Homework 4

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1	2	3	4	5
E	C	E	B	B

6.

(1)

$$\begin{cases} \frac{dSAC}{dq} = 4q - 6K = 0 \\ \frac{dSAC}{dK} = -6q + 18K - 18 = 0 \end{cases}$$

$\therefore \begin{cases} K = 2 \\ q = 3 \end{cases}$ $\therefore SAC_{min} = 18 - 36 + 36 - 36 + 24$
$= 6$

$\therefore$ Long-run average cost is 6

The capitel will each company invest will be 2.

(2)

$$P = 12 - \frac{Q}{50}$$

In a perfectly competitive equilibrium,

$P = MC = AC_{min} = 6$

$\therefore 6 = 12 - \frac{Q}{50}$

$\therefore Q = 300$

$\therefore n = \frac{Q}{q} = 100$

$\therefore$ the market prize will be 6 and will be 100 Companies here,

(3) Suppose $k = 4$

$$SAC = 2q^2 - 24q + 96$$

$\because$ there is no profit

$\therefore$ there must be at SAC_{min}

$\therefore$ $\frac{dSAC}{dq}$ $4q - 24$

$$\therefore \begin{cases} q = 6 \\ P = SAC_{min} = 72 - 24 \times 6 + 96 = 24 \end{cases}$$

$\therefore Q = 600 - 20P = 120$

$$\therefore SSR: S = \frac{d(SAC \cdot q)}{dq} = 6q^2 - 48q + 96$$

$$D': P' = 112 - \frac{Q}{10} = 112 - \frac{20}{10}q = 112 - 2q$$

Let $SSR = D'$, and we need $q > 0$

we have $q = 8$

$\therefore P' = 96$

$$\therefore \pi = P' \cdot q - (SAC|_{q=8} \cdot q) = 8 \times 96 - 32 \times 8 = 512$$

$\therefore$ The price is 96, and the profit for each company is 512.

7.

(1)

$$P = 80 - Q$$

$$\therefore \begin{cases} MR = \dfrac{d(P\cdot Q)}{dQ} = 80 - 2Q \\ MC = \dfrac{dC(Q)}{dQ} = 2Q + 20 \end{cases}$$

Let $MR = MC$

$$\therefore Q^* = 15$$

$$\therefore \pi^* = P^* \cdot Q^* - C(Q^*)$$

$$= 65 \times 15 - 15^2 - 20 \times 15$$

$$= 450$$

$\therefore$ the optimal production for him is 15. And the associated profit is 450

(2) Suppose the payment from the goverment is in average t per unit of output.

$$\therefore \begin{cases} MR = \dfrac{d\,P\cdot Q}{dQ} + t = 80 - 2Q + t \\ MC = 2Q + 20 \end{cases}$$

Let $MR = MC$

we get $Q^* = \dfrac{60+t}{4} = 15 + \dfrac{t}{4}$ 3

$$\therefore P^* = 65 - \frac{t}{4}$$

$\therefore TS = CS + PS + GS$ (Goverment Surplus)

$$= \frac{1}{2}(15+\frac{t}{4})^2 + (65-\frac{t}{4})(15+\frac{t}{4}) - (15+\frac{t}{4})(35+\frac{t}{4}) - t(15+\frac{t}{4})$$

$$= (15+\frac{t}{4})(\frac{15}{2}+\frac{t}{8}+65-\frac{t}{4}-35-\frac{t}{4}-t)$$

$$= (15+\frac{t}{4})(\frac{75}{2}-\frac{11}{8}t)$$

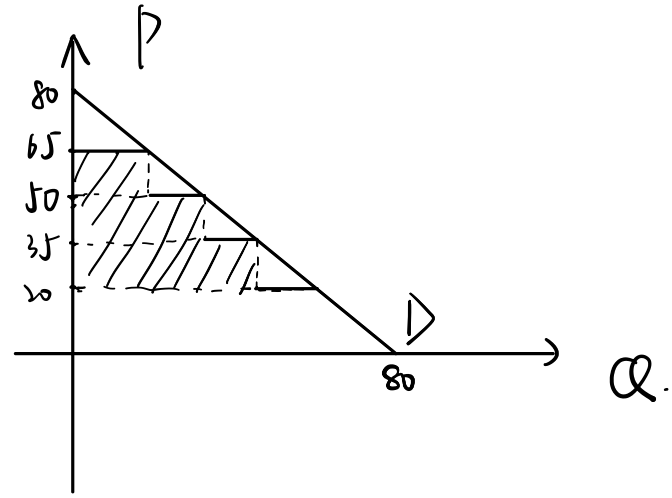
$$= -\frac{11}{32}t^2 - \frac{45}{4}t + \frac{75}{2}\times 15$$

$$\therefore t^* = \frac{\frac{45}{4}}{2\cdot\frac{11}{32}} < 0$$

$\therefore$ The TS_{max} exists when $t=0$

8.

(1)



(2)

if $n = 1$

$Q_1 = 80 - P_1 = 30$

$\pi^* = Q_1 \cdot (P_1 - MC)$

$= 900$

if $n = 2$

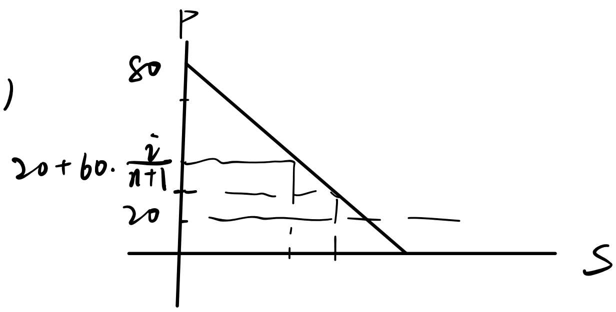
$P_1 = 40$. $\quad P_2 = 60$

$Q_1 = 80 - P_1 = 40 \quad Q_2 = 80 - P_2 = 20$

$\therefore \pi^* = Q_2 \cdot (P_2 - MC) + (Q_1 - Q_2) \cdot (P_1 - MC)$

$= 1200$

(3)



$$\begin{cases} P_i = 20 + \frac{i}{n+1} \cdot 60 \\ Q_i = 80 - P_i \\ \quad = 60 - \frac{i}{n+1} \cdot 60 \end{cases}$$

$$\Delta Q_i = Q_i - Q_{i+1} ,$$

$$= \frac{60}{n+1} \qquad i = 0, 1, \cdots n$$

$$\therefore \pi^* = \sum_{i=1}^{n} (P_i - MC) \cdot \Delta Q_i$$

$$= \sum_{i=1}^{n} \frac{60^2 i}{(n+1)^2}$$

$$= 1800 \frac{n}{(n+1)}$$