Written Exam in Learning Systems (CDT407)

Date	2012-01-09
Time	8:10 – 13:30
Allowed material	None
Examiner	Ning Xiong (Phone 151716 / 0768035832)
Grades	0-19 U (did not pass) 20-27 3 (pass) 28-32 4 (very good) 33-40 5 (excellent)

Good Luck!!!

1. Decision Tree Learning (3+3 p)

Construct a decision tree for the set of training instances in the left table with attributes x_1, x_2 and classes A, B.
a) Which attribute should be used at the root node and why?
b) Show how the examples D_1, …., D_6 are sorted down the tree and how they are classified.

Use the right table to look up the entropy for a subset of training instances. Take the entry that is closest to the decimal number for which you want to calculate the entropy (e.g. if you need to calculate the entropy of 0.33, take the entry for 0.3 in the table which is 0.9). It is sufficient to make approximate calculations rounded to one digit behind the decimal.

Training examples

No.	a_1	a_2	$c(a_1,a_2)$
D_1	F	F	B
D_2	F	T	A
D_3	T	T	B
D_4	F	T	A
D_5	T	T	B
D_6	T	F	B

p	entropy(p)
0.0	0.0
0.1	0.5
0.2	0.7
0.3	0.9
0.4	1.0
0.5	1.0
0.6	1.0
0.7	0.9
0.8	0.7
0.9	0.5
1.0	0.0

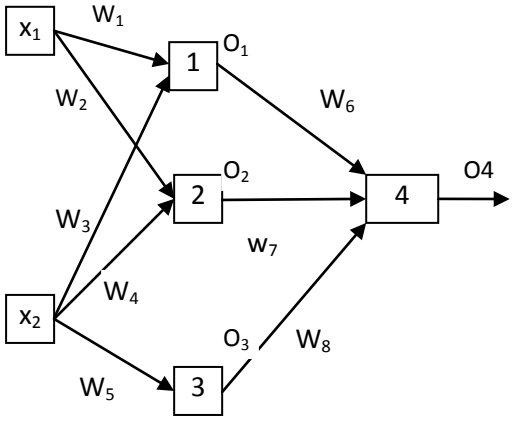
2. Artificial Neural Network (2+4p)

a) Consider a perceptron whose current weights are w_0=–2, w_1=1.0, w_2=2.0, where w_0 is a basic coefficient appearing in the linear combination of inputs and w_1 and w_2 are the connection weights for inputs x_1 and x_2 respectively. Suppose a training example (x_1=0.8,

x_2=0.4, t=1.0), how can you revise the values of w_0, w_1 and w_2 in terms of this training example? Please use the learning rate as 0.02.

b) Consider a neural network shown in the following figure. The outputs from the four units are given by

$$O_1 = sigmoid(w_1 x_1 + w_3 x_2 + c_1)$$
$$O_2 = sigmoid(w_2 x_1 + w_4 x_2 + c_2)$$
$$O_3 = sigmoid(w_5 x_2 + c_3)$$
$$O_4 = sigmoid(w_6 O_1 + w_7 O_2 + w_8 O_3 + c_4)$$



Now you are given a training example (x_{10}, x_{20}, t_0). Suppose that, under this training example, the outputs of the four units are O_{10}, O_{20}, O_{30} and O_{40} respectively, and the current values for weights of W_6, W_7 and W_8 are W_{60}, W_{70} and W_{80} respectively. Assume the learning rate is defined as γ. Your task is to update the weights and thresholds for the neuron units in light of this training example. Please write out the formulas to calculate ΔW_i and Δc_i in terms of the incremental BP algorithm.

3. Genetic algorithms (2+2+2+2p)

a) Suppose a population has six individuals whose fitness values are illustrated in the table as follows

Individual	Fitness
1	3
2	6
3	12
4	7
5	10
6	12

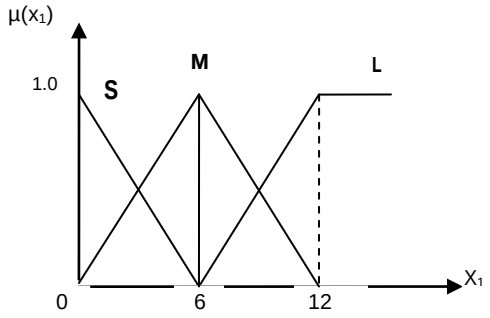
What are the probabilities of selection for these individuals?

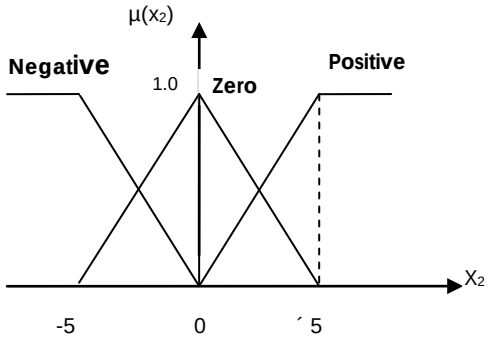
b) Now your task is to select individuals based on these selection probabilities. For this purpose a uniform random number from [0, 1] is created. Suppose this created number is

equal to 0.43, which individual in the population should be selected according to the roulette wheel scheme and why?
c) What is the role of mutation in genetic algorithms?
d) Now consider mutation on real numbers. It is done in terms of a normal density function. The cumulative probability values derived from this normal density function is given in the following table. Suppose a uniform random number is generated as 0.1128, how should you change the original real number (you need to indicate whether to increase or decrease the original real number and how much)?

y	P(x<=y)
……..	……..
-1.29	0.0985
-1.28	0.1003
-1.27	0.1020
-1.26	0.1038
-1.25	0.1056
-1.24	0.1075
-1.23	0.1093
-1.22	01112
-1.21	0.1131
-1.20	0.1151
……	……

4. Fuzzy systems and learning (3+2+1+2p)
Suppose a fuzzy classification system with two inputs x_1 and x_2. The fuzzy subsets *S, M, and L* correspond to input x_1, and fuzzy subsets *Negative*, *Zero*, and *Positive* correspond to input x_2. The fuzzy set membership functions of the inputs are depicted in the figures below.



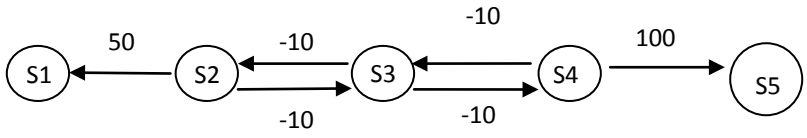


Now suppose there are four training examples as follows:

x_1	x_2	Class
2	-1	A
10	4	B
6	1	C
2	-2	B

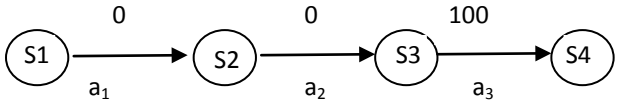
a) Please generate a fuzzy rule set from the above training examples using the Wang-Mendel algorithm.
b) What are firing strengths of the generated fuzzy rules given the input situation x_1=5, x_2=2?
c) What are the output fuzzy sets suggested by the generated fuzzy rules in the current situation with x_1=5, x_2=2?
d) How can you make final decision in terms of these fuzzy rules in the current situation with x_1=5, x_2=2?

5. Reinforcement Learning with model (4+2p)
Consider the environment with five states S1, …, S5 and its model is shown in the figure below. The states S1 and S5 are absorbing terminal states. In each non-terminal state, the agent can choose between two alternative actions →, ← that bring the agent to the neighboring state as indicated in the figure. The deterministic reward function is shown by the numbers beside the arrows corresponding to actions. The discounting factor is 0.8.



a) Suppose an initial policy as given by: π(S2)= ←, π(S3)= ←, π(S4)= →,
Please do policy iteration to yield an optimal policy of actions in this environment.
b) What are the optimal values of the states in this environment?

6. Reinforcement Learning without model (3+3p)
Given a sequence of the agent's interaction with the environment as shown in the figure as follows, where rewards are indicated above the arrows and actions are indicated below the arrows.



Let Q be the estimate of the best action values Q*. Assume the current Q estimates are given as follows:

$Q(S1,a_1)=7.0$, $Q(S1, a)=0$ for all other actions different from a_1
$Q(S2,a_2)=5.0$, $Q(S2, a)=0$ for all other actions different from a_2
$Q(S3,a_3)=3.0$, $Q(S3, a)=0$ for all other actions different from a_3
$Q(S4, a)=0$ for any action *a*

So far the numbers of visits for the state-actions pairs $(S1,a_1)$, $(S2,a_2)$, and $(S3,a_3)$ are N_1, N_2, and N_3 respectively. The discounting factor is 0.8.

a) How can you update the Q estimates using the Q-learning rule if the environment is deterministic?
b) How can you update the Q estimates using the Q-learning rule if the environment is stochastic?