Exercises in Learning Systems (DVA427)

Some questions from previous exams

1. Decision Tree Learning

Construct the decision tree for the set of training instances in the left table with attributes x_1, x_2 and classes +, –.

a) Which attribute should be used at the root node and why?

b) Show how the examples D_1, …., D_{10} are sorted down the tree and how they are classified.

Use the right table to look up the entropy for a subset of training instances. Take the entry that is closest to the decimal number for which you want to calculate the entropy (e.g. if you need to calculate the entropy of 0.33, take the entry for 0.3 in the table which is 0.9). It is sufficient to make approximate calculations rounded to one digit behind the decimal.

Training examples

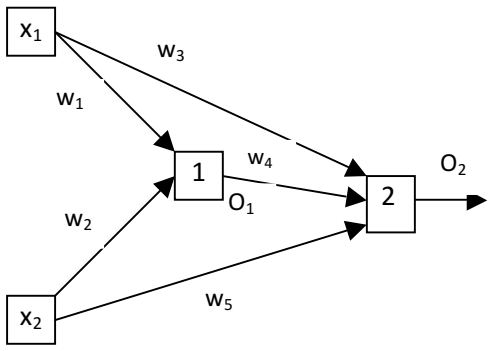
No.	x_1	x_2	$c(x_1,x_2)$
D_1	0	0	+
D_2	0	0	+
D_3	0	0	+
D_4	0	0	+
D_5	0	1	-
D_6	0	1	-
D_7	0	1	-
D_8	1	0	-
D_9	1	1	+
D_{10}	1	1	+

p	entropy(p)
0.0	0.0
0.1	0.5
0.2	0.7
0.3	0.9
0.4	1.0
0.5	1.0
0.6	1.0
0.7	0.9
0.8	0.7
0.9	0.5
1.0	0.0

2. Artificial Neural Network

Consider a neural network shown in the following figure. The outputs from the two units are given by

$$O_1 = sigmoid(w_1 x_1 + w_2 x_2 + c_1)$$
$$O_2 = sigmoid(w_3 x_1 + w_4 O_1 + w_5 x_2 + c_2)$$



Now you are given a training example (x_{10}, x_{20}, t_0). Suppose that, under this training example, the outputs of the two units are O_{10} and O_{20} respectively, and the current value for weight w_4 is w_{40}. The question is how to update the weights and thresholds for the neuron units in light of this training example. Assume the learning rate is γ, please write out the formulas to calculate Δw_i and Δc_i in terms of the incremental BP algorithm.

3. Genetic algorithms

a) Suppose a population has six individuals whose fitness values are illustrated in the table as follows

Individual	Fitness
1	6
2	4
3	5
4	10
5	12
6	13

What are the probabilities of selection for these individuals?

b) Next consider the issue of individual selection based on these selection probabilities. For this purpose a uniform random number from [0, 1] is created. Suppose this created number is equal to 0.27, which individual in the population should be selected according to the roulette wheel scheme and why?

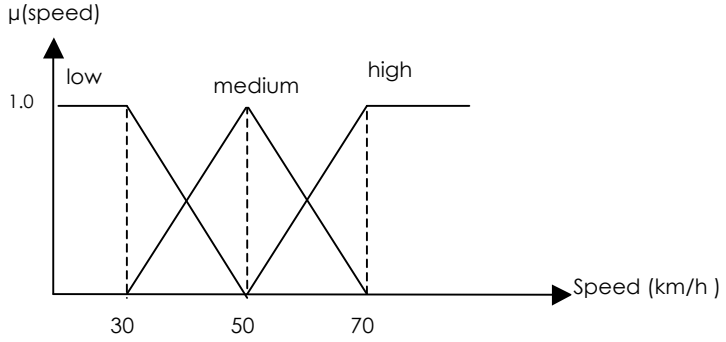
c) Why is it required to have mutation in genetic algorithms?

4. Fuzzy Rule-Based Reasoning

Consider three fuzzy rules to decide on the proper gears during driving a car in city traffic

R1: If speed = *low* Then gear two
R2: If speed= *medium* Then gear three
R3: If speed =*high* Then gear four

The membership functions for the linguistic terms *low*, *medium*, and *high* are defined as shown in the figure. Suppose the current speed is 54 km/hour.

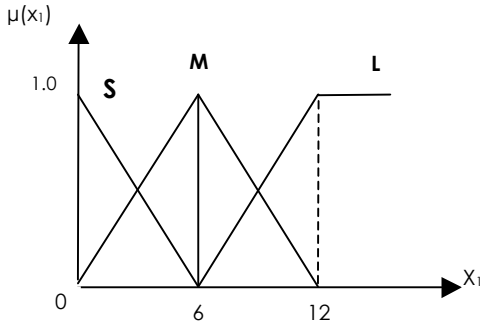


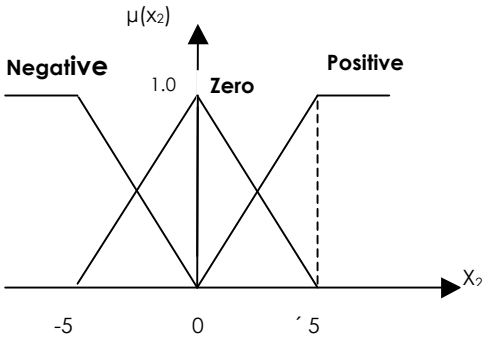
a) What are the firing strengths of these fuzzy rules under the current speed?
b) What are the output fuzzy sets suggested by the fuzzy rules in the current situation?
c) What are the overall output fuzzy set according to the whole fuzzy rule set?
d) What is your final decision and why?

e) How does a fuzzy rule distinguish from a crisp rule? How can you change the above fuzzy rules into crisp rules?

5. Fuzzy systems learning

Suppose a fuzzy classification system with two inputs x_1 and x_2. The fuzzy subsets *S, M, and L* correspond to input x_1, and fuzzy subsets *Negative*, *Zero*, and *Positive* correspond to input x_2. The fuzzy set membership functions of the inputs are depicted in the figures below.





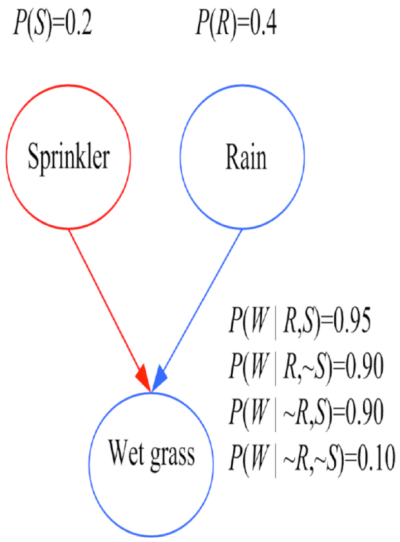
Now suppose there are four training examples as follows:

x_1	x_2	Class
2	-1	A
10	4	B
6	1	C
2	-2	B

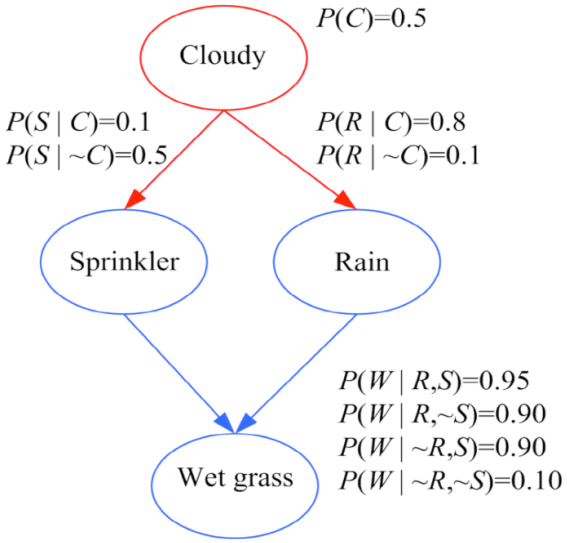
Please generate a fuzzy rule set from the above training examples using the Wang-Mendel algorithm.

6. Bayesian network and probabilistic reasoning

(1) Consider a Bayesian network with 3 nodes as follows, what is the probability for Rain if you see wet grass?

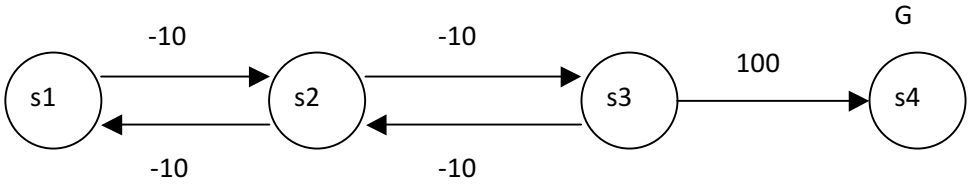


(2) Consider a Bayesian network with 4 nodes below, how do you calculate the probability of cloudy weather if grass is not wet?



7. Reinforcement Learning in Deterministic Environments

Consider the deterministic environment of four states s1, …., s4 as shown in the figure. The state s4 is an absorbing terminal state. In state s1 there is only one way of move till right state s2, while for states s2 and s3 the agent can choose between two alternative actions (right or left move). The agent obtains a reward of +100 when entering into the terminal state s4 and a penalty of -10 for all other moves. Let the discount factor be 0.8.



a) Suppose a policy π of actions as follows:

$\pi(S1) = \rightarrow$, $\pi(S2) = \leftarrow$, $\pi(S3) = \rightarrow$,

What are the values of the states under this policy? For states S1 and S2, you only need to write formulas for calculation rather than exact results.

b) What are the optimal values of the states s1, s2 and s3? How do you get the optimal actions at states from these optimal state values?

c) Suppose that the previous estimates for Q* values are zero for all state-action pairs, how can you update some estimates of the Q* values using the recorded sequence s1 → s2 → s3 → s4?

8. Reinforcement Learning in Stochastic Environments

Write out the Q-learning rule in a stochastic environment. What is its difference with the Q-learning rule for a deterministic environment? Please also justify the Q-learning rule you have presented.