Homework 4

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1. **In a perfectly competitive industry, the supply curve of each firm is given by $S = \frac{P}{2}$. If a firm produces 6 units of output, what is the total variable cost?**
   A. 34.
   B. 36.
   C. 54.
   D. 72.
   E. There is not enough information to determine the total variable cost.

Answer: B

TVC=36, TR=72, TC unknown.

2. **In a market with the demand given by $Q = 10 - P$, Fiori is a monopolist with a marginal cost of \$2 and no fixed cost. If the marginal cost rises to \$4, by how much will the price of Fiori rise?**
   A. \$3.
   B. \$2.
   C. \$1.
   D. \$0; the firm is already charging the monopoly price.
   E. None of the above.

Answer: C

3. **Suppose a monopolist Fiori would receive a payment from the government for each unit of his output that is consumed by his customers. Fiori faces a constant marginal cost and the payment that he could receive for each unit of output is higher than his marginal cost of production in magnitude. But to obtain the payment on a unit of the output from the government, somebody has to consume it. If Fiori is rational, which of the following must be true?**
   A. He will pay customers to consume his product.
   B. If he sells at a positive price, the demand must be inelastic at that price.
   C. He will sell at a price where the demand is elastic.
   D. He will give the good away.
   E. None of the above.

Answer: B

4. **A monopolist Fiori has a constant marginal cost of \$2 per unit and no fixed cost. He faces separate markets in the United States and England. He can set one price P_1 for the U.S. market and another price P_2 for the English market. If the demand in the United States is given by $Q_1 = 6,000 - 600P_1$ and the demand in England is given by $Q_2 = 2,400 - 400P_2$, the price of the product in the United States will**
A. be higher than the price in England by \$4.
B. be higher than the price in England by \$2.
C. equal the price in England.
D. be lower than the price in England by \$2.
E. be lower than the price in England by \$4.

Answer: B

q1=2400 p1=6

q2=800 p2=4

5. **A price-discriminating monopolist Fiori sells in two separate markets such that goods sold in one market are never resold in the other. It charges $p_1 = \$5$ in one market and $p_2 = \$10$ in the other market. At these prices, the price elasticity of demand in the first market is -1.4 and the price elasticity of demand in the second market is -0.1. Which of the following actions is sure to raise the profit of Fiori?**
A. Lower p_2.
B. Raise p_2.
C. Raise both p_1 and p_2.
D. Raise p_1 and lower p_2.
E. Raise p_2 and lower p_1.

Answer: B

6. **Suppose a market of Brownie is perfectly competitive. Currently all companies are identical in size and face the same short-run average cost of**

$$SAC = 2q^2 - 6Kq + 9K^2 - 18K + 24 \tag{1}$$

where q represents the quantity of the output and K represents the amount of capital invested.

(a) **Calculate the long-run average cost for each company. How much capital will each company invest at the long-run equilibrium?**

LAC is the enveloping line of SAC.

Take SAC's minimum,

$$\frac{\partial SAC}{\partial K} = -6q + 18K - 18 = 0 \Rightarrow K = \frac{q}{3} + 1 \tag{2}$$

Therefore, the long-run average cost is

$$LAC = q^2 - 6q + 15 \tag{3}$$

The long-run equilibrium is $q = 3$, so that each company invest $K = 2$ at the long-run equilibrium.

(b) **Suppose the demand of brownie is given by $Q = 600 - 50P$. Calculate the market price and the number of companies at the long-run equilibrium.**

As the market of Brownie is a perfectly competitive market, there's no economic profit,

$$\begin{gathered} P = LAC_{min} = 6 \\ q = 3 \end{gathered} \tag{4}$$

In the whole market,

$$Q = 300 \tag{5}$$

Therefore, the number of companies,

$$N = \frac{Q}{q} = 100 \tag{6}$$

(c) **Suppose each firm has invested 4 units of capital and achieved a shortrun equilibrium with no profit, but the demand for brownie suddenly changes from $Q = 600 - 20P$ to $Q = 1,120 - 10P$. Calculate the market price and the profit for each company in the new short-run equilibrium.**

$$SAC = 2q^2 - 24q + 96 \tag{7}$$

In this short-run equilibrium, there's no profit,

$$\begin{gathered} q_1 = 6 \\ P_1 = SAC_{min} = 24 \\ N = \frac{Q_1}{q_1} = 20 \end{gathered} \tag{8}$$

The tatal cost of all companies,

$$TC = Q * SAC(\frac{Q}{20}) \tag{9}$$

The supply curve,

$$P = \frac{\partial TC}{\partial Q} = \frac{3}{200}Q^2 - \frac{12}{5}Q + 96 \tag{10}$$

When the demand suddenly changes, the price of each company changes and no company leaves or enters this market, supply curve doesn't change,

$$\begin{gathered} S = D \Rightarrow Q_2 = 160, P_2 = 96 \\ \pi_0 = q_2(P_2 - SAC(q_2)) = 512 \end{gathered} \tag{11}$$

Above all, the market price is 96 and the profit for each company is 512.

7. **Thompson is a profit-maximizing monopolist. The market demand that he faces is given by $Q = 80 - P$, and his cost is given by $C(Q) = Q^2 + 20Q$.**

   (a) **Find out the optimal production of Thompson and the associated profit.**

$$\begin{aligned} \pi &= PQ - C(Q) \\ &= -2Q^2 + 60Q \end{aligned} \tag{12}$$

   Thus, the optimal production is 15, the associated profit is 450.

   (b) **Suppose Thompson would receive a payment from the government for each unit of his output that is sold out. The government intends to maximize the total surplus. Calculate the average of this payment from the government per unit of output.**

   Suppose x is the unit subsidy.

$$\begin{cases} P = 2Q + 20 - x \\ P = 80 - 2Q \end{cases} \Rightarrow \begin{cases} Q = \frac{1}{4}x + 15 \\ P = -\frac{1}{2}x + 50 \end{cases} \Rightarrow P' = -\frac{1}{4}x + 65 \tag{13}$$

   Calculate the government surplus:

$$GS = -Qx = -\frac{1}{4}x^2 - 15x \tag{16}$$

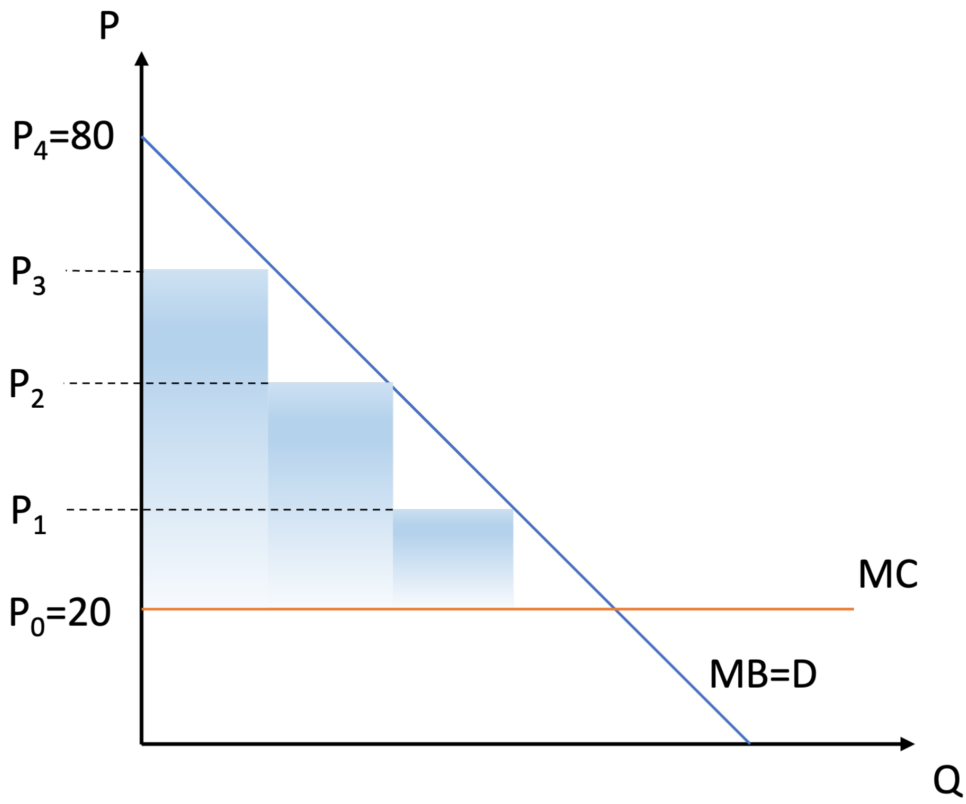
   The total surplus:

$$\begin{aligned} TS &= CS + PS + GS \\ &= ((80 - (20 - x)) + ((-\frac{1}{4}x + 65) - (-\frac{1}{2}x + 50)) \times (\frac{1}{4}x + 15) + (-\frac{1}{4}x^2 - 15x) \\ &= -(\frac{1}{4}x + 15)(\frac{3}{8}x - \frac{75}{2}) \end{aligned} \tag{17}$$

   TS is maximized to 600 when $x = 20$.

8. **The product Fiori is monopolized by Mr. Brown. The demand for Fiori is given by $Q = 80 - P$. Suppose Mr. Brown faces a constant marginal cost of 20 and no fixed cost. Now Mr. Brown price discriminates: he sets $(n + 2)$ prices, where $20 = P_0 < P_1 < \ldots < P_{\mathrm{n}} < P_{\mathrm{n+1}} = 80$. For each consumer, if his willingness to pay is higher than P_i but below $P_{i+1} (0 \le i \le n)$, he pays P_i to purchase the product.**

   (a) **Graph a figure to show the profit of Mr. Brown if $n = 3$.**



The shadowed area is Mr. Brown's profit.

(b) **Calculate the maximum profit of Mr. Brown for $n = 1$ and $n = 2$.**

$n = 1$:

$$\pi_1 = (P_1 - P_0)(80 - P_1) \tag{18}$$

Which maximum is 900.

$n = 2$:

$$\pi_2 = (P_2 - P_1)(80 - P_2) + (P_1 - P_0)(80 - P_1) \tag{19}$$

Use the first-order condition and do the calculations,

$$\begin{cases} P_1 = 40 \\ P_2 = 60 \end{cases} \tag{20}$$

π_2 maximum is 1200.

(c) **Calculate the maximum profit of Mr. Brown for any n.**

$$\pi_n = \sum_{i=1}^{n} (P_i - P_{i-1})(80 - P_i) \tag{21}$$

First-order condition,

$$\frac{\partial \pi_n}{\partial P_i} = P_{i+1} - 2P_i + P_{i-1} = 0 \tag{22}$$

Add all equations together,

$$P_1 + P_n = 100 \tag{23}$$

Add $1 \sim n-1$ equations together,

$$P_2 + 2P_n = 180 \tag{24}$$

Add $1 \sim n-2$ equations together,

$$P_3 + 3P_n = 260 \tag{25}$$

e.t.c.

$$\begin{aligned} P_n + nP_n = 100 + 80(n-1) \Rightarrow P_n = \frac{80n+20}{n+1} \\ \Rightarrow P_i + iP_n = 100 + 80(i-1) \Rightarrow P_i = 20 + \frac{60}{n+1}i \end{aligned} \tag{26}$$

Therefore,

$$\pi_n = \frac{1800n}{n+1} \tag{27}$$