Homework 4

Name _____ Student ID # _____

1	2	3	4	5
B	C	B	B	B

6. (1) $SAC = 2q^2 - 6kq + 9k^2 - 18k + 24$

$$\begin{cases} \frac{\partial SAC}{\partial q} = 4q - 6k = 0 \\ \frac{\partial SAC}{\partial k} = -6q + 18k - 18 = 0 \end{cases} \Rightarrow \begin{cases} q^* = 3 \\ k^* = 2 \end{cases}$$

$SAC_{min} = 18 - 36 + 36 - 36 + 24 = 6$

$LAC = 6, k^* = 2$

(2) $Q = 600 - 50P$

$P^* = LAC = 6$

$Q^* = 600 - 50P^* = 300$

$n = \frac{Q^*}{q^*} = \frac{300}{6} = 50$

(3) $k = 4$

$Q = 600 - 20P \rightarrow Q' = 1120 - 10P'$

$SAC = 2q^2 - 24q + 96$

$SAC_{min} = 72 - 144 + 96 = 24$

$P = 24$

$Q = 200 - 2P = 120$

$n = \frac{Q}{q} = \frac{120}{6} = 20$

$STC = 2q^3 - 24q^2 + 96q$

$\Rightarrow SMC = 6q^2 - 48q + 96$

Supply: $P = SMC$

$$\begin{cases} Q' = 1120 - 10P' \\ P' = 6\left(\frac{Q'}{20}\right)^2 - 48\left(\frac{Q'}{20}\right) + 96 \end{cases}$$

$$\Rightarrow \begin{cases} Q'^* = 160 \\ P'^* = 96 \end{cases}$$

$q'^* = \frac{Q'^*}{n} = 8$

$STC = SAC \cdot q'^* = 256$

$TR = P'^* \cdot q'^* = 768$

$\pi = TR - STC = 512$

$\therefore P'^* = 96, \pi = 512$

7. (1) $C(Q) = Q^2 + 20Q$

$\Rightarrow MC = C'(Q) = 2Q + 20$

$P = 80 - Q$

$TR = P \cdot Q = -Q^2 + 80Q$

$MR = \frac{dTR}{dQ} = -2Q + 80$

$MR = MC$

$\Rightarrow 2Q + 20 = -2Q + 80$

$\Rightarrow Q^* = 15$

$\Rightarrow P^* = 80 - 15 = 65$

$C(Q^*) = 15^2 + 20 \times 15 + 225 + 300 = 525$

$TR = 65 \times 15 = 975$

$\pi = TR - C(Q^*) = 450$

$\Rightarrow Q^* = 15, \pi = 450$

(2) Suppose payment per unit = Sub

Demand: $Q = 80 - P$

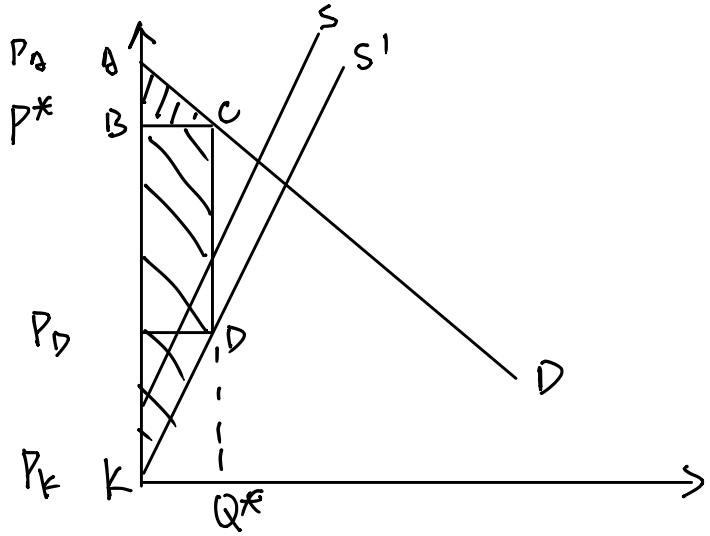
Supply: $P = 2Q + 20$

Supply': $P = 2Q + 20 - Sub$

Monopolist wants to maximize profit with payment in this case.

$$\begin{cases} MC = 2Q + 20 - Sub \\ MR = -2Q + 80 \end{cases}$$

$$\Rightarrow \begin{cases} Q^* = 15 + \frac{1}{4} Sub \\ P^* = 80 - Q^* = 65 - \frac{1}{4} Sub \end{cases}$$



Gov't wants to reaches TS max:

$P_B = 80$, $P_K = 20 - Sub$

$P_D = 2Q^* + 20 - Sub = 50 - \frac{1}{2} Sub$

$CS = S_{\triangle ABC}$, $PS = S_{BCPK}$

$TS = CS + PS - Subsidy$

$= S_{\triangle ABC} + S_{BCPK} - Sub \cdot Q^*$

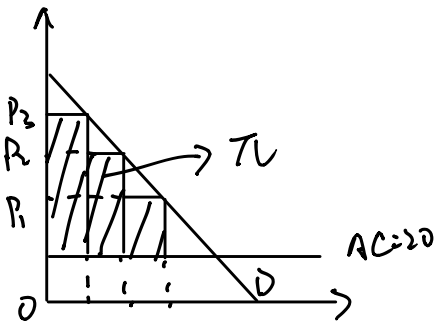
$= \frac{1}{2} Q^* (P^* - P_D + P_A - P_K) - Sub \cdot Q^*$

$= \frac{1125}{2} + \frac{15}{4} Sub - \frac{3}{32} Sub^2$

when $Sub = 20$, TS reaches maximum.

∴ Average payment per unit equals to $20.

8. (1)



(2) ① $n=1$

$Q_1 = 80 - P_1$

$\pi = Q_1 \cdot (P_1 - 20)$

$= -P_1^2 + 100P_1 - 1600$

$P_1 = 50,\ \pi_{max} = 900$

② $n=2$

$Q_1 = 80 - P_1$, $Q_2 = 80 - P_2$

$\pi_1 = Q_1 \cdot (P_1 - 20) = -P_1^2 + 100P_1 - 1600$

$\pi_2 = (Q_2 - Q_1) \cdot (P_2 - 20) = P_1P_2 - P_2^2 - 20P_1 + 20P_2$

$\pi = \pi_1 + \pi_2 = P_1P_2 - P_1^2 - P_2^2 + 80P_1 + 20P_2 - 1600$

$$\begin{cases} \frac{\partial \pi}{\partial P_1} = P_2 - 2P_1 + 80 = 0 \\ \frac{\partial \pi}{\partial P_2} = P_1 - 2P_2 + 20 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} P_1 = 60 \\ P_2 = 100 \end{cases}$$

$\pi = 2400 - 3600 - 1600 + 6000 - 4800 - 1600 = 1200$

(2) $Q_1 = 80 - P_1$, $Q_2 = 80 - P_2$, ...

$\pi_1 = -P_1^2 + 100P_1 - 160$

$\pi_2 = P_1P_2 - P_2^2 - 20P_1 + 20P_2$

$\pi_i = (Q_i - Q_{i-1}) \cdot (P_i - 20)$

$= (P_{i-1} - P_i)(P_i - 20)$

$= P_iP_{i-1} - 20P_{i-1} - P_i^2 + 20P_i \quad (2 \le i \le n-1)$

$\pi_{i+1} = P_{i+1} \cdot P_i - 20P_i - P_{i+1}^2 + 20P_{i+1}$

$$\begin{cases} \frac{\partial \pi}{\partial P_1} = -2P_1 + P_2 + 80 = 0 \\ \frac{\partial \pi}{\partial P_i} = P_{i-1} - 2P_i + P_{i+1} = 0 \quad (2 \le i \le n-1) \\ \frac{\partial \pi}{\partial P_n} = P_{n-1} - 2P_n + 20 = 0 \end{cases}$$

$\Rightarrow P_i = 80 - \frac{60i}{n+1} \quad (1 \le i \le n)$

$\pi_{max} = 1800\frac{n}{n+1}$