Intermediate Microeconomics (Fall 2023)
Homework 3 Answer Key

1 − 5 DDBAA

6.

1) **Suppose the real wage rate is given by w and the real rental rate is given by r.**

⇒ $\pi = K^{\alpha}L^{1-\alpha} - rK - wL$

To maximize π given K, L > 0

$\frac{\partial \Pi}{\partial K} = \alpha \left(\frac{L}{K}\right)^{1-\alpha} - r = 0$ and $\frac{\partial \Pi}{\partial L} = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha} - w = 0$

⇒ $r = \alpha \left(\frac{L}{K}\right)^{1-\alpha}$, $w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$ and $\pi = K^{\alpha}L^{1-\alpha} - rK - wL = 0$

2)

Method I:

If Fiori earns a positive profit, he will double K and L to obtain more profit; if Fiori faces a negative profit, he will shut down.

Method II:

With constant returns to scale, F (αK, αL) = α F (K, L)

Take partial derivative with respect to α on both sides

⇒ $F(K, L) = \frac{\partial F}{\partial \alpha} = K\frac{\partial F}{\partial K} + L\frac{\partial F}{\partial L}$

Suppose the real wage rate is given by w and the real rental rate is given by r.

⇒ π = F (K, L) − rK – wL

To maximize π given K, L > 0

$\frac{\partial \Pi}{\partial K} = \frac{\partial F}{\partial K} - r = 0$ and $\frac{\partial \Pi}{\partial L} = \frac{\partial F}{\partial L} - w = 0$

⇒ $r = \frac{\partial F}{\partial K}$, $w = \frac{\partial F}{\partial L}$ and π = F (K, L) − rK − wL = 0

7.

1) $\sqrt{KL} = 10$

$\Rightarrow L = \frac{100}{K}$

$\Rightarrow C = K + L + |K - 5| = K + \frac{100}{K} + |K - 5|$

To minimize C given K > 0

- **If K > 5**

  $\Rightarrow C = 2K + \frac{100}{K} - 5 \geq 20\sqrt{2} - 5$

  $\Rightarrow K^* = 5\sqrt{2}$ **(> 5, works)**

- **If K ≤ 5**

  $\Rightarrow C = \frac{100}{K} + 5 \geq 25 > 20\sqrt{2} - 5$

Therefore,

The optimal choice is given by (K, L) = $(5\sqrt{2}, 10\sqrt{2})$**, and**

Cost = $20\sqrt{2} - 5$

2) When x decreases to 0,

Total Cost = wK + rL

⇒ The situation is closer to a long-run case.

8.

1) $\pi = P * Q - C = (100 - Q) * Q - 20 * Q = -Q^2 + 80Q \leq 1{,}600$

   $\Rightarrow Q^* = 40$ and $P^* = 60$

2) $P_1 = 100 - Q_1$, $P_2 = 200 - Q_2 - P_1 = Q_1 - Q_2 + 100$

   $\Rightarrow \pi = P_1 * Q_1 - C_1 + P_2 * Q_2 - C_2$

   $= Q_1 * (100 - Q_1) - 20 * Q_1 + Q_2 * (Q_1 - Q_2 + 100) - 20 * Q_2$

   $= -Q_1^2 - Q_2^2 + Q_1Q_2 + 80Q_1 + 80Q_2$

   $\Rightarrow \frac{\partial \Pi}{\partial Q_1} = -2Q_1 + Q_2 + 80 = 0$ and $\frac{\partial \Pi}{\partial Q_2} = -2Q_2 + Q_1 + 80 = 0$

   $\Rightarrow Q_1 = 80$, $Q_2 = 80$, $P_1 = 20$, $P_2 = 100$ and $\pi = 6{,}400$