EXAM 1: Topic List

This is an outline of the topics you need to know for the exam.

- Know the distance formula and the equation of the sphere. (12.1)
- Perform addition, subtraction, and scalar multiplication of vectors. (12.2)
- Find the magnitude of a vector and be able to convert it to a unit vector. (12.2)
- Compute the dot product of two vectors. (12.3)
- If θ is the angle between vectors $\vec{a}$ and $\vec{b}$, then:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos(\theta)$$

- Two (nonzero) vectors $\vec{a}$, $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$. (12.3)
- Be able to compute the cross-product of two vectors. (12.4)
- Know properties of cross product: $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}, \vec{b}$ with direction determined by right-hand rule and has length $\|\vec{a}\|\|\vec{b}\| \sin(\theta)$. (12.4)
- Two nonzero vectors $\vec{a}$, $\vec{b}$ are parallel if and only if $\vec{a} \times \vec{b} = \vec{0}$. (12.4)
- Find the equation of the line L in vector equation or parametric equation (12.5) for the following cases:
  - Given a point and the direction vector.
  - Given two points on the line.
  - Given a point and the equation of another line that is parallel to the line we want to find.
- Find the direction vector of a line, such as from the vector equation or parametric equation of another line. (12.5)
- Find the standard equation of the plane in for the following cases through identification of a point in the plane and a normal vector. (12.5)
  - Given a point and two vectors in the plane.
  - Given three points in the plane.
  - Given a point and a line in the plane.
  - Given two lines in the plane.
- For sections 12.6 and 14.1, you should know the common surfaces (both the equations and the graphs) and how to use traces to visualize the surface.
- Compute the first and second order partial derivatives and equality of mixed derivatives. (14.3)
- Find the equation of the tangent plane of $f(x, y)$ at a given point (x_0, y_0) using:

$$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

  and use it for linear approximation. (14.4)
- Use the chain rule to take the derivative. (14.5)
- Compute directional derivatives and find the direction and/or the magnitude of the greatest rate of change at any point. (14.6)

- If f is a differentiable function of x, y, then f has a directional derivative in the direction of any unit vector $\vec{u} = \langle a, b \rangle$ and:

$$D_{\vec{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b$$

  In terms of the vector operation, we could have:

$$D_{\vec{u}} f(x, y) = \langle f_x, f_y \rangle \cdot \langle a, b \rangle$$

- Identify points as local/global minimum/maximum or saddle points. (14.7)
- Find local minimum/maximum or saddle points through the second derivative test. (14.7)
- Find maximum and minimum of functions of two variables on closed bounded domains (14.7)
- Apply the method of Lagrange multipliers to find maximum and minimum of functions of two and three variables under constraints. (14.8)
- Use the second derivative test to classify critical points. (14.7)

  Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let:

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

  1. If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
  2. If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
  3. If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

  In case 3, the point (a, b) is the saddle point of f and the graph of f crosses its tangent plane at (a, b).

  If $D = 0$, the test gives no information: f could have a local maximum or local minimum at (a, b), or (a, b) could be a saddle point of f.

  - It is helpful to remember D in terms of the determinant:

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2$$