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Peking University
Intermediate Microeconomics
Fall 2023
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Homework 3
Due: Friday, December 8

Instructions:

1. Print your name on the answer sheet.
2. This homework assignment consists of 5 multiple-choice questions with each one worth 4 points and 3 short-answer questions for 80 points, 100 points total. Make sure you have a complete question set.
3. Please write down all your answers on the answer sheet. Answers written on the question sheet will NOT be graded.
4. The space provided on the answer sheet should be sufficient for your answer. If you need additional space, attach a blank paper.
5. Please write neatly. If I cannot read an answer, you will receive no credit for it.
6. Show enough of your work so that I can tell how you arrived at the answer. You will receive credit for sound reasoning. Partial credit will be awarded wherever I deem there is sufficient justification.
7. When drawing graphs, make sure to label everything, including the axes. It is not particularly important to draw your graphs with perfect precision.
8. Turn in the answer sheet ONLY.

D

1. When the price of Fiori falls and the substitution effect is in a negative direction, then Fiori is a(an)

A. luxury good.
B. inferior good.
C. Giffen good.
D. None of the above is necessarily correct, and the substitution effect should always be in a positive direction in this case.

D

2. Fiori's preferences can be represented by a utility function $2x + y = 12$ $x = 4$

(4, 4) $U_1 = 4$

$$U(x, y) = \min\{x, y\}$$

He faces prices of (2, 1), and his income is 12. Then the prices change to (3, 1). What are the compensating and equivalent variations?

16 − 12 12 − 9

(3, 3) $U_2 = 3$

A. 3, 3
B. 3, 4
C. 4, 4
D. 4, 3

B

3. Fiori uses two factors of production. Irrespective of how much of each factor is used, both factors always have positive marginal products, which imply that

A. isoquants are relevant only in the long-run.
B. isoquants have negative slope.
C. isoquants are convex.
D. isoquants can become vertical or horizontal.
E. None of the above.

A

4. In order for a taxicab to be operated in New York City, it must have a medallion on its hood. The medallion is expensive, but can be resold, and is therefore

A. a fixed cost.
B. a variable cost.
C. an opportunity cost.
D. a sunk cost.

A

5. Suppose that Fiori faces a production function of f $(x_1, x_2) = 3x_1 + x_2$. If the factor prices are 9 for Factor 1 and 4 for Factor 2, how much will it cost him to produce 50 units of output?

A. 150
B. 175
C. 200
D. 875
E. 1,550

6. **Suppose Fiori uses labor and capital to produce brownies and the production exhibits constant returns to scale. Meanwhile, the real rental and the real wage are both constant. In this case, Fiori has made the optimal choice to maximize his profit, given K, L > 0.**

   1) **If Fiori faces a standard Cobb-Douglas production function of $P(K, L) = K^{\alpha}L^{1-\alpha}$, prove that Fiori earns no profit.**

   2) **Prove that Fiori earns no profit in general.**

(1) $MP_K = \alpha K^{\alpha-1}L^{1-\alpha}$, $MP_L = (1-\alpha)K^{\alpha}L^{-\alpha}$

Let P, w, r be the price of output, laber, capital

profit $= P \cdot K^{\alpha} \cdot L^{1-\alpha} - wL - rK$

$$\frac{MP_K}{MP_L} = \frac{w}{r}$$

7. **Suppose Thompson owns a factory of producing Fiori and faces a production function of**

$$f(K, L) = \sqrt{KL}$$

where K and L do not have to be integers. Currently, the wage and rent are both \$1 and Thompson needs to produce 10 units of Fiori. Meanwhile, there is an extra cost to alter the amount of capital used in production, and additional \$x is required for each unit of capital deviated from K = 5, which was the former optimal level of capital used.

1) **Suppose x = 1. Figure out the optimal production choice for Thompson and the associated cost.**
2) **When x decreases to 0, is the situation closer to a short-run case where the capital is fixed at 5 or to a long-run case?**

1) $\min_{k,L} k+L+|k-5|$, s.t. $\sqrt{kL}=10 \Rightarrow kL=100$

① for $k>5$

$L_1 = k+L+k-5-\lambda(kL-100)$

$$\begin{cases} \frac{\partial L_1}{\partial k} = 2-\lambda L = 0 \\ \frac{\partial L_1}{\partial L} = 1-\lambda k = 0 \\ \frac{\partial L_1}{\partial \lambda} = -(kL-100)=0 \end{cases} \Rightarrow \begin{cases} k = 5\sqrt{2} \\ L = 10\sqrt{2} \end{cases} \Rightarrow cost = 2k+L-5 = 20\sqrt{2}-5$$

② for $k \leq 5$

$kL-100=0 \Rightarrow L_{min} = \frac{100}{k_{max}} = 20$

$(k+L+5-k)_{min} = (L+5)_{min} = 25 > 20\sqrt{2}-5$

so the optimal choice is $\begin{cases} k = 5\sqrt{2} \\ L = 10\sqrt{2} \end{cases}$, $cost = 20\sqrt{2}-5$

2) $cost = k+L$

$MP_k = \frac{\sqrt{L}}{2\sqrt{k}}$, $MP_L = \frac{\sqrt{k}}{2\sqrt{L}}$

$\frac{MP_k}{MP_L} = \frac{1}{1} \Rightarrow k=L$

$cost = k+L$, is closer to a long-run case.

8. **Profiteer Thompson operates a brownie factory. This year, he faces a cost of C = 20 ∗ Q and a demand of Q = 100 − P.**

   1) **Figure out the optimal quantity to produce, the price of brownies and profit gained by Thompson.**

   2) **Thompson decides to operate for at least two years. Instead of maximizing a one-year profit, he intends to maximize the profit of two consecutive years in total. Suppose his cost remains the same for the following year, but the demand changes to Q = 200 − P_1 − P_2, where P_1 is the price of brownies for this year and P_2 is the price for the following year. Figure out the optimal quantities and prices for both years and the total profit earned.**

1) $\text{profit} = f(P) = PQ - C = P(100-P) - 20(100-P) = -P^2 + 120P - 2000$

$$\frac{\partial f(P)}{\partial P} = -2P + 120 = 0 \Rightarrow P = 60 \Rightarrow Q = 40$$

$$\because \frac{\partial^2 f(P)}{\partial P^2} = -2 < 0$$

$$\therefore f(x)_{max} = f(60) = 1600$$

so the optimal quantity is 40, the price is 60, and the profit is 1600.

2) $\text{profit} = g(P_1, P_2) = P_1Q_1 + P_2Q_2 - C_1Q_1 - C_2Q_2$

$$= P_1(100-P_1) + P_2(200 - P_1 - P_2) - 20(100-P_1) - 20(200-P_1-P_2)$$

$$= -P_1^2 - P_2^2 - P_1P_2 + 140P_1 + 220P_2 - 6000$$

$$\begin{cases} \dfrac{\partial g(P_1,P_2)}{\partial P_1} = -2P_1 - P_2 + 140 = 0 \\ \dfrac{\partial g(P_1,P_2)}{\partial P_2} = -2P_2 - P_1 + 220 = 0 \end{cases} \Rightarrow \begin{cases} P_1 = 20 \\ P_2 = 100 \end{cases} \Rightarrow \begin{cases} Q_1 = 80 \\ Q_2 = 80 \end{cases}$$

$$A = \frac{\partial^2 g}{\partial P_1^2} = -1, \quad B = \frac{\partial^2 g}{\partial P_1 \partial P_2} = -1, \quad C = \frac{\partial^2 g}{\partial P_2^2} = -1$$

$$\because AC - B^2 = 1 > 0 \text{ and } A < 0$$

$$\therefore g(P_1, P_2)_{max} = g(20, 100) = 6400$$

So the optimal choices are $\begin{cases} Q_1 = 80 \\ Q_2 = 80 \end{cases}$, $\begin{cases} P_1 = 20 \\ P_2 = 100 \end{cases}$

the total profit is 6400.