Homework 2

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1	2	3	4	5
C	D	A	B	A

6.

总效用为:

1) $U(x) = 4\log(1+x) + 10 - 2x$

$U'(x) = \frac{4}{1+x} - 2$

令 $u'(x) > 0$, $x < 1$, $U(x) \uparrow$

令 $u'(x) < 0$, $x > 1$, $U(x) \downarrow$

$\therefore$ $u(x)$在 $x=1$ 时取得最大值.

$u(1)_{max} = 4\log 2 + 8$, Fiorio purchase 1 units of candy

2) 此时 $U(x) = 4\log(1+x) + 1 - 2x$, $0 < x \le \frac{1}{2}$

$U'(x) = \frac{4}{1+x} - 2$

同1), 当 $x = \frac{1}{2}$ 时, $U(x)_{max} = U(\frac{1}{2})$

此时 Fiori purchase $\frac{1}{2}$ units of candy

3) P_c - Q_c^d , 设m为手中美元数.

$$\begin{cases} 3 = Q_c^d \cdot P_c + Q_c^m \cdot 1 \\ \max: 4\log(1+Q_c^d) + Q_c^m \end{cases}$$

联立得: $U(x) = 4\log(1+Q_c^d) + 3 - Q_c^d \cdot P_c$

set up the Lagrangian

$L(\lambda) = 4\log(1+Q_c^d) + 3 - Q_c^d P_c - \lambda(Q_c^d \cdot P_c + Q_c^m - 3)$

$\frac{\partial L}{\partial Q_c^d} = \frac{4}{1+Q_c^d} - P_c - \lambda P_c = 0$ 解得：$\frac{4}{1+Q_c^d} = P_c$

$\frac{\partial L}{\partial Q_c^m} = -\lambda = 0$ $Q_c^d = \frac{4}{P_c} - 1$

$\frac{\partial L}{\lambda} = Q_c^d \cdot P_c + Q_c^m - 3 = 0$

7.

1) max: $u(x_1, x_2) = \sqrt{x_1} + 2\sqrt{x_2}$

$p_1 x_1 + p_2 x_2 = m$

set up the Lagrangian

$$L = \sqrt{x_1} + 2\sqrt{x_2} - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= \frac{1}{2\sqrt{x_1}} - \lambda p_1 = 0 \\ \frac{\partial L}{\partial x_2} &= \frac{1}{\sqrt{x_2}} - \lambda p_2 = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{1}{4x_1} &= \lambda^2 p_1^2 \\ \frac{1}{x_2} &= \lambda^2 p_2^2 \end{aligned} \Rightarrow 4x_1 p_1^2 = x_2 p_2^2$$

$$\frac{\partial L}{\lambda} = p_1 x_1 + p_2 x_2 - m = 0$$

$$x_1 = \frac{x_2 p_2^2}{4p_1^2}$$

$$x_2 = \frac{4x_1 p_1^2}{p_2^2}$$

代回得: ① $p_1 x_1 + \frac{4x_1 p_1^2}{p_2} - m = 0$

$$x_1 = \frac{m}{\left(p + \frac{4p_1^2}{p_2}\right)}$$

② $\frac{x_2 p_2^2}{4p_1} + p_2 x_2 - m = 0$

$$x_2 = \frac{m}{\left(p_2 + \frac{p_2^2}{4p_1}\right)}$$

2) max: $u(x_1, x_2) = \int_0^{x_1^a x_2^b} f(x)\,d(x)$

$$p_1 x_1 + p_2 x_2 = m$$

set up the Lagrangian

$$L = \int_0^{x_1^a x_2^b} f(x)\,d(x) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$$\frac{\partial L}{x_1} = f(x_1^a x_2^b) \cdot a x_1^{a-1} \cdot x_2^b - \lambda p_1 = 0$$

$$\frac{\partial L}{x_2} = f(x_1^a x_2^b) \cdot b x_2^{b-1} \cdot x_1^a - \lambda p_2 = 0$$

$$\frac{\partial L}{\lambda} = p_1 x_1 + p_2 x_2 - m = 0$$

$$\frac{f(m) \cdot a x_1^{a-1} x_2^b}{p_1} = \frac{f(m)\, b x_2^{b-1} \cdot \cancel{x_1}^a}{p_2}$$

3) $u(x_1, x_2) = \min\{x_1 + 2x_2, 2x_1 + x_2\}$

$p_1x_1 + p_2x_2 = m$

set up the Lagrangian

$L = \min\{x_1 + 2x_2, 2x_1 + x_2\} - \lambda(p_1x_1 + p_2x_2 - m)$

① if $x_1 + 2x_2$ is minimized:

$L_1 = (x_1 + 2x_2) - \lambda(p_1x_1 + p_2x_2 - m)$

$\frac{\partial L_1}{x_1} = 1 - \lambda p_1 = 0$

$\frac{\partial L_1}{x_2} = 2 - \lambda p_2 = 0$ $\Rightarrow \lambda = \frac{1}{p_1} = \frac{2}{p_2}$

$\frac{\partial L_1}{\lambda} = m - p_1x_1 - p_2x_2 = 0$ $\Rightarrow \frac{m}{p_1} = x_1 + 2x_2$

$\frac{m}{p_2} = 2x_1 + x_2$

解出: $\begin{cases} x_1 = 0 \\ x_2 = \frac{m}{p_2} \end{cases}$

② if $2x_1 + x_2$ is minimized:

$L_2 = (2x_1 + x_2) - \lambda(p_1x_1 + p_2x_2 - m)$

$\frac{\partial L_2}{x_1} = 2 - \lambda p_1 = 0$

$\frac{\partial L_2}{x_2} = 1 - \lambda p_1 = 0$ $\Rightarrow \lambda = \frac{2}{p_1} = \frac{1}{p_2}$

$\frac{\partial L_2}{\lambda} = m - p_1x_1 - p_2x_2 = 0$ $\Rightarrow \frac{m}{p_1} = 2x_1 + x_2$

$\frac{m}{p_2} = x_1 + 2x_2$

解出: $\begin{cases} x_1 = \frac{m}{p_1} \\ x_2 = 0 \end{cases}$

③ when $2x_1 + x_2 = 2x_2 + x_1$:

$m = (p_1 + p_2)x$

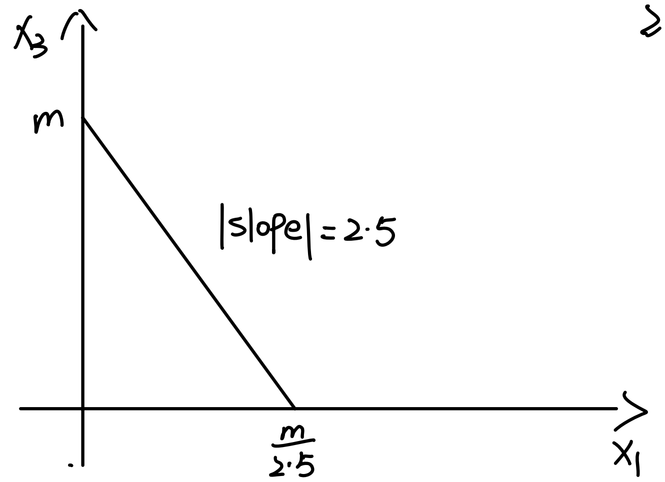
$x_1 = x_2 = \frac{m}{p_1 + p_2}$

8.

1) $x_1 = x_2$, budget constraint: $x_1 p_1 + x_2 p_2 + x_3 p_3 = m$

$$\begin{cases} 2.5x_1 + x_3 = m, & x_3 \le 4 \quad (a) \\ 2.5x_1 + (x_3 - 4)\cdot 4 + 4 = m, & x_3 > 4 \quad (b) \end{cases}$$

① $m \le 4$, only exists (a)



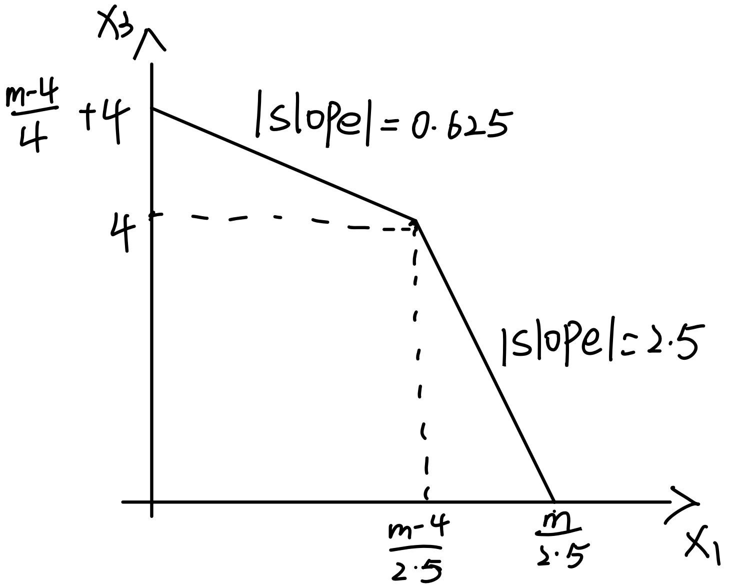
$2.5x_1 + 4x_3 - 12 = m.$

$4x_3 = -2.5x_1 + m + 12.$

$\frac{-2.5}{4}$

-2.5

② $m > 4$, (a), (b) conditions all exist



2) max: $u(x_1, x_2, x_3) = \sqrt{x_1 x_2} \cdot x_3$

$\underline{\text{st}}$ $m = 0.5x_1 + 2x_2 + x_3$

set up the Lagrangian

$$L = \sqrt{x_1 x_2} \cdot x_3 - \lambda(0.5x_1 + 2x_2 + x_3 - m)$$

$$\frac{\partial L}{\partial x_1} = \frac{x_3\sqrt{x_2}}{2\sqrt{x_1}} - 0.5\lambda = 0 \Rightarrow \lambda = \frac{x_3\sqrt{x_2}}{\sqrt{x_1}} \quad ①$$

$$\frac{\partial L}{\partial x_2} = \frac{x_3\sqrt{x_1}}{2\sqrt{x_2}} - 2\lambda = 0 \Rightarrow \lambda = \frac{x_3\sqrt{x_1}}{4\sqrt{x_2}} \quad ②$$

$$\frac{\partial L}{\partial x_3} = \sqrt{x_1 x_2} - \lambda = 0 \Rightarrow \lambda = \sqrt{x_1 x_2} \quad ③$$

use ①、③ $\Rightarrow x_1 = x_3$

use ①、② $\Rightarrow x_1 = 4x_2$

use ②、③ $\Rightarrow x_3 = 4x_2$

$$\frac{\partial L}{\partial \lambda} = 0.5x_1 + 2x_2 + x_3 - m = 0$$

$$\therefore \begin{cases} x_1 = \frac{m}{2} \\ x_2 = \frac{m}{8} \\ x_3 = \frac{m}{2} \end{cases}$$