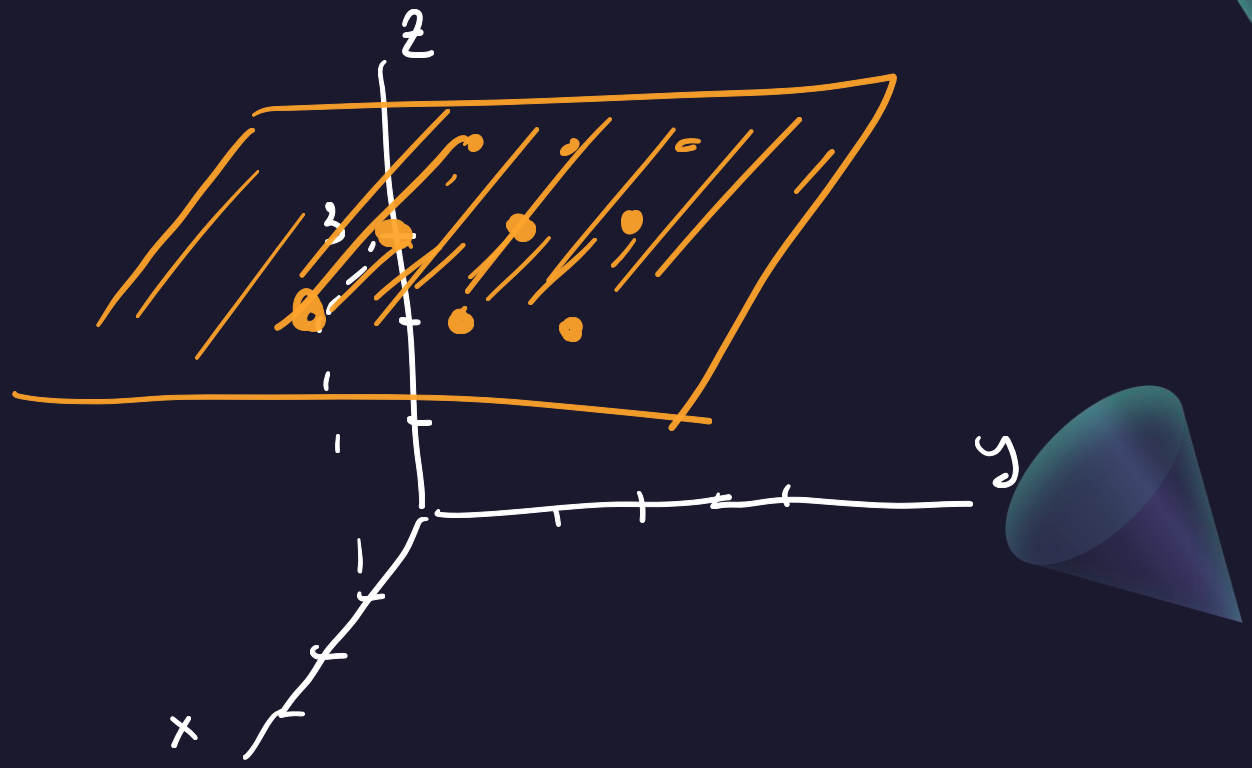
Example: Plot the points (3, -2, 1) and (-2, 3, -5)



Surfaces and Solids

Example: Sketch the surface in $\mathbb{R}^3$ given by $z = 3$.

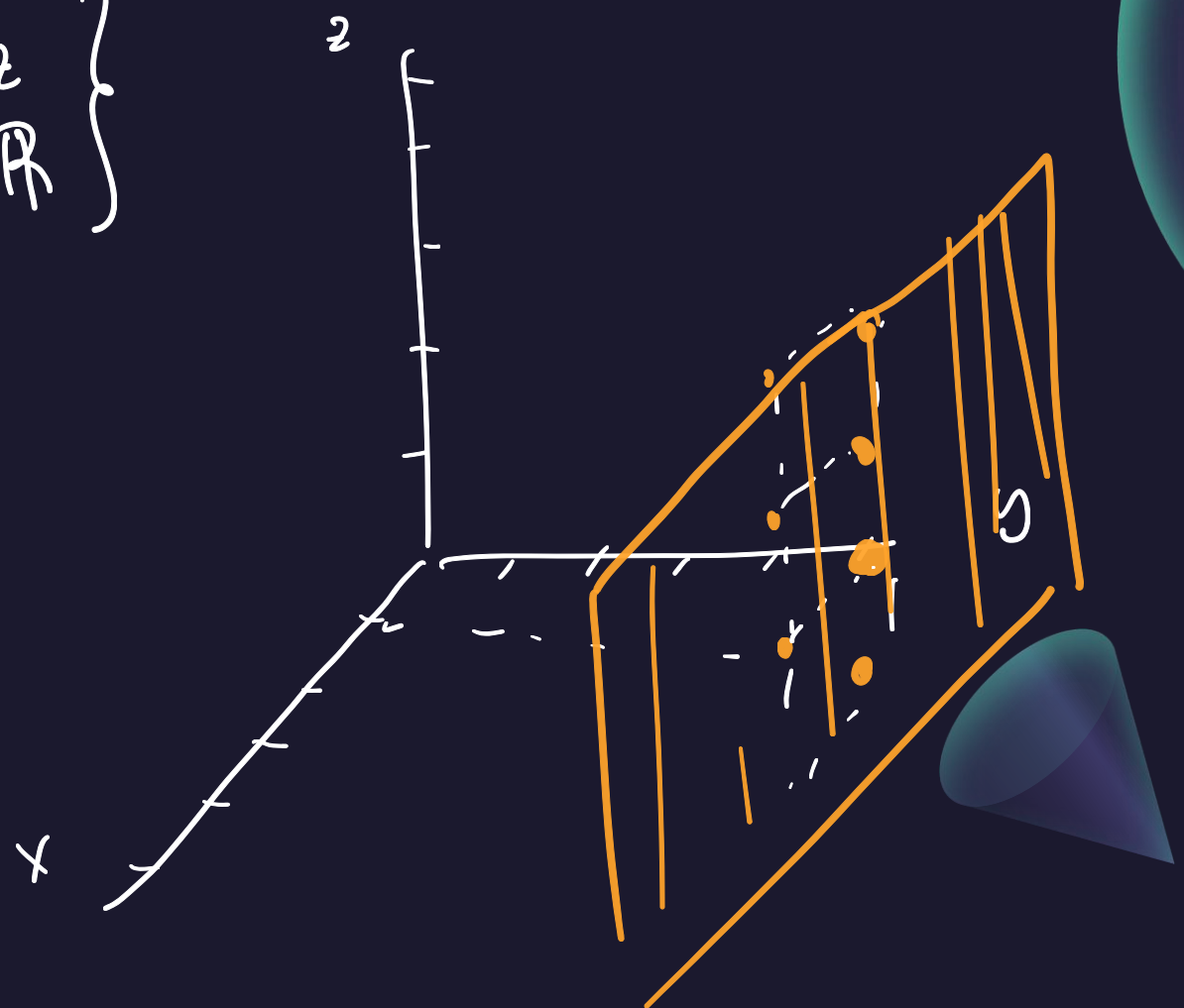
$$S = \{(x, y, 3) : x, y \in \mathbb{R}\}$$



Surfaces and Solids

Example: Sketch the surface in $\mathbb{R}^3$ given by $y = 5$.

$$\{(x, 5, z) : x, z \text{ in } \mathbb{R}\}$$



The surface in $\mathbb{R}^3$ given by $y = x$ is ...

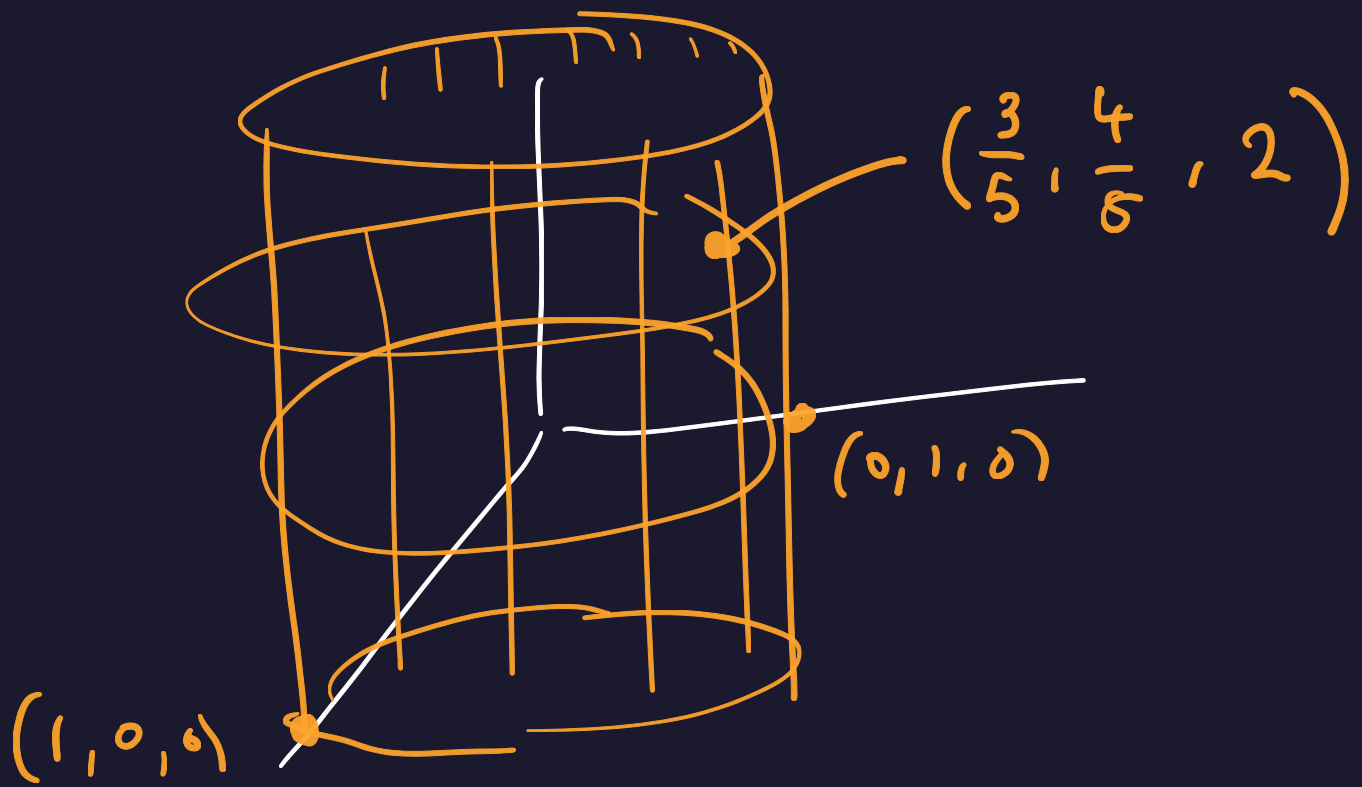
Surfaces and Solids

Example: Sketch the surface in $\mathbb{R}^3$ given by $y = x$.

$\{(x, x, z) : x, z \text{ in } \mathbb{R}\}$

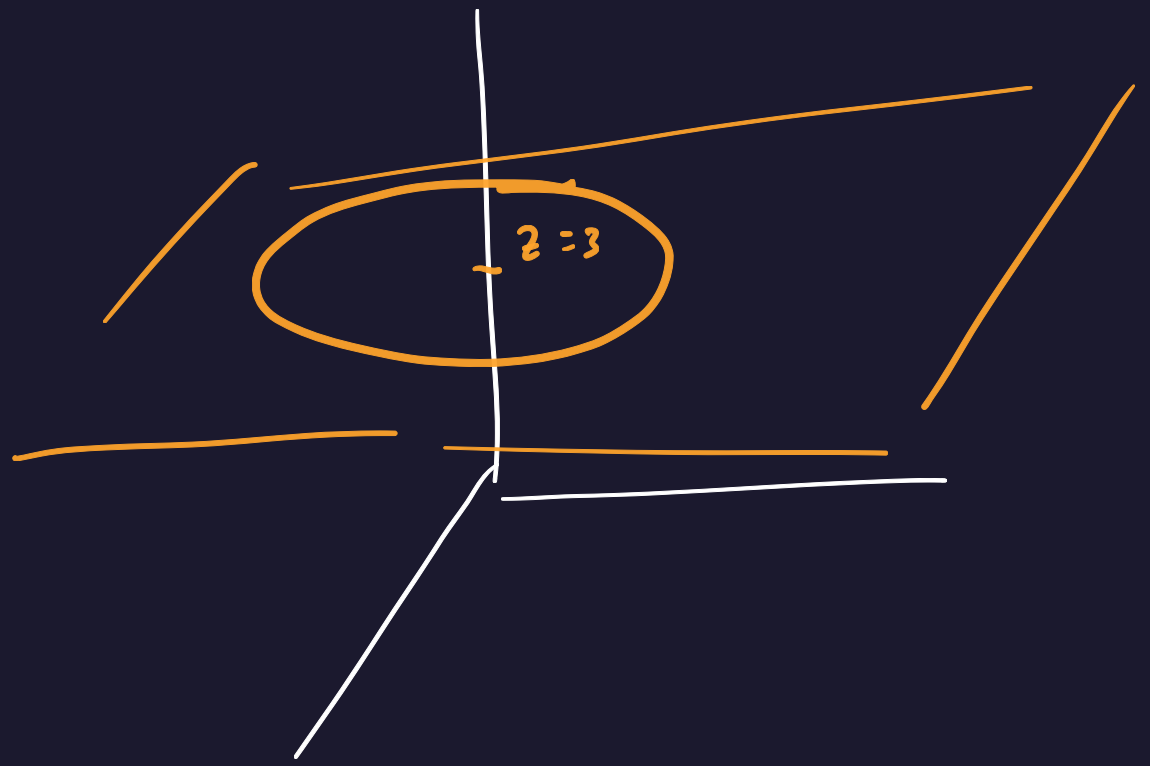
Surfaces and Solids

Example: Sketch the surface in $\mathbb{R}^3$ given by $x^2 + y^2 = 1$.



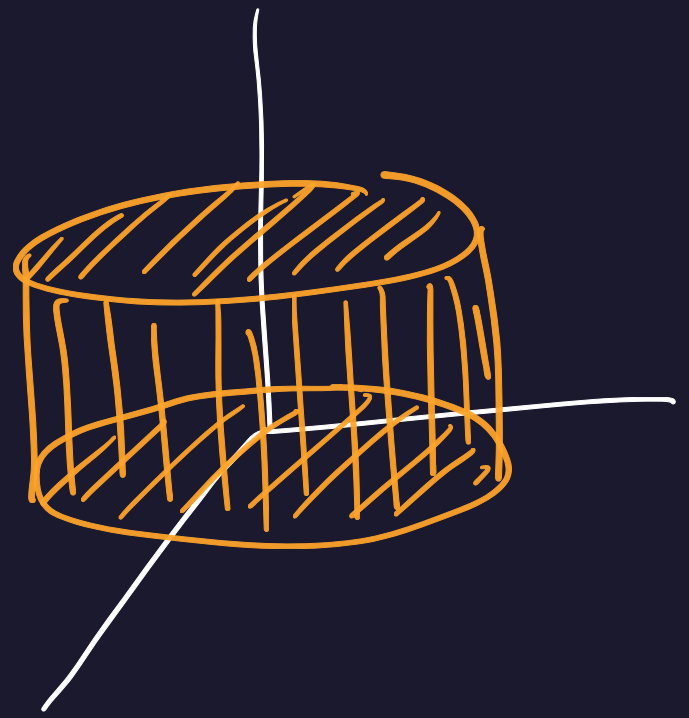
Example: Sketch the surface in $\mathbb{R}^3$ given by

$$x^2 + y^2 = 1 \quad \text{AND} \quad z = 3.$$

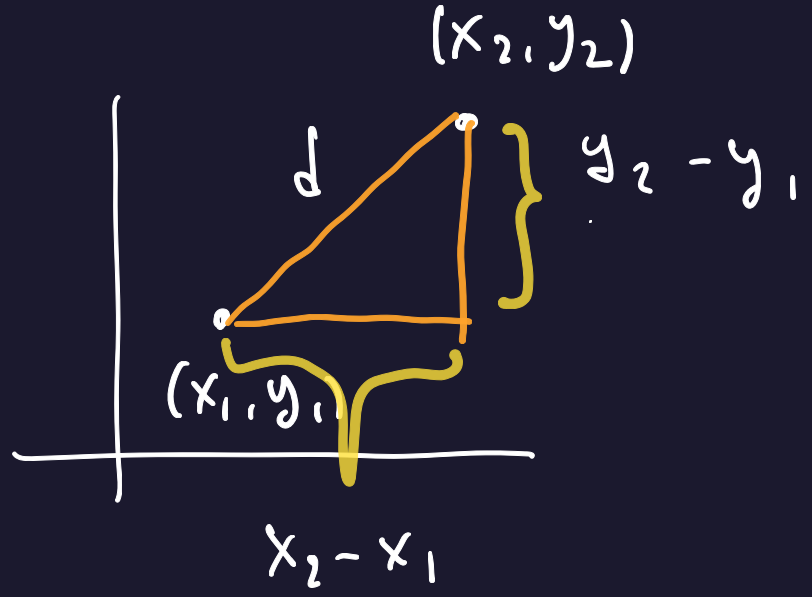


Example: Sketch the surface in $\mathbb{R}^3$ given by

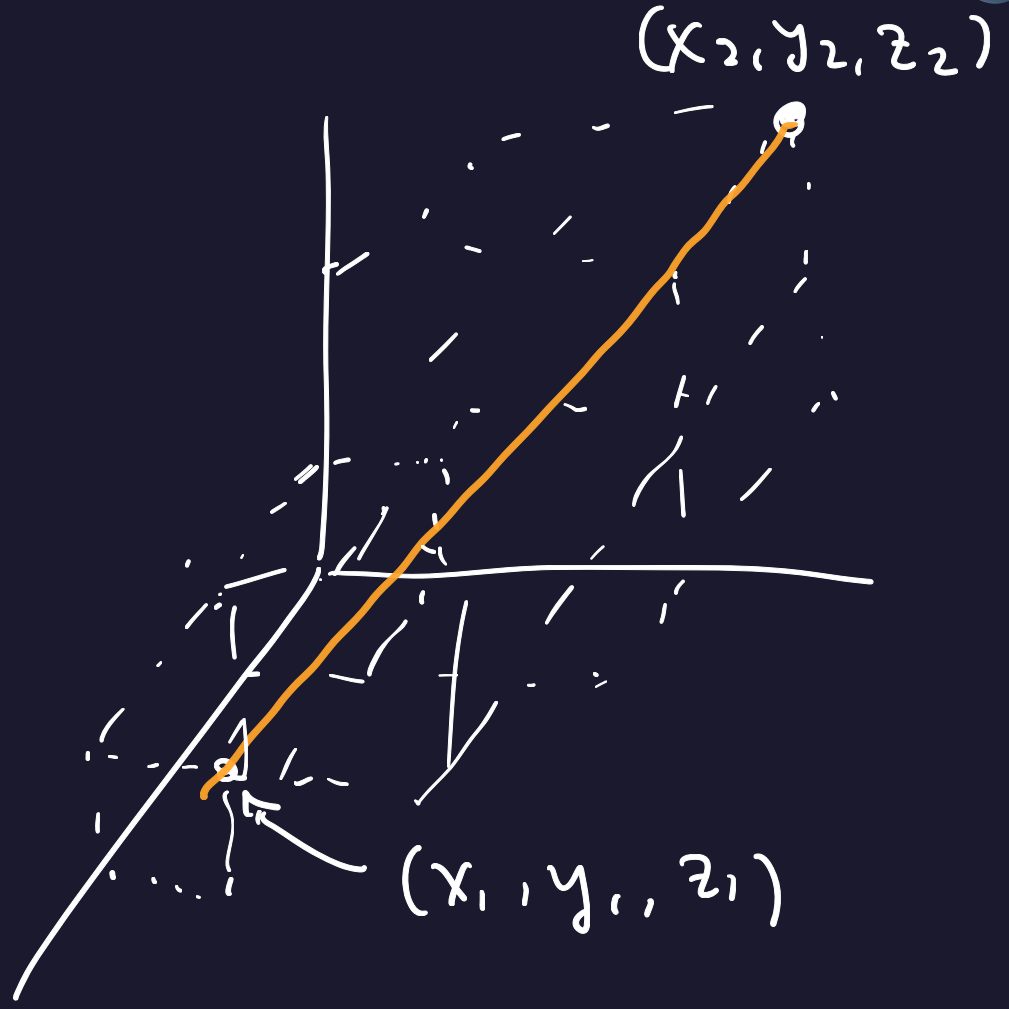
$$x^2 + y^2 \leq 1 \quad \text{AND} \quad 0 \leq z \leq 3.$$



Distance Formulas



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Distance Formula in Three Dimensions

The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example: Find the distance from (1, 0, 2) to (-1, 3, 0)

$$d = \sqrt{(1-(-1))^2 + (0-3)^2 + (2-0)^2}$$

$$= \sqrt{2^2 + 3^2 + 2^2}$$

$$= \sqrt{4 + 9 + 4} = \boxed{\sqrt{17}}$$

Example: Find the equation of a sphere with center at (1, 0, 2) and radius 3.

$$d = 3 = \sqrt{(x-1)^2 + (y-0)^2 + (z-2)^2}$$

$$3 = \sqrt{(x-1)^2 + y^2 + (z-2)^2}$$

$$9 = (x-1)^2 + y^2 + (z-2)^2$$

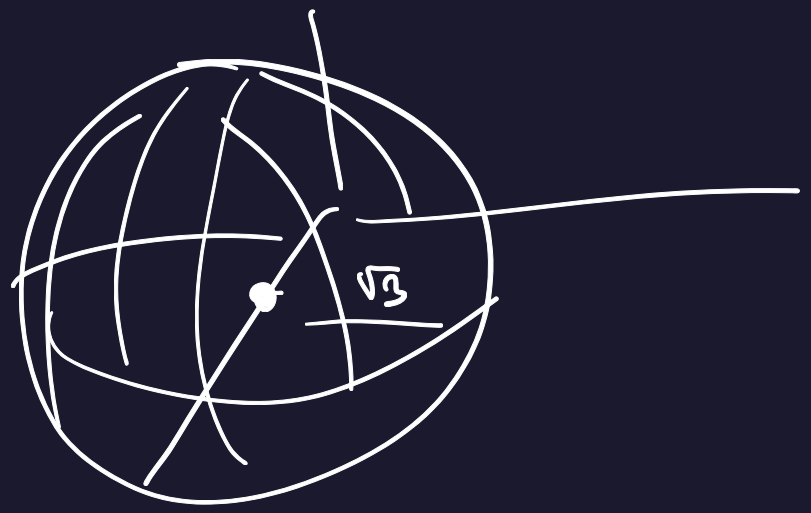
The equation (x-1)^2+y^2+z^2 = 3 is...

Example: Sketh the graph of $(x-1)^2 + y^2 + z^2 = 3$.

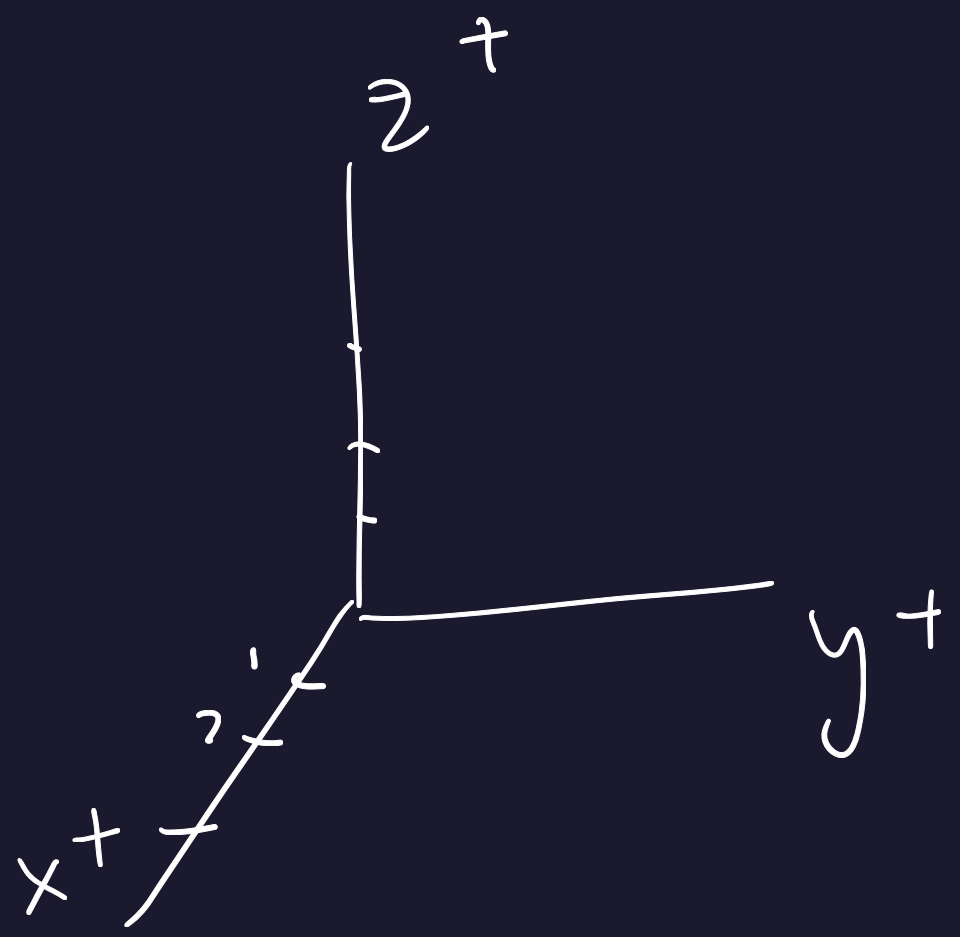
Center is $(1, 0, 0)$

R^2

$R = \sqrt{3}$



Questions?



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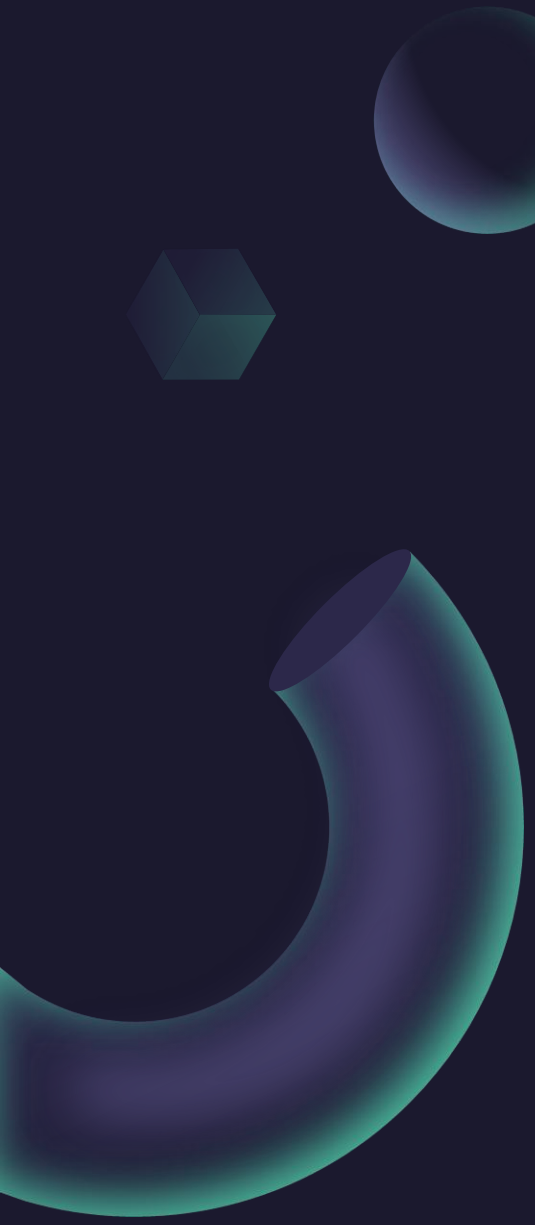
Day 1 – Part 2

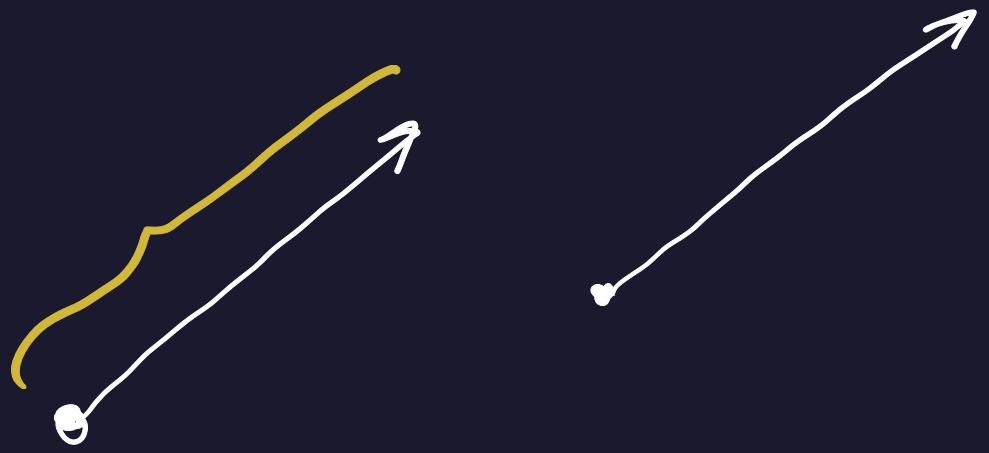


Today!

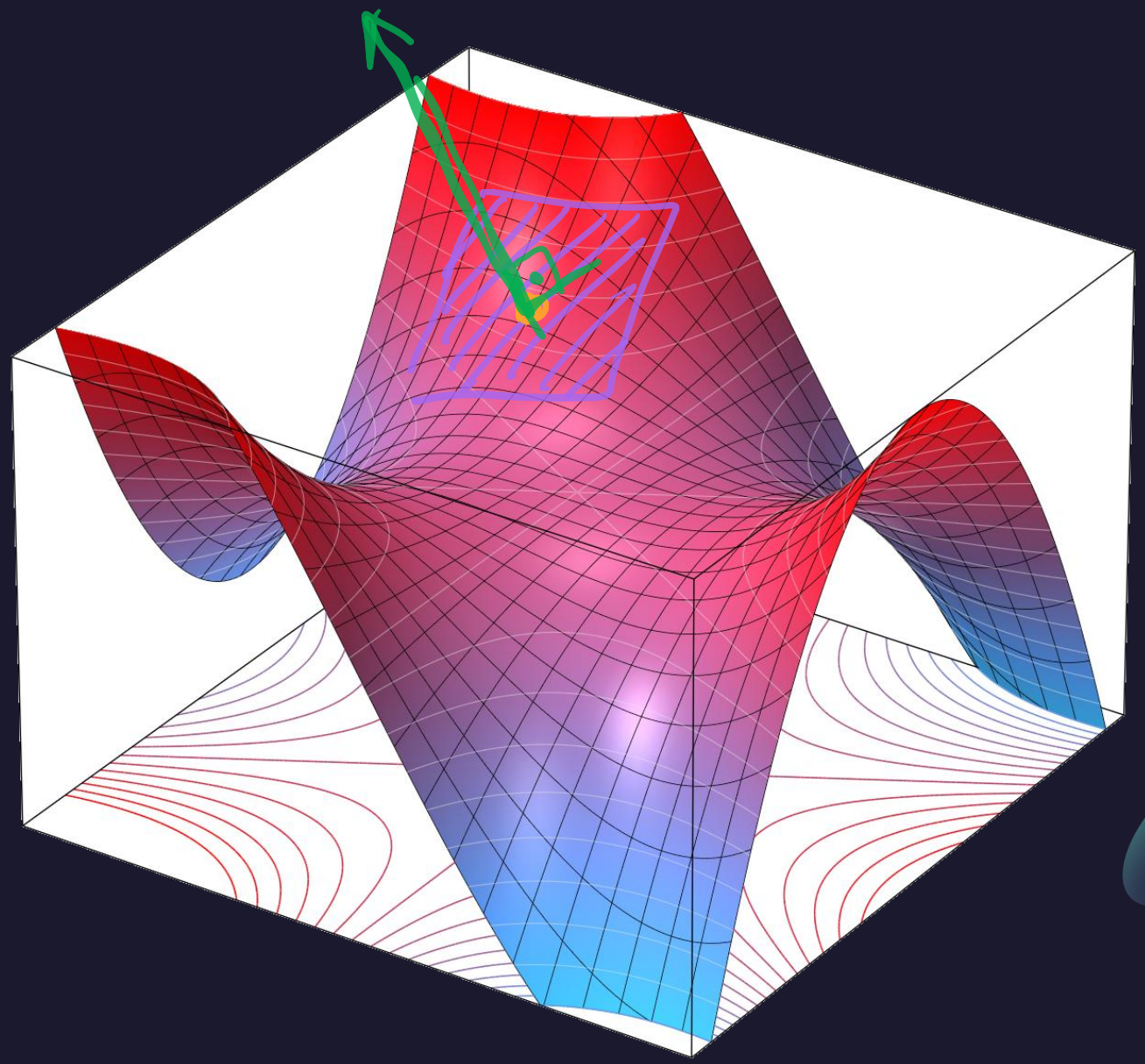
- Vectors
  - Vector addition and scalar multiplication
  - Components and length
  - Properties
  - Applications

Vectors

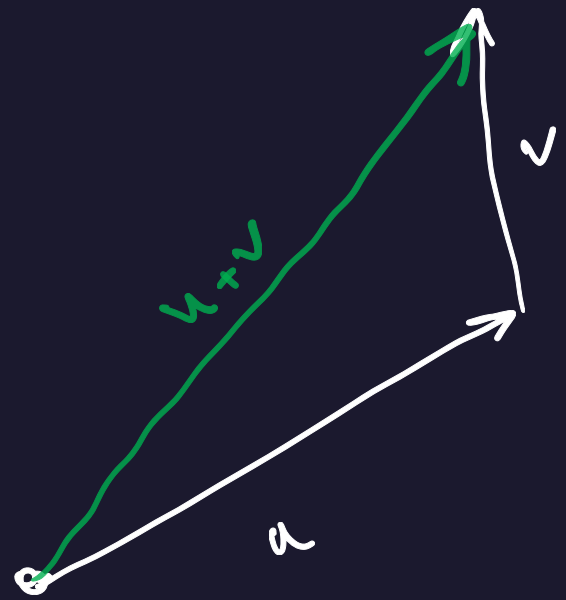
A **vector** is a mathematical object with both magnitude (size) and direction, represented as a directed line segment (arrow).



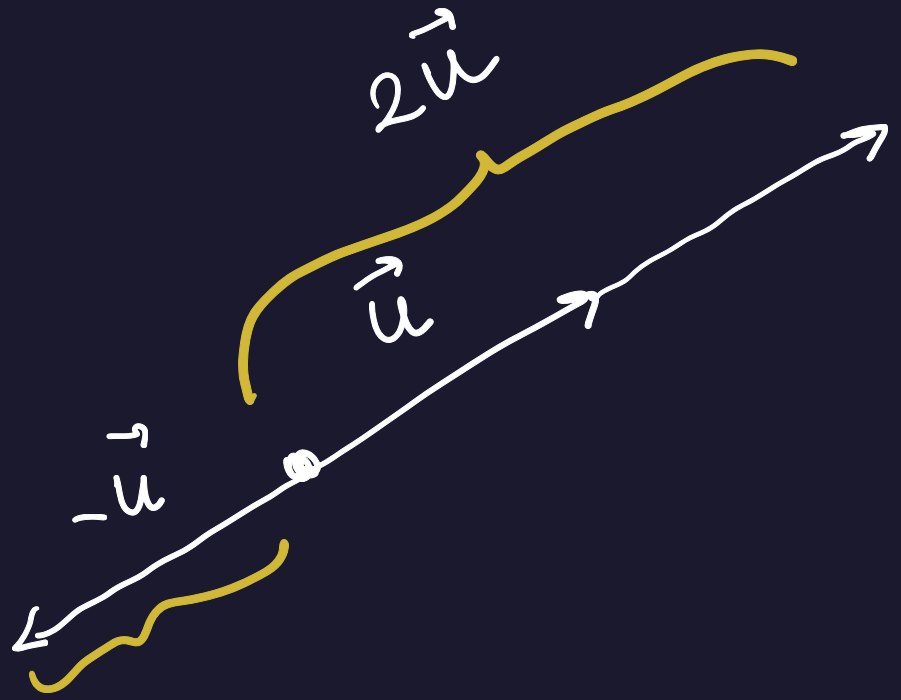
Why Vectors?



Vector Addition

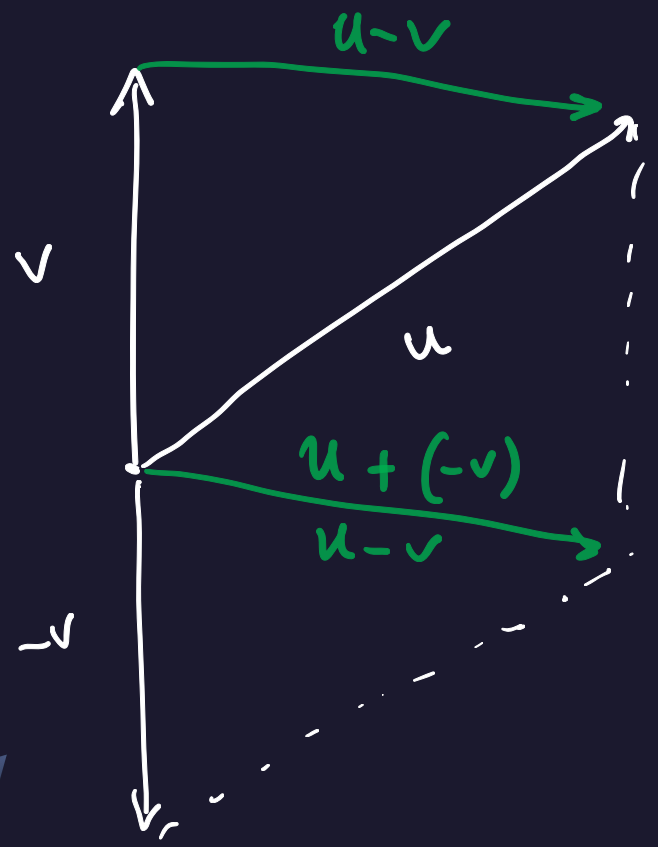


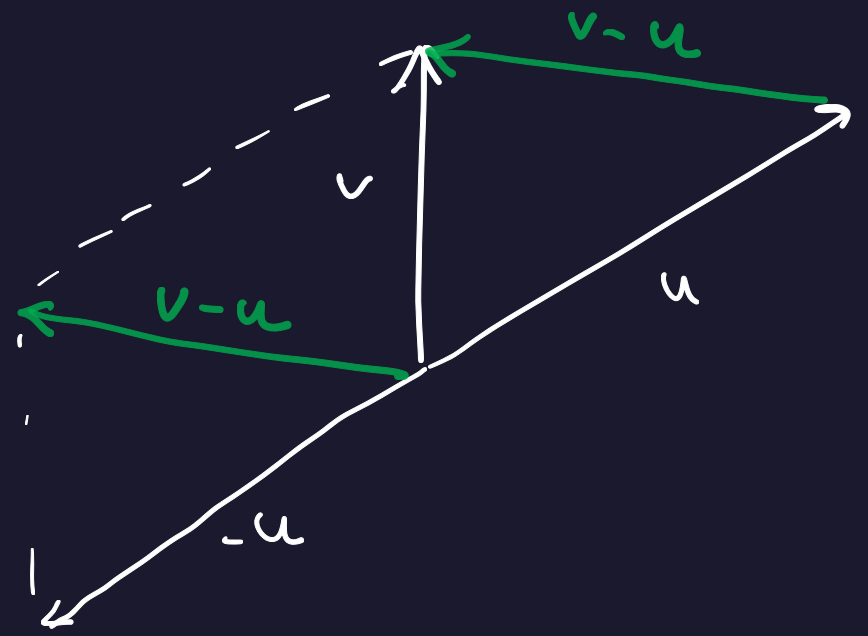
Vector Scalar Multiplication



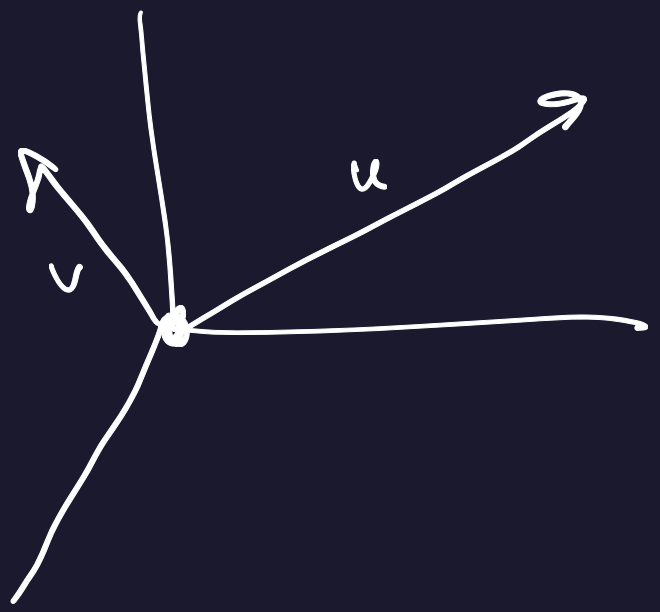
Difference of Vectors

$u - v$, $v - u$





Vectors in Coordinates



$$u = (x_0, y_0)$$

$$v = (x_1, y_1)$$

$$u + v = (x_0 + x_1, \; y_0 + y_1)$$

Example: Let $u = (2,3)$ and $v = (-1,1)$. Find $u+2v$ and $u-2v$.

Example: Let u = (2,3,0) and v = (-1,1,2).
Find u+v and u-v.

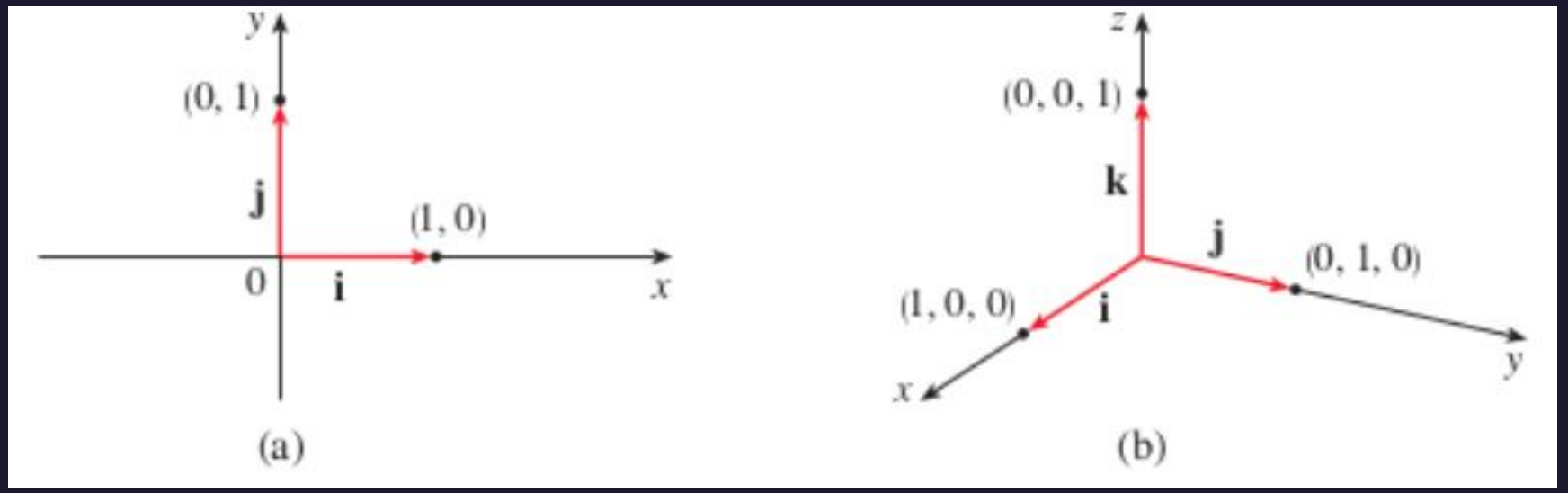
Length of a Vector

Example: Let a = (4, 0, 3) and b = (-2, 1, 5).
Find the lengths of a+b and a-b.

Properties of Vectors

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$



Standard Basis Vectors

- i = (1, 0, 0)
- j = (0, 1, 0)
- k = (0, 0, 1)

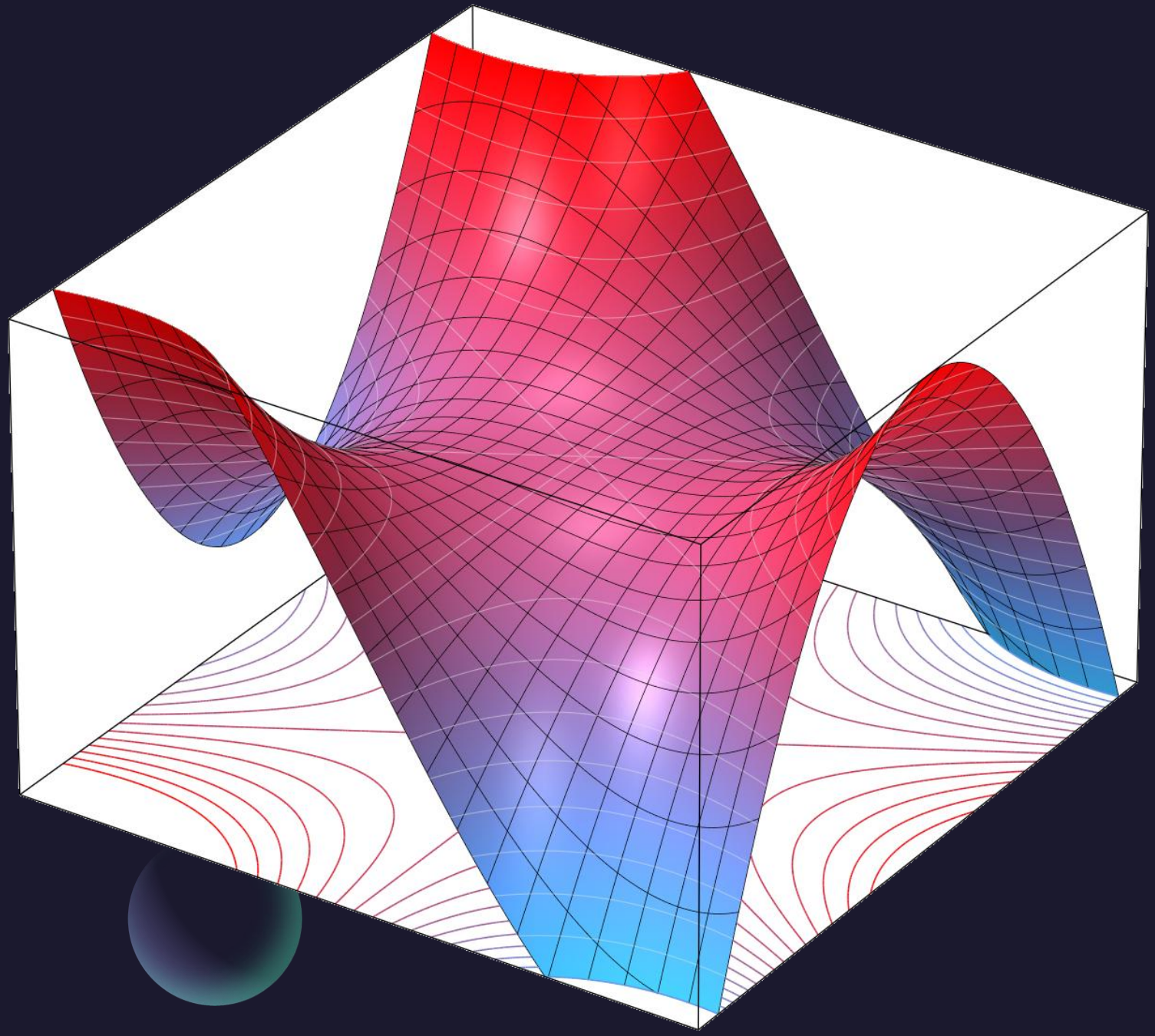
Example: Let a = 4i+3k and b = -2i + j + 5k.
Find the lengths of 2a+b and 2a-b in terms of i, j, and k.

Example (an application of vectors):

A woman launches a boat from the south shore of a straight river that flows directly west at 4 mi/h. She wants to land at the point directly across on the opposite shore. If the speed of the boat (relative to the water) is 8 mi/h, in what direction should she steer the boat in order to arrive at the desired landing point?

Thank you

Until next time.



Questions?