Intermediate Microeconomics (Fall 2023)
Homework 2 Answer Key

1 − 5 CBBBE

6.

1) $\max_{X} 4\log(1+X) + 10 - 2X$

such that 0 ≤ X ≤ 5

FOC:

$\frac{4}{1+X} - 2 = 0$

⇒ $X = 1$

⇒ Buy 1 unit

⇒ $u_{max} = 4\log 2 + 8 \approx 10.77$

2) $\max_{X} 4\log(1+X) + 1 - 2X$

such that 0 ≤ X ≤ 0.5

FOC:

$\frac{4}{1+X} - 2 = 0$

⇒ $X = 1$

⇒ Buy 0.5 units

⇒ $u_{max} = 4\log 1.5 \approx 1.62$

3) Suppose the price of candy is P

$$\max_{X} 4\log(1+X) + 3 - PX$$

such that $\begin{cases} PX \le 3 \\ X \ge 0 \end{cases}$

FOC:

$$\frac{4}{1+X} - P = 0$$

$$\Rightarrow X = \frac{4}{P} - 1$$

$\because X \ge 0 \Rightarrow 0 \le P \le 4$

$PX \le 3 \Rightarrow P \ge 1$

$\therefore 1 \le P \le 4$

If P > 4,

then buy 0 unit of candy.

If 0 < P < 1,

then buy $\frac{3}{P}$ units of candy.

Therefore,

$$\text{Demand for candy} = \begin{cases} 0 & \text{when } P > 4 \\ \frac{4}{P} - 1 & \text{when } 1 \le P \le 4 \\ \frac{3}{P} & \text{when } 0 < P < 1 \end{cases}$$

7.

1) $\frac{\partial u}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}}$ and $\frac{\partial u}{\partial x_2} = x_2^{-\frac{1}{2}}$

$$\Rightarrow \begin{cases} \dfrac{\frac{1}{2} x_1^{-\frac{1}{2}}}{x_2^{-\frac{1}{2}}} = \dfrac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = \dfrac{m}{p_1 + \frac{4\,p_1^2}{p_2}} \\ x_2 = \left(\dfrac{1}{p_2} - \dfrac{1}{p_2 + 4\,p_1}\right) m \end{cases}$$

2) f (x) is positive

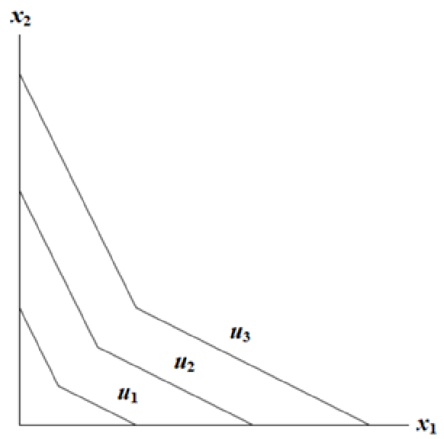
$\Rightarrow$ u (x_1, x_2) is a monotonic transformation of v (x_1, x_2) = $x_1^a x_2^b$

$$\Rightarrow \begin{cases} x_1 = \dfrac{a}{a+b} \dfrac{m}{p_1} \\ x_2 = \dfrac{b}{a+b} \dfrac{m}{p_2} \end{cases}$$

3) Draw the indifference curves and figure out the demand function.

$$(x_1, x_2) = \begin{cases} (0, \frac{m}{p_2}) & \text{when } \frac{p_1}{p_2} > 2 \\ (t, \frac{m - tp_1}{p_2}) & \text{when } \frac{p_1}{p_2} = 2 \\ (\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}) & \text{when } \frac{1}{2} < \frac{p_1}{p_2} < 2 \\ (\frac{m - tp_2}{p_1}, t) & \text{when } \frac{p_1}{p_2} = \frac{1}{2} \\ (\frac{m}{p_1}, 0) & \text{when } \frac{p_1}{p_2} < \frac{1}{2} \end{cases}$$

where $t \in [0, \frac{m}{p_1 + p_2}]$



8.

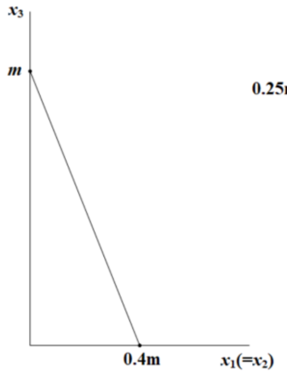
1) If $m \le 4$

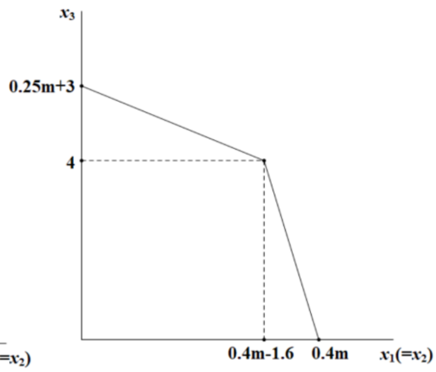
$$0.5x_1 + 2x_2 + x_3 = m$$

$$\Rightarrow 2.5x_1 + x_3 = m$$

If $m > 4$

$$\begin{cases} 2.5x_1 + x_3 = m & \text{when } x_3 \le 4 \\ 2.5x_1 + 4x_3 - 12 = m & \text{when } x_3 > 4 \end{cases}$$





2) $\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$

$\Rightarrow x_1 = 4x_2$

⇒ Consume 4 units of food and 1 unit of clothing as a bundle

Define $x_4 = \frac{x_1}{4} = x_2$

Then,

The price of x_4**:** $p = 4$

$u(x_3, x_4) = 2x_3x_4$

The budget line is given by $\begin{cases} 4x_4 + x_3 = m & \text{when } x_3 \le 4 \\ 4x_4 + 4x_3 - 12 = m & \text{when } x_3 > 4 \end{cases}$

① **If the budget line** $4x_4 + x_3 = m$ **is tangent to the indifference curve,**

then $x_3 = 4x_4 = \frac{m}{2} \le 4$

$\Rightarrow m \le 8$

② **If the budget line** $4x_4 + 4x_3 - 12 = m$ **is tangent to the indifference curve,**

then $x_3 = x_4 = \frac{m + 12}{8} > 4$

$\Rightarrow m > 20$

③ **If** $8 < m < 20$**,**

then $x_3 = 4$ **and** $x_4 = \frac{m}{4} - 1$

Therefore,

$$(x_1, x_2, x_3) = \begin{cases} (\frac{m}{2}, \frac{m}{8}, \frac{m}{2}) & \text{when } m < 8 \\ (m - 4, \frac{m}{4} - 1, 4) & \text{when } 8 \le m \le 20 \\ (\frac{m+12}{2}, \frac{m+12}{8}, \frac{m+12}{8}) & \text{when } m > 20 \end{cases}$$