Intermediate Microeconomics (Fall 2023)
Homework 4 Answer Key

1 − 5 BCBBB

6.

1) $\frac{\partial SAC}{\partial K} = -6q + 18K - 18 = 0$

$\Rightarrow 3K^* = q + 3$

$\Rightarrow LAC = q^2 - 6q + 15$

$\frac{\partial LAC}{\partial q} = 2q - 6 = 0$

$\Rightarrow q^* = 3$

$\Rightarrow K^* = 2$

2) **In the long-run equilibrium**

$P^* = LAC_{min} = 6$

$\Rightarrow Q^* = 300$ **and** $q^* = 3$

⇒ **There are 100 companies in the market**

3) When $K = 4$

$SAC = 2q^2 - 24q + 96$

$\Rightarrow q^* = 6$

$\Rightarrow P^* = SAC(6) = 24$

$\Rightarrow Q^* = 120$

$\Rightarrow$ There are 20 companies in the market

After the change in demand, assume the price is p

$\Rightarrow q = \frac{1}{20} Q = 56 - 0.5p$

$\Rightarrow p = 112 - 2q$

$\Rightarrow 6q^2 - 48q + 96 = MC = MR = 112 - 2q$

$\Rightarrow q^* = 8$

$\Rightarrow p^* = 96$

$\Rightarrow SAC = 32$

$\Rightarrow \pi = (96 - 32) * 8 = 512$

7.

1) **MC = 2Q + 20 and MR = 80 − 2Q**

   $\Rightarrow Q^* = 15$

   $\Rightarrow P^* = 65$

   $\Rightarrow \pi = 450$

2) **When the total surplus reaches its maximum**

   $2Q + 20 = MC = P = 80 - Q$

   $\Rightarrow Q^* = 20$

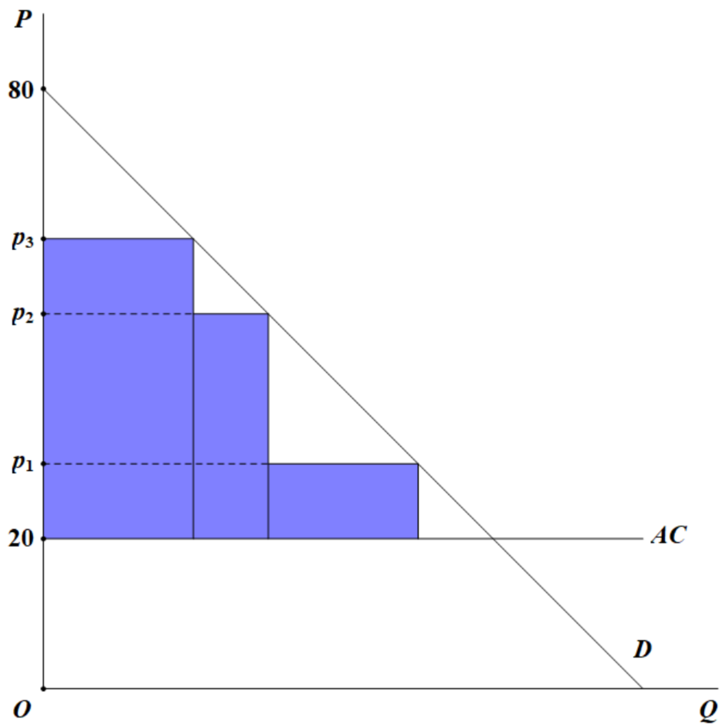
   Suppose the payment is t

   $\Rightarrow 2Q + 20 = 80 - 2Q + t$

   $\Rightarrow t = 20$

8.

1) Mr. Brown's profit is the blue area.



2) See (3).

3) Suppose $p_{i+1} - p_i = x_i$, where $i = 0, 1, 2, \cdots, n$.

$\Rightarrow x_0 + x_1 + \cdots + x_n = p_{n+1} - p_0 = 60$

Max $\pi = \frac{1}{2} * (60 * 60 - x_0^2 - x_1^2 - \cdots - x_n^2)$

subject to $x_0 + x_1 + \cdots + x_n = 60$

Method I:

$\pi = \frac{1}{2} * (60 * 60 - x_0^2 - x_1^2 - \cdots - x_n^2)$

$\leq 1{,}800 - \frac{(x_0 + x_1 + \cdots + x_n)^2}{2 * (n + 1)}$

$= 1{,}800 * \frac{n}{n + 1}$

Method II:

$L = \frac{1}{2} * (60 * 60 - x_0^2 - x_1^2 - \cdots - x_n^2) - \lambda (x_0 + x_1 + \cdots + x_n - 60)$

$\Rightarrow \frac{\partial L}{\partial x_i} = x_i - \lambda = 0$, where $i = 0, 1, 2, \cdots, n$.

$\Rightarrow x_0 = x_1 = \cdots = x_n = \lambda = \frac{60}{n + 1}$

$\Rightarrow \pi = 1{,}800 * \frac{n}{n + 1}$

Method III:

$x_0 = 60 - x_1 - \cdots - x_n$

$\Rightarrow \pi = \frac{1}{2} * [60 * 60 - (60 - x_1 - \cdots - x_n)^2 - x_1^2 - \cdots - x_n^2]$

$\Rightarrow \frac{\partial \pi}{\partial x_i} = - 2 * (60 - x_1 - \cdots - x_n) - 2x_i = 0$, where $i = 1, 2, \cdots, n$.

$\Rightarrow x_0 = x_1 = \cdots = x_n = \frac{60}{n + 1}$

$\Rightarrow \pi = 1{,}800 * \frac{n}{n + 1}$