Exam 1 Practice Problems - Spring 2026

1. Let $\vec{a} = \langle 1, 1, 4 \rangle$ and $\vec{b} = \langle c, 3, 4 \rangle$, where c is an unknown constant.
   (a) Find the value of c so that $\vec{a}$ and $\vec{b}$ are orthogonal.
   (b) With the value of c from part (a), find $\vec{a} \times \vec{b}$.

2. Find the equation of a line that passes through $(1, 2, 3)$ and is perpendicular to the plane $x - y + 3z = 5$.

3. Find the equation of a plane through the origin, $(0, 1, 2)$ and $(3, 0, 1)$.

4. Let $f(x, y)$ be a function satisfying $f(4, 3) = 5$ and $\nabla f(4, 3) = \langle 6, 8 \rangle$.
   (a) Find the equation of the tangent plane to f at $(4, 3)$.
   (b) Use the linear approximation of $f(x, y)$ at $(4, 3)$ to approximate $f(5, 2)$.
   (c) What is the rate of change of the function at $(4, 3)$ when moving towards the origin?
   (d) Which direction maximizes the rate of change of f at $(4, 3)$?

5. Let $f(x, y) = \sqrt{x^2 + y^2} \cdot \ln(2x)$.
   (a) Find the domain of f.
   (b) Verify by direct computation that $f_{xy} = f_{yx}$ (also known as *Clairaut's Theorem*).

6. Let $f(x, y) = (x^2 - y^2)e^y$ and let $g(t) = \cos(t)$ and $h(t) = \sin(t)$. Use the chain rule to compute the derivative with respect to t of the function $f(g(t), h(t))$.

7. Let f be a continuous function of two variables which is twice differentiable with the following table of values.

	$f(x,y)$	$f_x(x,y)$	$f_y(x,y)$	$f_{xx}(x,y)$	$f_{xy}(x,y)$	$f_{yy}(x,y)$
$(-1, 2)$	11	0	0	1	5	3
$(1, 4)$	-5	1	0	2	0	4
$(-2, -1)$	6	0	0	-3	0	-1
$(-4, -1)$	0	2	2	1	0	1
$(1, -3)$	2	3	0	-2	5	2

   (a) Which points are critical points? **Select ALL that apply.**
      A. $(-1, 2)$

B. $(1, 4)$

C. $(-2, -1)$

D. $(-4, -1)$

E. $(1, -3)$

(b) Classify each critical point as a local maximum, local minimum or saddle point or explain why there is not enough information to tell.

8. Use the method of *Lagrange Multipliers* to find the maximum and the minimum of $f(x, y) = x^2 + y$ over the ellipse $x^2 + 2y^2 = 8$.