NAME ______________________________

Peking University
Intermediate Microeconomics
Fall 2023
Dr. Jin Qin

Homework 2
Due: Friday, November 24

Instructions:

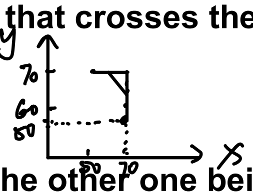
1. Print your name on the answer sheet.
2. This homework assignment consists of 5 multiple-choice questions with each one worth 4 points and 3 short-answer questions for 80 points, 100 points total. Make sure you have a complete question set.
3. Please write down all your answers on the answer sheet. Answers written on the question sheet will NOT be graded.
4. The space provided on the answer sheet should be sufficient for your answer. If you need additional space, attach a blank paper.
5. Please write neatly. If I cannot read an answer, you will receive no credit for it.
6. Show enough of your work so that I can tell how you arrived at the answer. You will receive credit for sound reasoning. Partial credit will be awarded wherever I deem there is sufficient justification.
7. When drawing graphs, make sure to label everything, including the axes. It is not particularly important to draw your graphs with perfect precision.
8. Turn in the answer sheet ONLY.

C **1. Satisfaction from consumption is maximized when**

A. marginal cost equals zero.
B. marginal benefit equals zero.
C. marginal benefit equals marginal cost.
D. marginal benefit is maximum.

J Q.

B **2. Dr. J gives 4 homework assignments for the Intermediate Microeconomics course. Suppose he drops each student's two lower scores and uses the average score of the other two assignments to determine the final homework score. Naughty Brown is taking this class and gets a 60 and a 70 in the first two assignments. Let x be his score on the third assignment and y be his score on the fourth assignment. If we draw his indifference curves for the scores on the third and fourth assignment with x on the horizontal axis and y on the vertical axis, then his indifference curve that crosses the point of (x, y) = (50, 70) is** 70 points



65, 65
= 65 X

A. a line segment between (0, 120) and (120, 0).
B. two line segments with one being vertical and the other one being horizontal.
C. three line segments with one being vertical, one being horizontal, and the other one that links (70, 50) and (50, 70).
D. three line segments with one being vertical, one being horizontal, and the other one that links (70, 60) and (60, 70).

B **3. Pencils sell for 10 cents and pens sell for 50 cents. Suppose Fiori, whose preferences satisfy all the basic assumptions, buys 5 pens and one pencil each semester. With this consumption bundle, $|MRS_{\text{Pencil Pen}}| = 3$. Which of the following is true?**

A. Fiori could optimize his utility by buying more pens and fewer pencils.
B. Fiori could optimize his utility by buying more pencils and fewer pens.
C. Fiori could optimize his utility by buying more pencils and more pens.
D. Fiori could optimize his utility by buying fewer pencils and fewer pens.

B **4. If $P_X = P_Y$, then when the consumer maximizes utility,**

A. X must equal Y.
B. MU_X must equal MU_Y.
C. $MRS_{XY} = 0$.
D. X and Y must be substitutes.

E **5. Fiori derives utility from consuming iced tea and lemonade. For the bundle he currently consumes, the marginal utility he receives from iced tea is 16, and the marginal utility he receives from lemonade is 8. Instead of consuming this bundle, Fiori should**

P_1 and P_2 ?

A. buy more iced tea and less lemonade.
B. buy more lemonade and less iced tea.
C. buy more iced tea and lemonade.
D. buy less iced tea and lemonade.
E. None of the above is necessarily correct.

6. **Fiori is a candy-loving kid. X units of candy provides him a utility of u (X) = 4 log (1 + X) units, while holding $1 gives him 1 unit of utility. X does not have to be an integer.**
   1) **If Fiori has $10 and the unit price of candy is $2, how many units of candy will Fiori purchase to maximize his utility? What is the maximized utility?**
   2) **How about if Fiori has $1 and the unit price of candy is $2?**
   3) **Derive Fiori's demand for candy when he has $3.**

1) Let y be the number of dollar Fiori holding.

$MU_x = \frac{4}{1+x}$, $MU_y = 1$ $(U(Y) = y)$

$$|MRS| = \frac{MU_x}{MU_y} = \frac{4}{1+x}$$

To max utility, $|MRS| = \frac{P_x}{P_y} \Rightarrow \frac{4}{1+x} = \frac{2}{1} \Rightarrow x = 1$

$y = 10 - 2x = 8$

$U_{max} = 4\log 2 + 8$

2) $y = 1 - 2x \geq 0 \Rightarrow x \leq \frac{1}{2} < 1$

$\frac{4}{1+x_{max}} > \frac{P_x}{P_y}$, so Fiori want to have as many candies as possible.

$\therefore x = \frac{1}{2}$, $y = 0$. $U_{max} = 4\log\frac{3}{2}$

3) $P_{candy} \cdot x + y = 3$

$|MRS| = \frac{P_x}{P_y} \Rightarrow \frac{4}{1+x} = \frac{P_{candy}}{1}$

$\Rightarrow x = \frac{4}{P_{candy}} - 1$

$y = 3 - x = 4 - \frac{4}{P_{candy}} \geq 0 \Rightarrow P_{candy} \geq 1$

$$\therefore x = \begin{cases} \frac{4}{P_{candy}} - 1, & \text{for } P_{candy} \geq 1 \\ \frac{3}{P_{candy}}, & \text{otherwise} \end{cases}$$

7. **Fiori is a utility-maximizing consumer. He has an income m > 0 to allocate between two goods (1 and 2). For each good, Fiori faces a constant price of p_1 and p_2. For each of the following utility function, derive Fiori's optimal demand x_1 (p_1, p_2, m) and x_2 (p_1, p_2, m).**

   const

   1) **u (x_1, x_2) = $\sqrt{x_1} + 2\sqrt{x_2}$** $\frac{\partial}{\partial x_1}\left[F(x_1^a x_2^b) - F(0)\right] = f(x_1^a x_2^b)\cdot a x_1^{a-1}$
   2) **u (x_1, x_2) = $\int_0^{x_1^a x_2^b} f(x)dx$, where a, b > 0 and f(x) is positive**
   3) **u (x_1, x_2) = min {$x_1 + 2x_2$, $2x_1 + x_2$}**

1) $L = \sqrt{x_1} + 2\sqrt{x_2} + \lambda(P_1 x_1 + P_2 x_2 - m)$

$$\begin{cases} \frac{\partial L}{\partial x_1} = \frac{1}{2\sqrt{x_1}} + \lambda P_1 = 0 \\ \frac{\partial L}{\partial x_2} = \frac{1}{\sqrt{x_2}} + \lambda P_2 = 0 \\ \frac{\partial L}{\partial \lambda} = P_1 x_1 + P_2 x_2 - m = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{mP_2}{P_1P_2 + 4P_1^2} \\ x_2 = \frac{4mP_1}{P_2^2 + 4P_1P_2} \end{cases}$$

2) because $a, b > 0$ and $f(x)$ is positive

so to max $u(x_1, x_2)$, we just need to max $x_1^a x_2^b$

$L = x_1^a x_2^b + \lambda(P_1 x_1 + P_2 x_2 - m)$

$$\begin{cases} \frac{\partial L}{\partial x_1} = a x_1^{a-1} x_2^b + \lambda P_1 = 0 \\ \frac{\partial L}{\partial x_2} = b x_1^a x_2^{b-1} + \lambda P_2 = 0 \\ \frac{\partial L}{\partial \lambda} = P_1 x_1 + P_2 x_2 - m = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{am}{aP_1 + bP_1} \\ x_2 = \frac{bm}{aP_2 + bP_2} \end{cases}$$

$\frac{a}{P_1} x_1^{a-1} x_2^b = \frac{b}{P_2} x_1^a x_2^{b-1}$ $\quad x_2 = \frac{P_1 b}{P_2 a} x_1$

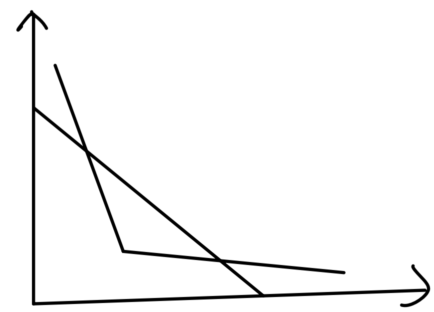
$P_2 a x_2 = P_1 b x_1$

$a(m - P_1 x_1) = P_1 b x_1$

3) ① $x_1 + 2x_2 \le 2x_1 + x_2 \Rightarrow x_1 \ge x_2$, $u = x_1 + 2x_2$

x_1 and x_2 are perfect substitutes

$x_1 P_1 + x_2 P_2 = m$



?

$$\begin{cases} x_1 = \frac{m}{P_1 + P_2} \\ x_2 = \frac{m}{P_1 + P_2} \end{cases}$$

8. Naughty Brown spends his monthly TA income m on food (x_1), clothing (x_2) and KTV (x_3). The price of food is constant at $p_1 = 0.5$, and the price of clothing is constant at $p_2 = 2$, while the KTV currently offers a deal: every month, $p_3 = 1$ per hour for the first four hours and $p_4 = 4$ per hour for the following hours. Naughty Brown's utility function is given by $u(x_1, x_2, x_3) = \sqrt{x_1 x_2} \cdot x_3$.

1) Assume that Naughty Brown buys the same amount of food and clothing this month. Derive his budget constraint and plot it with x_1 (= x_2) on the horizontal axis and x_3 on the vertical axis. Label everything clearly.

2) Derive Naughty Brown's demand function of x_1 (m), x_2 (m) and x_3 (m).

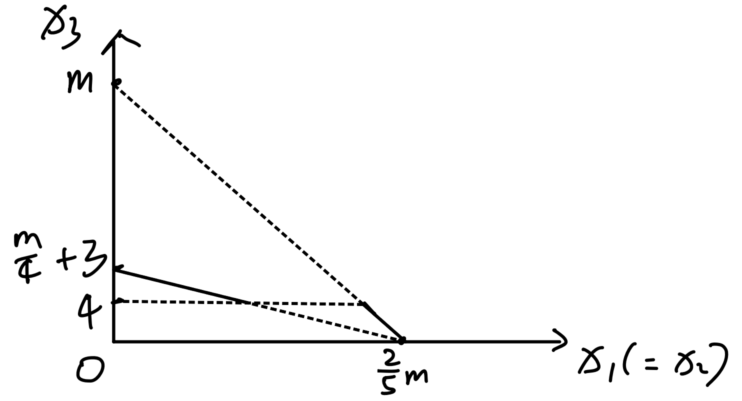
1) ① for $0 \le x_3 \le 4$

$$p_1 x_1 + p_2 x_2 + p_3 x_3 = m$$
$$2.5 x_1 + x_3 = m$$
$$x_1 = \frac{2}{5} m - \frac{2}{5} x_3$$

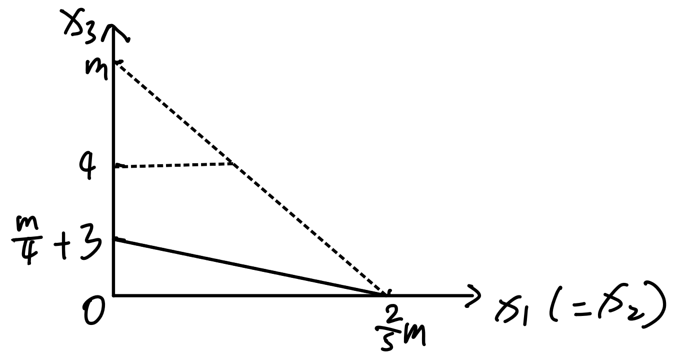
② for $x_3 > 4$

$$p_1 x_1 + p_2 x_2 + p_4 (x_3 - 4) + 4 = m$$
$$2.5 x_1 + 4 x_3 = m + 12$$
$$x_1 = \frac{2}{5} m - \frac{8}{5} x_3 + \frac{24}{5}$$

$\therefore$ budget constrain: $\begin{cases} 2.5 x_1 + x_3 = m, & \text{for } 0 \le x_3 \le 4 \\ 2.5 x_1 + 4 x_3 = m + 12, & \text{for } x_3 > 4 \end{cases}$



for $m > 4$



for $0 \le m \le 4$

2) ① for $0 \le m \le 16$,

$$L = \sqrt{x_1 x_2}\, x_3 - \lambda(0.5x_1 + 2x_2 + x_3)$$

$$\begin{cases} \frac{\partial L}{\partial x_1} = x_3\sqrt{x_2}\cdot\frac{1}{2\sqrt{x_1}} - 0.5\lambda = 0 \\ \frac{\partial L}{\partial x_2} = x_3\sqrt{x_1}\cdot\frac{1}{2\sqrt{x_2}} - 2\lambda = 0 \\ \frac{\partial L}{\partial x_3} = \sqrt{x_1 x_2} - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0.5x_1 + 2x_2 + x_3 = m \end{cases} \Rightarrow \begin{cases} x_1 = \frac{m}{2} \\ x_2 = \frac{m}{8} \\ x_3 = \frac{m}{2} \end{cases}$$

$$4\sqrt{x_2}\cdot\frac{1}{\cancel{2}\sqrt{x_1}} = \sqrt{x_1}\cdot\frac{1}{\cancel{2}\sqrt{x_2}} \Rightarrow 4x_2 = x_1$$

$$\cancel{4}\, x_3 \frac{\sqrt{x_1 x_2}}{\lambda\ 4\lambda} - \cancel{4}\lambda x_1 = 0 \Rightarrow x_3 = x_1$$

$$\frac{1}{2}x_1 + 2\cdot\frac{1}{4}x_1 + x_1 = m$$

② for $m > 16$

i. for $0 \le x_3 \le 4$

$$L = \sqrt{x_1 x_2}\, x_3 - \lambda(0.5x_1 + 2x_2 + x_3 - m)$$

from (1) we have $x_1 = \frac{m}{2}$, $x_2 = \frac{m}{8}$, $x_3 = \frac{m}{2}$

$x_3 = \frac{m}{2} \le 4 \Rightarrow m \le 8$ which is conflict with $m > 16$

so when $x_3 = 4$, we have U_{max}

$$L = \sqrt{x_1 x_2}\cdot 4 - \lambda(0.5x_1 + 2x_2 + 4 - m)$$

$$\begin{cases} \frac{\partial L}{\partial x_1} = 2\sqrt{x_2}\cdot\frac{1}{\sqrt{x_1}} - \frac{1}{2}\lambda = 0 \\ \frac{\partial L}{\partial x_2} = 2\sqrt{x_1}\cdot\frac{1}{\sqrt{x_2}} - 2\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0.5x_1 + 2x_2 + 4 - m = 0 \end{cases} \Rightarrow \begin{cases} x_1 = m - 4 \\ x_2 = \frac{m-4}{4} \\ x_3 = 4 \end{cases}$$

$$\frac{4\,\cancel{8}\sqrt{x_2}}{\sqrt{x_1}} = \frac{\cancel{2}\sqrt{x_1}}{\sqrt{x_2}}$$

$$4x_2 = x_1$$

$$2x_2 + 2x_2 + 4 - m = 0$$

$$U\left(m-4, \frac{m-4}{4}, 4\right) = \frac{1}{2}(m-4)\cdot 4 = 2m - 8$$

ii. for $x_3 > 4$

$$L = \sqrt{x_1 x_2} \cdot x_3 - \lambda(0.5x_1 + 2x_2 + 4x_3 - 12 - m)$$

$$\begin{cases} \frac{\partial L}{\partial x_1} = \sqrt{x_2}\, x_3 \cdot \frac{1}{2\sqrt{x_1}} - \frac{1}{2}\lambda = 0 \\ \frac{\partial L}{\partial x_2} = \sqrt{x_1}\, x_3 \cdot \frac{1}{2\sqrt{x_2}} - 2\lambda = 0 \\ \frac{\partial L}{\partial x_3} = \sqrt{x_1 x_2} - 4\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0.5x_1 + 2x_2 + 4x_3 - 12 - m = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{m+12}{2} \\ x_2 = \frac{m+12}{8} \\ x_3 = \frac{m+12}{8} \end{cases}$$

$(x_3 > 4 \Rightarrow m > 20)$

$4x_2 = x_1 \quad 4x_3 = x_1$

$\frac{1}{2}x_1 + \frac{1}{2}x_1 + x_1 - 12 - m = 0$

if $16 < m \le 20$ $\begin{cases} x_1 = m-4 \\ x_2 = \frac{m-4}{4} \\ x_3 = 4 \end{cases}$

$$U\left(\frac{m+12}{2}, \frac{m+12}{8}, \frac{m+12}{8}\right) = \frac{1}{4}(m+12)\left(\frac{m+12}{8}\right) = \frac{1}{32}(m+12)^2 \quad (m > 20)$$

because $2m - 8 \le \frac{1}{32}(m+12)^2$ hold for any m.

$64m - 8 \times 32 \le m^2 + 24m + 12^2$

$m^2 - 40m + 400 \geq 0$

$1600 - 1600 = 0$

so $U\left(\frac{m+12}{2}, \frac{m+12}{8}, \frac{m+12}{8}\right) \geq U\left(m-4, \frac{m-4}{4}, 4\right) \quad (m > 20)$

$\therefore$ for $0 \le m \le 16$, $x_1 = \frac{m}{2}$, $x_2 = \frac{m}{8}$, $x_3 = \frac{m}{2}$

for $16 < m \le 20$, $x_1 = m - 4$, $x_2 = \frac{m-4}{4}$, $x_3 = 4$

for $m > 20$, $x_1 = \frac{m+12}{2}$, $x_2 = \frac{m+12}{8}$, $x_3 = \frac{m+12}{8}$