第二次作业

刬线法

1: Out

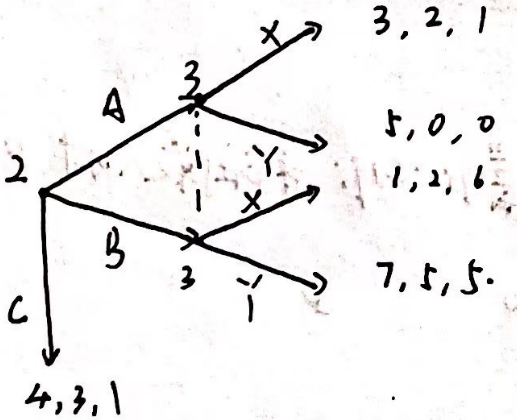
2 \ 3	X	Y
A	2, 2, 2	2, 2, 2
B	2, 2, 2	2, 2, 2
C	2, 2, 2	2, 2, 2

1: In

2 \ 3	X	Y
A	3, 2, 1	5, 0, 0
B	1, 2, 6	7, 5, 5
C	4, 3, 1	4, 3, 1

(Out, B, X)　(In, C, X)

(2)



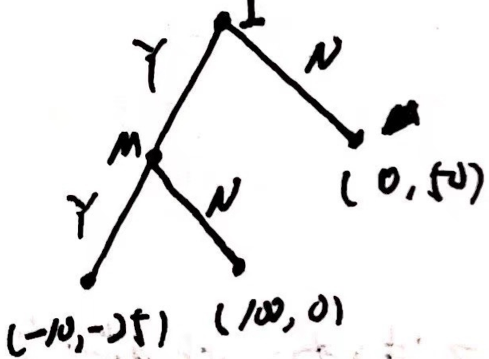
考虑子博弈，无论2选A还是B，C都会选X

故2会选C，此时1会选In

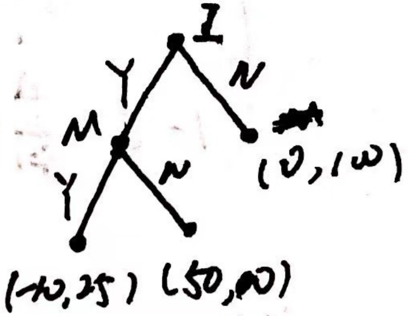
故(In, C, X)

2. (1) I表示伊丽莎白，M代表玛丽

① 孩子是I的



② 孩子是M的



(2)

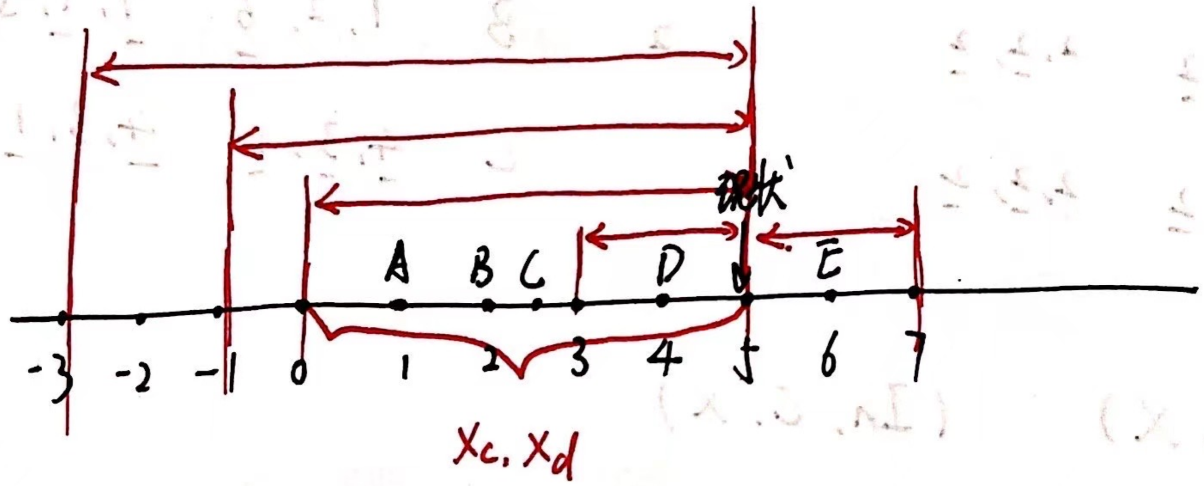
3.

逆向归纳法，若进入(5)，则 $0<X<5$ 会通过；否则会维持原状，故 x_c, x_d 应满足在 $(0,5)$ 区间

在(4)阶段，若要获得更多选票，则应更靠近2.5，由于 x_c 为 x_d 修正，故 x_c 后提出，故 $x_c=2.5$

(2)阶段若 x_d 被通过，则会被改至2.5，故对D、E更远，因此D，E不会通过，故 x_d 一定不会被

通过，故D提不提都一样，因此D不提方案，博弈结束



4.

在第二阶段，设有n个企业进入了市场

对第i个企业，设产量为q_i，则利润$W_i=(1-\sum_{i=1}^{n}q_i-c)q_i$，利润最大时，$\forall i$

$$\frac{\partial W_i}{\partial q_i}=1-c-\sum_{i=1}^{n}q_i-q_i=0$$

即

$$\begin{cases}1-c-\sum_{i=1}^{n}q_i-q_1=0 & ① \\ \vdots & \vdots \\ 1-c-\sum_{i=1}^{n}q_i-q_n=0 & ⓝ\end{cases}$$

联立有 $q_1=\cdots=q_n=\frac{1-c}{n+1}$

① 当$c\geq 1$时，$q_i=0$，故在第一阶段，若进入收益为-0.04，不进入为0，此时无企业进入

② 当$c<1$时 $p=1-nq_i=\frac{1+nc}{n+1}$，总利润$W_i=\frac{(1-c)^2}{(n+1)^2}$ $(\forall i)$ 故进入收益为$\frac{(1-c)^2}{(n+1)^2}-F$，进入为0

此时有$\lfloor 4-5c\rfloor$家企业进入，产量为$\frac{1-c}{\lfloor 5-5c\rfloor}$，价格为$\frac{1+\lfloor 4-5c\rfloor c}{\lfloor 5-5c\rfloor}$

5.

(1) ① 考虑甲的混合策略 $\frac{3}{4}A+\frac{1}{4}B$，设乙的策略为 $p_1A+p_2B+(1-p_1-p_2)C$

则甲的收益为 $\frac{3}{4}\times3p_1+\frac{3}{4}\times0p_2+\frac{3}{4}\times5(1-p_1-p_2)+\frac{1}{4}\times2p_1+\frac{1}{4}\times1p_2+\frac{1}{4}\times3(1-p_1-p_2)$

$=\frac{9}{2}-\frac{7}{4}p_1-\frac{17}{4}p_2$

甲选C的收益为 $p_1+4(1-p_1-p_2)=4-3p_1-4p_2$

故C为甲的永非最优反应 （作差为 $\frac{1}{2}+\frac{5}{4}p_1-\frac{1}{4}p_2\geq\frac{1}{2}-\frac{1}{4}=\frac{1}{4}\geq0$）

② 考虑~~设~~乙的混合策略 $\frac{1}{2}A+\frac{1}{2}B$，设甲的策略为 $qA+(1-q)B$

则乙的收益为 $\frac{1}{2}q+\frac{1}{2}(1-q)+\frac{1}{2}\times2(1-q)=\frac{3}{2}-q$.

乙选C的收益为 $\frac{1}{2}-\frac{1}{2}q$.

故C是乙的永非最优反应

因此甲的A、B，乙的A.B为可理性化的

(5)
(2)

		乙 A	乙 B	乙 C
甲	A	3,1	0,0	5,0
甲	B	2,1	1,2	3,1
甲	C	1,2	0,1	4,4

(A, A) (B, B)

由(1)得，甲的混合策略中不含C，乙的混合策略中不含C

设甲 $qA+(1-q)B$，乙 $pA+(1-p)B$

~~甲的收益为 $(2p-1)$~~ ~~$(2q-1)$~~

甲的收益为 $(2p-1)q+1-p$，故甲的最优反应为 $\begin{cases} q=0 & p<\frac{1}{2} \\ q\in[0,1] & p=\frac{1}{2} \\ q=1 & p>\frac{1}{2} \end{cases}$

乙的收益为 $(2q-1)p+2(1-q)$，故乙的最优反应为 $\begin{cases} p=0 & q<\frac{1}{2} \\ p\in[0,1] & q=\frac{1}{2} \\ p=1 & q>\frac{1}{2} \end{cases}$

故混合策略均衡为 $(\frac{1}{2}A+\frac{1}{2}B, \frac{1}{2}A+\frac{1}{2}B)$

5. （3）

- Play (B, R) in the first stage.
- If the first-stage outcome is (B, R), then play (T, L) in the second stage.
- If the first-stage outcome is not (B, R), then play the MSNE r^* in the second stage.

Proof Let's analyze if the former punishment scheme played by every agent induces both players to not deviate from outcome (B,R), with associated payoff (4, 4), in the first stage of the game. To clarify our discussion, we separately examine the incentives to deviate by players 1 and 2,

Player 1. Let's take as given that player 2 adopts the previous punishment scheme:

- If player 2 sticks to (B, R), then player 1 obtains $u_1 = 4 + 3 = 7$, where 4 reflects his payoffs in the first stage of the game, when both cooperate in (B, R); and 3 represents his payoffs in the second stage, where outcome (T, L) arises according to the above punishment scheme.
- If, instead, player 1 deviates from (B, R), then his optimal deviation is to play (T, R) which yields a utility level of 5 in the first period, but a payoff of $\frac{3}{2}$ in the second period (the punishment implies the play of the MSNE in the second period). Therefore, player 1 does not have incentives to deviate since his payoff from selecting the cooperative outcome (B, R), 4 + 3 = 7, exceed his payoff from deviating $5 + \frac{3}{2} = \frac{13}{2}$.

5. （3）

<u>Player 2.</u> Taking as given that player 1 sticks to the punishment scheme:

- If player 2 plays (B, R) in the first stage, then she obtains an overall payoff of $u_2(B,R) = 4+1 = 5$. As we can see, player 2 doesn't have incentives to deviate, because his best response function is in fact $BR_2(B) = R$ when we take as given that player 1 is playing B.

Therefore, no player has incentives to deviate from (B, R) in the first stage of the game. As a consequence, the efficient payoff (4, 4) can be sustained in the first stage of the game in a pure strategy SPNE strategy profile.

6.

(1)

垄断价格要求最大化 $(a-p)(p-c)$，即垄断价格为 $p=\frac{a+c}{2}$，垄断利润为 $\frac{(a-c)^2}{4}$

每家企业利润为 $\frac{(a-c)^2}{4n}$

此后，若一直维持垄断价格，则收益为 $\frac{(a-c)^2}{4n}(1+\delta+\cdots)=\frac{(a-c)^2}{4n(1-\delta)}$；若偏离，则该期可获得全部利益，即 $\frac{(a-c)^2}{4}$，要自发维持，应满足

$$\frac{(a-c)^2}{4n(1-\delta)} \geq \frac{(a-c)^2}{4}$$

即 $\delta \geq 1-\frac{1}{n}$

(2)

若滞后一期，则偏离与偏离后一期可拿到全部利润，故偏离收益为 $\frac{(a-c)^2}{4}+\frac{(a-c)^2}{4}\delta$

若自发维持，应满足

$$\frac{(a-c)^2}{4n(1-\delta)} \geq \frac{(a-c)^2}{4}+\frac{(a-c)^2}{4}\delta$$

即 $\delta \geq \sqrt{1-\frac{1}{n}}$