Exercise 5

Production function is given by Q = F (K, L) = $K^{\frac{1}{2}}L^{\frac{1}{2}}$, w = \$10, r = \$40, and the producer would like to spend \$2,000 on this product. Put labor on the horizontal axis and capital on the vertical axis, find the optimal production for this producer.

$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{2}K^{\frac{1}{2}}L^{-\frac{1}{2}}$ and $MP_K = \frac{\partial Q}{\partial K} = \frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}$

$\Rightarrow |MRTS| = \frac{MP_L}{MP_K} = \frac{\frac{1}{2}K^{\frac{1}{2}}L^{-\frac{1}{2}}}{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}} = \frac{K}{L}$

Input Price Ratio = $\frac{w}{r} = \frac{10}{40} = \frac{1}{4}$

Input Price Ratio = |MRTS|

$\Rightarrow \frac{1}{4} = \frac{K}{L}$

$\Rightarrow L = 4K$ ①

Isocost line: wL + rK = c

$\Rightarrow$ 10L + 40K = 2000

$\Rightarrow$ L + 4K = 200 ②

Combine ①② into a system

$$\begin{cases} L = 4K \\ L + 4K = 200 \end{cases}$$

$\Rightarrow$

$$\begin{cases} L^* = 100 \\ K^* = 25 \end{cases}$$

Optimal production: $Q^* = F(K^*, L^*) = 25^{\frac{1}{2}} * 100^{\frac{1}{2}} = \sqrt{25 * 100} = 50$