

## Exam in Learning Systems (CDT407)

Date	2013-01-15
Time	14:10 – 18:30
Allowed material	None
Examiner	Ning Xiong (Phone 151716)
Grades	0-19 U (did not pass) 20-26 3 (pass) 27-33 4 (very good) 34-40 5 (excellent)

**Good Luck!!!**

### 1. Decision Tree Learning (3+3p)

Construct a decision tree for the set of training instances in the left table with attributes  $a_1$ ,  $a_2$  and classes  $+$ ,  $-$ .

- Which attribute should be used at the root node and why?
- Show how the examples  $D_1, \dots, D_6$  are sorted down the tree and how they are classified.

Use the right table to look up the entropy for a subset of training instances. Take the entry that is closest to the decimal number for which you want to calculate the entropy (e.g. if you need to calculate the entropy of 0.33, take the entry for 0.3 in the table which is 0.9). It is sufficient to make approximate calculations rounded to one digit behind the decimal.

### Training examples

No.	$a_1$	$a_2$	$c(a_1, a_2)$
$D_1$	F	F	+
$D_2$	F	T	-
$D_3$	F	T	-
$D_4$	T	T	+
$D_5$	T	T	+
$D_6$	T	F	-

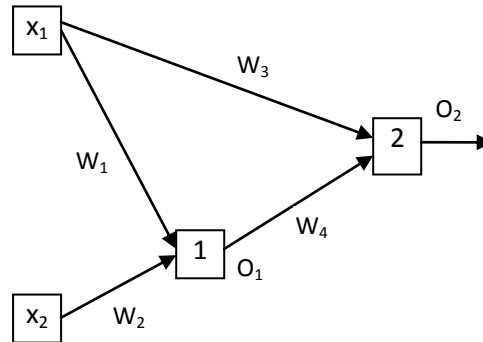
p	entropy(p)
0.0	0.0
0.1	0.5
0.2	0.7
0.3	0.9
0.4	1.0
0.5	1.0
0.6	1.0
0.7	0.9
0.8	0.7
0.9	0.5
1.0	0.0

### 2. Artificial Neural Network (2+4p)

- Write a perceptron to represent logical OR function.

(Hint: draw the data points to see how to separate them)

b) Consider a neural network as follows



The outputs from the two units are given by

$$O_1 = \text{sigmoid}(w_1x_1 + w_2x_2 + c_1)$$

$$O_2 = \text{sigmoid}(w_3x_1 + w_4O_1 + c_2)$$

Now you are given a training example  $(x_{10}, x_{20}, t_0)$ . Suppose that, under this training example, the outputs of the two units are  $O_{10}$  and  $O_{20}$  respectively, and the current values for weights of  $W_3$  and  $W_4$  are  $w_{30}$  and  $w_{40}$  respectively. The question is how to update the weights and thresholds for the neuron units in light of this training example? Assume the learning rate is  $\gamma$ , please write out the formulas to calculate  $\Delta w_{ij}$  and  $\Delta c_i$  in terms of the incremental BP algorithm.

### 3. Genetic algorithms (3+2+2p)

a) Suppose a population has six individuals whose fitness values are illustrated in the table as follows

Individual	Fitness
1	12
2	5
3	8
4	9
5	3
6	13

What are the probabilities of selection for these individuals?

- b) Consider individual selection based on these selection probabilities and using the roulette wheel scheme. For this purpose a uniform random number from  $[0, 1]$  is created to decide which individual to select. Now suppose that individual 4 has been selected, please judge the interval in which the generated uniform random number is located.
- c) Next consider mutation on real numbers for real-valued strings. It is done in terms of a normal density function. The cumulative probability values derived from this normal density function is given in the following table. Suppose a uniform random number is generated as 0.0316, how should you change the original real number (you need to indicate whether to increase or decrease the original real number and how much)?

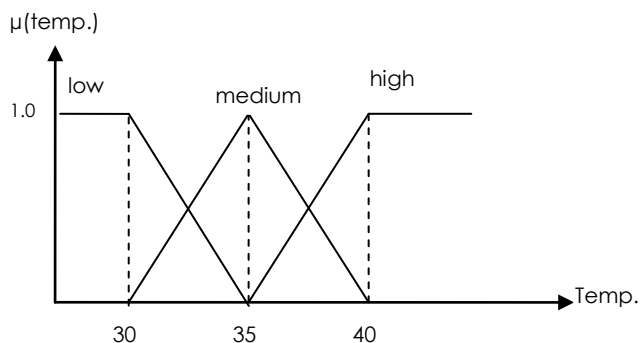
y	P(x≤y)
.....	.....
-1.89	0.0294
-1.88	0.0301
-1.87	0.0307
-1.86	0.0314
-1.85	0.0322
-1.84	0.0329
-1.83	0.0336
-1.82	0.0344
-1.81	0.0351
-1.80	0.0359

#### 4. Fuzzy rule-based reasoning (2+2+1+1p)

Consider three fuzzy rules to decide on stress levels of a subject in terms of his/her finger temperature

- R1: If finger temperature = *low* Then relaxed  
R2: If finger temperature = *medium* Then normal  
R3: If finger temperature = *high* Then stressed

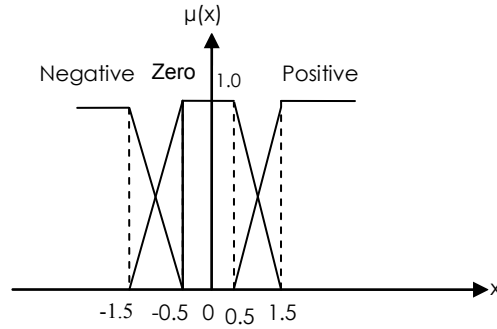
The membership functions for the linguistic terms *low*, *medium*, and *high* are defined as shown in the figure. Suppose the finger temperature of the subject is measured as 36 degree.



- What are the firing strengths of these fuzzy rules under this measured temperature?
- What are the output fuzzy sets suggested by the fuzzy rules in the current situation?
- What are the overall output fuzzy set according to the whole fuzzy rule set?
- What is your final decision and why?

#### 5. Fuzzy systems learning (3p)

Assume a simple fuzzy system with input  $x$  and output  $y$ . The membership functions of fuzzy sets for the input  $x$  are described in the figure below. The output  $y$  is a discrete value from  $\{A, B, C\}$ .



**Fig.:** Fuzzy sets membership functions for input  $x$

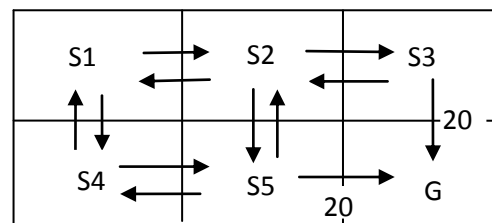
Given the four training examples as follows:

$x_1$	Class
2	A
-0.8	B
0.7	B
-1.8	C
1.3	B

Please generate a fuzzy rule set from the above training examples using the Wang-Mendel algorithm.

#### 6. Reinforcement Learning – Basic Concepts (3+2+1p)

Consider the deterministic grid world as shown in the figure with the absorbing goal state  $G$ . In each non-terminal state, the agent can choose between actions  $\downarrow$ ,  $\uparrow$ ,  $\leftarrow$ ,  $\rightarrow$  that brings the agent to the neighboring state. The intermediate rewards are 20 for the transitions entering the goal state  $G$  and 0 for all other transitions. Let the discounting factor be 0.9.



a) Suppose a policy  $\pi$  of actions as follows:

$$\pi(S1)=\downarrow, \pi(S2)=\rightarrow, \pi(S3)=\downarrow, \pi(S4)=\uparrow, \pi(S5)=\uparrow.$$

What are the values of the states under this policy?

b) What are the optimal values of states,  $V^*(S)$ , under this environment?

c) How do you make optimal action at state  $S2$  based on optimal values of states?

#### 7. Reinforcement Learning – Q learning (3+3p)

a) Given an episode of state-action pairs as follows, in which the immediate reward 10 was obtained when state  $s_2$  was driven to state  $s_3$  by action  $a_2$ . At all other transitions the immediate rewards were zero. The state  $s_3$  is the absorbing state. The discounting factor is 0.9.

$$(s_0, a_0) \xrightarrow{0} (s_1, a_1) \xrightarrow{0} (s_2, a_2) \xrightarrow{10} s_3$$

Assume all the state-action pairs along the sequence had been visited once before the episode. The current estimates of  $Q^*$  values for legal actions at states  $s_0$ ,  $s_1$ , and  $s_2$  are all equal to 2.0. How can you update the estimates of  $Q^*$  values for these state-action pairs using this episode information? Please present your results for deterministic and stochastic environments respectively.

8. Extra question: Have you completed one more lab than what is compulsory?  
(If your answer is yes, you will get two bonus points.)