1. (1)

	乙 C	乙 D
甲 $A^{15}A^{4}$	9.5, 18	6, 7
甲 $A^{15}B^{4}$	11.5, 9	7, 9
甲 $B^{15}A^{4}$	6, 9	7, 9
甲 $B^{15}B^{4}$	8, 0	8, 11

$\frac{15+4}{2}=9.5$

$\frac{15+8}{2}=11.5$ $\frac{18+0}{2}=9$ $\frac{6+8}{2}=7$ $\frac{7+11}{2}=9$

$\frac{8+4}{2}=6$

(2) 画 BR

BNE: $(A^{15}B^{4}, C)$ 和 $(B^{15}B^{4}, D)$

2.

对 firm2 在 $c_2=6$ 时

$\pi_2=(p_2-6)(22-2p_2+p_1)$

$\frac{\partial \pi_2}{\partial p_2}=0 \Rightarrow p_2^{(6)}=\frac{34+p_1}{4}$

$c_2=14$ 时

$\pi_2=(p_2-14)(22-2p_2+p_1)$

$\frac{\partial \pi_2}{\partial p_2}=0 \Rightarrow p_2^{(14)}=\frac{50+p_1}{4}$

firm 1

对 firm 1

$\pi_1=\frac{1}{2}(p_1-10)(22-2p_1+p_2^{L})+\frac{1}{2}(p_1-10)(22-2p_1+p_2^{H})$

$\frac{\partial \pi_1}{\partial p_1}=0=22-2p_1+\frac{p_2^{L}+p_2^{H}}{2}-2(p_1-10)$

$p_1=\frac{42+\frac{p_2^{L}+p_2^{H}}{2}}{4}$

$=\frac{42+\frac{42+p_1}{4}}{4}$ $\frac{42\times 5}{15}$

得 $p_1^*=14$

$p_2^{(6)}=12$ $p_2^{(14)}=16$

3.

Part (a). First, note that the probability of winning the object is unaffected, since, for a symmetric bidding function $b_i(v_i) = a \cdot v_i$ for bidder i, where $a \in (0, 1)$, the probability that bidder i wins the auction against another bidder j is

$$prob\big(b_i > b_j\big) = prob\big(b_i > a \cdot v_j\big) = prob\left(\frac{b_i}{a} > v_j\right) = \frac{b_i}{a}$$

where the first equality is due to the fact that $b_j = a \cdot v_j$, the second equality rearranges the terms in the parenthesis, and the last equality makes use of the uniform distribution of bidders' valuations. Therefore, bidder i's expected utility from participating in this auction by submitting a bid b_i when his valuation is v_i is given by

$$EU_i(b_i|v_i) = \frac{b_i}{a} \times (v_i - b_i)^{\alpha}$$

where, relative to the case of risk-neutral bidders analyzed in Exercise 1, the only difference arises in the evaluation of the net payoff from winning, $v_i - b_i$, which it is now evaluated with the concave function $(v_i - b_i)^{\alpha}$. Taking first-order conditions with respect to his bid, b_i, yields

$$\frac{1}{a}(v_i - b_i)^{\alpha} - \frac{b_i}{a}\alpha(v_i - b_i)^{\alpha-1} = 0,$$

and solving for b_i, we find the optimal bidding function,

$$b_i(v_i) = \frac{v_i}{1+\alpha}.$$

T4. $b_i = a v_i$, b, v 均为均匀分布

在 i 出价 b_i 中标条件下，第3高价格概率分布

$$F(x) = (n-1)\left(1-\frac{x}{b_i}\right)\left(\frac{x}{b_i}\right)^{n-2} + \left(\frac{x}{b_i}\right)^{n-1}$$

第3高价格条件期望

$$E[b_{3高} \mid b_i最大] = \int_0^{b_i} x\,dF(x) = xF(x)\Big|_0^{b_i} - \int_0^{b_i} F(x)\,dx$$

$$= b_i - \int_0^{b_i} (n-1)\left(\frac{x}{b_i}\right)^{n-2} dx + \int_0^{b_i} (n-2)\left(\frac{x}{b_i}\right)^{n-1} dx$$

$$= \frac{n-2}{n} b_i$$

由收益等价定理，最高价格拍卖中 $b = \frac{n-1}{n} v_i$

则 $\frac{n-2}{n} b_i = \frac{n-1}{n} v_i$

$$b_i = \frac{n-1}{n-2} v_i$$